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A Comparison of Manchester Symbol Tracking Loops for Block V Applications

J. K. Holmes
Radio Frequency and Microwave Subsystems Section

The linearized tracking errors of three Manchester (biphase-coded) symbol tracking loops are compared to determine which is appropriate for Block V receiver applications. The first is a nonreturn-to-zero (NRZ) symbol synchronizer loop operating at twice the symbol rate (NRZ × 2) so that it operates on half-symbols. The second near optimally processes the mid-symbol transitions and ignores the between-symbol transitions. In the third configuration, the first two approaches are combined as a hybrid to produce the best performance. Although this hybrid loop is the best at low symbol signal-to-noise ratios (SNRs), it has about the same performance as the NRZ × 2 loop at higher SNRs (> 0-dB E_S/N_0). Based on this analysis, it is tentatively recommended that the hybrid loop be implemented for Manchester data in the Block V receiver. However, the high data rate case and the hardware implications of each implementation must be understood and analyzed before the hybrid loop is recommended unconditionally.

I. Introduction

Three symbol-synchronization (sync) loops have been studied with the object of determining which structure provides the best tracking performance in terms of the minimum tracking error variance of the linearized loop:

(1) The nonreturn-to-zero (NRZ) digital data transition tracking loop (DTTL), which operates at twice the Manchester symbol rate (or at the equivalent NRZ symbol rate).

(2) A symbol-sync loop based on a near-optimal processing of the mid-symbol transition. The between-symbol transitions are ignored by this loop.

(3) A hybrid of loops (1) and (2). The mid-symbol transition processing is based on the second candidate loop and the between-symbol transition processing is based on the DTTL, in which a transition is estimated from the half-symbol on either side of the between-symbol transition.

Other possibilities exist, but these three seemed most relevant and more readily analyzable.

To make the analysis somewhat simpler to accomplish, the assumption was made that the symbol tracking loops are continuous in time and amplitude. Thus, the results given here would apply to the Block V digital receiver only at the low and medium symbol rate cases, and not to the high symbol rate case where as few as three samples can occur per half-symbol.

II. Analysis of DTTL Tracking Error Variance Operating at $2R_S$

In this section the continuous-time linearized closed-loop tracking performance (expressed in fractions of symbol time) is estimated. The symbol tracking loop under consideration is an NRZ DTTL, which works on the NRZ symbol transitions to detect the timing error. Figure 1 shows a block diagram of the NRZ DTTL operating at twice the symbol rate ($2R_S$) so that it is suitable for tracking Manchester (biphase) signal formatting. Note that because false lock can occur on any Manchester-coded symbol tracking loop, a false-lock detector must be used with Manchester data.

In Fig. 1, $\hat{\tau}$ denotes the symbol loop estimate of the symbol stream transmission delay and W is the window size in seconds used for the error-detection window.

Basically the loop performs one integration over one complete half-symbol (T_H sec) and another across the time where the transition could occur. When the transition is mid-symbol, a transition always occurs; when the transition is at the end of the symbol, a transition may or may not occur.

The input is modeled as an infinite sequence of Manchester symbols with transitions determined by the estimate of the half-symbol sequence a_k. In addition, the thermal noise corrupts the symbol stream.

Thus, the received signal is modeled as

$$y(t) = \sqrt{P} \sum_{k=-\infty}^{\infty} b_k q(t - kT - \tau) + n(t) \tag{1}$$

where

- $q(t)$ is one Manchester symbol
- $n(t)$ is modeled as white Gaussian noise (WGN) with spectral density $N_0/2$
- b_k is a random binary valued (± 1) symbol sequence
- T_H is the symbol half-period ($T = 2T_H$)
- T is the symbol period
- P is the data power in the received signal
- τ is the time delay of the signal
- $\hat{\tau}$ is the time-delay estimate of the symbol sync loop

The relationship between the half-symbol a_k and full-symbol data b_k sequence is given by

$$\left.\begin{aligned} a_k &= b_{k/2}, & k \text{ even} \\ a_k &= -b_{k/2-1/2}, & k \text{ odd} \end{aligned}\right\} \tag{2}$$

and is illustrated in Fig. 2. The general approach used in [1] is followed and it is assumed for convenience that $\tau = 0$.

The inphase channel produces an output pulse sequence estimate, which, for $\tau - \hat{\tau}$ close to zero, is essentially given by

$$\hat{a}_k = SGN\left(a_k\sqrt{P}T_H + \int_{kT_H}^{(k+1)T_H} n(t)dt\right), \quad (\tau - \hat{\tau} \cong 0) \tag{3}$$

where k is the index on half-symbols. The output of the transition indicator is given by

$$I_k = \frac{\hat{a}_k - \hat{a}_{k-1}}{2} \tag{4}$$

At the end of the kth pulse time, the midphase channel produces the following output[1] when $\tau - \hat{\tau} \cong 0$ (and $\tau = 0$):

$$U_k = 2a_{k-1}\sqrt{P}(\tau - \hat{\tau}) + \int_{kT_H - W/2}^{kT_H + W/2} n(t)dt\ , \ |\tau - \hat{\tau}| \leq W/2 \tag{5}$$

Consequently, the timing-error estimate $\hat{\varepsilon}$, which is the loop estimate of the timing error $\varepsilon = \tau - \hat{\tau}$, is given by

$$\hat{\varepsilon}(t) = \sum_{k=-\infty}^{\infty} I_k\ U'_k\ p(t - (k+1)T_H) \tag{6}$$

which changes every T_H seconds. Now $p(t)$ is a half-Manchester pulse of unity amplitude (see Fig. 8) and U'_k is the ($T_H - W/2$)-sec delayed version of U_k. It is used to align the midphase and inphase channels in time. This error signal is constant over T_H sec in Eq. (6). Using Eqs. (4) and (5) in Eq. (6), the expression for the loop error signal is obtained:

[1] The timing error is neglected in the noise term but included in the error-signal term.

$$\hat{\varepsilon}(t) = \sum_{k=-\infty}^{\infty} \left\{ a_k \left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2} \right) 2\sqrt{P}(\tau - \hat{\tau}) + \left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2} \right) \int_{kT_H - W/2}^{kT_H + W/2} n(t)dt \right\} \times p(t - (k+1)T_H) \tag{7}$$

It will be shown that the mean value of $\hat{\varepsilon}$ is given by $\alpha\varepsilon$ (linear) for small values of ε; the resulting additive noise process is denoted by $N(t)$. Both can be determined from Eq. (7) since the noise is a random amplitude pulse sequence process. It is assumed that ε is small in the following discussion. Then the timing estimate $\hat{\tau}$ is given by

$$\hat{\tau} \cong \frac{KF(s)}{s} [\alpha\varepsilon + N(t)] \tag{8}$$

where $F(s)$ is the loop filter expressed in Heaviside operator symbolism $(1/s)X(s)$ denotes $\int_0^t x(t')dt'$ and represents the effect of the voltage-controlled oscillator (VCO).

Since by definition of the error ε

$$\hat{\tau} = \tau - \varepsilon \tag{9}$$

Using Eq. (9) in Eq. (8) yields

$$\varepsilon(t) = \left(\frac{K'F(s)/s}{1 + K'F(s)/s} \right) \left(\frac{N(t)}{\alpha} \right) \tag{10}$$

where $F(s)$ is the loop filter function viewed as a Heaviside operator and the $1/s$ comes from the VCO. The terms that depend upon s comprise the closed-loop transfer function; it is denoted by $H(s)$, so that Eq. (10) becomes

$$\varepsilon(t) = H(s) \left(\frac{N(t)}{\alpha} \right) \tag{11}$$

where again $H(s)$ is viewed as a Heaviside operator operating on the noise term following it.

Next it is necessary to characterize the noise process $N(t)$ and the constant α. First consider the computation of $E[\hat{\varepsilon}|\varepsilon]$. For small timing errors, it will be assumed that the value of $\hat{a}_k$ is statistically independent of the integrated noise process (integrated from $kT_H - W/2$ to $kT_H + W/2$). Of course, this is not true but it has been demonstrated by simulation to be a reasonable approximation [2]. With this assumption, Eq. (7) can be used to obtain

$$E[\hat{\varepsilon}|\varepsilon] \cong 2\sqrt{P}(\varepsilon) \left[\overbrace{\frac{1}{2}(1)\left\{(1 - PE_H)^2 - PE_H^2\right\}}^{\text{mid-symbol transition}} + \overbrace{\frac{1}{2}\left(\frac{1}{2}\right)\left\{(1 - PE_H)^2 - PE_H^2\right\}}^{\text{adjacent symbol transition}} \right] \tag{12}$$

where the two leftmost 1/2 factors in the rectangular brackets are due to the probability of the transition being a mid-symbol transition or an adjacent symbol transition. The factor of unity following the first factor of 1/2 accounts for the fact that there is always a mid-symbol transition. The factor of 1/2 following the second leftmost factor of 1/2 is based on the assumption that there is a probability of 1/2 that there is a transition at the end of the symbol. Finally, PE_H is the probabilty of a half-symbol error and is given by

$$PE_H = \int_{\sqrt{2R_H}}^{\infty} \frac{1}{\sqrt{2R_H}} e^{-z^2/2}\, dz = Q(\sqrt{2R_H}) \tag{13}$$

where

$$R_H = \frac{E_H}{N_0} = \frac{PT_H}{N_0} \tag{14}$$

where

- P is the data power
- N_0 is the one-sided noise spectral density at the symbol sync input
- T_H is one-half the symbol duration

Hence, from Eqs. (12) and (13)

$$E[\hat{\varepsilon}|\varepsilon] \cong \frac{3}{2}\sqrt{P}\varepsilon \left(1 - 2Q\sqrt{2R_H}\right) \tag{15}$$

This can be rewritten as

$$E[\hat{\varepsilon}|\varepsilon] = \frac{3}{2}\sqrt{P}\varepsilon\,\mathrm{erf}(\sqrt{R_H}) = \frac{3}{2}\sqrt{P}\varepsilon\left(2\int_0^{\sqrt{R_H}} \frac{1}{\sqrt{\pi}}e^{-t^2}dt\right) \tag{16}$$

Thus, α of Eq. (8) is given by (α = the slope of the S-curve at $\varepsilon = 0$)

$$\alpha = \frac{3}{2}\sqrt{P}\,\mathrm{erf}(\sqrt{R_H}) \tag{17}$$

Now the noise spectral density of $N_E(t)$ is obtained from the process generated by

$$N_E(t) = \sum_{k=-\infty}^{\infty} \int_{kT_H-W/2}^{kT_H+W/2} n(t)\,dt \left[\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right] \times p(t-(k+1)T_H) \tag{18}$$

where, as before, $p(t)$ is a unit amplitude pulse of duration T_H sec long. Again assume that $\hat{a}_k$ is independent of $n(t)$, and note that the cyclostationary process $N_E(t)$ can be made stationary by averaging over time [3]. Thus

$$R(\zeta) = \frac{1}{T}\int_0^T E[N_E(t)N_E(t+\zeta)]\,dt \tag{19}$$

is the autocorrelation function of a stationary process derived from the corresponding cyclostationary process. An evaluation of Eq. (19) obtains

$$\begin{aligned} R(\zeta) &= \frac{N_0 W}{2}\left(1-\frac{|\zeta|}{T_H}\right) E\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2 && \text{for } |\zeta| \le T_H \\ &= 0 && \text{elsewhere} \end{aligned} \tag{20}$$

where it is assumed for analytic convenience that the noise process $n(t)$ over W sec is statistically independent of the symbol estimate. Consider the term inside the expectation. It can be evaluated by

$$E\left[\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2\right] = \frac{1}{2}E\left[\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2\Bigg|_{\substack{\text{mid-symbol}\\ \text{transition}}}\right] + \frac{1}{2}E\left[\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2\Bigg|_{\substack{\text{adjacent-symbol}\\ \text{transition}}}\right] \tag{21}$$

An evaluation obtains

$$E\left[\left(\frac{\hat{\alpha}_k - \hat{\alpha}_{k-1}}{2}\right)^2\right] = \overbrace{\frac{1}{2}\left[1\cdot(1-PE_H)^2 + 1\cdot PE_H^2\right]}^{\text{mid-symbol transition}} + \overbrace{\frac{1}{4}\left[1\cdot 2PE_H(1-PE_H)\right] + \frac{1}{4}\left[(1-PE_H)^2 + PE_H^2\right]}^{\text{adjacent symbol transition}} \tag{22}$$

or, simplifying

$$E\left[\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2\right] = \frac{3}{4} - PE_H(1-PE_H) \tag{23}$$

Note that when $PE_H \to 0$, the expectation approaches 3/4 as anticipated, and when $PE_H \to 1/2$, the expectation approaches 1/2 as anticipated. Since

$$1 - 2PE_H = \mathrm{erf}(\sqrt{R_H}) \tag{24}$$

one obtains

$$E\left[\left(\frac{\hat{a}_k - \hat{a}_{k-1}}{2}\right)^2\right] = \frac{3}{4} - \frac{1}{4}\left(1 - \mathrm{erf}^2\left(\sqrt{R_H}\right)\right) \tag{25}$$

Therefore

$$R(\zeta) \cong \frac{N_0 W}{2}\left(1-\frac{|\zeta|}{T_H}\right)\left\{\frac{3}{4}-\frac{1}{4}\left(1-\operatorname{erf}^2(\sqrt{R_H})\right)\right\} \quad \text{for } |\zeta| \le T_H$$

$$= 0 \quad \text{elsewhere} \quad (26)$$

Integrating $R(\zeta)$ from $-\infty$ to ∞ yields the noise spectral density at $f = 0$, which is

$$\mathscr{S}_N(0) = \left(\frac{N_0 W}{2}\right)\frac{3}{4}\left[1-\frac{1}{3}\left(1-\operatorname{erf}^2(\sqrt{R_H})\right)\right]T_H \quad (27)$$

Thus, the absolute, linearized, tracking-error variance is obtained from Eq. (11) to be

$$\sigma_\varepsilon^2 = \frac{2B_L \mathscr{S}_N(0)}{\alpha^2}\ \sec^2 \quad (28)$$

where $B_L = \int_0^\infty |H(j2\pi f)|^2 df$ is the one-sided loop noise bandwidth. Hence, from Eqs. (17), (27), and (28)

$$\frac{\sigma_\varepsilon^2}{T_H^2} = \frac{1}{3} - \frac{B_L W\left[1-\frac{1}{3}\left(1-\operatorname{erf}^2\left(\sqrt{R_H}\right)\right)\right]}{R_H \operatorname{erf}^2(\sqrt{R_H})},$$

$$\text{(fraction of a symbol)}^2 \quad (29)$$

Let

$$T = 2T_H \quad (30)$$

to relate the half-symbol to the full-symbol duration (T).

Translating to the full-symbol period T yields the desired result for arbitrary window size.

$$\frac{\sigma_\varepsilon^2}{T^2} = \frac{1}{6}\,\frac{W B_L\left[1-\frac{1}{3}\left(1-\operatorname{erf}^2\left(\sqrt{\frac{R}{2}}\right)\right)\right]}{R\operatorname{erf}^2\left(\sqrt{\frac{R}{2}}\right)},$$

$$\text{(fraction of a symbol)}^2 \quad (31)$$

where $R = PT/N_0 = E_S/N_0$.

The value $\sigma_\varepsilon^2/\left((B_L T)T^2\right)$ is plotted in Fig. 3 for $W = T/4$ versus R, the full-symbol SNR (E_S/N_0), where E_S is the data symbol energy and N_0 is the one-sided noise spectral density. The other loops will be discussed in the following sections. Taking the somewhat arbitrary window $W = T/4$, Eq. (31) becomes

$$\frac{\sigma_\varepsilon^2}{T^2} = \frac{1}{24}\,\frac{B_L T\left[1-\frac{1}{3}\left(1-\operatorname{erf}^2\left(\sqrt{\frac{R}{2}}\right)\right)\right]}{R\operatorname{erf}^2\left(\sqrt{\frac{R}{2}}\right)}\ (\text{sec/sec})^2 \quad (32)$$

The relationship $W = T/4$ is used for all three loop window sizes, since at high data rates this would probably be the minimum size.

III. Analysis of the Near-Optimum Mid-Transition–Tracking Manchester Symbol Synchronizer

This section describes a symbol synchronizer that is motivated by the optimum structure [4,5,6] for the mid-symbol transition and ignores the adjacent-symbol transition. Figure 4 illustrates this symbol synchronizer.

The received signal plus noise is modeled by Eq. (1) as

$$y(t) = \sqrt{P}\sum_{k=-\infty}^{\infty} b_k q(t - kT - \tau) + n(t) \quad (33)$$

with $n(t)$ modeled as white Gaussian noise having spectral density $N_0/2$.

In order to analyze the symbol synchronizer depicted in Fig. 4, it is advantageous to segment the noise process into four contiguous regions as shown in Figure 5. The bottom portion of Fig. 5 illustrates the symbol sync reference mid-symbol transition point integration region (solid lines) and the actual mid-symbol transition point of the received signal (dashed lines). The pulse function $q(t)$ is a complete Manchester symbol and b_k is the data sign ($b_k = \pm 1$) from Eq. (1). The X-channel signal (see Fig. 4) can be written at the end of symbol b_k, as

$$X = \sqrt{P}\Big[b_k T - (b_{k-1} + 3b_k)\varepsilon + (N_1 + N_2 - N_3 - N_4)\Big], \quad \varepsilon \ge 0 \quad (34a)$$

$$X = \sqrt{P}\Big[b_k T - (b_{k+1} + 3b_k)\varepsilon$$

$$+ (N_1 + N_2 - N_3 - N_4)\Big], \qquad \varepsilon < 0 \quad (34b)$$

where it is assumed that $|\varepsilon|$ ($\varepsilon = \tau - \hat{\tau}$) is less than W. Additionally, it is assumed that ε is small in the computations that follow.

The Y channel produces the signal at the end of its integration time given by

$$Y = \left[2\sqrt{P} b_k \varepsilon + (N_2 - N_3)\right] \qquad (35)$$

The product $Z = XY = \hat{\varepsilon}$ is the estimate of the timing error over one pulse time and for small ε is given by

$$\hat{\varepsilon}(t) = 2PT\varepsilon g(t) + \sqrt{P} b_k T(N_2 + N_3)g(t)$$

$$+ (N_1 + N_2 - N_3 - N_4)(N_2 + N_3)g(t) \quad (36)$$

where $g(t)$ is the pulse function taking on the value of one at each loop update (see Fig. 8). This pulse function is constant over T sec and thus acts as a sample-and-hold function for the loop error signal.

Equation (36) gives the conditional mean value of $\hat{\varepsilon}$ given ε over one update period as

$$E[\varepsilon|\varepsilon] = 2PT\varepsilon \qquad (37)$$

since

$$E[(N_1 + N_2 - N_3 - N_4)(N_2 + N_3)] = 0 \qquad (38)$$

because

$$E\left[N_2^2\right] = E\left[N_3^2\right] \qquad (39)$$

If the total noise term is denoted by N_T, where

$$N_T = \overbrace{\sqrt{P}T(N_2 + N_3)}^{N_A} + \overbrace{(N_1 + N_2 - N_3 - N_4)(N_2 + N_3)}^{N_B} \qquad (40)$$

then it can be shown that the two noise components N_A and N_B are independent. That is,

$$E[N_A N_B] = \sqrt{P}TE[(N_2 + N_3)$$

$$\times (N_1 + N_2 - N_3 - N_4)(N_2 + N_3)] = 0 \qquad (41)$$

since odd moments of zero mean Gaussian random variables are zero.

To obtain the loop equation, the estimate of the error is written as

$$\hat{\varepsilon} = 2PT\varepsilon + N_T g(t) \qquad (42)$$

The loop timing estimate is given by

$$\hat{\tau} = \frac{KF(s)}{s}\hat{\varepsilon} = \frac{KF(s)}{s}[2PT\varepsilon + N_T g(t)] \qquad (43)$$

where K is the loop gain of the symbol synchronizer and $F(s)$ is the loop filter expressed as a Heaviside operator. Using

$$\tau - \hat{\tau} = \varepsilon \qquad (44)$$

and for convenience $\tau = 0$ in Eq. (43) yields

$$\varepsilon(t) = H(s)\left[\frac{N_T g(t)}{2PT}\right] \qquad (45)$$

where $H(s)$ is the closed-loop transfer function and

$$H(s) = \frac{KF(s)/s}{1 + KF(s)/s} \qquad (46)$$

When the loop noise bandwidth B_L is small compared to the symbol rate, the linearized tracking error can be approximated by

$$\sigma_\varepsilon^2 = \frac{2B_L \mathscr{S}_{N_T}(0)}{4P^2T^2} \qquad (47)$$

where $\mathscr{S}_{N_T}(0)$ is the spectral density at $f = 0$ of the cyclostationary process $N_T(t)$, with $N_T(t)$ defined below.

Thus, it is necessary to evaluate the spectral density of the noise process at $f = 0$. The noise process can be written as

$$N_T(t) = \sum_{k=-\infty}^{\infty} N_T(k) g(t - kT) \tag{48}$$

and thus the $N_T(t)$ process is cyclostationary. It can be made stationary by averaging over time. Thus, the autocorrelation function of the stationary equivalent process is given by

$$R(\zeta) = \frac{1}{T} \int_0^T E[N_T(t) N_T(t+\zeta)]\, dt \tag{49}$$

Thus

$$\left.\begin{aligned} R_{N_T}(\zeta) &= \sigma^2{}_{N_T}\left(1 - \tfrac{|\zeta|}{T}\right) \quad \text{for } |\zeta| \le T \\ &= 0 \qquad\qquad\qquad\quad \text{otherwise} \end{aligned}\right\} \tag{50}$$

Since

$$\mathscr{S}_{N_T}(0) = \int_{-\infty}^{\infty} R(\zeta) d\zeta = T\sigma^2_{N_T} \tag{51}$$

one has for Eq. (47) that

$$\sigma^2_\varepsilon = \frac{B_L T \sigma^2_{N_T}}{2P^2T^2} \tag{52}$$

Evaluating $\sigma^2_{N_T}$ yields

$$\sigma^2_{N_T} = \frac{N_0 W P T^2}{2} + \frac{N_0^2 W T}{4} \tag{53}$$

Therefore, using Eq. (53) in Eq. (52), with $R = PT/N_0$, the normalized tracking error is given by

$$\frac{\sigma^2_\varepsilon}{T^2} = \frac{B_L W}{4R}\left[1 + \frac{1}{2R}\right] (\text{fraction of a chip})^2 \tag{54}$$

The normalized tracking error is plotted in Fig. 3. In the comparison, W was set equal to $T/4$ to be consistent with the other two loops considered here. Thus, Eq. (54) becomes

$$\frac{\sigma^2_\varepsilon}{T^2} = \frac{B_L T}{16R}\left[1 + \frac{1}{2R}\right] \tag{55}$$

IV. Analysis of the Mid-Symbol– and Adjacent-Symbol–Tracking Manchester Symbol Synchronizer

Figure 6 shows the symbol synchronizer discussed in this section. The functions $h(t)$ and $1 - h(t)$ are shown in Fig. 7. The upper branch performs the same function as the previous section and operates on the mid-symbol transitions. However, in addition, the lower section operates on the adjacent-symbol transition points. The lower and upper sections are used to update the loop at twice the symbol rate, unlike the loop discussed in the previous section, which is updated every symbol time.

A. Mid-Symbol Error Detection

First consider the upper two branches of the mid-symbol error detector illustrated in Fig. 6. A symbol and loop timing diagram is illustrated in Fig. 7. Initially, W_B and W_M denote the between-symbol and mid-symbol windows, respectively. Again the signal is modeled as described in Eq. (1).

The signal denoted X in Fig. 6 is given (for small ε) by

$$\begin{aligned} X &= \sqrt{P}(b_k T - (b_{k-1} + 3b_k)|\varepsilon|) \\ &\quad + (N_{00} + N_1 + N_2 - N_3 - N_4 - N_5) \end{aligned} \tag{56}$$

where the integration regions are as indicated in Fig. 8. The Y-channel output (for small $\varepsilon \ge 0$) is given by

$$Y = \left(2\sqrt{P} b_k \varepsilon + N_2 + N_3\right) \tag{57}$$

where

$$N_{00} = \int_0^{W_{B/2}} n(t)\, dt \tag{58}$$

with the time origin taken at the point where the synchronizer loop starts the kth symbol for simplicity of notation. In addition, there are the definitions

$$N_1 = \int_{W_B/2}^{T/2-W_M/2} n(t)\,dt \quad (59)$$

$$N_2 = \int_{T/2-W_M/2}^{T/2} n(t)\,dt \quad (60)$$

$$N_3 = \int_{T/2}^{T/2+W_M/2} n(t)\,dt \quad (61)$$

$$N_4 = \int_{T/2+W_M/2}^{T-W_B/2} n(t)\,dt \quad (62)$$

$$N_5 = \int_{T-W_B/2}^{T} n(t)\,dt \quad (63)$$

$$N_6 = \int_{T}^{T+W_B/2} n(t)\,dt \quad (64)$$

Thus, the first error signal Z_1 is given by

$$Z_1 = XY = \hat{\varepsilon}_1$$

and from Eqs. (56) and (57), for small $|\varepsilon|$

$$\hat{\varepsilon}_1 = 2PT\varepsilon + \overbrace{\sqrt{P}b_kT(N_2+N_3)}^{N_A}$$

$$+ \overbrace{(N_{00}+N_1+N_2-N_3-N_4-N_5)(N_2+N_3)}^{N_B} \quad (65)$$

over $T/2$ seconds. Thus, showing the explicit time dependence of the error signal with time

$$\hat{\varepsilon}_1(t) = \sum_{k=-\infty}^{\infty} \big(2PT\varepsilon(k) + N_A(k) + N_B(k)\big)h(t-kT) \quad (66)$$

where $h(t)$ is defined to be unity in the region $t\varepsilon(0,T/2)$ and zero in the region $t\varepsilon(T/2,T)$. The functions $h(t)$ and $1-h(t)$ are plotted in Fig. 8 along with $p(t)$, $g(t)$, and $q(t)$. The quantities $\varepsilon(k)$, $N_A(k)$, and $N_B(k)$ are the values of the respective variables at the kth symbol time.

Equation (66) is the contribution of the mid-symbol error detector composed of the upper two branches in Fig. 6. Furthermore, from Eq. (65) the mean value of $\hat{\varepsilon}_1$ over one symbol time is given by

$$E[\hat{\varepsilon}_1|\varepsilon] = 2PT\varepsilon \quad (67)$$

B. Adjacent-Symbol Error Detection

Now consider the adjacent-symbol transition detector depicted in the lower half of Fig. 6:

$$U_k' = 2b_k\sqrt{P}(\tau-\hat{\tau}) + N_5 + N_6 \quad (68)$$

For the upper branch of the adjacent-symbol detector, the detected half-symbols are given by

$$\hat{a}_{2K+1} = \mathrm{sgn}\left[-b_k(T/2-2|\varepsilon|) + N_3 + N_4 + N_5\right] \quad (69)$$

and

$$\hat{a}_{2k+2} = \mathrm{sgn}\left[b_{k+1}(T/2-2|\varepsilon|) + N_6 + N_7\right] \quad (70)$$

In Fig. 6 a scale factor of $\beta\sqrt{PT}$ has been included in the upper branch of the lower half of the figure. Its purpose is to make the units the same ($\sqrt{PT}$) for the upper and lower halves, with a scale factor of β ($0 \le \beta$) used to adjust the relative proportion of each error signal.

Hence, the upper branch of the between-half-symbol transition detector denoted by I_k can be expressed in terms of the a_k sequence as

$$I_k = \left(\frac{-\hat{a}_{2k+1} + \hat{a}_{2k+2}}{2}\right)\beta\sqrt{PT} \quad (71)$$

The estimate of the error signal from the lower half is therefore given by

$$\hat{\varepsilon} = U_k I_k \quad (72)$$

or

$$\hat{\varepsilon}_2 = \left(2b_k\sqrt{P}(\tau - \hat{\tau}) + N_5 + N_6\right)$$

$$\times \left(\frac{-\hat{a}_{2k+1} + \hat{a}_{2k+2}}{2}\right)\beta\sqrt{P}T \qquad (73)$$

To obtain the conditional mean value of $\hat{\varepsilon}_2$ conditioned on ε, a simplifying assumption is made. First, neglect the small correlation of $(N_5 + N_6)$ with $\hat{a}_{2k+1}$ and $\hat{a}_{2k+2}$. Second, assume that ε is very small in magnitude. Thus, letting $\varepsilon = \tau - \hat{\tau}$ obtains

$$E[\hat{\varepsilon}_2|\varepsilon] = 2\beta PT\left(\frac{1}{2}\right)\left\{(1 - PE_H)^2 - PE_H^2\right\} \qquad (74)$$

or

$$E[\hat{\varepsilon}_2|\varepsilon] = \beta PT\varepsilon(1 - 2PE_H) \qquad (75)$$

where the symbol error rate PE_H is the same as Eq. (16), so that

$$1 - 2PE_H = \operatorname{erf}\left(\sqrt{R/2}\right) \qquad (76)$$

where it was assumed that $\varepsilon = 0$ in the expression for PE_H. Thus,

$$E[\hat{\varepsilon}_2|\varepsilon] = \beta PT\varepsilon \operatorname{erf}\left(\sqrt{R/2}\right) \qquad (77)$$

C. Tracking Performance of the Combined Loop Signal

The total error signal that drives the loop filter $F(s)$ of Fig. 5 is given by

$$\hat{\varepsilon}(t) = \frac{1}{2}\left[\beta PT \operatorname{erf}\left(\sqrt{R/2}\right) + 2PT\varepsilon\right]$$

$$+ \sum_{k=-\infty}^{\infty}\{N_1(kT)h(t - kT)$$

$$+ N_2(kT)[1 - h(t - kT)]\} \qquad (78)$$

where the first term in Eq. (78) is the mean value of $\varepsilon(t)$, and where the second term is the noise process with ε assumed to be zero. The noise terms are given by

$$N_1(kT) = \sqrt{P}b_kT(N_2 + N_3)$$

$$+ (N_{00} + N_1 + N_2 - N_3 - N_4 - N_5)(N_2 + N_3) \qquad (79)$$

and

$$N_2(kT) = \beta\sqrt{PT}\left(\frac{-\hat{a}_{2k+1} + \hat{a}_{2k+2}}{2}\right)(N_5 + N_6) \qquad (80)$$

The symbol synchronizer forms the estimate $\hat{\tau}$ of the received signal delay and can be expressed by

$$\hat{\tau} = \frac{KF(s)}{s}\hat{\varepsilon}$$

$$= \frac{KF(s)}{s}\left\{PT + \frac{1}{2}\beta PT \operatorname{erf}\left(\sqrt{\frac{R}{2}}\right)\right\}\varepsilon + \frac{KF(s)}{s}\{N(t)\} \qquad (81)$$

where

$$N(t) = \sum_{k=-\infty}^{\infty}\{N_1(kT)h(t - kT) + N_2(kT)[1 - h(t - kT)]\} \qquad (82)$$

where K is the loop gain including the phase detector gain, and the ratio $F(s)/s$ is the loop filter expressed in the Heaviside polynomial divided by the filtering effect of the VCO $(1/s)$. Noting that $\varepsilon = \tau - \hat{\tau}$ and assuming that $\tau = 0$ for convenience leads to

$$\varepsilon(t) = H(s)\left[\frac{N(t)}{PT + \frac{\beta}{2}PT \operatorname{erf}\left(\sqrt{\frac{R}{2}}\right)}\right] \qquad (83)$$

where $H(s)$ is the closed-loop transfer function of the symbol synchronizer loop. Following the usual practice, it is assumed that the one-sided loop noise bandwidth B_L is much smaller than the symbol rate, so that the variance of the linearized tracking error in Eq. (83) can be determined from the expression

$$\sigma_\varepsilon^2 = \frac{2B_L\mathscr{S}_N(0)}{\left[PT + \frac{\beta}{2}PT \operatorname{erf}\left(\sqrt{\frac{R}{2}}\right)\right]^2} \qquad (84)$$

where $\mathscr{S}_N(0)$ is the spectral density of the noise process $N(t)$. To evaluate the spectral density, the autocorrelation function of the noise is determined. Since the noise process is cyclostationary, time is averaged over one period to obtain a stationary process.

$$R_N(\tau) \equiv \frac{1}{T}\int_0^T R_N(t+\tau, t)\,dt$$

$$= E\int_0^T \Bigg\{ \sum_{k=-\infty}^{\infty} \{N_1(kT)h(t+\tau-kT)$$

$$+ N_2(kT)[1-h(t+\tau-kT)]\}$$

$$\times \sum_{\ell=-\infty}^{\infty} \{N_1(\ell T)h(t-\ell T)$$

$$+ N_2(\ell T)[1-h(t-\ell T)]\} \Bigg\} dt \tag{85}$$

assuming for convenience that $E[N_1(kT)N_2(\ell T)] \cong 0$ for all k and ℓ (even though there is a small correlation between them). Letting $\ell = k+m$ obtains

$$R_N(\tau) = \frac{1}{T}\int_0^T \Bigg\{ \sum_{m=-\infty}^{\infty} R_{N_1}(mT)$$

$$\times \sum_{k=-\infty}^{\infty} h(t+\tau-kT)h(t-(k+m)T) \Bigg\} dt$$

$$+ \frac{1}{T}\int_0^T \Bigg\{ \sum_{m=-\infty}^{\infty} R_{N_2}(mT) \sum_{k=-\infty}^{\infty} [1-h(t+\tau-kT)]$$

$$\times [1-h(t-(k+m)T)] \Bigg\} dt \tag{86}$$

Additionally, $R_{N_1}(mT) = 0$ for all $m \neq 0$ since $N_1(kT)$ and $N_1((k+1)T)$ are based on integrations over disjoint time intervals. Furthermore, $R_{N_2}(mT) \cong 0$ for all $m \neq 0$ since $N_2(kT)$ and $N_2((k+1)T)$ only have $W_B/2 \ll T$ sec in common over the adjacent (full) symbol times. Using these two conditions obtains

$$R_N(\tau) = \sigma_{N_1}^2 \frac{1}{T}\int_0^T h(t+\tau)h(t)dt + \sigma_{N_2}^2$$

$$\times \frac{1}{T}\int_0^T [1-h(t+1)][1-h(t)]\,dt \tag{87}$$

Completing the averaging,

$$R_N(t) = \frac{\sigma_{N_1}^2}{T}R(\tau) + \frac{\sigma_{N_2}^2}{T}R(\tau) \tag{88}$$

where

$$R(\tau) = \int_0^T h(t+\tau)h(t)\,dt \tag{89}$$

and is illustrated in Fig. 9. Therefore, the spectral density of the noise process is given by

$$\mathscr{S}_N(f) = \frac{\sigma_{N_1}^2}{T}|S(f)|^2 + \frac{\sigma_{N_2}^2}{T}|S(f)|^2 \tag{90}$$

where

$$S(f) = \int_0^T e^{-i\omega\tau}d\tau = \frac{T}{2}e^{-i\omega T/4}\,\frac{\sin(\pi f T/2)}{(\pi f T/2)} \tag{91}$$

so that

$$|S(f)|^2 = \frac{T^2}{4}\,\frac{\sin^2(\pi f T/2)}{(\pi f T/2)^2} \tag{92}$$

From Eqs. (88) and (90), $\mathscr{S}_N(0)$ is given by

$$\mathscr{S}_N(0) = \left(\frac{\sigma_{N_1}^2 + \sigma_{N_2}^2}{4}\right)T \tag{93}$$

Thus, to evaluate the tracking-error variance it is necessary to evaluate $\sigma_{N_1}^2$ and $\sigma_{N_2}^2$. First, $\sigma_{N_1}^2$ is determined. From Eq. (79),

$$N_1 = \sqrt{P}b_kT(N_2+N_3)$$

$$+ (N_{00} + N_1 + N_2 - N_3 - N_4 - N_5)(N_2+N_3) \tag{94}$$

Since the two terms are uncorrelated and have zero mean values, the variance of N_1 is given by the sum of the variances in Eq. (94). If the first grouping of Eq. (94) is denoted as N_A and the second grouping as N_B, then $E[N_A^2]$ and $E[N_B^2]$ can be evaluated. Consider the former:

$$E\left[N_A^2\right] = PT^2E\left[(N_2+N_3)^2\right] = PT^2\frac{N_0}{2}W_M \tag{95}$$

Now consider the latter term with

$$E\left[N_B^2\right] = E\left[N_2^2N_{00}^2\right] + E\left[N_2^2N_1^2\right] + E\left[N_2^4\right] + E\left[N_2^2N_3^2\right]$$

$$+ E\left[N_2^2N_4^2\right] + E\left[N_2^2N_5^2\right] - 4E\left[N_2^2N_3^2\right]$$

$$+ E\left[N_3^2N_{00}^2\right] + E\left[N_3^2N_1^2\right] + E\left[N_3^2N_2^2\right]$$

$$+ E\left[N_3^4\right] + E\left[N_3^2N_4^2\right] + E\left[N_3^2N_5^2\right] \tag{96}$$

After simplifying, one obtains

$$E\left[N_B^2\right] = \frac{N_0^2W_MT}{4} \tag{97}$$

and

$$\sigma_{N_1}^2 = PT^2\frac{N_0}{2}W_M + \frac{N_0^2W_MT}{4} \tag{98}$$

Now consider the computation of $\sigma_{N_2}^2$. From Eq. (80),

$$N_2 = \beta\sqrt{PT}\left[\frac{-\hat{a}_{2k+1}+\hat{a}_{2k+2}}{2}\right](N_5+N_6) \tag{99}$$

To evaluate N_2, the small correlations between N_5 and $\hat{a}_{2k+1}$ and N_6 and $\hat{a}_{2k+2}$ are neglected so that

$$\sigma_{N_2}^2 = \beta^2PT^2E\left[N_5^2+N_6^2\right]E\left[\left(\frac{-\hat{a}_{2k+1}+\hat{a}_{2k+2}}{2}\right)^2\right] \tag{100}$$

or

$$\sigma_{N_2}^2 = \frac{N_0}{4}W_B\beta^2PT^2 \tag{101}$$

since the transition detector term has an average value of 1/2. So,

$$\frac{\sigma_\epsilon^2}{T^2} = \frac{\beta_LT\left[1+2R\left(1+\frac{\beta^2}{2}\right)\right]}{32R^2\left[1+\frac{\beta}{2}\,\mathrm{erf}\left(\sqrt{\frac{R}{2}}\right)\right]^2}\ \text{(fraction of a symbol)}^2 \tag{102}$$

Notice that when $\beta = 0$, this result is the same as Eq. (55), as it should be! Since β is a parameter, it can be varied to minimize Eq. (102). Figure 3 illustrates the results for this symbol sync loop plotted versus R in decibels. For this loop, $\beta\sqrt{PT}$ must be known a priori to obtain optimum performance. However, the parameter β is not very sensitive. For example, at $R = -12$ dB, $\beta_{\text{opt}} = 1.75$ yields a normalized tracking error of 7.52 sec^2/sec^2, and at the value $\beta = 1$, the normalized tracking error becomes 7.73 sec^2/sec^2. However, at $\beta = 0$ (mid-transition detector only), the normalized tracking error becomes 8.84, the same as the second symbol sync loop considered. In fact, when $\beta = 0$, the two curves are identical as noted above.

Therefore, since T would be known precisely a priori, and since P would be known to within 10 to 20 percent, it seems that setting $\beta = 1$ would allow very close to optimum (β_{opt}) performance. Furthermore, β_{opt} is equal to approximately 1 at $E_S/N_0 \geq 7$ dB, so that under most reasonable conditions of the Block V receiver setting $\beta = 1$ is optimum.

V. Conclusion

All three symbol-synchronization tracking loops offer fairly similar performance. The hybrid loop called optimum Manchester is better (low tracking error) for $R \leq$

0 dB than the other two loops. However, for $R \geq 0$, the NRZ$\times$2 loop and the hybrid optimum Manchester loop are essentially equal in performance. For the hybrid optimum Manchester loop to work, the power of the signal has to be estimated to provide the weighting $\beta\sqrt{PT}$ in Fig. 6 with β set equal to unity.

Although the hybrid Manchester loop is optimum, it is not clear that the extra hardware requirement of this loop is warranted. It is necessary to compare the actual estimated tracking losses for each loop based on the requirements to determine if the complexity of the hybrid loop is justified and if it is best at high data rates.

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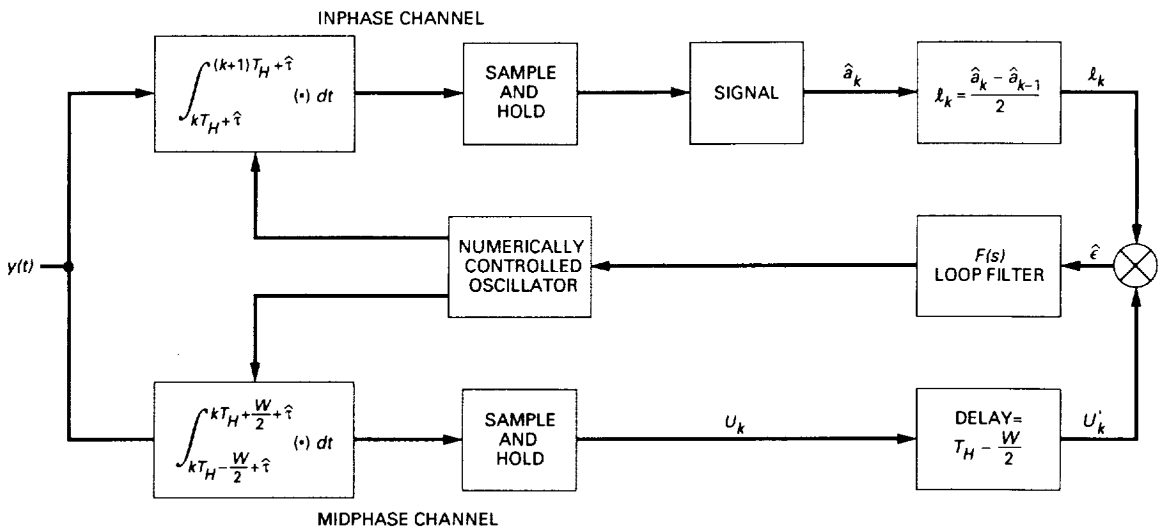
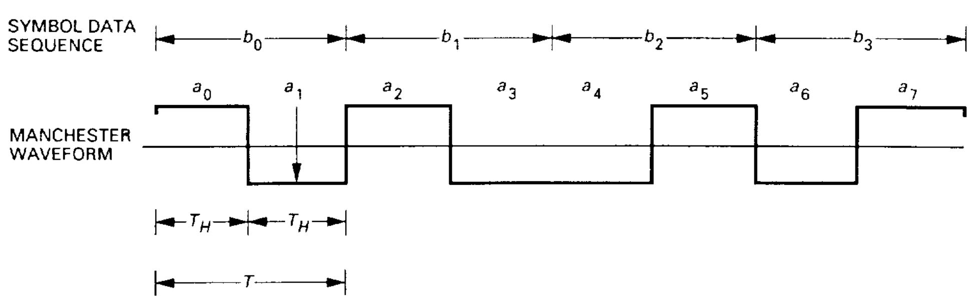


Fig. 1. The NRZ × 2 Manchester symbol synchronizer.



SYMBOL DATA SEQUENCE: $b_0 = 1, b_1 = 1, b_2 = -1, b_3 = -1, \ldots$

PULSE SEQUENCE: $a_0 = 1, a_1 = -1, a_2 = 1, a_3 = -1, a_4 = -1, a_5 = 1, a_6 = -1, a_7 = 1, \ldots$

Fig. 2. The relationship between the full-symbol data sequence and the Manchester half-symbols.

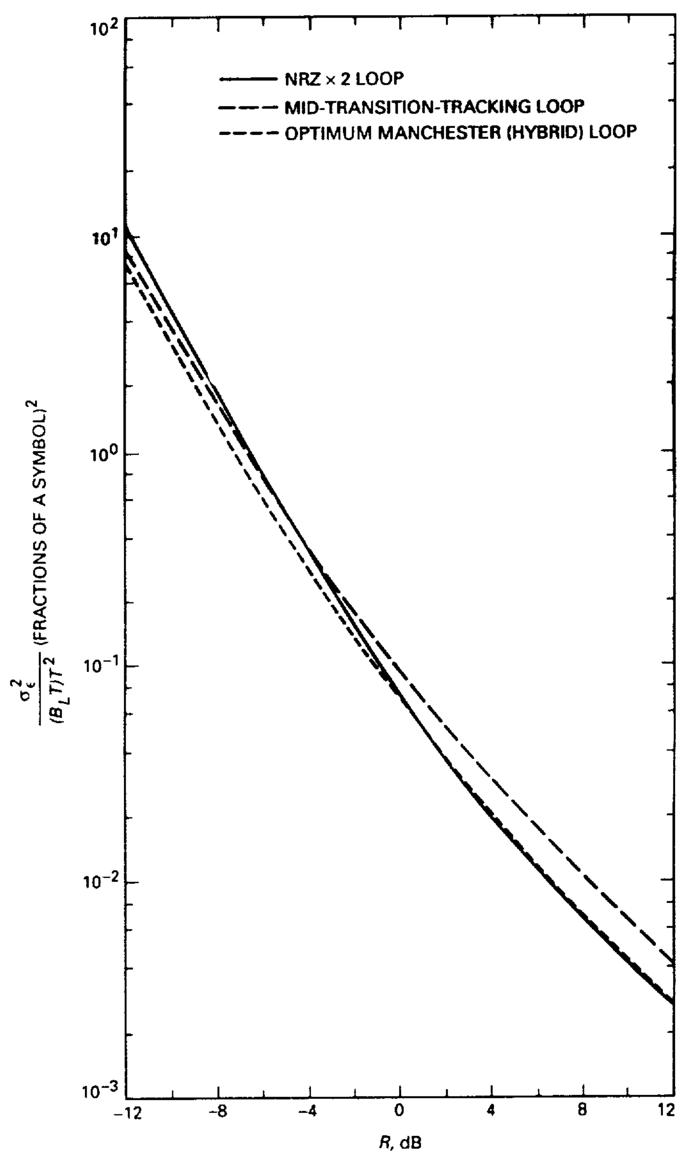


Fig. 3. A comparison of the three Manchester symbol-synchronization loops.

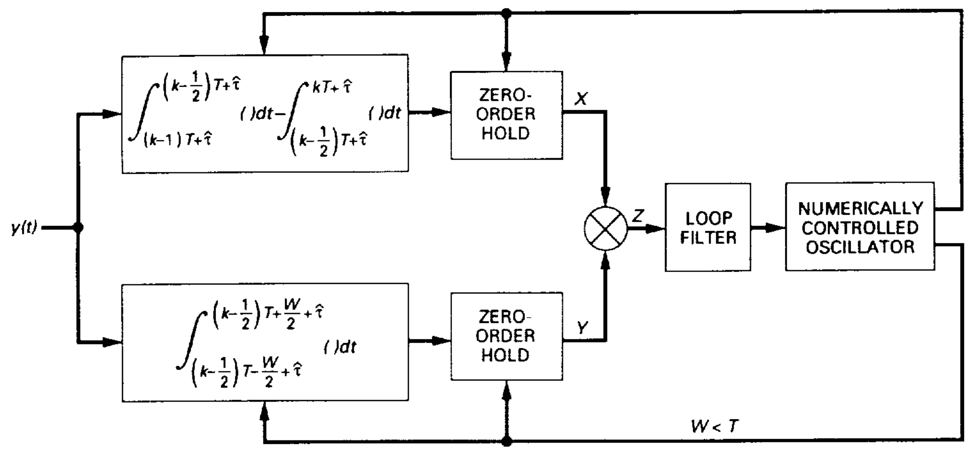


Fig. 4. The mid-transition-tracking Manchester symbol synchronizer.

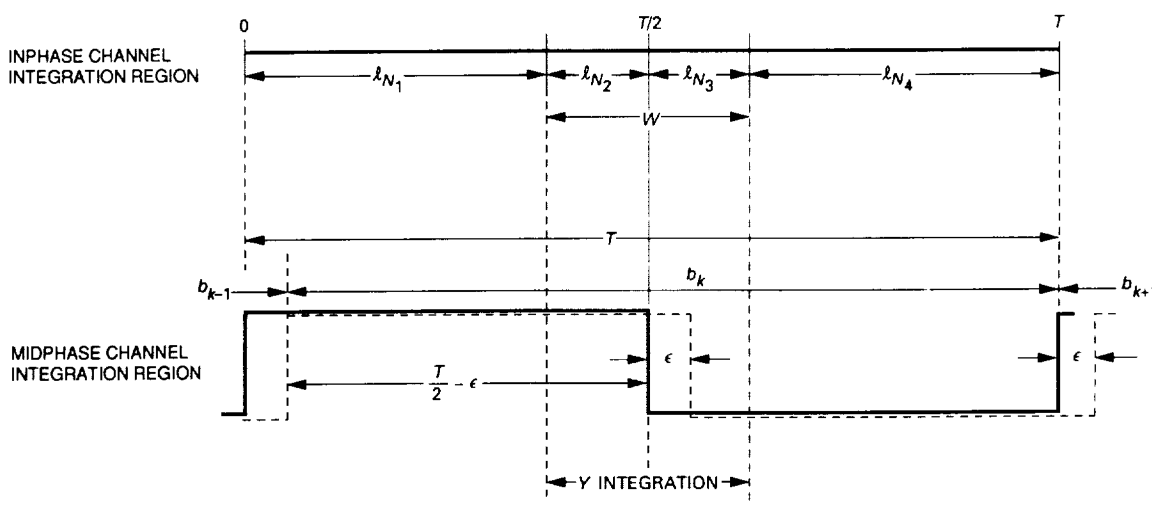


Fig. 5. The inphase channel and the midphase channel integration region for the Manchester symbol synchronizer (shown for $W/2 > \epsilon > 0$).

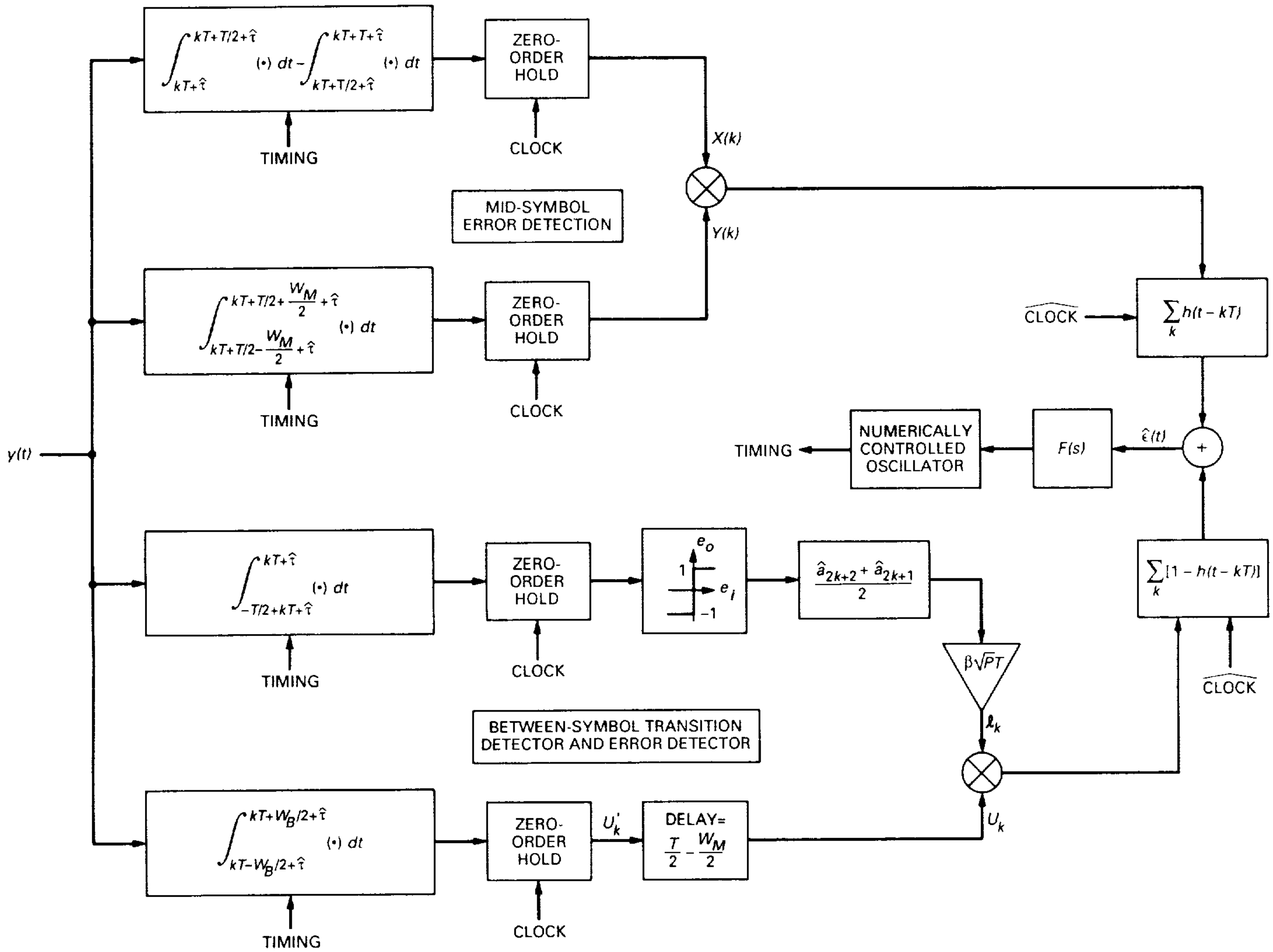


Fig. 6. A Manchester symbol synchronizer for tracking mid-symbol and adjacent-symbol transitions.

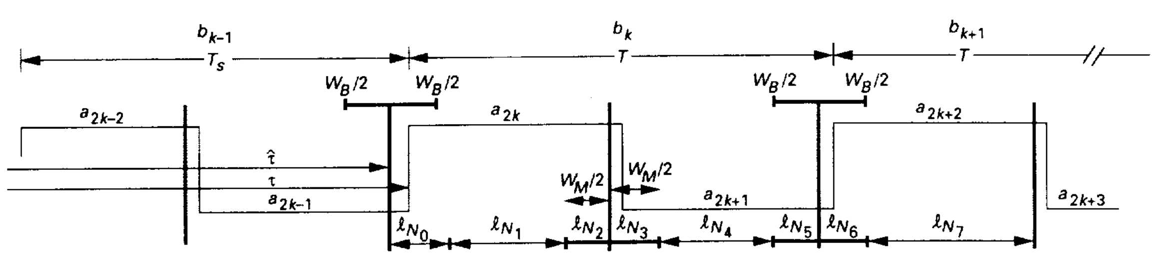


Fig. 7. Symbol and loop timing diagram. Loop timing is indicated in dark lines and received signal is indicated in light lines.

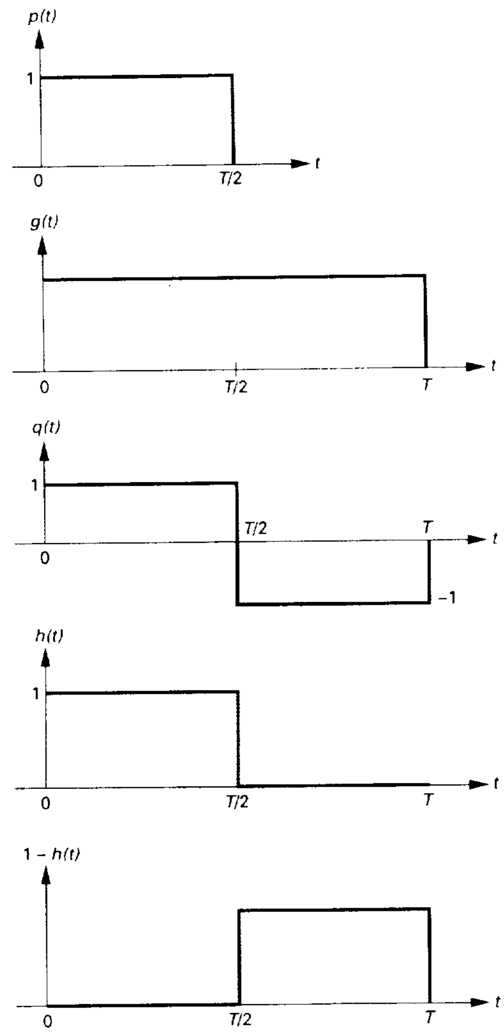


Fig. 8. Repetitive pulses used in the noise analysis.

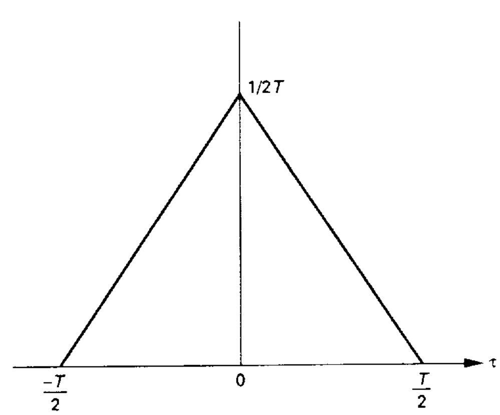


Fig. 9. Autocorrelation function for the *h*(*t*) multiplexing symbols.