N91-20510

MODELING OF CONNECTIONS BETWEEN SUBSTRUCTURES

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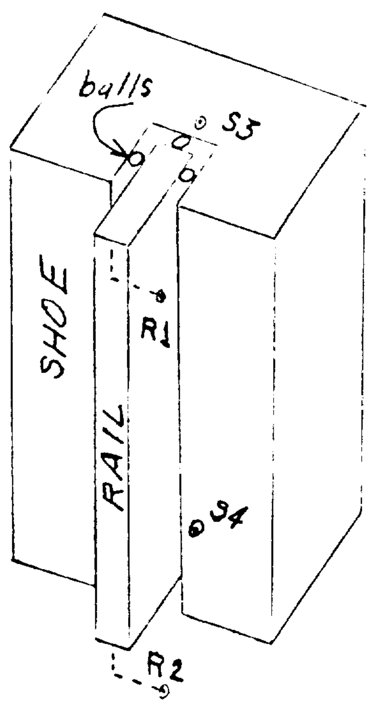
BUTLER ANALYSES

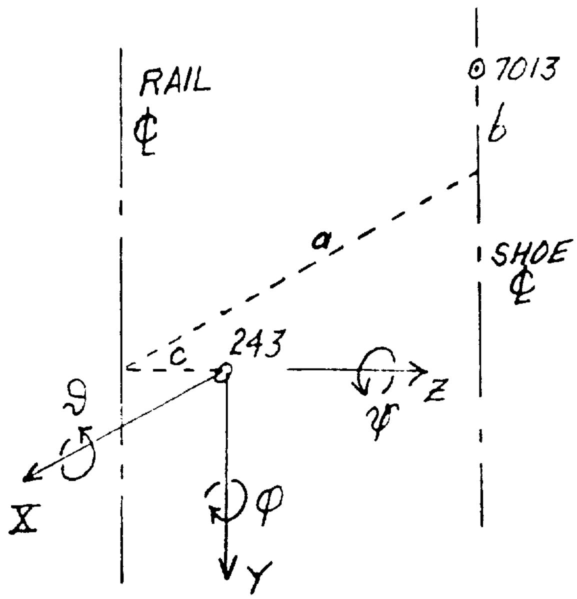
The focus of this paper is on joints that are only partially connected such as slip joints in bridges and in ship superstructures or sliding of a grooved structure onto the rails of a mating structure as shown in the sketch.



In substructure analysis it is desireable to organize each substructure so as to be self contained for purposes of validity checking. If part of the check is to embrace a connection, then all of the elements of the interface that it sees in its mate should be included within its model. In the case of the groove/rail structure, shown above, it will enhance the checking if the rail points, to which the shoe points will connect, are duplicated in the substructure with the shoe. Thus a complete job of checking out the shoe substrucure can be done in Phase 1 with statics and eigenvalues and not protract the checking procedure of basic substructures into Phase 2.

To implement such a scheme, referring to the sketch, points R1 & R2 are included in the shoe model. The connection from S3 to R1 and from S4 to R2 are made in Phase 1 and now become available for complete chekout of the shoe substructure, including its mating with the rail. To make this example general, postulate that the planes through the four points are not parallel to the coordinate planes. In effect there are offsets. Generally, one likes to plan to avoid having out-of-plane offsets, butexigencies do crop up which forces the analyst to face up to such realities. Often such interface connections involve MPC's or elastic ties. In any case a requirement of Substructure Analysis is that points that are to be connected in Phase 2 must be available in Phase 2; i.e. they cannot be condensed out or constrained out in Phase 1. Therefore, if an MPC is used, the connecting points must be the independent degrees of freedom in the MPC relationship.

The needs of this joint are that there will be no relative translation in either transverse direction and no relative rotation about the long axis of the rail. In terms of the indicated coordinate system, translations in x and z directions must be constrained together and rotations about y must be constrained together. Just a pair of connecting points will be used herein to carry on the discussion. A sketch will be used to assist in the discussion of making the connection by means of multi-point constraints.



Include rail point 243 in the Shoe Model. When Phase 2 COMBINE operation is invoked, NASTRAN will recognize that rail 243 = shoe 243. As remarked above, since point 243 is going to be commanded to connect in Phase 2, it must therefore be an active available point for joining; and must therefore be an independent point in an MPC relationship. Now following the needs of this joint, constrain point 7013(X,Z,Φ) to 243(X,Z,Φ). The constraint equations for translations in X and Z are:

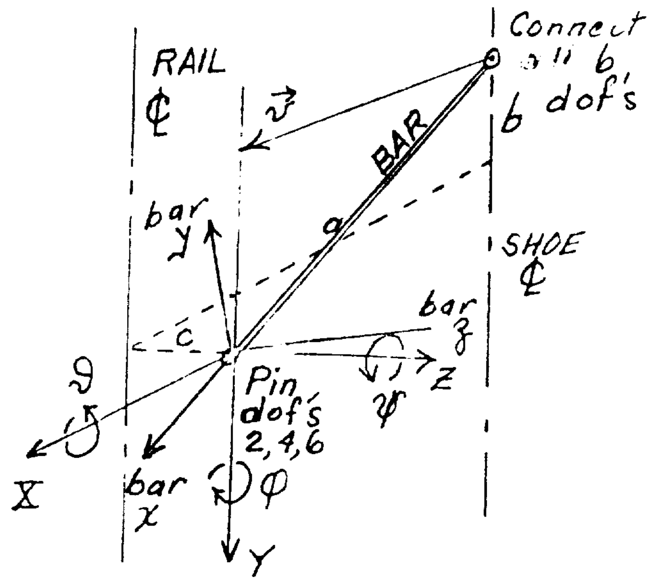
$$7013(X) = 243(X) - c \times 243(\Phi) + b \times 243(\Psi)$$
$$7013(Z) = 243(Z) + a \times 243(\Phi) - b \times 243(\Theta).$$

But 243(Ψ) and 243(Θ) are rail rotations which are not sensed by the shoe. If 243(4,6) are included in the shoe model they would be independent shoe rotations which will engage in the MPC relationship but would have no elastic path out to other parts of the shoe. Thus, if nominal mass were added to these rail points to keep the eigenvalue matrix from being singular, an eigenvalue check for rigid body modes would show the shoe model to fail. One might argue, why not leave the rotations in until they are connected during COMBINE, then they are no longer disjoint. I cannot afford to leave the 243(4,6) rotations in the shoe model, because after connecting with the rail these rail rotations must not be transmitted back to the shoe. Moments in the shoe/rail configuration about the two transverse axes are produced only by couples of forces not by local rotational bending. This rules out the use of MPC's during Phase 1 in this case. There are other cases of connections between substructures in which MPC's in Phase 1 would work. The case in which there were no transverse offsets would work. A NASTRAN run of a simple model demonstrates these results in Appendix A.

The alternative is to make a stiff elastic connection, but not so stiff as to cause matrix ill-conditioning. If a bar instead of elastic scalars is used, it will be modeled so as to be fully connected in all 6 degrees of freedom at the shoe end, but only partially connected at the rail end. At the rail end it must allow for sliding along the rail and not transmit rotations to the shoe about the rail transverse axes. This implies that pin flags must be used at the rail and to inhibit these freedoms.

This stiff bar connection can be implemented the wrong way or the right way. One gets trapped into modeling the wrong way by forgetting that pin flags are applied to bar coordinates not to the displacement coordinates. I fell into this trap and will show you what happens. Then I will follow it up with the correct way to model it.

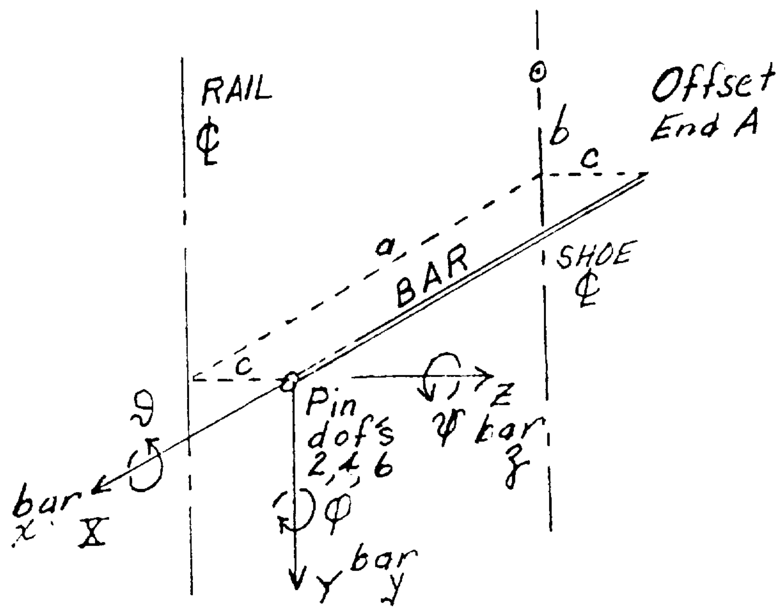
BAR CONNECTION
WRONG WAY



Include the rail grid points in the shoe model and apply SPC's at the GRID level in d.o.f.'s 2,4,6. Connect the shoe point to the rail point with a stiff bar. Note that the connection from shoe GP to rail GP produces bar coordinates that are skewed with respect to the displacement coordinates. Thus when bar element coordinate 2 is pinned, a component of force still develops at the rail end of the bar in the Y displacement coordinate direction, and so the eigenvalue check for rigid body modes fails once again. The listing in Appendix B of a simple model, incorporating this wrong approach, shows the constraint forces in the rigid body modes in freedoms T1, R1, & R3 to be non-negligible. Then the elastic mode shows large constraint forces in these freedoms.

BAR CONNECTION

RIGHT WAY



Offset the bar at the shoe end so as to terminate the bar at the rail end so as to be perpendicular to all displacement coordinates at the rail end. This connection passes the eigenvalue check for rigid body modes. Appendix C is a listing of a simple demonstration problem of the joint modeled the right way. Note that the constraint forces in freedoms T2, R1, & R3 are negligible in rigid body modes as well as in elastic modes.

CONCLUSIONS

This paper has demonstrated that complete checkout of a basic substructure can be done under the special circumstances of a sliding connection with offsets. Stiff bar connections make this possible so long as the bar coordinates are aligned with the displacement coordinates at the sliding surface.

APPENDIX A
RUN WITH MPC CONNECTION

```
                    ID   OFFSET,CONNECT
                    APP  DISP
                    SOL  3,0
                    DIAG  8,21,22
                    TIME  10
                    CEND
FREE-BODY MODAL STUDY OF SHOE/RAIL CONNECTIONS WITH MPC'S      JANUARY  30, 1991   UAI/NASTRAN VERSION 11.1A       PAGE      2
ALIGNED WITH OFFSETS. RAIL END INDEPENDENT.


                          C A S E    C O N T R O L    P A C K E T    E C H O

                    TITLE = FREE-BODY MODAL STUDY OF SHOE/RAIL CONNECTIONS WITH MPC'S
                    SUBTITLE = ALIGNED WITH OFFSETS. RAIL END INDEPENDENT.
                    OUTPUT
                       DISP = ALL
                       ELFORCES = ALL
                    $   SPCFORCES = ALL
                    SUBCASE 1
                       LABEL = ALL 6 DOF'S RETAINED ON BOTH ENDS.
                       MPC = 20
                       METHOD = 3
                    BEGIN BULK


                              S O R T E D    B U L K    D A T A    E C H O
           SORTED
           COUNT    ....1... ....2... ....3... ....4... ....5... ....6... ....7... ....8... ....9... ...10...
             1-     CBAR     1        1        71       72       1.0      1.0      0.0
             2-     CMASS2   132      0.1      13       2                                            SHOE
             3-     CMASS2   142      0.1      14       2                                            31Y
             4-     CMASS2   715      0.1      71       5                                            14Y
             5-     CMASS2   725      0.1      72       5                                            71PHI
             6-     EIGR     3        INV      0.0      10.0     8        8        6        1.-3     72PHI
             7-     +ALLMODE MAX                                                                     +ALLMODE
             8-     GRID     13       0        3.0      2.0      2.0      0
             9-     GRID     14       0        3.0      17.0     2.0      0                          RAIL1PT
            10-     GRID     71       0        0.0      0.0      0.0      0                          RAIL2PT
            11-     GRID     72       0        0.0      15.0     0.0      0                          SHOE1PT
            12-     MAT1     1        3.+7              0.28     2.4-4                               SHOE2PT
            13-     MPC      20       71       1        1.0      13       1        -1.0
            14-     +UP1BAR           13       5        2.0      13       6        -2.0              +UP1BAR
            15-     MPC      20       71       3        1.0      13       3        -1.0
            16-     +UP3BAR           13       4        2.0      13       5        -3.0              +UP3BAR
            17-     MPC      20       71       5        1.0      13       5        -1.0
            18-     MPC      20       72       1        1.0      14       1        -1.0              +UP5BAR
            19-     +LO1BAR           14       5        2.0      14       6        -2.0              +LO1BAR
            20-     MPC      20       72       3        1.0      14       3        -1.0
            21-     +LO3BAR           14       4        2.0      14       5        -3.0              +LO3BAR
            22-     MPC      20       72       5        1.0      14       5        -1.0
            23-     PARAM    COUPMASS 7                                                              +LO5BAR
            24-     PBAR     1        1        1.0      1.0      1.0      1.0
            25-     PBAR     2        1        100.0    100.0    100.0    100.0
                    ENDDATA
```

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIAN FREQUENCY	CYCLIC FREQUENCY	GENERALIZED MASS	GENERALIZED STIFFNESS
1	3	0.000000E+00	0.000000E+00	0.000000E+00	1.521215E-03	0.000000E+00
2	4	0.000000E+00	0.000000E+00	0.000000E+00	4.247887E-03	0.000000E+00
3	5	0.000000E+00	0.000000E+00	0.000000E+00	1.607912E-03	0.000000E+00
4	6	0.000000E+00	0.000000E+00	0.000000E+00	2.451029E-03	0.000000E+00
5	7	0.000000E+00	0.000000E+00	0.000000E+00	1.061112E-03	0.000000E+00
6	9	0.000000E+00	0.000000E+00	0.000000E+00	1.012346E-01	0.000000E+00
7	1	9.563905E-12	3.092556E-06	4.921956E-07	4.928215E-02	4.713298E-13
8	2	1.161892E-08	1.077911E-04	1.715549E-05	3.912935E-03	4.546408E-11
9	8	1.562500E+07	3.952847E+03	6.291152E+02	2.222214E-02	3.472210E+05

MODE NUMBER = 1   FREQUENCY = 0.000000E+00 HZ.   EIGENVALUE = 0.000000E+00

R E A L   E I G E N V E C T O R   N O .   1

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	4.885226E-01	-3.069099E-02	1.000000E+00	0.0	2.000755E-02	0.0
14	GRID	-1.334166E-03	-3.410552E-03	-6.272650E-01	0.0	2.000755E-02	0.0
71	GRID	4.485075E-01	1.761184E-01	1.060023E+00	-1.084843E-01	2.000755E-02	3.265712E-02
72	GRID	-4.134927E-02	1.761184E-01	-5.672423E-01	-1.084843E-01	2.000755E-02	3.265712E-02

MODE NUMBER = 2   FREQUENCY = 0.000000E+00 HZ.   EIGENVALUE = 0.000000E+00

R E A L   E I G E N V E C T O R   N O .   2

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-6.913960E-02	3.166597E-02	1.000000E+00	0.0	-1.377827E-01	0.0
14	GRID	-3.550368E-01	3.518357E-03	9.440481E-02	0.0	-1.377827E-01	0.0
71	GRID	2.064258E-01	5.781769E-04	5.866519E-01	-6.037301E-02	-1.377827E-01	1.905981E-02
72	GRID	-7.947137E-02	5.781769E-04	-3.189434E-01	-6.037301E-02	-1.377827E-01	1.905981E-02

MODE NUMBER = 3   FREQUENCY = 0.000000E+00 HZ.   EIGENVALUE = 0.000000E+00

R E A L   E I G E N V E C T O R   N O .   3

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	8.612644E-01	-1.069826E-02	-3.312686E-01	0.0	-2.891179E-03	0.0
14	GRID	-5.001400E-01	-1.188542E-03	1.000000E+00	0.0	-2.891179E-03	0.0
71	GRID	8.670468E-01	4.258756E-03	-3.399421E-01	8.875124E-02	-2.891179E-03	9.076030E-02
72	GRID	-4.943576E-01	4.258756E-03	9.913265E-01	8.875124E-02	-2.891179E-03	9.076030E-02

MODE NUMBER = 4 FREQUENCY = 0.000000E+00 HZ. EIGENVALUE = 0.000000E+00

R E A L E I G E N V E C T O R N O . 4

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	1.000000E+00	3.279515E-03	-5.439819E-01	0.0	-1.890255E-03	0.0
14	GRID	-6.589970E-02	3.642190E-04	-6.497411E-01	0.0	-1.890255E-03	0.0
71	GRID	1.003780E+00	-1.787680E-03	-5.496526E-01	-7.050612E-03	-1.890255E-03	7.105998E-02
72	GRID	-6.211919E-02	-1.787680E-03	-6.554118E-01	-7.050612E-03	-1.890255E-03	7.105998E-02

MODE NUMBER = 5 FREQUENCY = 0.000000E+00 HZ. EIGENVALUE = 0.000000E+00

R E A L E I G E N V E C T O R N O . 5

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-2.679756E-01	-4.753040E-03	-7.319843E-02	0.0	-3.266693E-03	0.0
14	GRID	1.000000E+00	-5.290496E-04	2.902177E-01	0.0	-3.266693E-03	0.0
71	GRID	-2.614422E-01	2.310795E-03	-8.299851E-02	2.422774E-02	-3.266693E-03	-8.453170E-02
72	GRID	1.006533E+00	2.310795E-03	2.804176E-01	2.422774E-02	-3.266693E-03	-8.453170E-02

MODE NUMBER = 6 FREQUENCY = 0.000000E+00 HZ. EIGENVALUE = 0.000000E+00

R E A L E I G E N V E C T O R N O . 6

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-1.127395E-05	-1.111122E-01	2.954264E-05	0.0	-8.683184E-07	0.0
14	GRID	9.360020E-05	1.000000E+00	-1.109201E-05	0.0	1.072533E-06	0.0
71	GRID	-9.537311E-06	-4.564906E-07	2.693768E-05	-2.320808E-06	-8.683184E-07	-6.732830E-06
72	GRID	9.145513E-05	-4.564906E-07	-7.874410E-06	-2.320804E-06	1.072533E-06	-6.732828E-06

MODE NUMBER = 7 FREQUENCY = 4.921956E-07 HZ. EIGENVALUE = 9.563905E-12

R E A L E I G E N V E C T O R N O . 7

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	1.000000E+00	6.632150E-01	7.963101E-01	0.0	1.036273E-01	0.0
14	GRID	2.864973E-01	7.369055E-02	-3.282914E-01	0.0	1.036273E-01	0.0
71	GRID	7.927454E-01	2.984467E-01	1.107192E+00	-7.497343E-02	1.036273E-01	4.756685E-02
72	GRID	7.924265E-02	2.984467E-01	-1.740939E-02	-7.497343E-02	1.036273E-01	4.756685E-02

ALIGNED WITH OFFSETS. RAIL END INDEPENDENT. SUBCASE 1
ALL 6 DOF'S RETAINED ON BOTH ENDS.

MODE NUMBER = 8 FREQUENCY = 1.715549E-05 HZ. EIGENVALUE = 1.161892E-08

R E A L E I G E N V E C T O R N O . 8

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-2.487581E-01	-1.668238E-03	-4.699167E-01	0.0	-1.139360E-02	0.0
14	GRID	-9.521336E-03	-1.851357E-04	2.772007E-01	0.0	-1.139360E-02	0.0
71	GRID	-2.259709E-01	1.000000E+00	-5.040975E-01	4.980783E-02	-1.139360E-02	-1.594912E-02
72	GRID	1.326586E-02	1.000000E+00	2.430200E-01	4.980783E-02	-1.139360E-02	-1.594912E-02

MODE NUMBER = 9 FREQUENCY = 6.291152E+02 HZ. EIGENVALUE = 1.562500E+07

R E A L E I G E N V E C T O R N O . 9

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-6.666663E-01	7.100735E-08	1.000000E+00	0.0	-3.333327E-01	0.0
14	GRID	6.666647E-01	-6.390596E-07	-9.999965E-01	0.0	3.333327E-01	0.0
71	GRID	-7.944661E-07	-1.626112E-10	1.744123E-06	-7.029677E-07	-3.333327E-01	-3.188153E-07
72	GRID	-7.893768E-07	1.631945E-10	1.717579E-06	6.817177E-07	3.333327E-01	3.147009E-07

APPENDIX B
RUN WITH WRONG BAR CONNECTION

GRID POINT SINGULARITY TABLE SPC 0 MPC 0

POINT ID.	TYPE	SINGULARITY ORDER	LIST OF COORDINATE COMBINATIONS THAT WILL REMOVE SINGULARITY STRONGEST COMBINATION	WEAKER COMBINATION	WEAKEST COMBINATIO
71	G	1	5	4	
72	G	1	5	4	

6 ROOTS BELOW 1.973921E+01

EIGENVALUE ANALYSIS SUMMARY (INVERSE POWER METHOD)

NUMBER OF EIGENVALUES EXTRACTED 7
NUMBER OF STARTING POINTS USED 1
NUMBER OF STARTING POINT MOVES 0
NUMBER OF TRIANGULAR DECOMPOSITIONS 1
TOTAL NUMBER OF VECTOR ITERATIONS 34
REASON FOR TERMINATION 7*
LARGEST OFF-DIAGONAL MODAL MASS TERM 0.13E-06
. . . 6

MODE PAIR . . . .
. . . 4

NUMBER OF OFF-DIAGONAL MODAL MASS
TERMS FAILING CRITERION 0
(* 1 OR MORE ROOT OUTSIDE FR.RANGE.
SEE NASTRAN U.M. SECTION 2.3.3)

REAL EIGENVALUES

```
*****************************************************************
*                                                               *
*                                                               *
*  NASTRAN INFORMATION MESSAGE 3308, LOWEST EIGENVALUE FOUND    *
*  AS INDICATED BY THE STURM'S SEQUENCE OF THE DYNAMIC MATRIX   *
*                                                               *
*        (THIS MESSAGE CAN BE SUPPRESSED BY DIAG 37)            *
*****************************************************************
```

MODE NO.	EIGENVALUE	CYCLIC FREQUENCY	GENERALIZED MASS	GENERALIZED STIFFNESS
1	-7.963921E-08	4.491420E-05	2.016630E-01	-1.606029E-08
2	2.852769E-08	2.688150E-05	1.250841E-01	3.568360E-09
3	9.751177E-08	4.969911E-05	1.946527E-01	1.898093E-08
4	1.989545E-07	7.098997E-05	1.088651E-01	2.165920E-08
5	2.305249E-07	7.641507E-05	2.362906E-01	5.447087E-08
6	3.321909E-07	9.173054E-05	1.223466E-01	4.064244E-08
7	7.002442E+06	4.211578E+02	1.515952E-01	1.061536E+06

```
ID   OFFSET,CONNECT
APP  DISP
SOL  3,0
DIAG  8,21,22
TIME  10
CEND
```



```
C A S E   C O N T R O L   D E C K   E C H O

TITLE = FREE-BODY MODAL STUDY OF SHOE/RAIL CONNECTIONS WITH BARS
SUBTITLE = WRONG WAY WITH CONNECTOR BAR SKEWED TO RAIL.WANT 3 RB MODES.
OUTPUT
DISP = ALL
MPCFORCES = ALL
ELFORCES = ALL
SPCFORCES = ALL
SUBCASE 1
LABEL = BARS PINNED AT RAIL END. NO OFFSETS AT SHOE END.
METHOD = 3
BEGIN BULK
          S O R T E D   B U L K   D A T A   E C H O

---1--- +++2+++ ---3--- +++4+++ ---5--- +++6+++ ---7--- +++8+++ ---9--- +++10+++
CBAR    1       1       71      72      1.0     1.0     0.0             SHOE
CBAR    2       2       71      13      14                              +TIE UP
+TIE UP         246
CBAR    3       2       72      14      13                              +TIE DWN
+TIE DWN        246
CMASS2  131     0.1     13      4                                       31THETA
CMASS2  141     0.1     14      4                                       14THETA
CMASS2  711     0.1     71      4                                       71THETA
CMASS2  712     0.1     71      5                                       71PHI
CMASS2  721     0.1     72      4                                       72THETA
CMASS2  722     0.1     72      5                                       72PHI
EIGR    3       INV     0.0     1.0     6       6       3       1.-3    +ALLMODE
+ALLMODEMAX
GRID    13      0       3.0     2.0     2.0     0       246             RAIL1PT
GRID    14      0       3.0     17.0    2.0     0       246             RAIL1PT
GRID    71      0       0.0     0.0     0.0     0                       SHOE1PT
GRID    72      0       0.0     15.0    0.0     0                       SHOE1PT
MAT1    1       3.+7            0.28    2.4-4
PARAM   COUPMASS7
PBAR    1       1       1.0     1.0     1.0     1.0
PBAR    2       1       100.0   100.0   100.0   100.0
ENDDATA
```

SUBCASE 1 EIGENVALUE = -7.963921E-08

R E A L   E I G E N V E C T O R   N O .   1

PT ID.	T1	T2	T3	R1	R2	R3
13	3.308091E-01	0.0	5.615981E-01	0.0	9.189782E-02	0.0
14	-6.346744E-01	0.0	-9.552183E-01	0.0	9.189782E-02	0.0
71	3.910763E-01	-5.288029E-01	1.000000E+00	-1.011211E-01	6.503133E-02	6.436E-2
72	-5.744071E-01	-5.288029E-01	-5.168163E-01	-1.011211E-01	6.503133E-02	6.436E-2

EIGENVALUE = 2.852769E-08 R E A L   E I G E N V E C T O R   N O .   2

PT ID.	T1	T2	T3	R1	R2	R3
1	-1.806720E-01	0.0	3.554041E-01	0.0	-1.205854E-01	0.0
14	-7.829153E-01	0.0	1.000000E+00	0.0	-1.205854E-01	0.0
71	1.403818E-01	-1.730800E-01	4.690333E-02	4.297306E-02	-8.839797E-02	4.015E-2
72	-4.618614E-01	-1.730800E-01	6.914992E-01	4.297306E-02	-8.839797E-02	4.015E-2

EIGENVALUE = 9.751177E-08 R E A L   E I G E N V E C T O R   N O .   3

PT ID.	T1	T2	T3	R1	R2	R3
13	-7.289532E-01	0.0	2.853720E-01	0.0	-1.817658E-01	0.0
14	3.520157E-01	0.0	2.345846E-01	0.0	-1.817658E-01	0.0
71	-8.096893E-02	-6.163144E-01	-7.029006E-02	-3.385834E-03	-2.055022E-01	-7.2E-
72	1.000000E+00	-6.163144E-01	-1.210776E-01	-3.385834E-03	-2.055022E-01	-7.2E-

EIGENVALUE = 1.989545E-07 R E A L   E I G E N V E C T O R   N O .   4

PT ID.	T1	T2	T3	R1	R2	R3
13	3.177751E-01	0.0	1.764639E-02	0.0	-3.180602E-01	0.0
14	-4.354112E-01	0.0	3.301453E-01	0.0	-3.180602E-01	0.0
71	1.000000E+00	-9.989289E-02	-9.057981E-01	2.083326E-02	-2.929949E-01	5.02E-2
72	2.468137E-01	-9.989289E-02	-5.932992E-01	2.083326E-02	-2.929949E-01	5.02E-2

EIGENVALUE = 2.305249E-07 R E A L   E I G E N V E C T O R   N O .   5

PT ID.	T1	T2	T3	R1	R2	R3
13	9.812109E-01	0.0	1.000000E+00	0.0	-1.272332E-02	0.0
14	8.707730E-01	0.0	5.573768E-01	0.0	-1.272332E-02	0.0
71	7.983168E-01	4.514050E-01	8.229362E-01	-2.950821E-02	-2.407710E-02	7.363E-3
72	6.878789E-01	4.514050E-01	3.803130E-01	-2.950821E-02	-2.407710E-02	7.363E-3

EIGENVALUE = 3.321909E-07 R E A L   E I G E N V E C T O R   N O .   6

PT ID.	T1	T2	T3	R1	R2	R3
13	-7.600989E-01	0.0	1.000000E+00	0.0	-3.662173E-01	0.0
14	-3.776701E-01	0.0	3.752712E-01	0.0	-3.662173E-01	0.0
71	-1.461953E-01	2.568173E-01	-1.776728E-01	-4.164859E-02	-3.932845E-01	-2.6E-
72	2.362335E-01	2.568173E-01	-8.024015E-01	-4.164859E-02	-3.932845E-01	-2.6E-

EIGENVALUE = 7.002442E+06 R E A L   E I G E N V E C T O R   N O .   7

PT ID.	T1	T2	T3	R1	R2	R3
13	-6.670417E-01	0.0	1.000000E+00	0.0	-6.548009E-01	0.0
14	5.839078E-01	0.0	-8.773038E-01	0.0	5.701413E-01	0.0
71	6.416196E-01	1.446323E-03	-9.644380E-01	2.011386E-01	-5.263158E-01	1.14E-1
72	-5.595077E-01	7.309780E-03	8.305104E-01	5.448166E-02	6.091975E-01	4.62E-2

EIGENVALUE = -7.963921E-08 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	2.806213E+01	0.0	-2.590351E+01	0.0	-1.726900E+01
14	0.0	-1.530662E+01	0.0	-4.709728E+00	0.0	-3.139819E+00

EIGENVALUE = 2.852769E-08 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	2.551103E+00	0.0	1.177432E+00	0.0	7.849547E-01
14	0.0	-2.551103E+00	0.0	-1.118560E+01	0.0	-7.457070E+00

EIGENVALUE = 9.751177E-08 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	-1.275551E+00	0.0	1.876532E+00	0.0	1.251022E+00
14	0.0	5.102206E+00	0.0	1.103843E-01	0.0	7.358950E-02

EIGENVALUE = 1.989545E-07 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	9.566635E-01	0.0	3.017170E+00	0.0	2.011446E+00
14	0.0	-5.102206E+00	0.0	4.121012E+00	0.0	2.747341E+00

EIGENVALUE = 2.305249E-07 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	1.020441E+01	0.0	1.964840E+01	0.0	1.309893E+01
14	0.0	2.793968E-09	0.0	-5.077676E+00	0.0	-3.385117E+00

EIGENVALUE = 3.321909E-07 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	-2.551103E+00	0.0	-3.532296E+01	0.0	-2.354864E+01
14	0.0	-2.551103E+00	0.0	-1.236304E+01	0.0	-8.242024E+00

EIGENVALUE = 7.002442E+06 FORCES OF SINGLE-POINT CONSTRAINT

PT ID.	T1	T2	T3	R1	R2	R3
13	0.0	-3.846585E+01	0.0	-2.479790E+04	0.0	-1.653193E+04
14	0.0	-1.149591E+02	0.0	2.221343E+04	0.0	1.480896E+04

APPENDIX C

RUN WITH RIGHT BAR CONNECTION

```
                 ID   OFFSET,CONNECT
                 APP  DISP
                 SOL  3,0
                 DIAG  8,21,22
                 TIME  10
                 CEND
FREE-BODY MODAL STUDY OF SHOE/RAIL CONNECTIONS WITH BARS       PAGE     2
RIGHT WAY WITH CONNECTOR BAR NORMAL TO RAIL.

                          C A S E   C O N T R O L   P A C K E T   E C H O

                 TITLE = FREE-BODY MODAL STUDY OF SHOE/RAIL CONNECTIONS WITH BARS
                 SUBTITLE = RIGHT WAY WITH CONNECTOR BAR NORMAL TO RAIL.
                 OUTPUT
                    DISP = ALL
                    ELFORCES = ALL
                    SPCFORCES = ALL
                 SUBCASE 1
                    LABEL = BARS PINNED AT RAIL END. OFFSETS AT SHOE END.
                    METHOD = 3
                 BEGIN BULK

                                S O R T E D   B U L K   D A T A   E C H O
           SORTED
           COUNT      ....1... ....2... ....3... ....4... ....5... ....6... ....7... ....8... ....9... ...10...
             1-       CBAR     1        1        71       72       1.0      1.0      0.0               SHOE
             2-       CBAR     2        2        71       13       1.0      1.0      0.0               +TIE UP
             3-       +TIE UP           246      0.0      2.0      2.0
             4-       CBAR     3        2        72       14       1.0      1.0      0.0               +TIE DWN
             5-       +TIE DWN          246      0.0      2.0      2.0
             6-       CMASS2   131      0.1      13       4                                            31THETA
             7-       CMASS2   141      0.1      14       4                                            14THETA
             8-       CMASS2   711      0.1      71       4                                            71THETA
             9-       CMASS2   712      0.1      71       5                                            71PHI
            10-       CMASS2   721      0.1      72       4                                            72THETA
            11-       CMASS2   722      0.1      72       5                                            72PHI
            12-       EIGR     3        INV      0.0      10.0     6        6        3        1.-3     +ALLMODE
            13-       +ALLMODE MAX
            14-       GRID     13       0        3.0      2.0      2.0      0        246               RAIL1PT
            15-       GRID     14       0        3.0      17.0     2.0      0        246               RAIL1PT
            16-       GRID     71       0        0.0      0.0      0.0      0                          SHOE1PT
            17-       GRID     72       0        0.0      15.0     0.0      0                          SHOE1PT
            18-       MAT1     1        3.+7              0.28     2.4-4
            19-       PARAM    COUPMASS 7
            20-       PBAR     1        1        1.0      1.0      1.0      1.0
            21-       PBAR     2        1        100.0    100.0    100.0    100.0
                      ENDDATA
```

R E A L   E I G E N V A L U E S

MODE NO.	EXTRACTION ORDER	EIGENVALUE	RADIAN FREQUENCY	CYCLIC FREQUENCY	GENERALIZED MASS	GENERALIZED STIFFNESS
1	2	-8.597651E-07	9.272352E-04	1.475741E-04	8.360698E-02	-7.188236E-08
2	3	-3.708519E-07	6.089761E-04	9.692155E-05	7.466593E-02	-2.769000E-08
3	1	5.746691E-08	2.397226E-04	3.815303E-05	1.415073E-01	8.131985E-09
4	4	1.050890E-07	3.241743E-04	5.159394E-05	1.495328E-01	1.571424E-08
5	5	1.103310E-07	3.321611E-04	5.286507E-05	2.399001E-01	2.646841E-08
6	6	3.483668E-07	5.902261E-04	9.393740E-05	1.215011E-01	4.232696E-08

MODE NUMBER = 1 FREQUENCY = 1.475741E-04 HZ. EIGENVALUE = -8.597651E-07

R E A L   E I G E N V E C T O R   N O .   1

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-2.433693E-04	0.0	9.871816E-01	0.0	-3.738998E-01	0.0
14	GRID	-4.052280E-01	0.0	1.000000E+00	0.0	-3.738998E-01	0.0
71	GRID	8.015541E-01	4.693623E-03	-1.362268E-01	8.545591E-04	-3.738998E-01	2.699898E-02
72	GRID	3.965695E-01	4.693623E-03	-1.234084E-01	8.545591E-04	-3.738998E-01	2.699898E-02

MODE NUMBER = 2 FREQUENCY = 9.692155E-05 HZ. EIGENVALUE = -3.708519E-07

R E A L   E I G E N V E C T O R   N O .   2

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	7.450739E-02	0.0	9.888168E-02	0.0	2.797289E-01	0.0
14	GRID	-5.090036E-01	0.0	-3.656060E-01	0.0	2.797289E-01	0.0
71	GRID	-4.071489E-01	1.079729E-01	1.000000E+00	-3.096585E-02	2.797289E-01	3.890074E-02
72	GRID	-9.906599E-01	1.079729E-01	5.355123E-01	-3.096585E-02	2.797289E-01	3.890074E-02

MODE NUMBER = 3 FREQUENCY = 3.815303E-05 HZ. EIGENVALUE = 5.746691E-08

R E A L   E I G E N V E C T O R   N O .   3

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	2.404255E-01	0.0	2.897290E-01	0.0	1.577857E-01	0.0
14	GRID	6.952828E-01	0.0	5.630910E-01	0.0	1.577857E-01	0.0
71	GRID	-1.357936E-01	-3.387907E-01	7.266380E-01	1.822414E-02	1.577857E-01	-3.032382E-02
72	GRID	3.190636E-01	-3.387907E-01	1.000000E+00	1.822414E-02	1.577857E-01	-3.032382E-02

MODE NUMBER = 4 FREQUENCY = 5.159394E-05 HZ. EIGENVALUE = 1.050890E-07

R E A L   E I G E N V E C T O R   N O .   4

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	8.741319E-01	0.0	-1.770641E-01	0.0	-3.227559E-02	0.0
14	GRID	4.142550E-01	0.0	3.377019E-01	0.0	-3.227559E-02	0.0
71	GRID	1.000000E+00	7.083516E-01	-3.425264E-01	3.431774E-02	-3.227559E-02	3.065846E-02
72	GRID	5.401231E-01	7.083516E-01	1.722396E-01	3.431774E-02	-3.227559E-02	3.065846E-02

BARS PINNED AT RAIL END. OFFSETS AT SHOE END. SUBCASE 1

MODE NUMBER = 5 FREQUENCY = 5.286507E-05 HZ. EIGENVALUE = 1.103310E-07

R E A L E I G E N V E C T O R N O . 5

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	-9.356353E-01	0.0	1.000000E+00	0.0	-1.059662E-01	0.0
14	GRID	9.272855E-01	0.0	-2.696967E-01	0.0	-1.059662E-01	0.0
71	GRID	-9.720924E-01	5.534579E-01	8.513944E-01	-8.464644E-02	-1.059662E-01	-1.241947E-01
72	GRID	8.908284E-01	5.534579E-01	-4.183022E-01	-8.464644E-02	-1.059662E-01	-1.241947E-01

MODE NUMBER = 6 FREQUENCY = 9.393740E-05 HZ. EIGENVALUE = 3.483668E-07

R E A L E I G E N V E C T O R N O . 6

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	7.174048E-01	0.0	6.860800E-01	0.0	-9.937754E-02	0.0
14	GRID	8.860417E-02	0.0	-6.589323E-01	0.0	-9.937754E-02	0.0
71	GRID	1.000000E+00	-4.786835E-01	5.672824E-01	-8.966749E-02	-9.937754E-02	4.192004E-02
72	GRID	3.711993E-01	-4.786835E-01	-7.777300E-01	-8.966749E-02	-9.937754E-02	4.192004E-02

MODE NUMBER = 1 FREQUENCY = 1.475741E-04 HZ. EIGENVALUE = -8.597651E-07

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	-4.023161E-10	0.0	0.0	0.0	2.413897E-09

MODE NUMBER = 2 FREQUENCY = 9.692155E-05 HZ. EIGENVALUE = -3.708519E-07

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	-5.796661E-10	0.0	0.0	0.0	3.477997E-09

MODE NUMBER = 3 FREQUENCY = 3.815303E-05 HZ. EIGENVALUE = 5.746691E-08

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	4.518601E-10	0.0	0.0	0.0	-2.711161E-09

BARS PINNED AT RAIL END. OFFSETS AT SHOE END.

SUBCASE 1

MODE NUMBER = 4 FREQUENCY = 5.159394E-05 HZ. EIGENVALUE = 1.050890E-07

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	-4.568466E-10	0.0	0.0	0.0	2.741080E-09

MODE NUMBER = 5 FREQUENCY = 5.286507E-05 HZ. EIGENVALUE = 1.103310E-07

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	1.850646E-09	0.0	0.0	0.0	-1.110387E-08

MODE NUMBER = 6 FREQUENCY = 9.393740E-05 HZ. EIGENVALUE = 3.483668E-07

F O R C E S O F S I N G L E - P O I N T C O N S T R A I N T

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
13	GRID	0.0	-6.246573E-10	0.0	0.0	0.0	3.747944E-09