Department of Geodetic Science and Surveying

BASIC RESEARCH FOR THE GEODYNAMICS PROGRAM

Twenty-Fifth Semiannual Status Report
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NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771

The Ohio State University
Research Foundation
Columbus, Ohio 43212

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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor is Dr. David E. Smith, Code 620, and the Technical Officer is Dr. Herbert Frey, Code 621, both at Goddard Space Flight Center, Greenbelt, Maryland 20771.

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            Error Analysis Results for Earth Rotation Parameters," poster paper at
            NASA CDP Investigators' Meeting, Greenbelt, MD, Oct. 25–26, 1990.

1. SPACE VLBI FOR GEODESY AND GEODYNAMICS

1.1 Introduction

In the 1990's dedicated radio telescopes will be launched into Earth orbit and will be integrated in the *ground-based VLBI* networks. A straightforward extension from present ground-based VLBI to space is called *space VLBI*, which uses radio-antennas in space.

The first space VLBI mission that will be implemented in the near future is RADIOASTRON. Its trajectory is planned to be 3,000 km in perigee and 69,000 km in apogee. The satellite will carry a 10 m antenna. It will be launched in 1993. The second project is a Japanese orbiting VLBI mission called VSOP. Its trajectory is 1,000 km in perigee and 20,000 km in apogee. The expected launch date is 1995. Both projects are now in progress. Most probably a combined use of RADIOASTRON and VSOP observations will be performed in the future.

Space VLBI observables may be useful to improve Earth's gravity field and in the unification and connection of reference frames inherent in the space VLBI technique. The space VLBI missions offer and will provide new types of satellite observables (VLBI time delay, delay rate and differential VLBI tracking data) with high accuracy for these potential applications.

In the simplest version of the space VLBI technique, one station in orbit observes in conjunction with a second station on the ground. However, in practice we have a number of networks of ground antennas observing the common celestial radio sources simultaneously with a conventional VLBI technique. Moreover, joint observations of two or more space VLBI satellites will supposedly be performed in the future. Therefore, in our research work a combined use of simultaneous space and ground-based VLBI observations are considered from the geodetic and geodynamic points of view.

1.2 Purpose of the Research

The space VLBI observables may be useful to improve the Earth's gravity field and in the unification and connection of reference frames inherent in the space VLBI technique. The space VLBI missions offer and provide new types of satellite observables (VLBI time delay, delay rate and differential VLBI tracking data) with high accuracy for these potential applications. Therefore, we started to investigate potential possibilities of space VLBI missions for these areas. The purpose of the corresponding research work is to explore the feasibility of these potential applications and to provide sufficient background for the inclusion of space VLBI observables in geodetic data processing programs (e.g., GEODYN). The final goal of the research from the point of view of geodetic science should be to figure out whether or not space VLBI can provide real improvements in the two potential applications in geodesy and geodynamics.

The purpose of this research work is, as a first step, to derive the mathematical models of space VLBI observables suitable for least squares covariance analysis as well as to explore estimability problems inherent in the space VLBI system, including a detailed rank defect analysis and sensitivity analysis. An important aim is to carry out a comparative analysis of the mathematical models of the ground-based VLBI and space VLBI observables in order to describe the background in detail. Developing computer programs in order to check the relations, and for error assessment and sensitivity analysis later on, is also an important purpose of this work.

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1.3 Estimability of Geodetic and Geodynamic Parameters from Space VLBI

In order to investigate the estimability of different geodetic and geodynamic parameters from the space VLBI observables, the mathematical models for time delay and time delay rate observables of space VLBI have been analytically derived along with the partial derivatives with respect to the parameters. Rank defect analysis has been carried out both by analytical and numerical testing of linear dependencies between the columns of the normal matrix thus formed. Definite conclusions have been formed about the rank defects in the system.

1.3.1 Mathematical Models. The mathematical models for ground-to-space VLBI observables have been defined through equations formed by suitable modifications to the equations relating the ground-to-ground VLBI observables to the parameters. Our main purpose is not to derive explicit observations equations, but rather to develop qualitative expressions to demonstrate the relationship between the observables (time delay and delay rate) and the parameters (station and source coordinates, orbital parameters, earth rotation parameters—ERP's, and clock parameters), and to explore the estimability problems inherent in the system.

The mathematical models for time delay and delay rate observables for ground-to-space and for space-to-space VLBI observations have been derived and are included in a detailed report by Jozsef Adam (Estimability of Geodetic Parameters from Space VLBI Observables, Department of Geodetic Science and Surveying Rep. 406, in press).

1.3.2 Rank Defect Analysis. To carry out the analysis of rank deficiency of the normal matrix for estimating the set of parameters from the time delay ground-to-space observable, partial derivatives of the observables with respect to the parameters are derived analytically as a first step. Examining the analytical expressions for the partial derivatives, the four linear dependency relations between the columns of the design matrix have been derived as given in the abovementioned report.

Thus from the analytical method, the rank deficiency of the system is found to be four. In order to confirm this, numerical testing was carried out using software developed/ modified/obtained from various sources. The stations and radio sources and their coordinates, the orbital parameters of the space VLBI satellite and the basic configuration used in the test computations are tabulated in Tables 1–4 of the report by Adam.

To determine the rank deficiency of the normal matrix, eigenvalue analysis is done, the results of which for both the short-arc and full orbit cases are given in Table 5 of the report by Adam. These are also confirmed by doing Cholesky lower-upper triangular matrix decomposition of the matrix. It is obvious from the results that the rank deficiency of four, as derived analytically, is confirmed from these numerical tests.

1.3.3 Conclusions. From the analytical derivations and also from the numerical tests carried out, it is concluded that the rank deficiency of the normal equation systems in the case under investigation is four. A geometric interpretation for this rank deficiency can be given as follows: A linear dependence between various partial derivatives of the ground station coordinates and the earth rotation parameters indicate a rank deficiency of three due to a lack of absolute orientation of the network of ground stations with respect to the true-of-date celestial frame which cannot be sensed by the observables. The fourth rank deficiency, a linear dependency between partial derivatives with respect to the right ascension of a radio source and the ascending node of satellite orbit, is due to a lack of reference direction for the true-of-date celestial frame, i.e., due to the observables being

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insensitive to the orientation of the true-of-date inertial frame in right ascension. The origin of the reference frames is fixed by the given Keplerian orbital elements at some reference epoch T_0, thus rendering a rank deficiency of only four. For more detail, please refer to the report by Adam.

1.4 Sensitivity Analysis

After concluding the rank defect analysis of the system, numerical testing was taken up to determine the sensitivity of the space VLBI time delay observables to the determination of·different parameters of geodetic and geodynamic interest, including the ERP's, orbital parameters, etc. Computer programs have been developed/modified/obtained and used for such numerical testing.

As a first step, Cartesian coordinates of the space VLBI satellite at specified epochs and coordinates of the quasars are obtained and converted to Keplerian orbital elements. Input files with UT and GAST epochs at a given interval are created, and ERP's are predicted using the IERS formula for the prediction. The partial derivatives are numerically evaluated using a computer program developed to obtain elements of design matrix A from which the normal matrix $A^T A$ is computed, assuming unit weights.

Different combinations of number of epochs, number of stations and number of radio sources are used to investigate the effect of these factors on the sensitivity. These computations have been carried out for both the proposed space VLBI systems: VSOP and RADIOASTRON. Different combinations of parameters to be estimated have also been used in order to study the sensitivity to different combinations of parameters.

The numerical results of the different tests carried out indicate that the accuracy of the ERP's determined from the minimum configuration of three observing stations, three quasar sources and ten observations epochs, for an observation period of about six hours is as low as a few tens of milliarcseconds, which improves to the order of about 0.3 milliseconds for a configuration of 15 stations, nine sources and 2880 observations epochs, with an observation period of about two days.

It must be noted that these accuracy estimates are most optimistic, assuming a simple Keplerian orbit, no errors in the parameters held fixed, and before fixing the rank defect. These assumptions are only gross approximations and cannot be considered realistic. A more detailed sensitivity analysis, by fixing rank defects analytically and by fixing parameters and including error propagation and also by removing some of the approximations mentioned above, is in hand. The results from the preliminary simple analysis indicate accuracies marginally better than those obtained by the existing techniques for the determination of the ERP's; however, a more detailed investigation may be required before reaching definite conclusions.

1.5 Error Analysis for Estimation of Earth Rotation Parameters

From the sensitivity analysis carried out to analyze the effect of the different parameters, e.g., the number of observing stations, number of radio sources, number of observation epochs, total observation time, etc. on the accuracy of earth rotation parameters to be estimated from the space VLBI observables, the optimum number of these parameters were found to be:

Number of observation epochs	1800
Number of observing stations	15
Number of radio sources	3
Total observation time	45 hours

With these optimum number of parameters, a detailed error analysis to estimate the effect of errors in the parameters on the errors in the ERPs estimated from this data, using the simplified mathematical model, was taken up. A priori variances were assigned to the parameters: station coordinates, radio source coordinates, orbital parameters (Keplerian elements of the satellite orbit), and the clock parameters, and these errors were apropagated into the ERPs using the simplified mathematical model.

The necessary relationships and models for error propagation were derived and computer programs developed for computing these effects numerically, using the simulated orbit, the necessary data and the assumed a priori variances. Different values were chosen for the a priori variances assigned to different parameters, from the most optimistic case to the worst case.

The results obtained through this detailed error analysis were tabulated and plotted and were presented in the poster papaer titled "Space VLBI Error Analysis Results for Earth Rotation Parameters," presented at the NASA CDP Investigators' Meeting, Greenbelt, Maryland, October 25-26, 1990. (See the Appendix.)

From the results obtained, it can be seen that in the most realistic case—Case B (Page 17 of Appendix A)—the error in the ERPs estimated from the space VLBI observables is about 0.7 mas in ERPs X_p and Y_p, and about 0.06 ms in the change in the earth's rotation rate (UT1–UTC), which are still marginally better than the accuracy obtained by present techniques. Hence it is considered worthwhile to continue these investigations and to take up a detailed simulation analysis for the feasibility study of determination of the ERPs from the space VLBI observables. It can also be seen from the results of analysis of effect of orbit errors on ERPs (page 15 of Appendix A) that errors in orbit, holding the station positions fixed, are not propagated effectively to the errors in ERPs through the present model. Hence a detailed investigation of this also is in hand.

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2. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time, without compensation
Madhav N. Kulkarni, Graduate Research Associate, part time

3. TRAVEL

Ivan I. Mueller
    Ottawa, Canada   September 1–7, 1990
    To attend the 2nd International Symposium on Precise Positioning with the Global
    Positioning System—GPS '90 and present the present the keynote address .

Ivan I. Mueller
    Miami, Florida   October 14–15, 1990
    To attend International Symposium on Marine Positioning—INSMAP '90, and to
    present the keynote address. Partial project support.

Ivan I. Mueller
    Virginia Beach, Virginia   October 16–19, 1990
    To attend IAU Colloquium 127, Reference Systems, and to participate in IAU
    Working Group on Coordinate Frames and Origins. Partial project support.

Ivan I. Mueller
    Greenbelt, Maryland   October 24–27, 1990
    To attend NASA Crustal Dynamics Program Investigators' Meeting and to present a
    poster paper.

Ivan Mueller conducted meetings of the Planning Group for the IAG International GPS
Service at
    Ottawa, Canada   Sept. 2–5, 1990

and of the Steering Committee at
    Greenbelt, Maryland October 24, 1990

4. DEPT. OF GEODETIC SCIENCE AND SURVEYING REPORTS PUBLISHED TO DATE UNDER PROJECT SUPPORT

No.

262 The Observability of the Celestial Pole and Its Nutations
     by Alfred Leick, June, 1978

263 Earth Orientation from Lunar Laser Range-Differencing
     by Alfred Leick, June, 1978

284 Estimability and Simple Dynamical Analyses of Range (Range-Rate and Range-Difference) Observations to Artificial Satellites
     by Boudewijn H.W. van Gelder, December, 1978

289 Investigations on the Hierarchy of Reference Frames in Geodesy and Geodynamics
     by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter, August, 1979

290 Error Analysis for a Spaceborne Laser Ranging System
     by Erricos C. Pavlis, September, 1979

298 A VLBI Variance-Covariance Analysis Interactive Computer Program
     by Yehuda Bock, May, 1980

299 Geodetic Positioning Using a Global Positioning System of Satellites
     by Patrick J. Fell, June, 1980

302 Reference Coordinate Systems for Earth Dynamics: A Preview
     by Ivan I. Mueller, August, 1980

320 Prediction of Earth Rotation and Polar Motion
     by Sheng-Yuan Zhu, September, 1981

329 Reference Frame Requirements and the MERIT Campaign
     by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock, June, 1982

337 The Use of Baseline Measurements and Geophysical Models for the Estimation of Crustal Deformations and the Terrestrial Reference System
     by Yehuda Bock, December, 1982

338 On the Geodetic Applications of Simultaneous Range-Differencing to Lageos
     by Erricos C. Pavlis, December, 1982

340 A Comparison of Geodetic Doppler Satellite Receivers
     by Brent A. Archinal, November, 1982 (partial support)

348 On the Time Delay Weight Matrix in VLBI Geodetic Parameter Estimation
     by Yehuda Bock, July, 1983

351 Model Choice and Adjustment Techniques in the Presence of Prior Information
     by Burkhard Schaffrin, September, 1983

370 Positioning with NAVSTAR, the Global Positioning System
     by Ziqing Wei, October, 1986

375   Determination of Earth Rotation by the Combination of Data from Different Space
      Geodetic Systems
      by Brent A. Archinal, February, 1987

382   An Algorithm for Crustal Deformation Analysis
      by Huseyin Baki Iz, September, 1987

384   Baseline Estimation from Simultaneous Satellite Laser Tracking
      by George C. Dedes, October, 1987

394   Reference Coordinate Systems: An Update
      by Ivan I. Mueller, November, 1988

406   Estimability of Geodetic Parameters from Space VLBI Observables
      by Jozsef Adam, July, 1990

Publications and Presentations Since Mid-1985

Mueller, Ivan I., "Reference Coordinate Systems and Frames: Concepts and Realization,"
    Bulletin Géodésique 59, 2, 181-188, 1985.

Wei, Ziqing, "Positioning at The Ohio State University: Franklin County Results,"
    Positioning with GPS–1985," Proc. of the 1st International Symp. on Precise
    Positioning with the Global Positioning System, Rockville, Maryland, April 15-19,
    1985 (C.C. Goad, ed.,), pp. 509-520, National Geodetic Information Center, NOAA,
    Rockville, MD 20852.

Mueller, Ivan I., "Plans for a New Terrestrial Reference Frame: The MERIT/COTES
    Campaigns," 7th Annual Conf. on the NASA Geodynamics Program, May 27-31,
    1985, Baltimore, Maryland (invited).

Mueller, Ivan I., "IAG/COSPAR African Dopler Survey–Project ADOS," 7th Annual
    Conf. on the NASA Geodynamics Program, May 27-31, 1985, Baltimore, Maryland.

Mueller, Ivan I., "Terrestrial Reference Systems," NATO Workshop on Earth's Rotation:
    Solved and Unsolved Problems, June 11-13, 1985, Chateau de Bonas, Gers, France
    (invited).

Mueller, Ivan I. and Ziqing Wei, "MERIT Main Campaign: Reference Frame
    Intercomparisons," **Papers in Honor of Antal Tárczy-Hornoch 85**, Geodetic
    and Geophysical Research Institute of the Hungarian Academy of Sciences, Sopron,
    1985. Also, **Proc. of the International Conf. on Earth Rotation and the
    Terrestrial Reference Frame**, July 31–August 3, 1985, Columbus, Ohio, Dept.
    of Geodetic Science and Surveying, Ohio State Univ.

Wilkins, G.A. and I.I. Mueller, "On the Rotation of the Earth and the Terrestrial Reference
    System, Joint Summary Report of the IAU/IUGG Working Groups MERIT and
    COTES, **Bulletin Géodésique, 60**, 1986. Also, Spring Meeting of AGU, May
    19-23, 1986, Baltimore, Maryland (invited).

Mueller, Ivan I., "From 100 m to 100 mm in (About) 25 Years," keynote address, **Proc. 4th International Geodetic Symp. on Satellite Positioning**, April 28–May 2, 1986, Austin, Texas.

Wei, Ziqing, "Mathematical Models and Results Comparison for Various Relative Positioning Modes," **Proc. 4th International Geodetic Symp. on Satellite Positioning**, April 28–May 2, 1986, Austin, Texas.

Hilla, Stephen A., "Processing Cycle Slips in Nondifferenced Phase Data from the Macrometer V-1000 Receiver," **Proc. 4th International Geodetic Symp. on Satellite Positioning**, April 28–May 2, 1986, Austin, Texas.

Wilkins, G.A. and I.I. Mueller, "Rotation of the Earth and the Terrestrial Reference System," **EOS, Transactions of the American Geophysical Union, 67**, No. 31, Aug. 5, 1986. Also, in **CSTG Bulletin, No. 8, 1986.**

Mueller, Ivan I. and George A. Wilkins, "Earth Rotation and the Terrestrial Reference Fame," **Applications of Space Techniques for Geodesy and Geodynamics** (COSPAR Symp. 2, July 30, 1986), D. Reidel Publ. Also, 7th International Symp. on Recent Crustal Movements of the Earth, Tallinn, USSR, Sept. 8-13, 1986. Also at Pulkovo Observatory, Leningrad, USSR, Sept. 15, 1986.

Archinal, Brent A. "Combination of Data from Different Space Geodetic Systems for the Determination of Earth Rotation Parameters," 11th Crustal Dynamics Project Principal Investigators Meeting, October 2--24, 1986, Greenbelt, Maryland. Also, Proc. of IAU/IAG Symp. No. 128, **Earth Rotation and Reference Frames for Geodesy and Geodynamics**, Coolfont, West Virginia, October 20-24, 1986, D. Reidel Publ., 1987.

Iz, Huseyin Baki, "An Algorithm for Crustal Deformation Analysis," 11th Crustal Dynamics Project Principal Investigators Meeting, October 20-24, 1986, Greenbelt, Maryland.

Mueller, Ivan I., "Surveying with the Global Positioning System," Argentine Geophysical and Geodetic Association, 14th Scientific Meeting on Geophysics and Geodesy, October 27-31, 1986, Mendoza, Argentina.

Mueller, Ivan I., "Terrestrial and Celestial Reference Frames: Concepts and Realization," Argentine Geophysical and Geodetic Association, 14th Scientific Meeting on Geophysics and Geodesy, October 27-31, 1986, Mendoza, Argentina.

Mueller, Ivan I., "International Earth Rotation Service," Intercosmos Symp. on Use of Artificial Satellite Observations for Geodesy and Geophysics, May 18-23, 1987, Szentendre, Hungary.

Dedes, George C. and Ivan I. Mueller, "Differential Laser Observations for Baseline Determinations," 3rd International Conf. on Wegener-Medlas Project, May 25-27, 1987, Bologna, Italy.

Archinal, Brent A. and Ivan I. Mueller, "Further Considerations on Combining Earth Rotation Observations from Different Space Geodetic Systems," pres. at Symp. 4, Variations in Earth Rotation, XIX General Assembly of the IUGG, August 18-19, 1987, Vancouver, Canada. Also, **Bulletin Géodésique, 63**, 35-49, 1989.

Dedes, George C. and Ivan I. Mueller, "Baseline Estimation with Semidynamic and Geometric Satellite Methods," pres. at XIX General Assembly of the IUGG, August 18-19, 1987, Vancouver, Canada. Also, **Bulletin Géodésique, 63**, 99-114, 1989.

Dedes, George C., "Baseline Estimation from Simultaneous Satellite Laser Range Differencing," pres. at 11th Crustal Dynamics Project Principal Investigators Meeting, October 21-23, 1987, Goddard Space Flight Center, Greenbelt, Maryland.

Dedes, George C., "GPS Orbit Determination in the Personal Computer Environment," pres. at 11th Crustal Dynamics Project Principal Investigators Meeting, October 21-23, 1987, Goddard Space Flight Center, Greenbelt, Maryland.

Mueller, Ivan I., "Space Techniques for Geodesy and Geodynamics: Present and Future," pres. at Naval Postgraduate School, November 17, 1987, Monterey, California.

Mueller, Ivan I., "Reference Coordinate Systems: An Update," in **Theory of Satellite Geodesy and Gravity Field Determination**, F. Sansò and R. Rummel (eds.), Lecture Notes in Earth Sciences, No. 25, Springer-Verlag Publ., Heidelberg, 1989, 153-196.

Moritz, Helmut and Ivan I. Mueller, **Earth Rotation: Theory and Observation**, Ungar Publ., New York, 1987.

Kovalevsky, J. and Ivan I. Mueller, "Ch. 1, Introduction to Reference Frames"; Ivan I. Mueller and Richard H. Rapp, "Ch. 6, Horizontal and Vertical Geodetic Datums"; Ivan I. Mueller, "Ch 7, Conventional Terrestrial Reference Frames" and "Ch. 11, Transformations Between Celestial and Terrestrial Reference Frames"; in **Reference Frames**, J. Kovalevsky, I. Mueller, B. Kolaczek (eds.), Kluwer Publ., Dordrecht, 1989.

Dedes, G.C. and C.C. Goad, "Orbit Determination for the Global Positioning System Satellites and Its Potential Implementation in the Microcomputer Environment," invited paper, **Festschrift for Torben Krarup**, Danish Geodetic Institute, Copenhagen, 1989.

Mueller, Ivan I., "Basic Concepts and Realizations of Reference Systems," pres. to Commission 19 of the International Astronomical Union, General Assembly, Baltimore, MD, Aug. 5, 1988

Mueller, Ivan I., "Future Satellite System for Earth Science," pres. at 6th International Symp. on Geodesy and Physics of the Earth, Aug. 22-26, 1988, Potsdam, GDR.

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Dedes, G.C. and C.C. Goad, "Orbit Determination for the Global Positioning System Satellites and Its Potential Implementation in the Microcomputer Environment," Las Cruces Symp., March, 1989.

Mueller, Ivan I., "Ch. 7, Reference Coordinate Systems," in **Interdisciplinary Role of Space Geodesy**, I. Mueller and S. Zerbini (eds.), Lecture Notes in Earth Sciences, No. 22, Springer-Verlag Publ., Heidelberg, 1989.

Mueller, Ivan I., "Terrestrial and Celestial Reference Frames," pres. Mizusawa, Japan, March 15, 1989, and Tokyo National Observatory, March 17, 1989.

Mueller, Ivan I., "Reference Frames and Mapping," Proc. of the 1989 ASPRS/ACSM/AutoCarto 9 Convention,.Baltimore, MD, Apr. 2-7.

Mueller, Ivan I., "Interdisciplinary Role of Space Geodesy, A Report on the Worshop in Erice, Italy, July, 1988," pres. 18th Intercosmos Symp. on the Use of Artificial Satellite Observations for Geodesy and Geophysics, June 12-17, 1989.

Mueller, Ivan I., "Satellites for Geodesy and Geodynamics: Prospects for the Future," Meeting on Scientific and Technological Progress of the World, Budapest, Hungary, Aug. 21-25, 1989.

Adam, Jozsef and Ivan I. Mueller, "Space VLBI for Geodesy and Geodynamics," presented at American Geophysical Union Fall Meeting, San Francisco, December 4–8, 1989. Abstract publ. in **EOS, Trans. AGU, 70**, p. 1052, 1989.

Mueller, Ivan I., "Space Geodesy and Geophysics," presented to Commission of Geophysics and "125 Years of International Cooperation in Geodesy," presented to the Commission of Cartography of the Pan-American Institute of Geography and History XIV General Assembly, San Jose, Costa Rica, Feb. 27-28, 1990.

Mueller, Ivan I., Report to the Committee on Geodesy on the Proposed International GPS Service for Geodynamics, National Academy of Sciences, Washington, D.C., April 19, 1990.

Mueller, Ivan I., Report to the International Earth Rotation Service on the Proposed International GPS Service for Geodynamics, Paris, April 23, 1990.

Mueller, Ivan I., "Satellite Positioning and the IAG," keynote address, Proc. 2nd International Symposium on Precise Positioning with the Global Positioning System—GPS '90, Ottawa, Canada, Sept. 3-7, 1990.

Mueller, Ivan I., "Marine Positioning and the IAG," keynote address, Proc. International Symposium on Marine Positioning—INSMAP '90, Miami, Florida, October 15–19, 1990.

Kulkarni, Madhav, Ivan I. Mueller and Burkhard Schaffrin, "Space VLBI Error Analysis Results for Earth Rotation Parameters," poster paper at NASA CDP Investigators' Meeting, Greenbelt, MD, Oct. 25–26, 1990.

Space VLBI Error Analysis Results for Earth Rotation Parameters

Madhav Kulkarni, Ivan I. Mueller, Burkhard Schaffrin

Dept. of Geodetic Science & Surveying
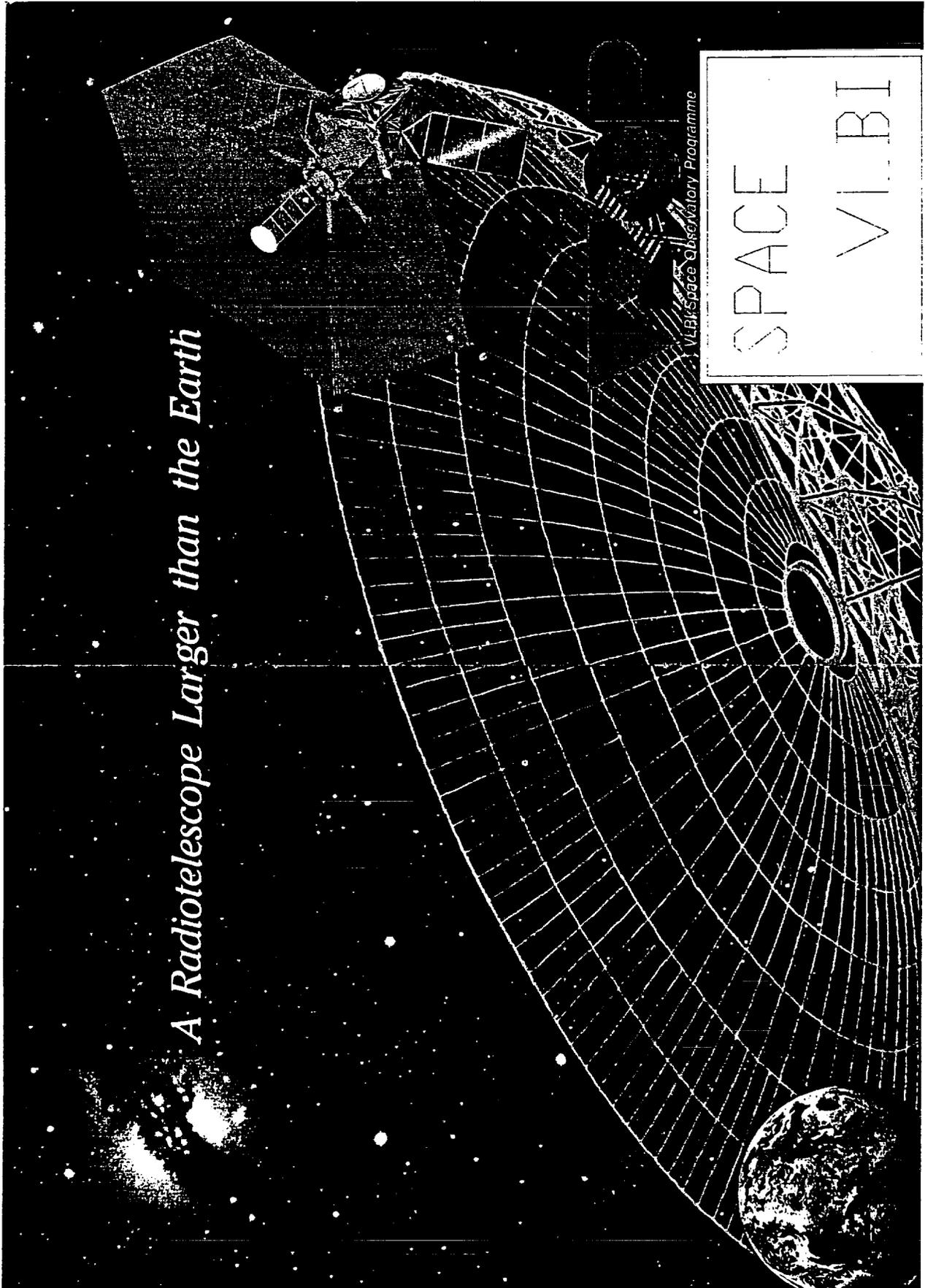The Ohio State University

NASA CDP Investigators' Meeting

Greenbelt, Maryland

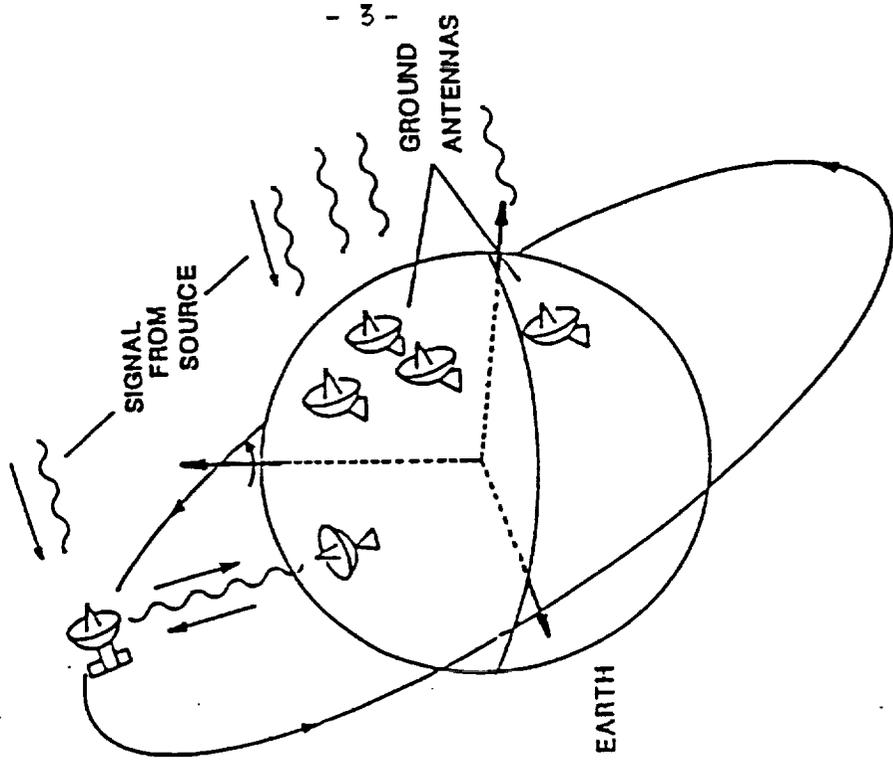October 25-26, 1990

A Radiotelescope Larger than the Earth
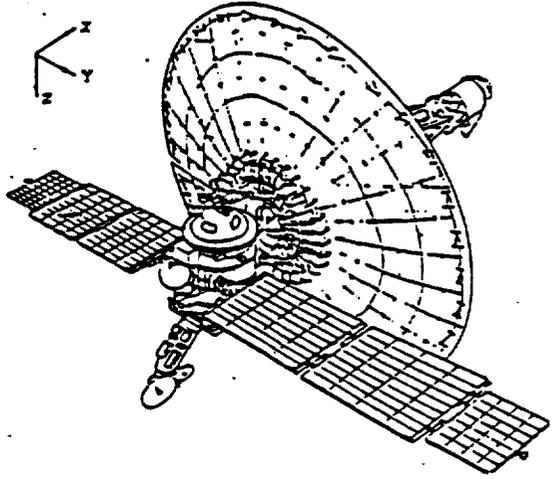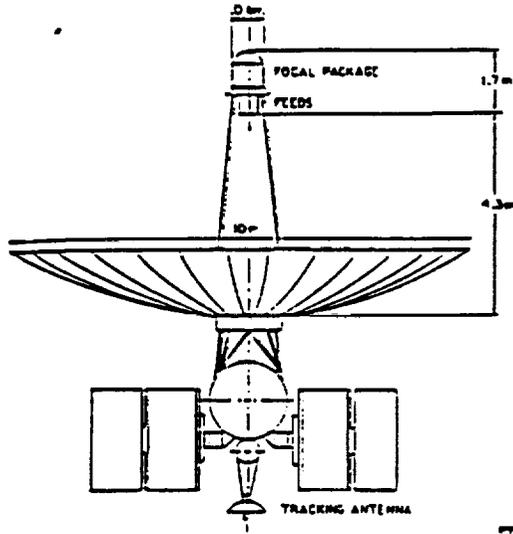
SPACE VLBI

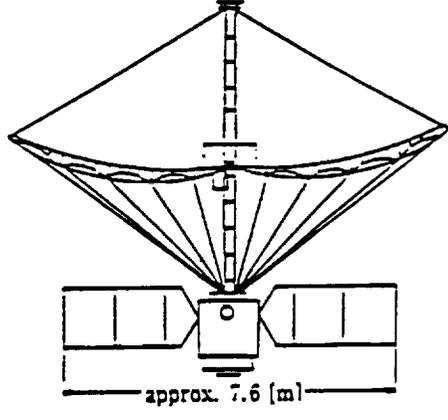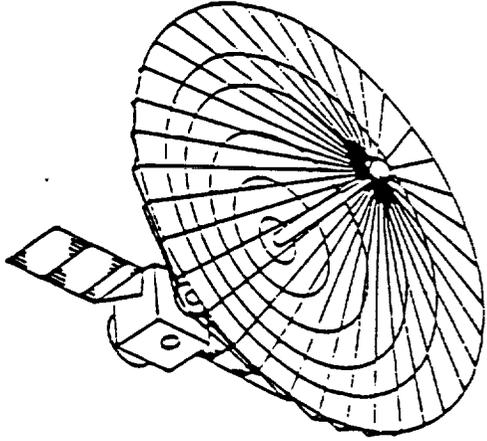VLBI Space Observatory Programme

Geodetic Applications of Space VLBI

- to improve the Earth's gravity field

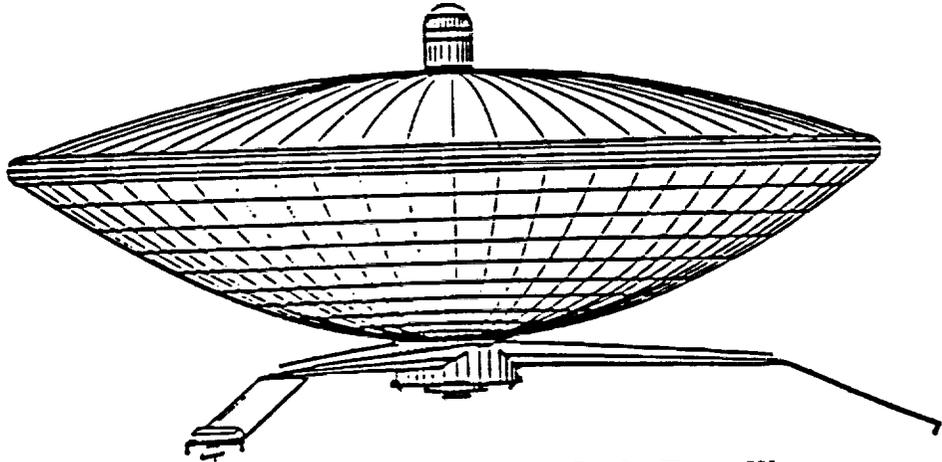- to connect reference frames inherent in the Space VLBI technique

The RADIOASTRON satellite.

The VSOP satellite.

The QUASAT satellite

Mathematical Model

Ground-to-Space VLBI Observables:

Time Delay

$$
d^I_{jk\ell} = - \left\{ \begin{bmatrix} X_j \\ Y_j \\ Z_j \end{bmatrix}^T R_2(-\xi)\, R_1(-\eta)\, R_3(\theta_\kappa) - \begin{bmatrix} X^I_k \\ Y^I_k \\ Z^I_k \end{bmatrix}^T \right\} \begin{bmatrix} \cos\delta_\ell \cos\alpha_\ell \\ \sin\delta_\ell \sin\alpha_\ell \\ \sin\delta_\ell \end{bmatrix} +
$$

$$
+ c\left[\Delta C^I_{0_{rj}} + \Delta C^I_{1_{rj}}\,(t_k - t_0) \right],
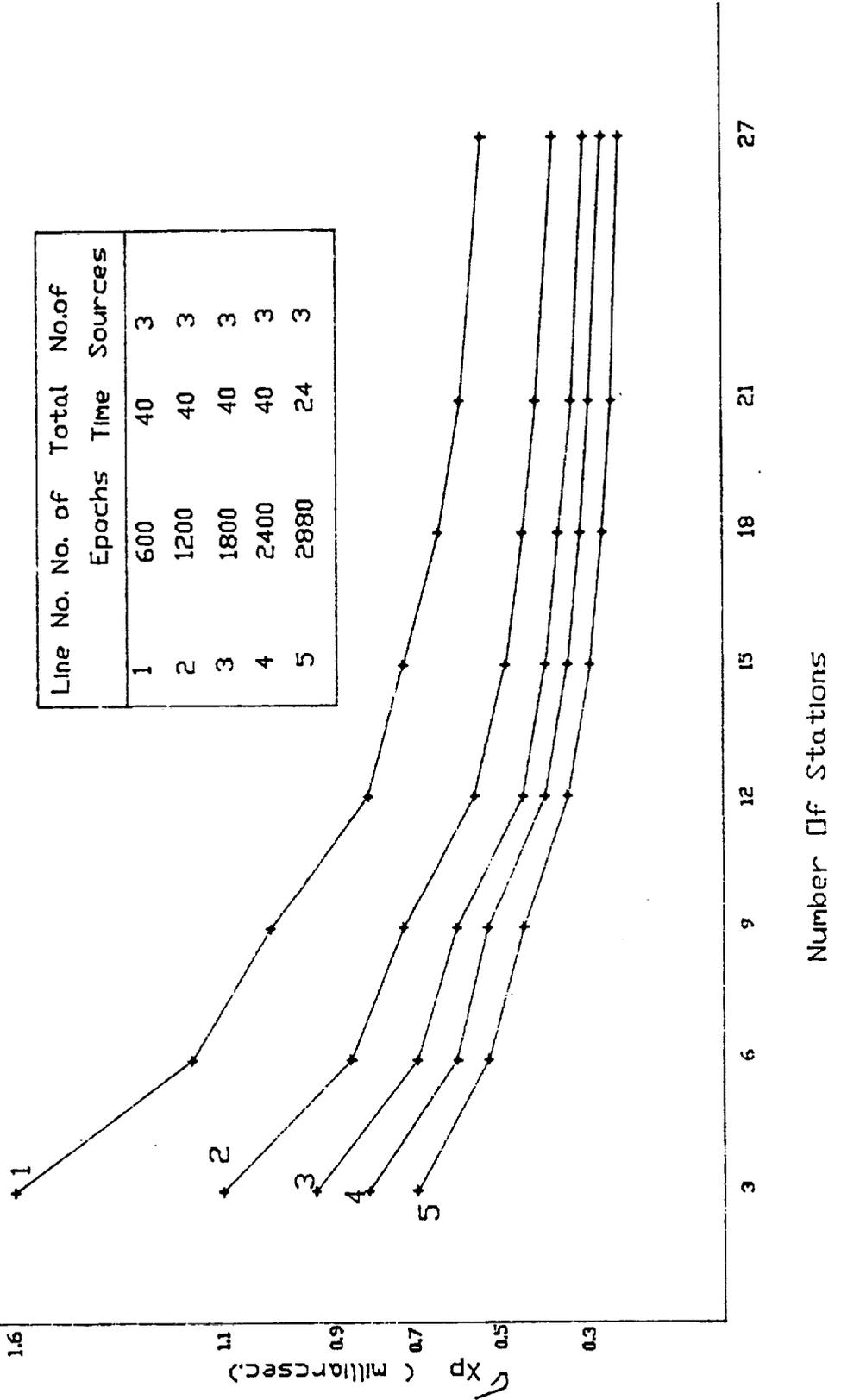$$

where

$X_j,\ Y_j,\ Z_j$ are the Earth-fixed coordinates of the station P_j,

$X^I_k,\ Y^I_k,\ Z^I_k$ are the coordinates of satellite S_I in a true-of-date geocentric inertial coordinate frame at the epoch t_k,

α_ℓ, δ_ℓ is the right ascension and declination of the ℓth radio source in the same as above true-of-date coordinate frame,

ξ, η are the polar motion components that relate the instantaneous rotation axis of the Earth with the average terrestrial pole,

θ_k is the Greenwich Apparent Sidereal Time (GAST) at epoch t_k,

c is the speed of light,

t_0 is the initial epoch of observation,

$\Delta C_{0_{rj}}^I, \Delta C_{1_{rj}}^I$ is the clock offset and drift between the reference clock at the telemetry/control station and the clock of station Pj, and

$R_i(\phi)$ is the rotation matrix for a right handed rotation ϕ about the axis i.

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Number Of Stations On Accuracy Of ERP Xp

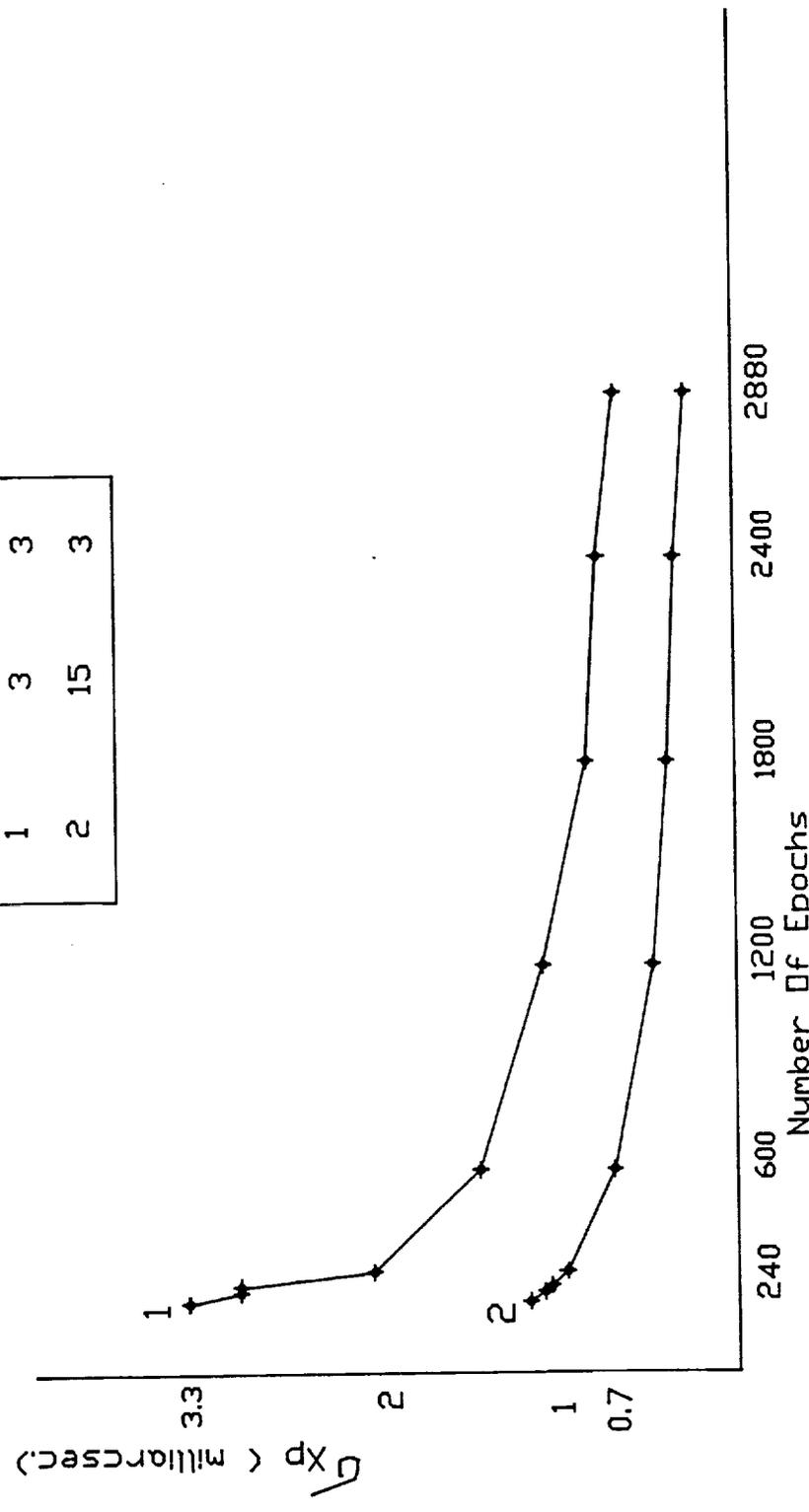| Line No.	No. of Epochs	Total Time	No.of Sources
1	600	40	3
2	1200	40	3
3	1800	40	3
4	2400	40	3
5	2880	24	3

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Number Of Sources On Accuracy Of ERP Xp

Line No.	No. Of Epochs	No. Of Stns.	Total Time
1	1200	3	40
2	1200	12	20
3	1200	15	20
4	2880	6	48
5	2880	15	48

σXp (milliarcsec.) — 1.13, 0.6, 0.53, 0.3

Number Of Quasar Sources — 3, 6, 9
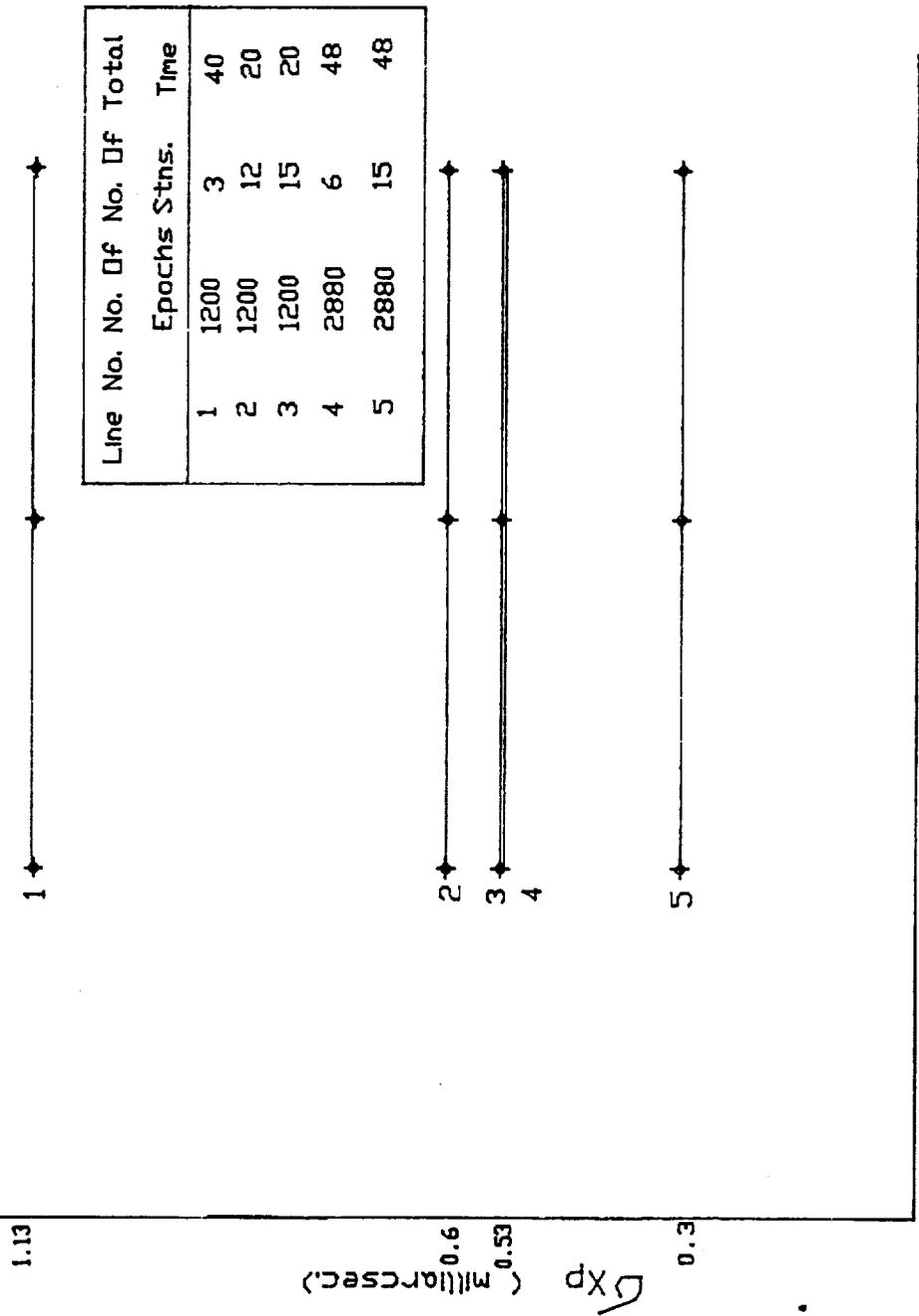
SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Number Of Observation Epochs On Accuracy Of ERP Xp

Line No.	Number Of Stns.	Sources
1	3	3
2	15	3

σ Xp (milliarcsec.)

Number Of Epochs

240 600 1200 1800 2400 2880

3.3 2 1 0.7

1

2

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Total Observation Time On Accuracy Of ERP XP

Line No.	Number Epochs	Of Sources	Stns.
1	210	3	15
2	240	3	15
3	258	3	15
4	300	3	15
5	576	3	15
6	720	3	15
7	960	3	15
8	1200	3	15
9	2880	3	3
10	2880	6	6
11	2880	6	12
12	2880	9	15

σ Xp (milliarcsec.)

1.25   1.0   0.5   0.25
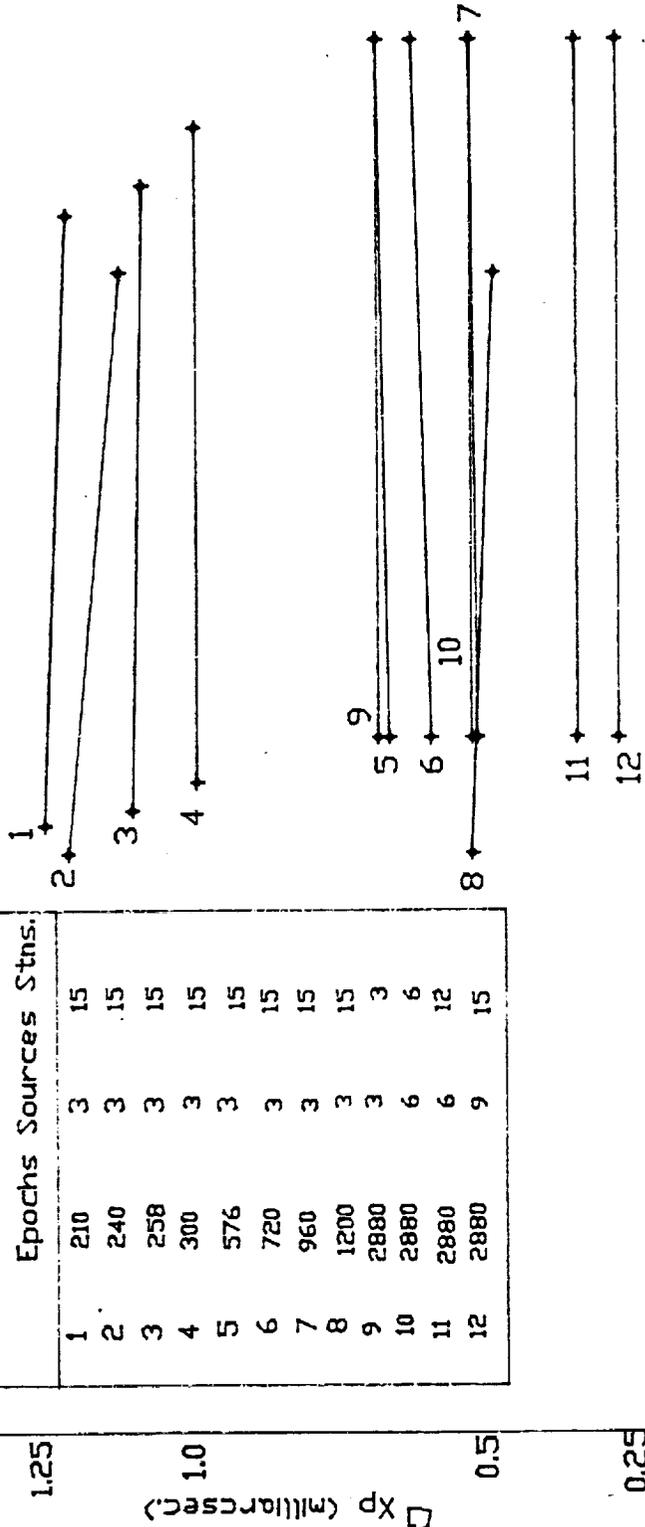
Total Observation Time (hrs.)

20   24   40   45   48
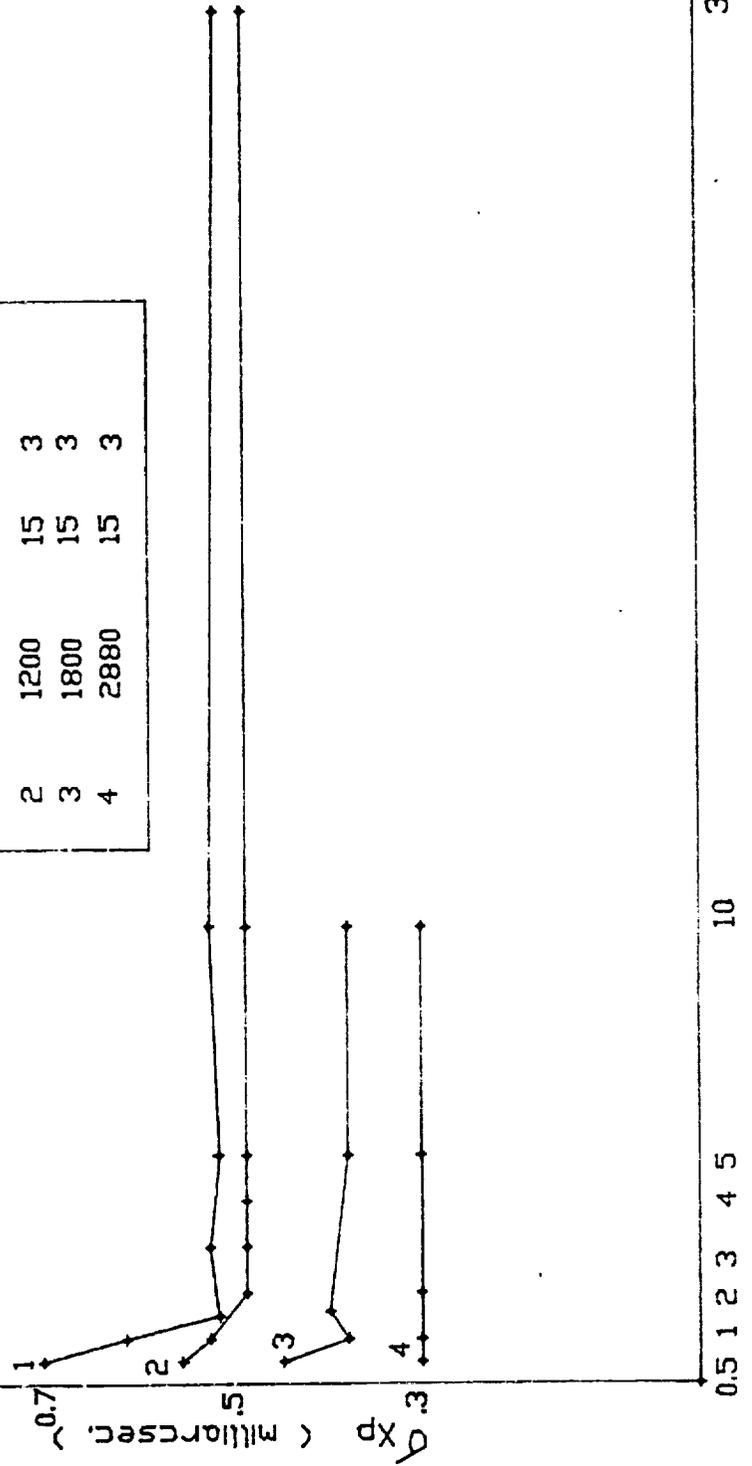
SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Observation Interval On Accuracy Of ERP Xp

A : Keeping Number Of Epochs Constant

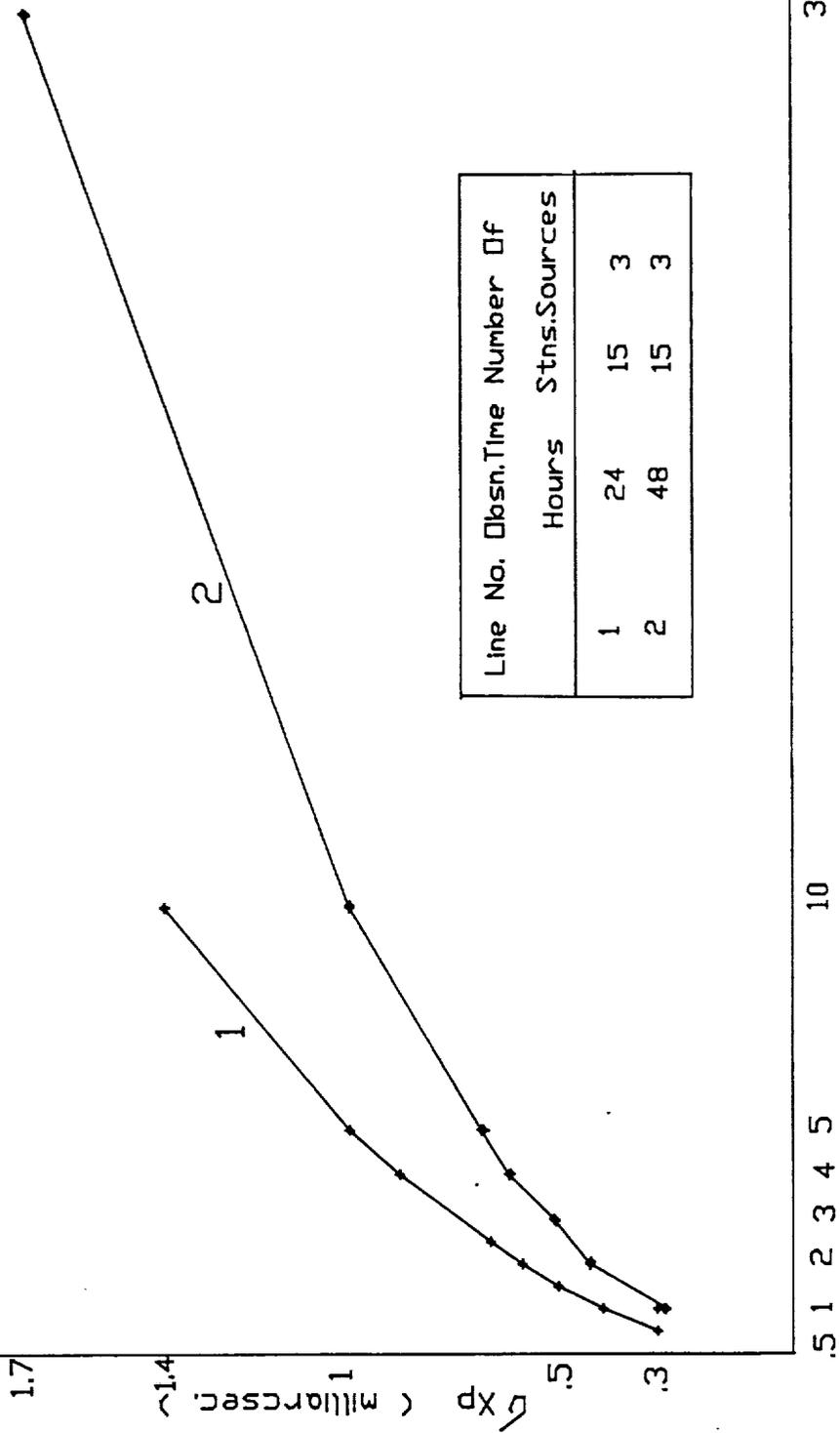| Line No.	Number Of Epochs	Stns.	Sources
1	960	15	3
2	1200	15	3
3	1800	15	3
4	2880	15	3

σ_{Xp} (milliarcsec)

0.7

.5

.3

0.5 1 2 3 4 5   10   30

Time Interval Between Observations (min.)

SPACE VLBI : SENSITIVITY ANALYSIS

Effect of Observation Interval On Accuracy Of ERP Xp

B : Keeping Total Observation Time Constant

2

1

Line No.	Obsn.Time Hours	Number Of Stns.	Sources
1	24	15	3
2	48	15	3

σ Xp (milliarcsec.)

1.7

1.4

1

.5

.3

.5 1 2 3 4 5   10   30
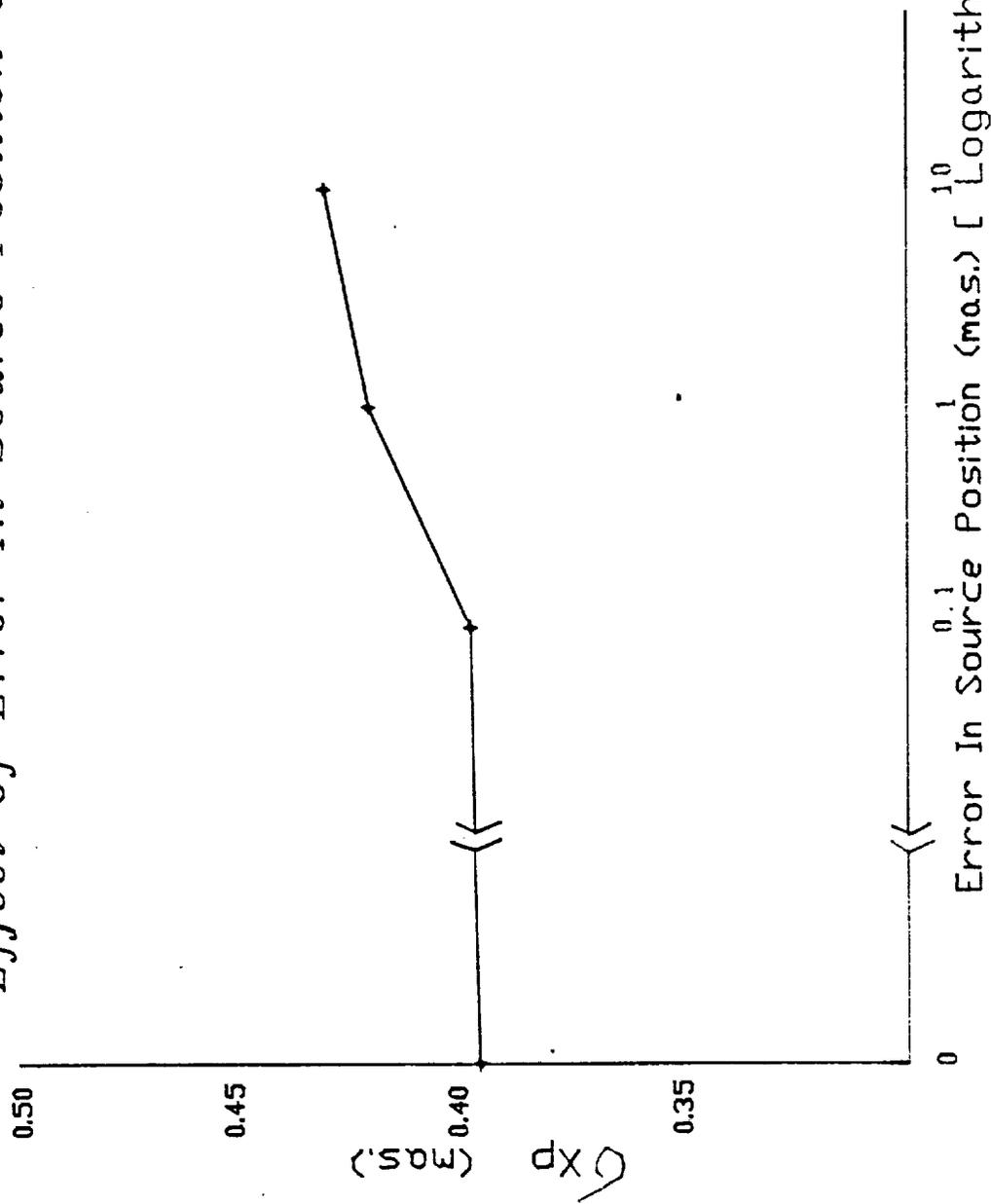
Time Interval Between Observations (min.)

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Error In Station Position On ERP Xp
[6 Coords. Of 3 Fiducial Stations, with $\varsigma = 5$ mm.]

σ Xp (mas.)

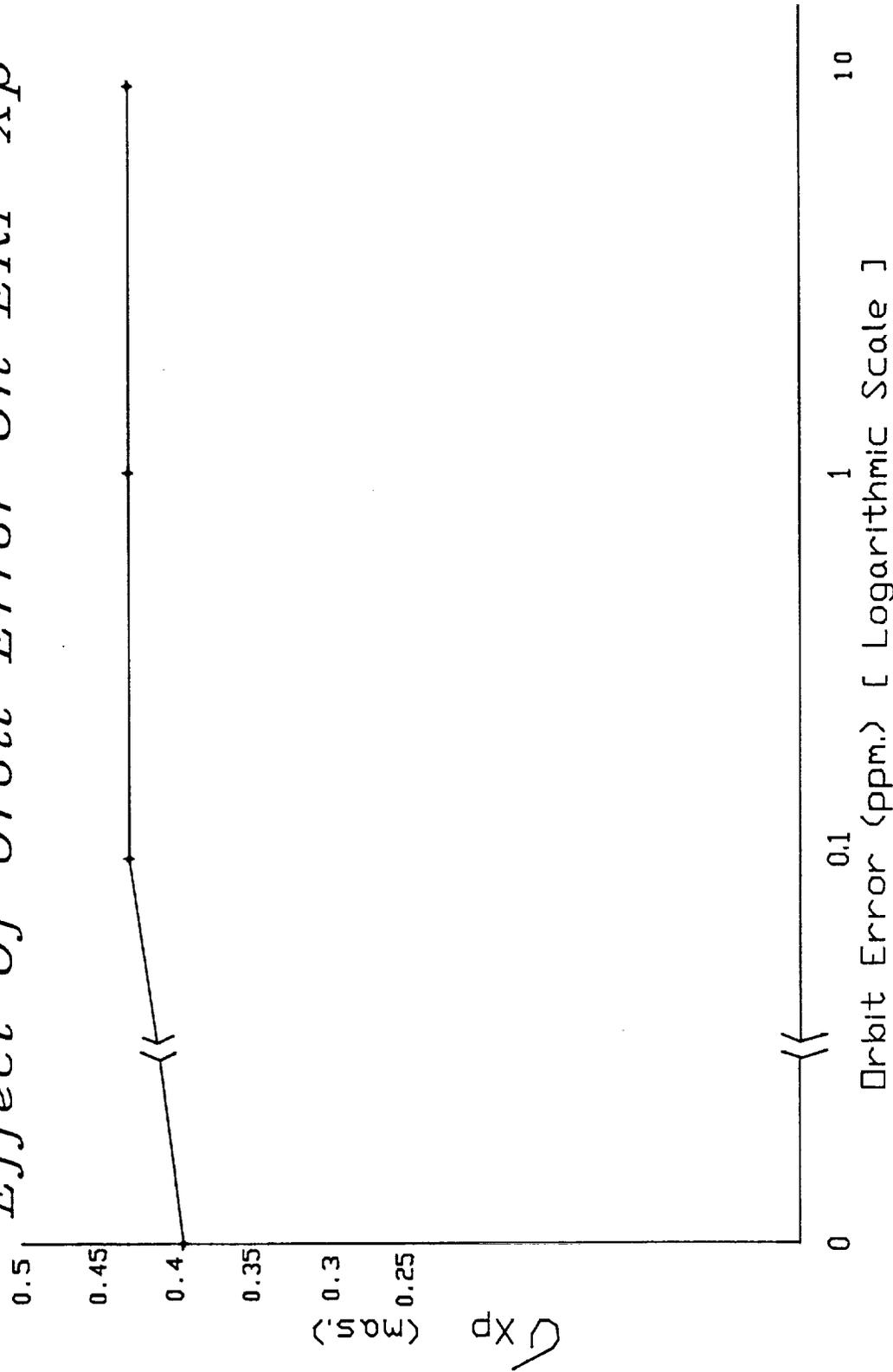Error in Station Position (cm.) [Logarithmic Scale]
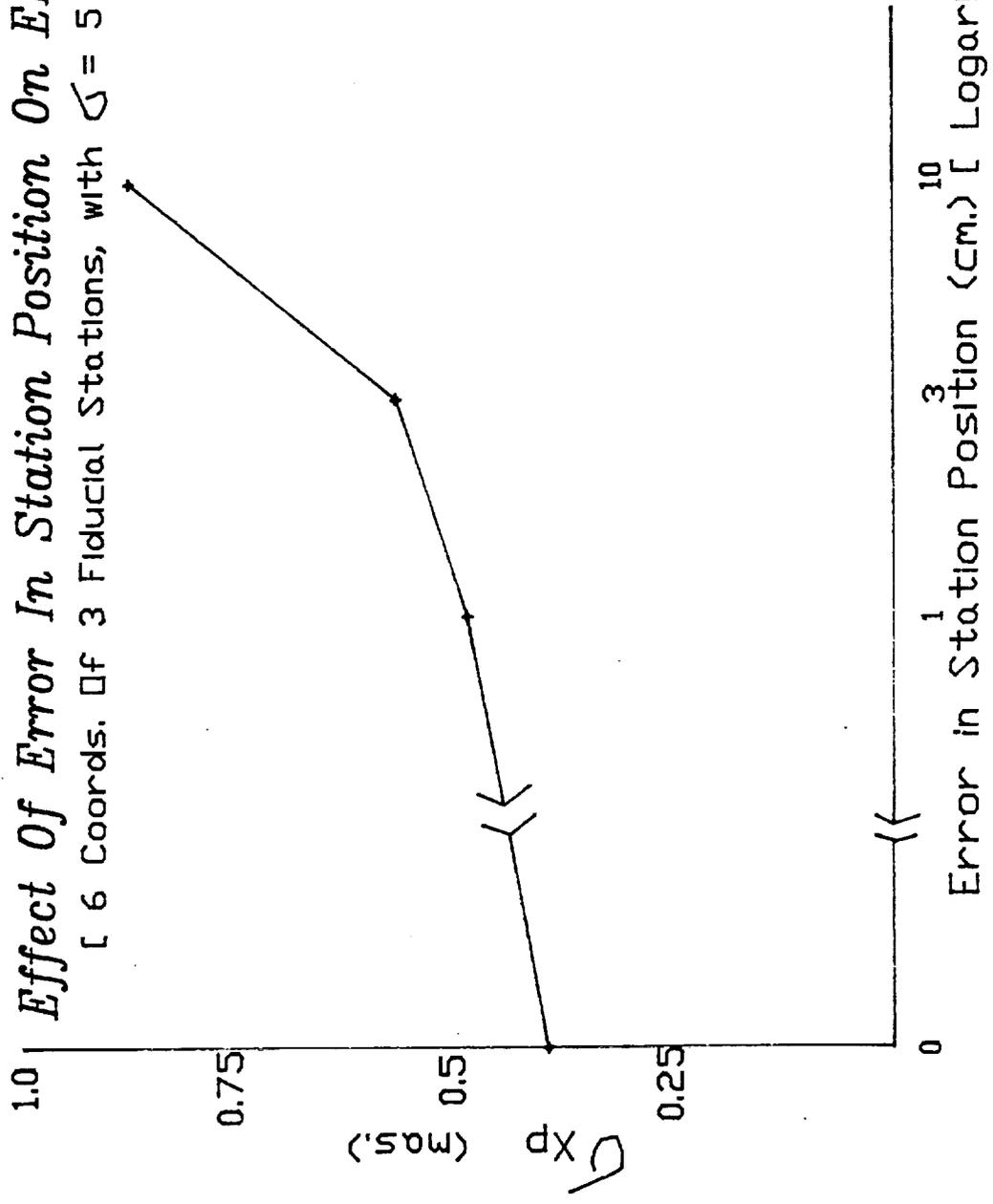
SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Error In Source Position On ERP Xp

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Orbit Error On ERP Xp

σ Xp (mas.)

Orbit Error (ppm.) [Logarithmic Scale]

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Errors In Clock Parameters On ERP Xp
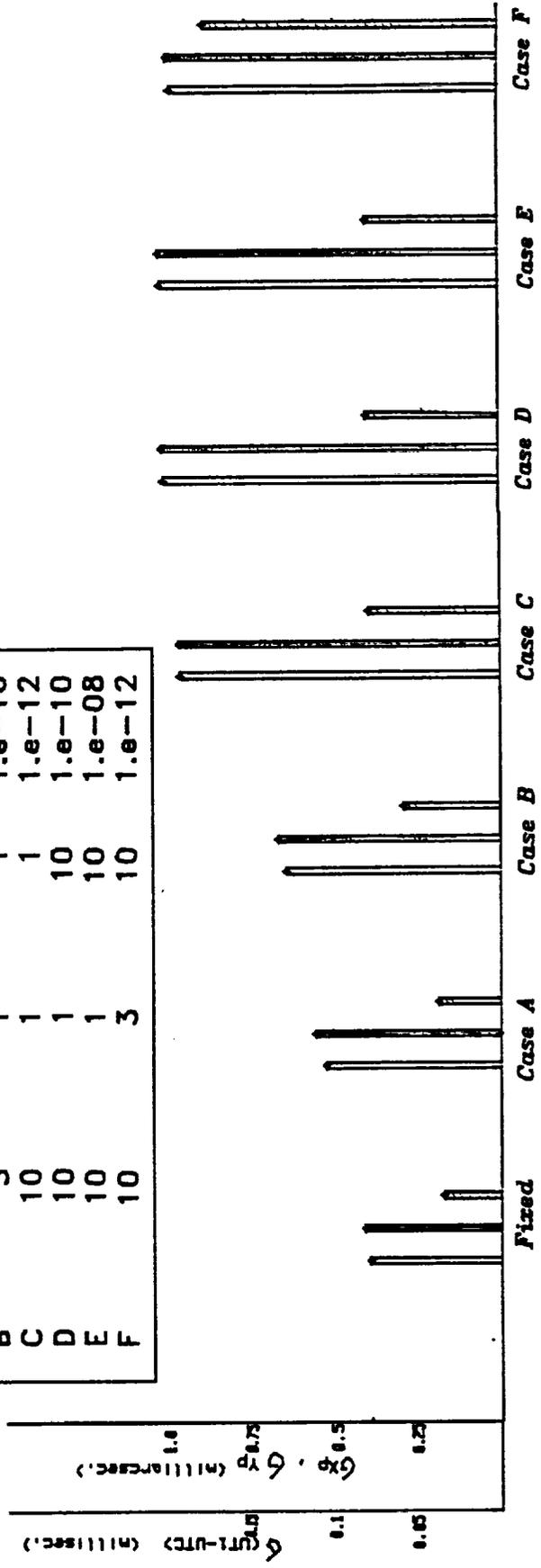

ΔXp (mas.)
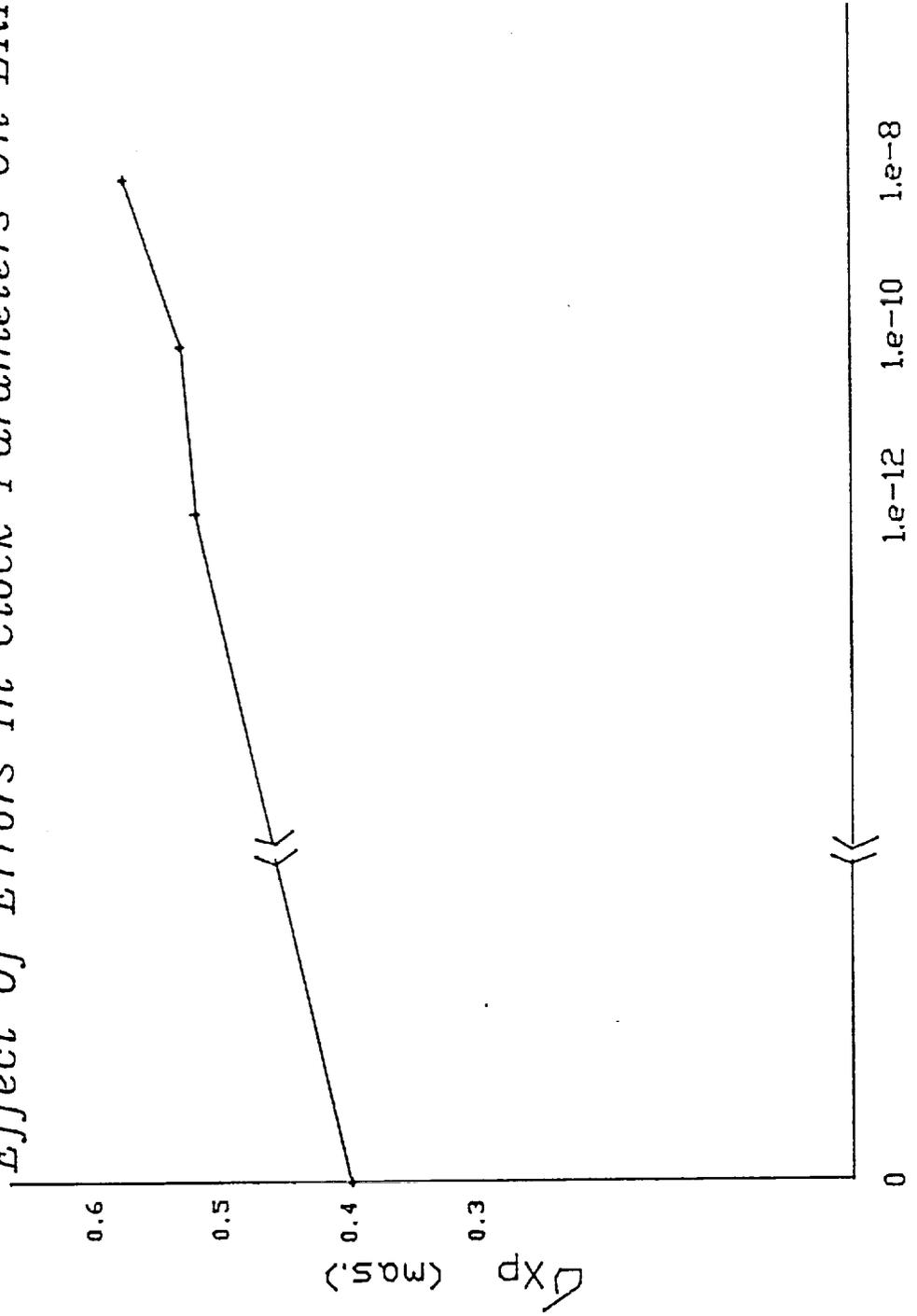
Error In Clock Parameters (sec, sec/day) [Logarithmic Scale]

SPACE VLBI : SENSITIVITY ANALYSIS

Effect Of Errors In Parameters On Accuracy Of ERPs : $X_p, Y_p, UT1-UTC$

Xp :

Yp :

UT1-UTC :

Errors In Parameters

Case	Station cm.	Source mas.	Orbit ppm.	Clock sec.
Fixed	0	0	0	0
A	1	0.1	0.1	1.e-12
B	3	1	1	1.e-10
C	10	1	1	1.e-12
D	10	1	10	1.e-10
E	10	1	10	1.e-08
F	10	3	10	1.e-12

Errors In Parameters

Fixed   Case A   Case B   Case C   Case D   Case E   Case F

$\sigma_{X_p}, \sigma_{Y_p}$ (milliarsec.)

1.0   0.75   0.5   0.25

$\sigma_{(UT1-UTC)}$ (millisec.)

0.1   0.05

NOTES :

1. Number Of Epochs = 1800,

   Number Of Stations = 15,

   Number Of Quasar Sources = 3,

   Total Observation Time = 45 Hours.

2. Plots For Other Two Earth Rotation

   Parameters Are Similar.