Task Title	**Analytical Solutions with Generalized Impedance Boundary Conditions**
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Period Covered	**Sept. 1990 - Feb. 1991**

ABSTRACT

Rigorous UTD (uniform Geometrical Theory of Diffraction) diffraction coefficients are presented for a coated convex cylinder simulated with generalized impedance boundary conditions. In particular, ray solutions are obtained which remain valid in the transition region and reduce uniformly to those in the deep lit and shadow regions. These involve new transition functions in place of the usual Fock-type integrals, characteristic to the impedance cylinder. A uniform asymptotic solution is also presented for observations in the close vicinity of the cylinder. As usual, the diffraction coefficients for the convex cylinder are obtained via a generalization of the corresponding ones for the circular cylinder.

OBJECTIVE

This task involves the use of higher order boundary conditions to generate new solutions in diffraction theory. In particular, diffraction coefficients will be developed for dielectric/magnetic layers and metal-dielectric junctions which are often encountered on airborne vehicles as terminations of coatings and conformal antennas. Solutions for both polarizations will be developed for fairly thick junctions and versatile computer codes will be written and tested. Creeping wave diffraction coefficients will be also developed for multilayered coated cylinders.

PROGRESS

1. Introduction

The problem of scattering by a smooth convex impedance cylinder has received much attention. Wang [1, 2] presented ray-optical solutions for the impedance and coated cylinders. His results are valid only in the deep lit and shadow regions and do not apply to the case where the observation point is in the transition region. Wait and Conda [3, 4] developed a solution which is valid in the transition region and for observation points on and off the surface. However, as pointed out by Pathak [5] it did not uniformly reduce to the ray solution [6, 7] exterior to the transition regions. Also, it is not valid on the portion of the surface in the transition region and these limitations were the primary motivation in Pathak's work [5] for the perfectly conducting convex cylinder. Recently, Kim and Wang [8] presented a solution applicable to a coated cylinder that remained valid in the transition region. They employed a heuristic approach to obtain the numerical values of the resulting transition integral applicable to a coated cylinder. Their solution is uniform but is not applicable to the close vicinity of the cylinder.

Here we develop a rigorous UTD solution of the diffraction by a coated cylinder simulated with generalized impedance boundary conditions. In addition, a uniform asymptotic solution is obtained which remains valid when the observation point is in close vicinity of the cylinder. An important aspect of the paper is also the use of second order generalized impedance boundary conditions (GIBC) for the simulation of the coating. Their derivation has already been given in [9] and [10] and are characterized by the inclusion of higher order field derivatives in their definition. Because of this they are less local which leads to an improved simulation (with respect to the standard impedance boundary condition - SIBC) of the coating in a manner analogous to the order of the highest derivative kept in the condition. Recently, they were successfully applied to a number of diffraction problems [11], [12] and have also been used in numerical simulations of multilayer coatings (see fig. 1) [13]. These applications provided a measure of the accuracy of the proposed GIBC and in particular accuracy criteria were derived in [13] for the second order conditions as a function of coating thickness and composition.

The UTD solution to be presented here parallels that given by Pathak [5] for the circular perfectly conducting cylinder. However, in the case of the coated cylinder the resulting UTD expressions are in terms of Fock-type integrals whose efficient evaluation is of primary interest. In the following we first present the eigenfunction solution based on

the second order GIBC simulation of a circular coated cylinder. By employing Watson's transformation this is written in integral form which is then cast in a ray representation. They ray solution is subsequently generalized to the case of a general convex cylinder. Finally, the evaluation of the Fock-type integrals is discussed and some results are presented which validate the accuracy of the GIBC eigenfunction and ray solutions. In the process, we demonstrate the improved accuracy of the GIBC solution over the corresponding SIBC solution, and it is also shown how the presented UTD solution can be extended to treat multilayered coated cylinders.

The details of the analysis are described in the report 025921-13-T which was recently submitted to the sponsor. Below we only attach a few results which demonstrate the accuracy and utility of the derived formulae.

2.   Numerical Results

The UTD expressions derived in the UM report 025921-17-T provide a complete set of equations for the computation of the total field in all regions of interest. Below, we present some calculated data which validate the accuracy of the derived expressions by comparison with data based on the moment method and eigenfunction solutions.

In figure 2 the eigenfunction solutions based on the GIBC and SIBC simulations are compared with the exact for a coated cylinder and this clearly demonstrates the improved simulation (with respect to the standard impedance boundary conditions - SIBC) achieved with the second order GIBC. To show the validity of the UTD solution in the case of the convex cylinder, a special case of an elliptical cylinder is considered in figure 3. Data based on the moment method are compared with those obtained from the UTD solution in conjunction with the second order low and high contrast boundary conditions.

Figure 4 verifies the asymptotic solution developed for the field point in the close vicinity of a convex cylinder. We remark, however, that the approximations used for the Hankel functions in the derivation of (42) and (43) become less accurate for some values of ε_r and μ_r associated with lossless coatings, and this can be avoided by using more accurate approximations for the Hankel functions. Finally, figure 5 demonstrates the use of GIBC in simulating multilayer coatings by simply redefining the material constants a_m and a_m' as discussed in [10, 13].

A difficulty in implementing the expressions derived in this paper was the evaluation of the Fock-type integrals $G(x,q)$, g_1 (D) and $g_2(D)$ as well as determination of the zeros corresponding to (21). The Fock-type integrals were evaluated by employing the

method described in [16] and the zeros of (21) were determined using the routine given in [20].

Summary

Rigorous ray solutions of the scattered fields were presented for a coated convex cylinder. These were developed in the context of the uniform geometrical theory of diffraction and specific expressions were given for the scattered fields in the lit, shadow and transition regions as well as for observations in the near vicinity of the cylinder. That is, UTD expressions were derived for all regions exterior to the coated cylinder. These are suited for engineering computations and are given in terms of the generalized Pekeris or Fock-type functions whose evaluation was efficiently performed via the Fourier Trapezoidal rule suggested by Pearson [16].

In comparison to the solution given by Kim and Wang [8], the ray representations given here are based on a second order generalized impedance boundary condition which permits the simulation of thin multilayered coating as demonstrated in the included examples. Also, in our implementation of the transition fields we employed a rigorous rather than a heuristic evaluation of the Fock-type integrals. Further, we have presented accurate field representations for observations on or near the vicinity of the coated cylinder and these can also be used for computing the radiated fields by a source or an aperture on the surface of the convex cylinder.

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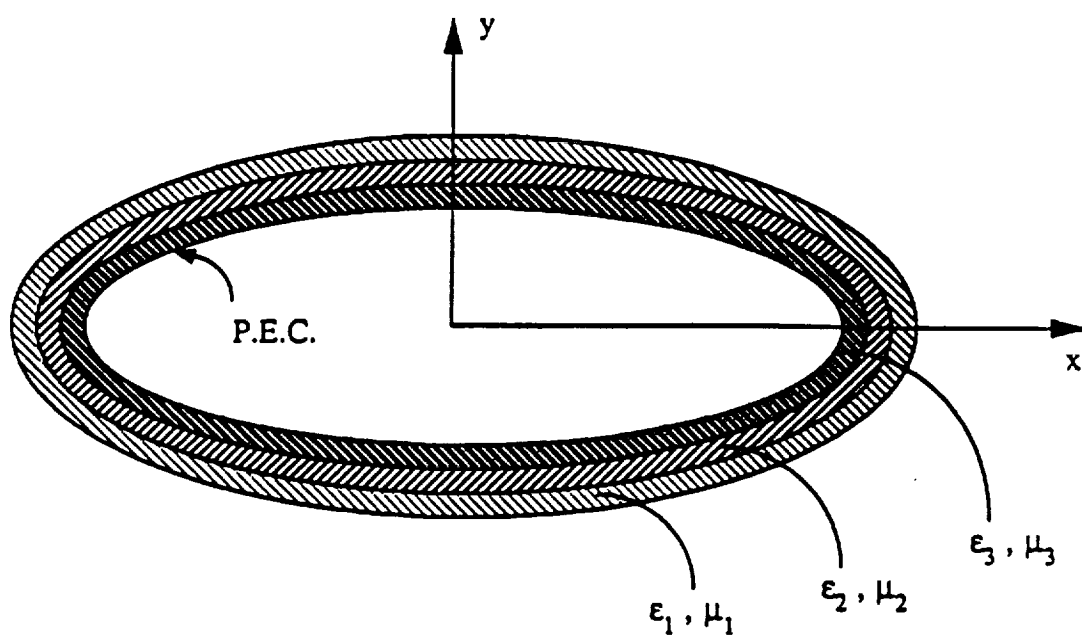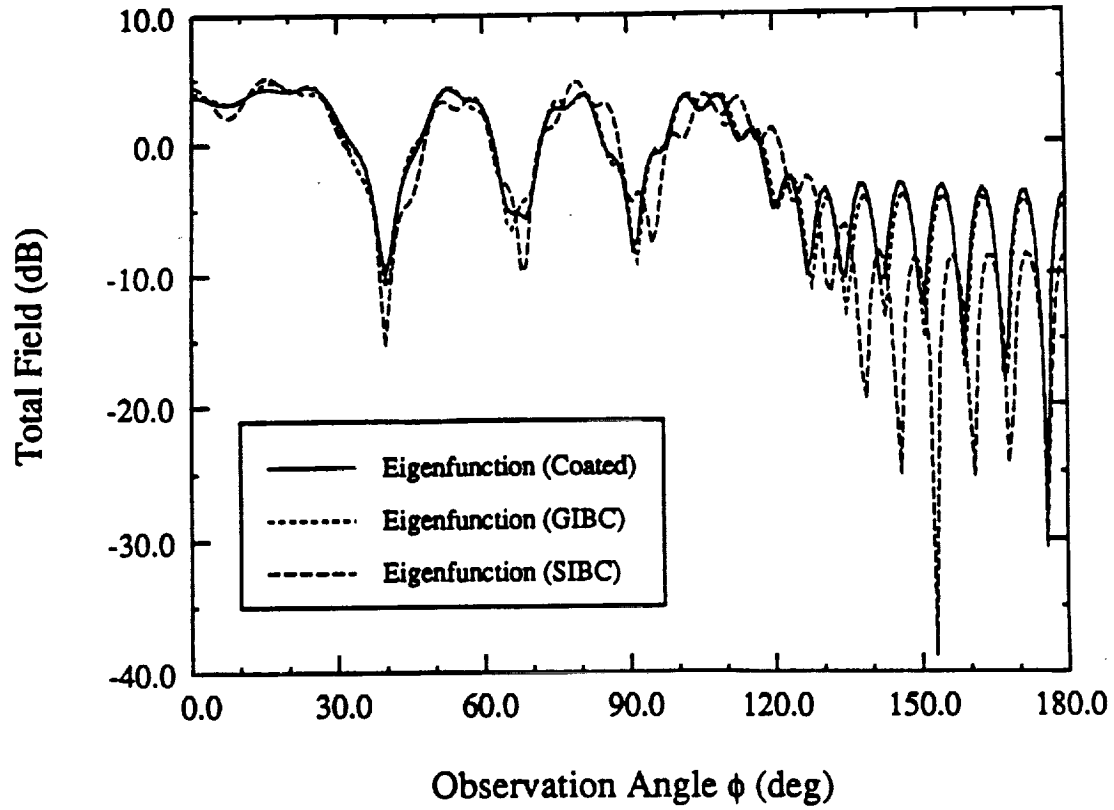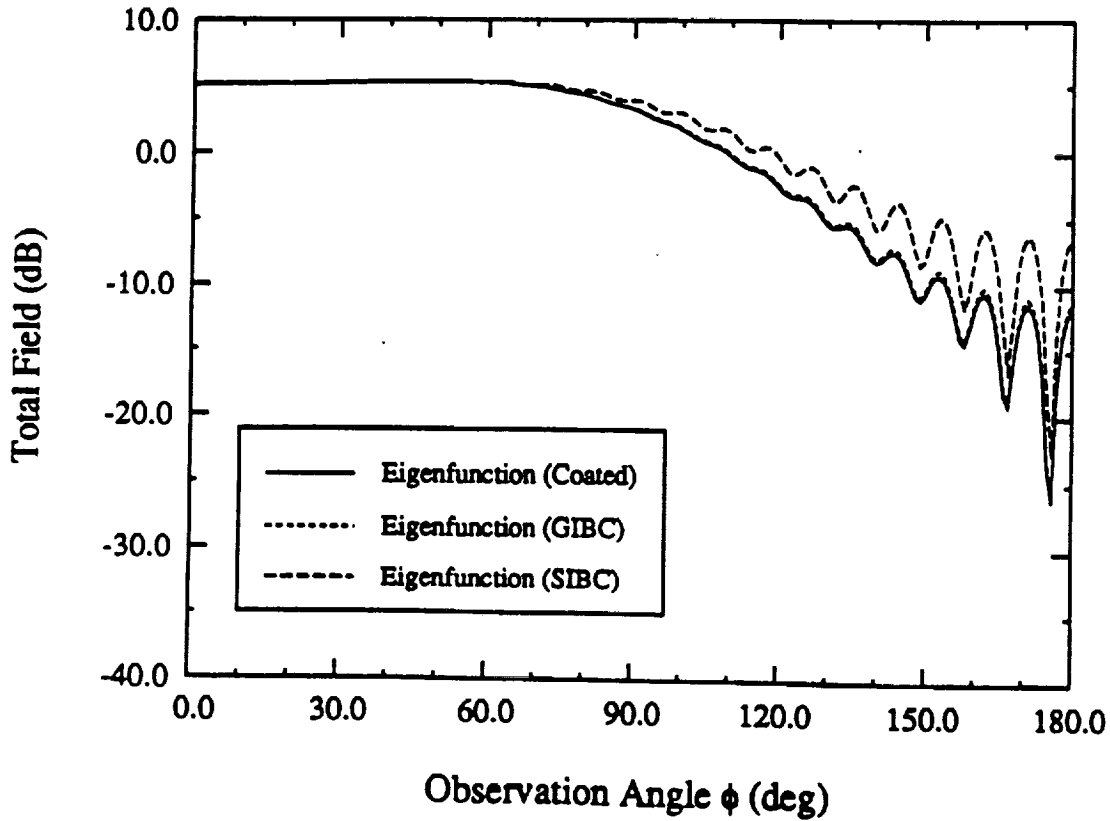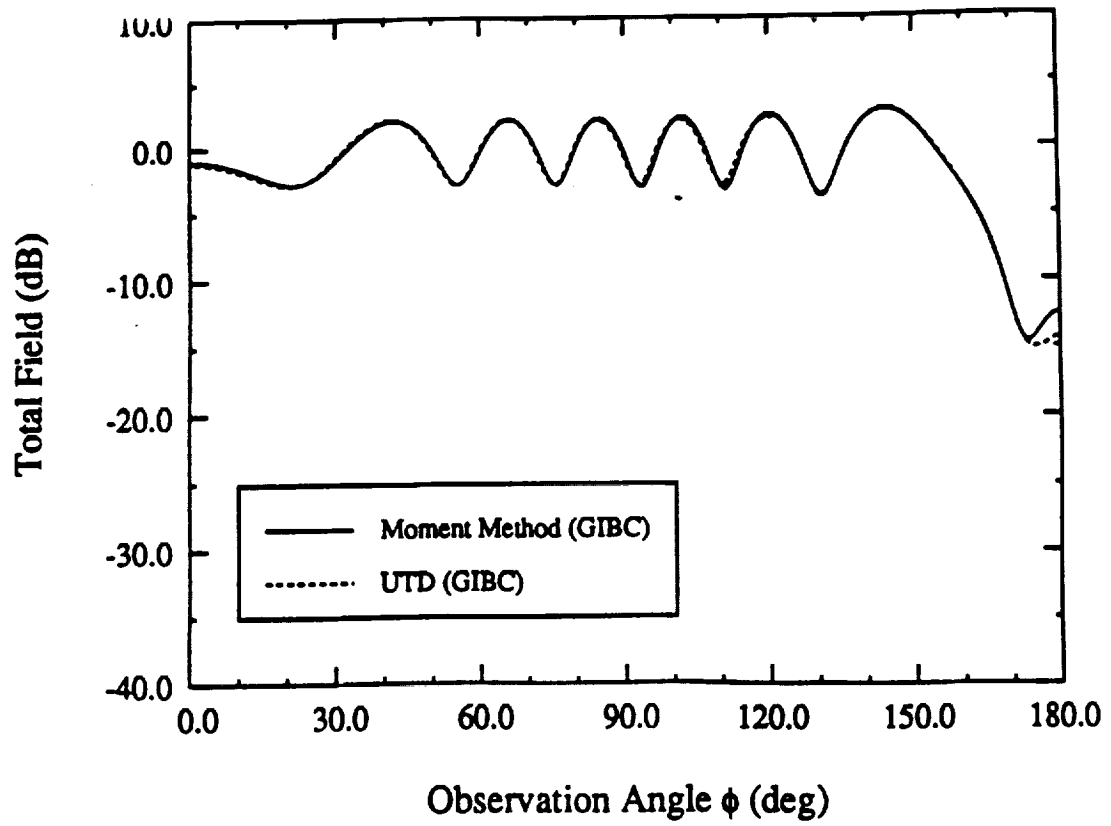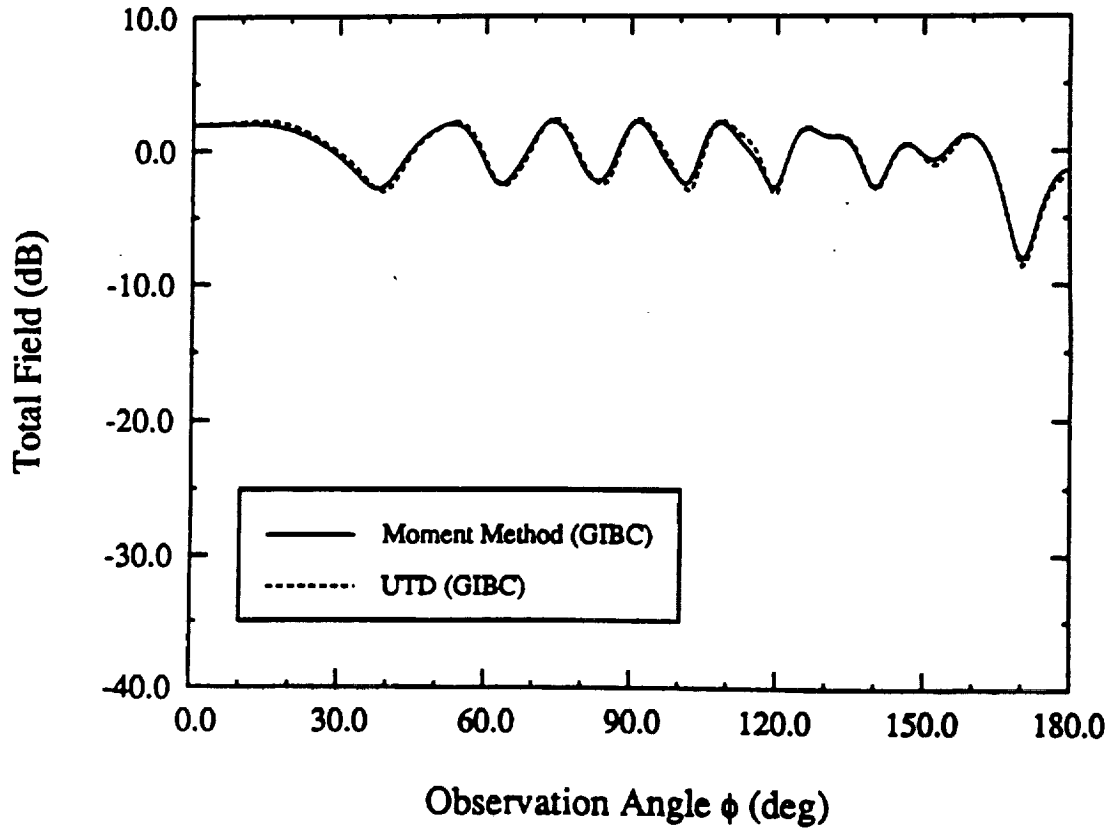
Fig. 1. Illustration of a three-layer coated cylinder.

Fig. 2: Bistatic H-polarization scattering pattern for a circular cylinder having $b = 3\lambda$, (a) $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$, $\rho = 5\lambda$ (b) $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$, $\rho = 3.05\lambda$.

Fig. 3: Bistatic scattering pattern for an elliptical cylinder having $a = 2\lambda$, $b = 1\lambda$, $\rho = 5\lambda$, $\phi_i = 0$ (a) E-polarization, $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$ (b) H-polarization, $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$.
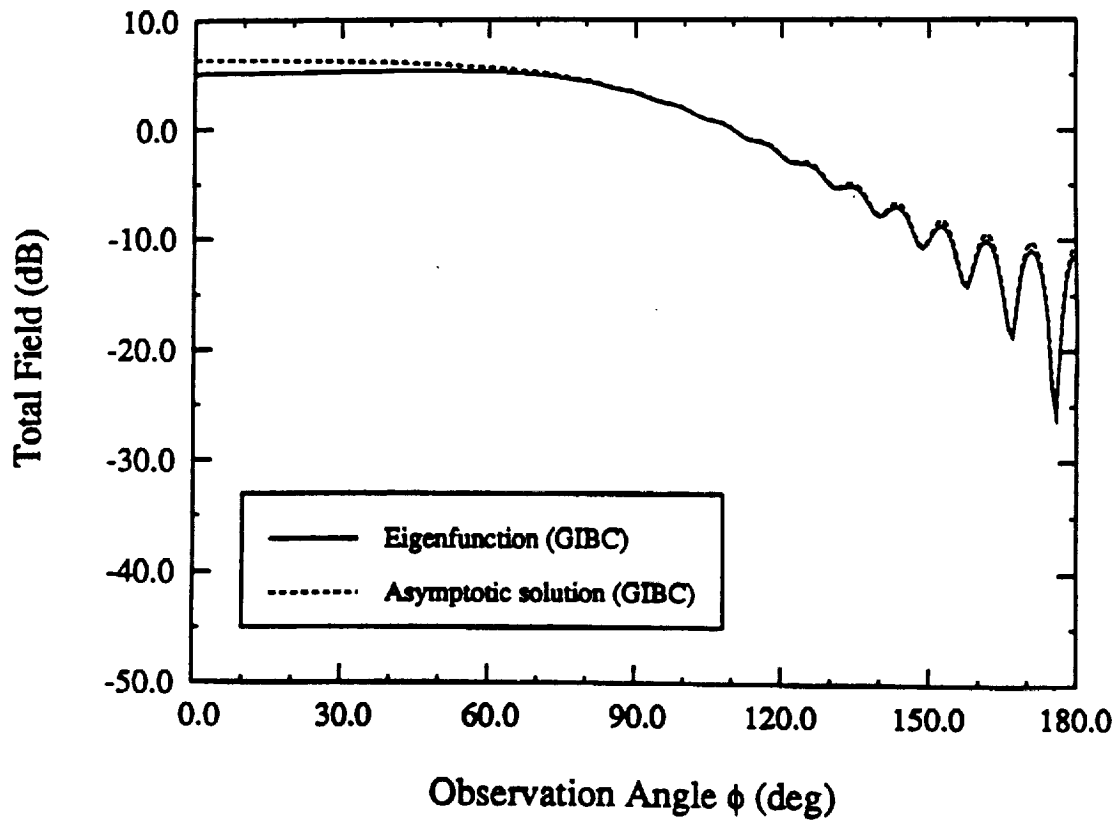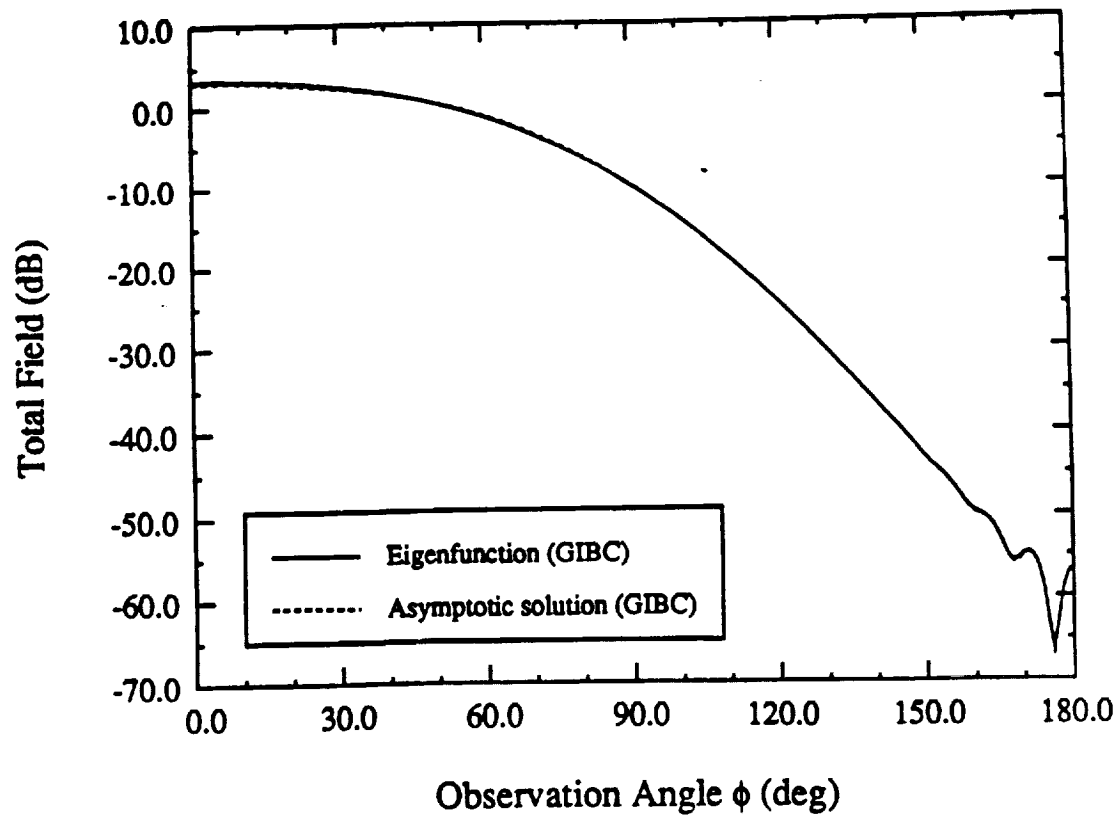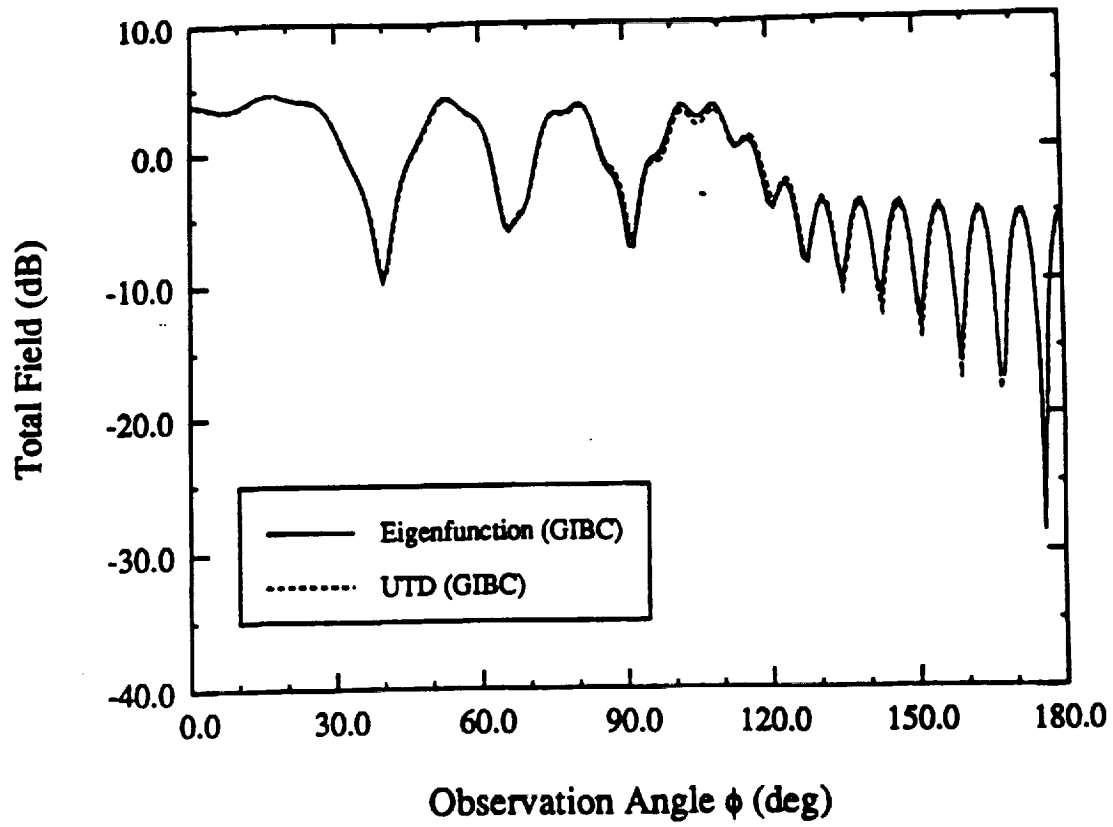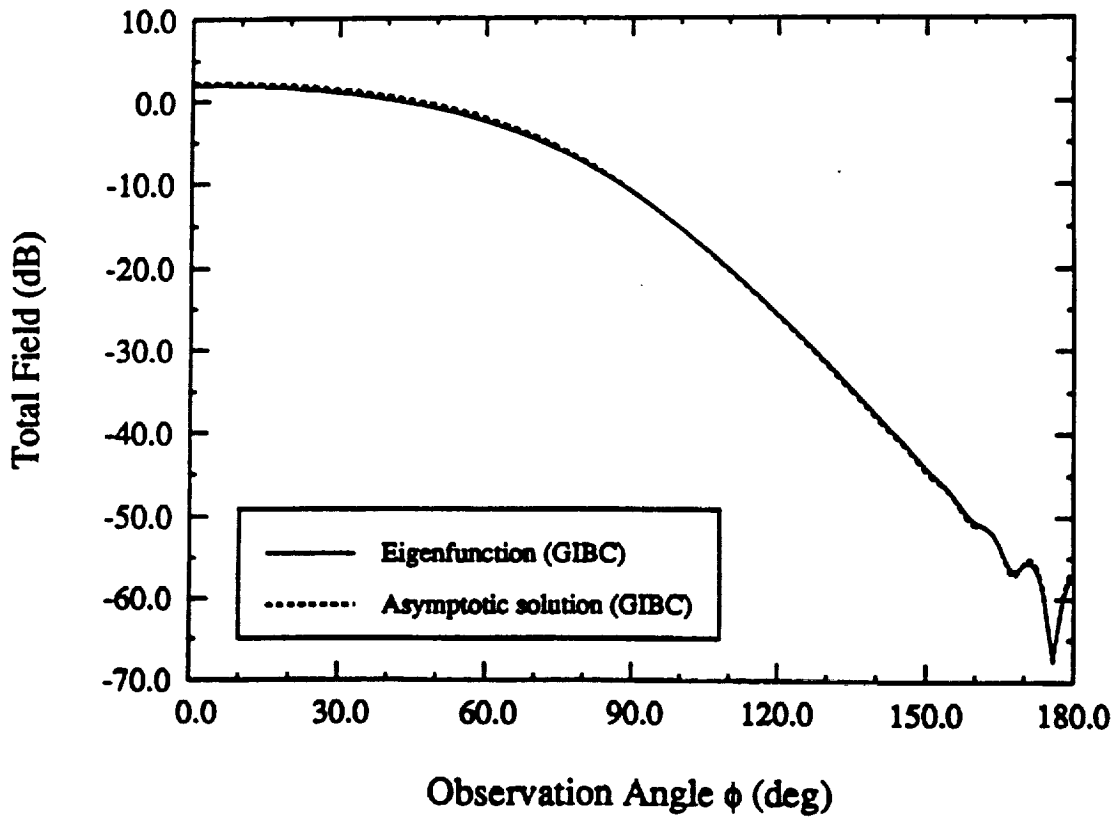
Fig. 4: Bistatic scattering pattern of a circular cylinder having $b = 3\lambda$, $\rho = 3.05\lambda$, $\phi_i = 0$ (a) E-polarization, $\epsilon_r = 4$, $\mu_r = 1$, $\delta = 0.07\lambda$ (b) H-polarization, $\epsilon_r = 8$, $\mu_r = 1$, $\delta = 0.2\lambda$.

Fig. 5: Bistatic scattering pattern of a three-layer coated circular cylinder having $b = 3\lambda$, $\phi_i = 0$, $\epsilon_{r1} = 3 - j0.1$, $\epsilon_{r2} = 4 - j0.3$, $\epsilon_{r3} = 7 - j1.5$, $\mu_{r1} = \mu_{r2} = \mu_{r3} = 1$, $\delta_1 = 0.01\lambda$, $\delta_2 = 0.02\lambda$, $\delta_3 = 0.03\lambda$ (a) H-polarization, $\rho = 5\lambda$ (b) E- polarization, $\rho = 3.05\lambda$.

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