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CONTROL ISSUES of MICROGRAVITY VIBRATION ISOLATION

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ABSTRACT

Active vibration isolation systems contemplated for microgravity space experiments may be designed to reach given performance requirements in a variety of ways. An analogy to passive isolation systems proves to be illustrative but lacks the flexibility as a design tool of a control systems approach and may lead to poor designs. For example, it is shown that a focus on equivalent stiffness in isolation system design leads to a controller that sacrifices robustness for performance. Control theory as applied to vibration isolation is reviewed and passive analogies discussed. The loop shaping trade–off is introduced and used to design a single degree of freedom feedback controller. An algebraic control design methodology is contrasted to loop shaping and critiqued. Multi–axis vibration isolation and the problems of decoupled single loop control are introduced through a two degree of freedom example problem. It is shown that center of mass uncertainty may result in instability when decoupled single loop control is used. This results from the ill conditioned nature of the feedback control design. The use of the Linear Quadratic Regulator synthesis procedure for vibration isolation controller design is discussed.

NOMENCLATURE

A	System dynamic matrix
a	Acceleration feedback coefficient
B	System input matrix
b	Rotational stiffness feedback cofficient
C(s)	Complimentary sensitivity function
c	Damping
d,D(s)	Direct disturbance force
$\bar{D}$(s)	Equivalent disturbance
f,F,F(s)	Actuator force
G(s)	Plant transfer function
H(s)	Feedback transfer function
I	Moment of inertia
k	Stiffness
M,M(s)	Control moment
n	Rotational damping feedback
P(s)	Umbilical precompensation transfer function
Q	State weighting matrix
q	Actuator placement
R	Control weighting matrix
S(s)	Sensitivity function
T(s)	Feedforward transfer function
u	Control vector
v(s)	Measurement noise
x,X(s)	Experiment position
$\bar{x}$	State vector

NOMENCLATURE (Continued)

y,Y(s)	Wall position
$\mathscr{Y}$	accelerometer measurements
z,Z(s)	Decoupled measurements
Δ	Center of Mass error
θ,Θ(s)	Angular position
ω_n	Natural frequency
ζ	Percent critical damping

Subscripts

a	Acceleration
cl	closed loop
p	position

I. INTRODUCTION

Active vibration isolation for microgravity space experiments has generated much interest lately. A variety of disturbances aboard manned space orbiters contaminate the desired microgravity environment. These accelerations cover a frequency band from DC to 100 Hz. Low frequency ($< 10^{-3}$ Hz) sources include drag, solar pressure oscillations, tidal effects, and gravity gradient forces. At the higher frequencies, manned activity, thruster firing, and orbiter systems contribute most significantly. A comprehensive treatment of the orbiter acceleration environment is presented in [1] from which Figure 1, a characterization of the environment is taken.

The need for the active isolation of materials processing and fluid science experiments in the frequency range .01 to 10 Hz has been demonstrated by Jones, Owen, and Owens [1,2,3]. Above this ragne passive isolation systems could be used. Below .01 Hz the rattlespace available for the experiment is not large enough to accommodate the relative motion. Therefore, these accelerations must be passed by the isolation system to the experiment.

Active isolation systems for microgravity and pointing applications have been designed and constructed by many investigators [3,4,5]. These systems generally use conventional P.I.D. control of a non–contacting actuator, either Lorentz or electromagnetic, to achieve low frequency disturbance attenuation. While an actual microgravity experiment may require umbilicals for cooling and power (at this point, it is not clear whether these functions can be performed otherwise as described in [4]) the isolation systems designed and tested so far preclude an umbilical from consideration. These systems achieve their performance by the very low stiffness made possible by low gain feedback of the relative position of the experiment to the mounting surface. Without an umbilical this stiffness may be set by the designer at will. However, when an umbilical is present, the umbilical stiffness presents a lower bound on achievable stiffness unless the feedback loop is used to introduce a negative stiffness. In this paper, the issues of control system design for the *generic* (i.e., with umbilical) microgravity experiment will be considered.

Previous research in the area of active microgravity vibration isolation has established the importance of the umbilical in control system design. Jones et. al [6] present a good preliminary examination of the single–degree–of–freedom control issues for intrusive and non–intrusive isolation systems. Grodsinsky [7] examined the use of acceleration and velocity feedback. Many of the issues these researchers have discussed are revisited here from a control theory perspective. Analysis of the six–degree–of–freedom problem in the literature has been restricted to one–loop–at–a–time design. Generally the effects of cross coupling between the various degrees of freedom have been ignored. Owens and Jones [2] have investigated the effect of cross coupling due to center of mass displacement for a single loop based controller. This work examines this important problem for the non–intrusive experiment platform case where relative position feedback is sufficient. The authors concluded that satisfactory performance can be achieved if the control loops are designed for the decoupled degrees of freedom and not autonomously for each local position. It should be noted that high gains are not required to achieve isolation for the umbilical–free case. An example is presented in this paper which shows that decoupled single loop design may not be sufficient for the generic isolation problem.

Any microgravity isolation system design should meet the following specifications for translational axes:

(1) Unity transmissibility from D.C. to 0.001 Hz so as to prevent the experiment from impacting its enclosure's walls.

(2) At least 40 Db attenuation above 0.1 Hz [3].

(3) Both stability and performance robustness under errors due to changes in umbilical/experiment properties, non–collocation or misalignment of sensors and actuators, center of mass uncertainties, and unmodeled cross coupling between the degrees of freedom.

Robustness refers to the ability of the control system to perform satisfactorily when the true plant varies from the nominal plant. Performance requirements of the type (2) for rotational degrees of freedom have not yet been specified to the knowledge of the authors.

In this paper we shall examine the control system issues associated with active microgravity vibration isolation. The purpose here is not to develop new control theory but to apply existing concepts to the problem. We hope that this paper will serve a ‹utorial function for vibration engineers involved with the microgravity problem. The thesis of this paper is that control system design, not passive isolator design familiar to vibration engineers, is the proper tool for anlaysis and synthesis. First, the control theory required for the examination is reviewed in Section II. Section III reviews passive isolation and applies it as an analogy to control system design. In Section IV classical loop shaping is applied to the isolation problem and a controller is designed. A discussion of the result and a passive system analogy follow. An example multi–degree–of–freedom system is explored in Section V and system robustness is examined. Section VI concludes with an examination of the Linear Quadratic Regulator for the isolation problem.

II. Control Theory Preliminaries

We examine here the prerequisite control theory for the examination to follow. While the actual isolation problem is multi–dimensional, a single–degree–of–freedom example will be examined first.

The one–degree–of–freedom microgravity vibration isolation problem, depicted in Figure 2, consists of an experiment of mass m connected by an umbilical and an actuator to a wall of the experiment enclosure. The umbilical is modeled here as a linear element with stiffness k and damping c. The wall's motion (displacement y) is transferred through the umbilical to the experiment resulting in its motion (displacement x). Direct disturbances may also act on the experiment due to the experiment's processes (e.g. motors, valves, shutters). While it may seem that there is no need to distinguish between umbilical and direct disturbances, they are indeed different. The distinction lies in the fact that the actuator influences through the experiments motion the force transmitted through the umbilical; direct disturbance forces, however, are independent of actuator force. This distinction carries through to both passive isolator performance and control system design.

The equation of motion for the experiment is

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky + d + f \tag{1}$$

where d is the direct disturbance force and f is the actuator force. We assume here that the spacecraft wall is of sufficient impedance so as to not be affected by the actuator force. Under Laplace transformation Eqn. (1) yields

$$X(s) = \left[\frac{cs + k}{ms^2 + cs + k}\right] Y(s) + \left[\frac{1}{ms^2 + cs + k}\right][D(s) + F(s)] \tag{2a}$$

or

$$\ddot{X}(s) = \left[\frac{cs + k}{ms^2 + cs + k}\right] \ddot{Y}(s) + \left[\frac{s^2}{ms^2 + cs + k}\right][D(s) + F(s)] \tag{2b}$$

and this is illustrated in Figures 3a and 3b, a block diagram for the isolation system. Here, H(s) is the feedback transfer function, T(s) is the feedforward transfer function, and $v_1(s)$, $v_2(s)$ are measurement noises. The actuator force is therefore a linear function of the wall and the experiment motion. The subscripts p and a throughout this paper refer to whether the model used is in position or acceleration form.

If the umbilical properties were known explicitly and measurement noise is sufficiently small, then transmitted disturbances can be rejected with only feedforward control. Note however that direct disturbances can only be attenuated through feedback. As always, the primary purpose of feedback here is to account for uncertainties, either in the disturbance or in the plant model.

The price paid for this property of feedback is the requirement that the feedback be stabilizing over the range of uncertainties in the *nominal plant*, the plant model assumed for design. The nominal stability of the closed loop system may be checked by a variety of methods, the most popular for single–input–single–output (SISO) systems being the Nyquist and Bode plots. Implicit in these methods are measures of system robustness. The Nyquist stability criterion can be generalized straightforwardly to multi–input–multi–output (MIMO) systems, however the robustness measures do not carry over as straightforwardly.

Both Figure 3a and 3b can be generically expressed in the form of Figure 4 where G(s) is the plant, P(s) is umbilical's pre–compensation of the wall disturbance and $\overline{D}(s)$ is the equivalent disturbance to the system. Figure 4 has been presented in unity feedback form so as to introduce the concept of loop shaping and the trade–offs inherent in control system design. Denote the transfer functions between $\overline{D}(s)$ and X(s), the *sensitivity function*, as

$$S(s) \equiv \frac{X(s)}{D(s)} = \frac{1}{1 + GH} \tag{3}$$

and between $v_2(s)$ and $x(s)$, the *complementary sensitivity function*, as

$$C(s) \equiv \frac{X(s)}{v_2(s)} = \frac{GH}{1 + GH} \tag{4}$$

Note that $S(s) + C(s) = 1$. Therefore, a feedback controller designed to attenuate external disturbances at a particular frequency

$$|S(j\omega_0)| << 1.0 \qquad |GH(j\omega_0)| >> 1.0$$

cannot attenuate the measurement noise signal at that frequency

$$|C(j\omega_0)| \approx 1.0$$

Likewise, a controller designed to reject a certain frequency measurement noise, $|C(j\omega_0)| << 1.0$, must pass the external disturbance at this frequency, $|S(j\omega_0)| \approx 1.0$. Classical design of control systems usually involves separating (if possible) the frequency spectrum into regions where input disturbances (measurement noise here) and output disturbances (external disturbance here) predominate. The methodology, known as loop shaping, consists of choosing H(s) so that GH is large and therefore S(s) is small at frequencies where output disturbances are dominant, and choosing H(s) so that GH is small and therefore C(s) is small at frequencies where input disturbances are dominant. This would be a relatively simple task if the designer only needed to be concerned with the magnitude of GH. However, stability of the feedback system requires that the argument of GH at crossover, where $|GH(j\omega_0)| \equiv 1.0$, be greater than -180°. That is, the system must have some phase margin. Since the phase of a transfer function is tied to its magnitude's (in dB) derivative with respect to frequency, as was shown by Bode [8], the loop shaping's results are fundamentally limited by the difference in frequency between the input and output disturbances. The designer may only change through shaping H(s) the magnitude in dB of GH at so fast a rate. Thus, the frequency bands where the magnitude of the sensitivity function and complimentary sensitivity function may be small must be separated in frequency by a crossover region of a certain width (which is dependent on G(s) as well as how small $|C(s)|$ and $|S(s)|$ must be).

The trade–off between rejection of input and output disturbances through feedback is also inherent in passive isolation systems. Suppose we are capable of choosing the umbilical stiffness and damping of Figure 2 so as to design a passive isolator. Note that the transfer function relations

$$\frac{\ddot{X}(s)}{\ddot{Y}(s)} = \frac{G(s)}{1 + G(s)} \qquad \frac{\ddot{X}(s)}{D(s)/m} = \frac{1}{1 + G(s)} \tag{5}$$

apply where

$$G(s) = \left(\frac{cs + k}{ms^2}\right)$$

From this, it is easy to see that direct disturbances act as output disturbances while wall accelerations act as input disturbances. The difference between designing an isolation mount for base disturbances and for direct disturbances is well known and understood by vibration engineers. A soft mount is appropriate for isolating against base disturbances while a stiff mount is appropriate for direct disturbances excitation. The loop shaping capability of springs and dampers is however very restricted. Indeed, one cannot shape the loop to yield an unstable system. An active control system may have its loop shaped to an arbitrary specification provided it is possible to meet the specification with a stable system. Here lies the chief advantage of designing an isolation system from a control paradigm: the interaction of the conflicting specifications, stability, and robustness is clear throughout the loop shaping procedure. Sensitivity and complimentary sensitivity functions are extendable to MIMO systems through the use of singular values.

Robustness in single–input–single–output controller design is measured by *gain* and *phase margins*. The gain margin is the range of gain that can be introduced into the loop while maintaining stability. Similarly, the phase margin is the amount of phase that can be introduced into the loop while maintaining stability. The practical importance of the margins is that the gain and phase of the nominal plant is not the same as that of actual plant. These margins may be easily determined from Nyquist or Bode plots. Loop shaping also implies that a compensator H(s) should not be so large as to extend the crossover frequency of the compensated system into the higher frequency range where nominal models are very inaccurate.

Robustness for MIMO systems can also be specified in terms of the simultaneous gain and phase variations that may be introduced into the loops while preserving stability. However, this description does not account for unmodeled coupling in the dynamics. Uncertainty may be represented in terms of an additive (in parallel) or multiplicative (in series) transfer function matrix appended to the plant. (While these are the most common there are other representations.) Using either uncertainty representation it can be easily shown by the small gain theorem that stability can be guaranteed if uncertainties in the plant are required to be bounded by a norm of the compensated plant. This is best represented in terms of the frequency dependent singular values of the plant and uncertainty transfer function matrices. This measure, however, is conservative since it allows cross coupling dynamics between channels that in actuality could never occur. The structured singular value methodology attempts to alleviate this conservatism through structuring the uncertainty model. Readers interested in a general treatment of MIMO stability and robustness should consult Ref. 9.

III. Passive Isolation: An Analogy

We now examine the design of an active vibration isolation system for microgravity space experiments from an analogy to passive isolators. Indeed, the primary reason for pursuing an active rather than passive system is not the increased flexibility in loop shaping but the limitations of active systems in attaining a stiffness low enough to meet the isolation requirements. This is true even when no umbilical is present.

For the generic system model of Eqn. (1) with the nominal values

$m = 220$ kg

$k = 20$ N/m

$c = 6.63$ N·s/m (5% of critical damping)

The transmissibility curve between base and experiment acceleration, shown in Figure 5 is given by

$$\frac{\ddot{X}(s)}{\ddot{Y}(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{6}$$

with

$$\omega_n \equiv \sqrt{k/m} = 0.3 \text{ rad/sec} = 0.048 \text{ Hz}$$

$$\zeta = \sqrt{c^2/4mk} \approx 0.05$$

Also depicted in Figure 5 are the transmissibility specifications (1) and (2) discussed in Section I. While the system satisfies the unity transmissibility criterion, note that the natural frequency is not low enough to meet the 40 dB attenuation requirement. The system is also deficient in the magnification of disturbances at and near the resonance. Clearly any modification to the umbilical's dynamics through feedback should include increased damping through a positive gain on experiment velocity. Feedback of inertial experiment velocity permits the damping coefficient ζ to be increased in the denominator of Eqn. (6) without changing it in the numerator. Thus, the resonance can be removed without affecting the roll–off rate (since the zero of Eqn. (6) is not changed).

If the umbilical were softer, say with k = 0.20 N/m, both specifications (1) and (2) could be met by the passive system. Unfortunately, an active system cannot lower the stiffness with positive gains on position feedback. An active system may be used to insert a negative stiffness spring in parallel with the umbilical. For example, for the nominal plant with the controller transfer functions of Figure 3a equal to

$$H_p(s) = -(6.0s + 19.8) \qquad T_p(s) = -(6.0s + 19.8) \qquad (7)$$

The natural frequency of t'e system is moved an order of magnitude lower. (Here, a negative damper has also been introduced so as to maintain the system's 5% critical damping for the purpose of comparison. If less negative damping is introduced in order to remove the resonance, even more negative stiffness must be introduced to meet the 40 dB specification.) Note that this vibration engineering approach, that is, lowering the stiffness, requires the near cancellation of the umbilical's stiffness with that introduced via feedback. If the negative stiffness exceeds that of the umbilical, the equivalent stiffness of the system will be negative and the system will be unstable. It is not surprising then that the introduction of negative stiffness via the controller has no robustness whatsoever. The design using Eqn. (7) has less than 0.1° phase margin. The root locus for the system, shown in Figure 6, clearly indicates this potential for instability. A focus on equivalent stiffness in isolation system design thus leads to control systems which sacrifice robustness for performance. In addition, a design which achieves isolation through lowering the system stiffness cannot attenuate direct disturbances over the same frequency band, as discussed in Section II.

From a vibration engineering viewpoint, an alternative means of achieving rejection of disturbances is to rigidly fasten it to an inertial structure. While there is no such structure in space, it is possible to achieve this result by a high positive gain feedback on experiment position. (The inertial position must be obtained by integrating an accelerometer reading twice. This does pose a problem since this procedure is marginally stable. However, this problem may be ameliorated through replacing the integrators with a second order low pass filter. The authors are aware of this method being employed successfully on a six–degree–of–freedom magnetic suspension isolation rig at NASA Lewis Research Center.) This acts as a very stiff spring tying the experiment to inertial space. The controller and resulting transfer functions in this case are

$$H_p(s) = 2000 \qquad T_p(s) = 0 \tag{8}$$

$$\frac{\ddot{X}(s)}{\ddot{Y}(s)} = \frac{6.63s + 20}{220s^2 + 6.63s + 2020}$$

$$\frac{\ddot{X}(s)}{D(s)/m} = \frac{220s^2}{220s^2 + 6.63s + 2020}$$

While this controller meets the 40 dB specification, it does not have unity transmissibility below 0.001 Hz. An experiment controlled in this fashion will collide into the wall. The feedforward transfer function may be adjusted to provide unit gain via

$$T_p(s) = \frac{-2000}{159s + 1}$$

This feedforward with the feedback term of Eqn. (8) effectively acts to base disturbances as a high relative stiffness up to 0.001 Hz changing to a large inertial stiffness. The resulting transmissibility $\ddot{X}(S)/\ddot{Y}(s)$ is presented in Figure 7. Note that since the feedback loop introduced no damping, the original resonance is still present although less damped and at a higher frequency. This may be corrected by adding an inertial damping into the feedback loop. While this design method may be used to meet the specifications with robustness it has three faults: (1) it requires inertial experiment position, inertial wall position, and inertial experiment velocity measurements which are problematic to obtain, (2) it requires very high gains in both feedforward and feedback loops to obtain attenuation, and (3) an extension of the method to multi–degree–of–freedom systems would be difficult. It is also possible that when a flexible wall is considered, rather than the infinite impedance structure assumed, that the system will be unstable.

As another method of fastening the experiment to inertial space, one may employ inertial damping via feedback. By feeding back the inertial experiment velocity with a high gain, it is almost possible to achieve both the 40 dB and unity transmissibility specification without resorting to feedforward. For example, with

$$H_p(s) = 1000\ s \qquad T_p(s) = 0$$

the resultant transmissibility are shown in Figure 8. Unfortunately, the roll–off rate here is approximately 20 dB/decade and therefore it is impossible to achieve both specifications simultaneously. This method has the advantage over the inertial spring of being a great deal simpler and requiring only one inertial measurement (experiment velocity which requires only one integration of accelerometer measurements).

Another passive analogy is the lowering of the natural frequency of the umbilical by increasing the experiment mass. An increased experiment mass would attenuate direct disturbances as well as those transmitted through the umbilical. In addition, at frequencies below the natural frequency of the umbilical, the isolation system would have unity transmissibility. Of course, for space applications any additional mass is very costly. To lower the natural frequency by an order of magnitude would require increasing the experiment mass by a factor of one hundred. Clearly, it is not practical to accomplish increased isolation through the addition of real mass. However, it is possible to increase the effective mass of the system through feedback. This will be examined in the next section, as this idea most properly evolves out of loop shaping.

To summarize, the passive isolation analogy to control system design yields some insight but falls short as a design tool on three counts: (1) it does not have the flexibility to shape the response with its simple analogical elements, stiffness, damping, and mass, so as to achieve the performance requirements, (2) it cannot be easily or effectively generalized to multi–degree–of–freedom problems, (3) it completely ignores the robustness problem inherent to active control systems. We advocate, therefore, that vibration engineers consider active isolation a controls problem and use an automatic controls framework for tackling it.

IV. The Control System Approach

A simple controller is now designed for the system described by Eqn. (1) and the nominal values. The authors refer the reader back to Figure 5 where transmissibility curve between experiment and wall accelerations (or positions) is presented again along with the design specifications (1) and (2). The goal is to design a feedback control $H_p(s)$ that results in the closed loop transfer function

$$G_{cl}(s) \equiv \frac{X(s)}{Y(s)} = \frac{G_p(s)P(s)}{1 + G_p(s)H_p(s)} \tag{9}$$

satisfying both constraints; that is

$$|G_{cl}(j\omega_o)| \approx 1.0 \qquad \frac{\omega_o}{2\pi} < 0.001 \text{ Hz}$$

$$|G_{cl}(j\omega_o)| < 0.01 \qquad \frac{\omega_o}{2\pi} > 0.1 \text{ Hz}$$

Here, $G_p(s)$ and $P(s)$ are as indicated in the block diagram of the system, Figure 3a. Note that the uncontrolled system $G_p(s)P(s)$ satisfies the first of these constraints.

therefore, $H_p(S)$ should be very small in the low frequency band so that the closed loop system will continue to satisfy the unit transmissibility specification. Therefore, this specification yields a condition like

$$| G\, H_p(j\omega_o) | < 0.01$$

$$| H_p(j\omega_o) | < 0.2 \qquad \frac{\omega_o}{2\pi} < 0.001 \text{ Hz}$$

At and above 0.1 Hz, the attenuation of the uncontrolled system is not sufficient. It is desirable to increase the attenuation by approximately two orders of magnitude. This may be accomplished by requiring $H_p(s)$ to be very large in this frequency range, approximately

$$| G\, H_p(j\omega_o) | > 100.$$

$$| H_p(j\omega_o) | > 2000. \qquad \frac{\omega_o}{2\pi} > 0.1 \text{ Hz}$$

These two design specifications on $H_p(s)$ are shown in Figure 9 along with a simple function satisfying these conditions,

$$H_p(s) = 5000\, s^2 \tag{10}$$

This controller design results in the closed loop transmissibility between experiment and wall accelerations which is plotted in Figure 10. Note that both specifications (1) and (2) are met. Inertial damping should be added to this design to eliminate the resonance. It is easily seen from a root locus plot that this design is robust with respect to changes in umbilical/experiment properties, Figure 11, and actuator finite bandwidth, Figure 12. In practice, this design would be improved by rolling off the controller gain. This limits the controller bandwidth so as to not affect possible unmodeled lightly–damped high frequency modes of the system (e.g. wall flexure). A controller design would probably also include a weak position integral feedback to provide a slow centering force so that accelerometer bias and noise does not result in wall collision.

The reader might object to the controller of Eqn. (10) since it is improper (i.e., has more zeros than poles). However, this controller is realizable. Note that $H_p(s)$ multiplies the position measurement to yield the control force. Since the factor s^2 in the time domain is equivalent to two differentiations with respect to time, Eqn. (10) prescribes constant gain acceleration feedback. This, as discussed earlier, increases the effective ma of the system. (Of course, if we modify Eqn. (10) to limit the controller bandwidth, then the mass analogy only holds within the band.)

While both transmitted and direct disturbances are attenuated, the experiment acceleration level will be approximately the same as the accelerometer measurement noise level. This results from the transmissibility between experiment acceleration and measurement noise being nearly one due to the high gain feedback. This is a fundamental issue as discussed in Section II; one must trade–off the rejection of disturbances to the system and the rejection of measurement noise. Since the disturbances may be up to one thousand times larger than the measurement noise (accelerometer resolution typically 1 μg) the controller is designed to reject disturbances. The performance of the control system is thus directly a function of the quality of the accelerometer.

Recently, an alternative approach to design of active vibration isolation control systems for microgravity experiments was presented in Reference [10]. A desired transmissibility ratio $G_{cl}(s)$ is specified along with the plant model $G_p(s)$

and P(s). Eqn. (9) is then solved via algebraic manipulation for the feedback controller $H_p(s)$ that yields the desired transmissibility (feedback of relative position is also allowed and may be used; if used, a second condition must then be specified for solution). While this approach resembles loop shaping in that it attempts to achieve a certain transmissibility, it is fundamentally different in that it does not properly consider the plant. The algebraic procedure in essence first eliminates the plant and then replaces it with one which will yield the desired transmissibility. As a control design procedure, this methodology has serious flaws: (1) the stability of the resulting system may be entirely dependent on perfect knowledge of the plant, (2) the procedure incorporates none of the known relationships and fundamental trade–offs between stability and attenuation; it implies that any specified transmissibility is achievable, and (3) for systems with right half plane poles/zeros, the methodology may attempt cancellation with right half plane zeros/poles. For a simple controls problem, the algebraic manipulation method may result in a good controller. However, for more difficult problems, the method is questionable. An extension of this methodology to multi–input–multi–output control would be plagued by many problems.

To summarize, controller design for single–degree–of–freedom vibration isolation problems is best performed through the classical control framework of loop shaping where the natural interplay between performance, stability and robustness are evident. For multiple degree of freedom isolation problems, recent advances in controller design, such as the extension of loop shaping principles via frequency weighting and singular values [11] seems to be most promising. In order to emphasize the question of coordination in control of MIMO systems, we next examine a multiple–degree–of–freedom isolation problem.

V. A Multiple–Degree–of–Freedom System

A common misunderstanding among many engineers unfamiliar with control system design is the nature of the differences between SISO and MIMO control problems. The relative ease with which the uninitiated comprehend the elimination of one error signal through negative error feedback yields the false impression that the MIMO control problem is little more than the feeding back of multiple error signals. This impression, however, is not totally groundless. Indeed, many MIMO controllers in use today were designed by a single–loop–at–a–time procedure. Design with this method can be quite difficult, time consuming, and non–intuitive. Robustness is difficult to check except by analyzing all the possible permutations to the nominal plant. The fundamental problem in MIMO design is the coordination of the control in coupled channels when the plant is not well known (poorly modeled or time varying).

Easily decoupled active vibration isolation control problems may be deceptively simple. Unmodeled cross–coupling due to inaccuracies in center of mass, sensor, and/or umbilical locations can result in poor performance and even instability. An example isolation problem illustrates. Figure 13 shows a two–degree–of–freedom isolation system composed of an isolated platform (width 0.5 meters and height 0.2 meters, depth unspecified), two accelerometers, two actuators, an umbilical, and a translating base. The platform may translate vertically or rotate about its center of mass. The actuators and accelerometers are positioned a distance of $q = 0.2$ m symmetrically about the assumed center of mass location. An umbilical of stiffness k (no damping) runs between this location and the base. The platform has mass m and inertia I. The equations of motion for the platforms translation $x(t)$ and rotation $\theta(t)$ are

$$m\ddot{x} + k\Delta\theta + kx = f_1 + f_2 + d_1$$
$$I\ddot{\theta} + k\Delta^2\theta + k\Delta x = (q + \Delta)f_2 - (q-\Delta)f_1 + d_2 \tag{11}$$

where d_1 and d_2 are the disturbances, and Δ is the error in the assumed center of mass. The accelerometer readings are

$$\mathcal{Y}_1 = \ddot{x} - (q - \Delta)\ddot{\theta}$$
$$\mathcal{Y}_2 = \ddot{x} + (q + \Delta)\ddot{\theta} \tag{12}$$

The nominal system ($\Delta = 0$) can be decoupled to one in terms of the degrees of freedom by the change in variables

$$F = f_1 + f_2$$
$$M = q(f_2 - f_1)$$
$$z_1 = (\mathcal{Y}_1 + \mathcal{Y}_2)/2 \tag{13}$$
$$z_2 = q\,(\mathcal{Y}_2 - \mathcal{Y}_1)/2$$

which are nominally the translational force, the moment, the translational acceleration, and the angular acceleration for the platform respectively. The nominal transfer function for the system are then

$$Z_1(s) = \left[\frac{s^2}{ms^2 + k}\right](F(s) + D_1(s))$$

$$Z_2(s) = \left[\frac{1}{I}\right](M(s) + D_2(s))$$

For translational motion, the natural frequency of the platform is $\sqrt{k/m}$. The rotational motion of the platform is free since the umbilical is attached to the center of mass. To compensate the nominal system, feedback can be designed for each mode of the system separately since the system is decoupled. Translational acceleration and velocity feedback is first used to add effective mass and damping

$$F(s) = -\left(a + \frac{c}{s}\right) Z_1(s) \tag{14}$$

This lowers the natural frequency of translational motion yielding the closed loop transfer function

$$Z_1(s) = \left[\frac{s^2}{(m+a)s^2 + cs + k} \right] D_1(s)$$

Next, angular deflection feedback is used to constrain low frequency rotational motion and some damping is provided

$$M(s) = -\left(\frac{n}{s} + \frac{b}{s^2}\right) Z_2(s) \tag{15}$$

yielding

$$Z_2(s) = \left[\frac{s^2}{Is^2 + ns + b} \right] D_2(s)$$

The following values are used to illustrate this example.

PLATFORM	CONTROL SYSTEM
$m = 400$ kg	$a = 31600$ kg
$k = 50$ N/m	$c = 1000$ N·s/m
$I = 10$ kg·m^2	$b = 0.015$ N·m
	$n = 0.2$ N·m·s

where the control system values are in effective units. This control design lowers the natural frequency of translational motion from 0.056 Hz to 0.006 Hz with 40% of critical damping. The controlled rotational motion has a natural frequency of 0.006 Hz with 26% of critical damping. This controller design would yield very effective isolation on the nominal system.

The actual closed loop transfer functions will be different from the nominal due to the error in the center of mass, Δ. The transmissibility can be derived from Eqns. (11–15) as follows

$$[ms^2 + k]X(s) + [k\Delta]\Theta(s) = F(s) + D_1(s)$$

$$[Is^2 + k\Delta^2]\Theta(s) + [k\Delta]X(s) = M(s) + \Delta F(s) + D_2(s)$$

$$Z_1(s) = [s^2]X(s) + [\Delta s^2]\Theta(s)$$

$$Z_2(s) = [s^2]\Theta(s)$$

$$F(s) = -[a + c/s]\, Z_1(s)$$

$$M(s) = -[n/s + b/s^2]\, Z_2(s)$$

yielding

$$\left[\frac{(m+a)s^2 + cs + k}{s^2}\right] Z_1(s) + [m\Delta]\, Z_2(s) = D_1(s)$$

$$\left[\frac{(as^2 + s + k)\Delta}{s^2}\right] Z_1(s) + \left[\frac{Is^2 + ns + b}{s^2}\right] Z_2(s) = D_2(s)$$

The poles of this system are given by the roots of the characteristic equation

$$[(m + a)s^2 + cs + k][Is^2 + ns + b] - [m\Delta][\Delta(as^2 + cs + k)] = 0 \qquad (16)$$

For the nominal plant, $\Delta = 0$, the roots of the Eqn. (16) result in the prescribed natural frequencies and critical dampings. However, as the center of mass error increases, the poles migrate and the system becomes unstable. For an error as small as 6 millimeters, instability occurs. A plot of the pole movement versus error in center of mass is shown in Figure 14. This sensitivity results from the ill conditioned character of the required controller. Ill conditioned here means that the controller's gain to an output signal varies strongly with the signal's direction. This results in a control system which is not robust to this model's uncertainty (center of mass) [12]. A proper MIMO controller design might remedy this problem. In any case, an analysis of the problem from a MIMO control perspective would indicate the potential instability and the nature of the trade–off between performance and robustness. (The authors note that increasing the damping and stiffness for the rotational mode improves the system robustness significantly, while changing the damping or effective mass for the translational mode has little effect.)

VI. Linear Quadratic Regulator for Isolation

MIMO control design, since it requires a high degree of coordination, must proceed by a synthesis procedure. One such method is Linear Quadratic Regulator (LQR) synthesis [13]. This produces a state feedback controller which is optimal with respect to the quadratic (two norm) performance function

$$J = \int_{-\infty}^{\infty} \bar{x}^T(j\omega)\, Q\, \bar{x}(j\omega) + u^T(j\omega)\, R\, u(j\omega)\, d\omega \qquad (17)$$

where Q and R are respectively the symmetric (usually diagonal) state and control weighting matrices, and $\bar{x}(j\omega)$ and $u(j\omega)$ are the Fourier transforms of the state and control vectors. The state (positions and velocities for vibration isolation) satisfies the differential equation

$$\dot{\bar{x}} = A\bar{x} + Bu$$

The quadratic performance function of LQR, Eqn. (17), is well suited to this problem since vibration isolation quality is usually measured in terms of root–mean–square. However, some modification of the performance function is necessary to apply this synthesis procedure to microgravity isolation controller design. The reader will note that state feedback for the isolation problem is feedback of experiment positions, velocities, angles, and angular velocities. Thus, LQR can only result in inertial stiffness and inertial damping feedback. As was shown in Section III, these isolation techniques cannot yield acceptable isolation performance. Thus, an LQR performance function of the form of Eqn. (17) will not yield a satisfactory controller. Note that the differential equation does not include a disturbance term. Consequently, the controller is optimal with respect to white noise. Since the power spectrum of the microgravity environment is not of this shape, the LQR controller will not be optimal with respect to rejection of the disturbance. Through the incorporation of a disturbance model (essentially a shaping filter) the LQR problem may be modified to yield an optimal disturbance accommodating (i.e. rejection) controller. This incorporates the addition of pseudo–states to the state variable model [14].

Closely related to disturbance accommodation is the concept of frequency weighted LQR performance functions [15]. Here, the Q and R matrices are chosen to be even rational functions of frequency. This results in the addition of pseudo–states to the state variable model. Through choice of the weighting functions, the designer can in essence shape the control loops [11]. This also permits the weighting of experiment acceleration. It should be noted that for successful application of LQR theory to the microgravity isolation problem frequency shaped cost functions must be used. Without this, the control resulting from the synthesis procedure would attenuate the vibration at frequencies below 0.001 Hz (non–unity transmissibility). The reader should note that the well known robustness characteristics of LQR controllers do not apply to most frequency shaped designs or to plants with unmodeled cross coupling.

VII. Conclusions

Successful active isolation for microgravity experiments can be achieved but only if the problem is analyzed from a controls perspective. A passive isolation analogy, while useful for an understanding of the control problem, is not an effective design tool. Design of active vibration control systems can best be carried out through loop shaping. For intrusive isolation platforms, this results in a high gain acceleration feedback design. A two–degree–of–freedom example was used to illustrate the instability that can result under unmodeled cross coupling when the control system is designed via decoupling/single loop design procedures. The source of this sensitivity was ill conditioning of the controller. The Linear Quadratic Regulator was examined for the isolation problem. For synthesis of an effective controller, the procedure must be modified to include loop shaping.

Acknowledgements

This work was supported in part by NASA Lewis Research Center and the Commonwealth of Virginia. Some of this research was performed at NASA LRC as part of the Summer Faculty Fellowship Program.

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Figure 1: The Microgravity Environment (from [1])

Figure 2: The One–Degree–of–Freedom Microgravity Vibration Isolation Problem

Figure 3: Displacement (a) and Acceleration (b) Isolation System Block Diagrams

Figure 4: Unity Feedback Form of Control System

Figure 5: Specifications (1) and (2) and Uncompensated Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$

Figure 6: Root Locus for Equivalent Stiffness Design with respect to Umbilical Stiffness Error

Figure 7: Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$ for Inertial Stiffness with Feedforward Design

Figure 8: Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$ for Inertial Damping Design

Figure 9: Designs Specifications and $H_p(s)$

Figure 10: Resultant Transmissibility for Loop Shaped Design

Figure 11: Root Locus of Loop Shaped Design with respect to Umbilical Stiffness Error

Figure 12: Root Locus of Loop Shaped Design with respect to Actuator Finite Bandwidth, ω_b = actuator pole break frequency

Figure 13: Two–Degree–of–Freedom Active Isolation System

Figure 14: Root Locus of Two–Degree–of–Freedom with respect to Center of Mass Error Δ

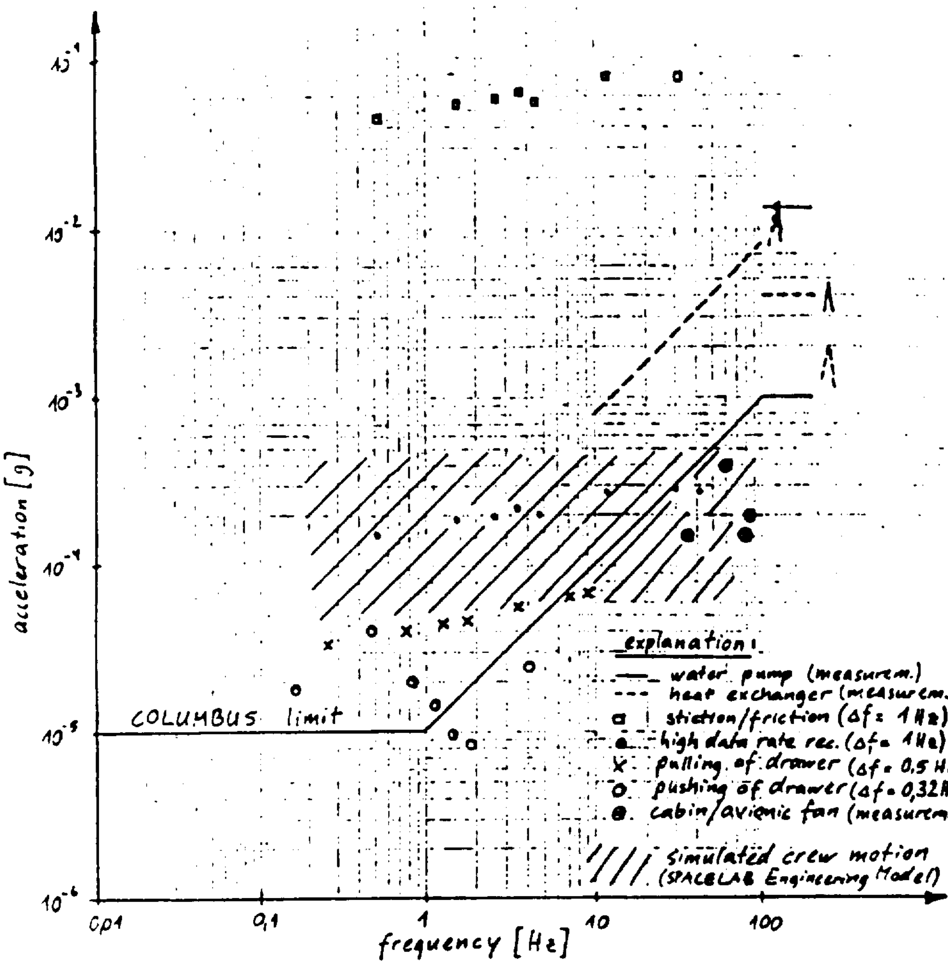

Figure 1: The Microgravity Environment (from [1])

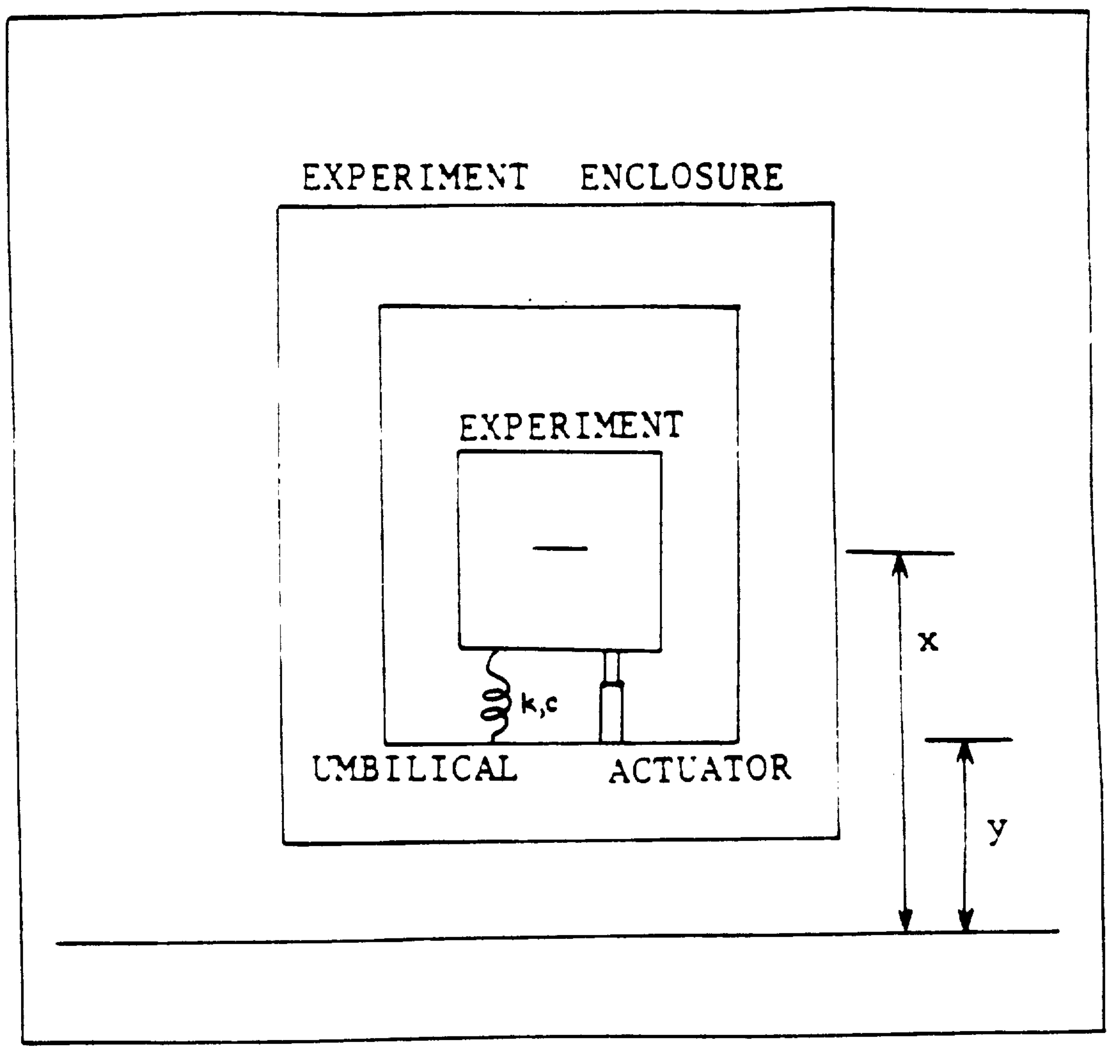

Figure 2: The One–Degree–of–Freedom Microgravity Vibration Isolation Problem

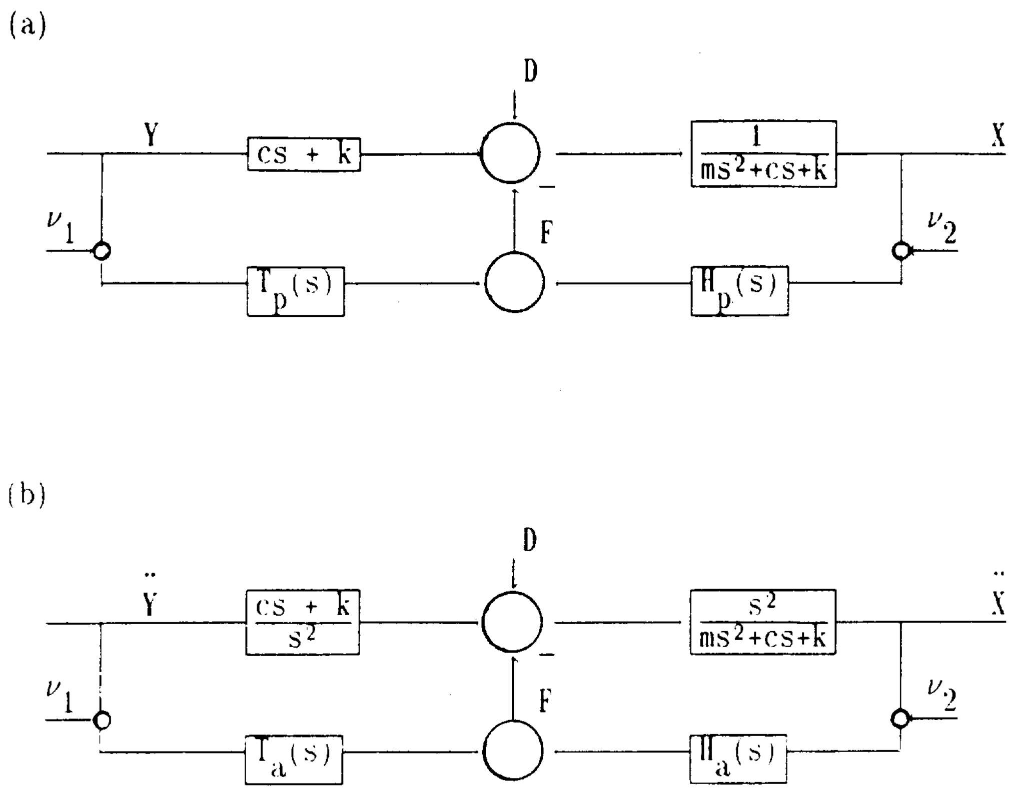

Figure 3: Displacement (a) and Acceleration (b) Isolation System Block Diagrams

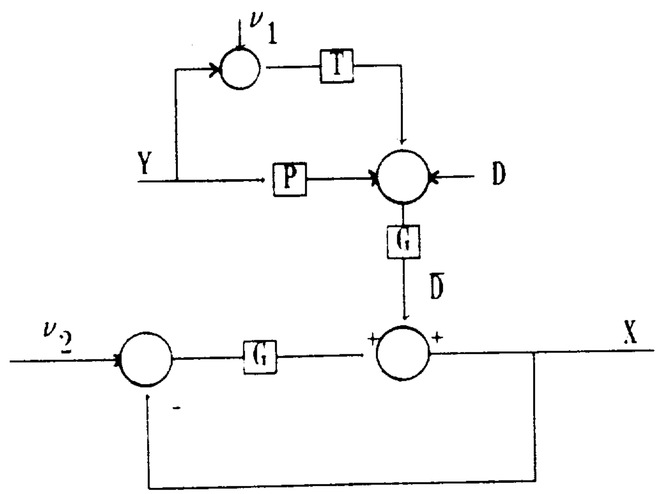

Figure 4: Unity Feedback Form of Control System

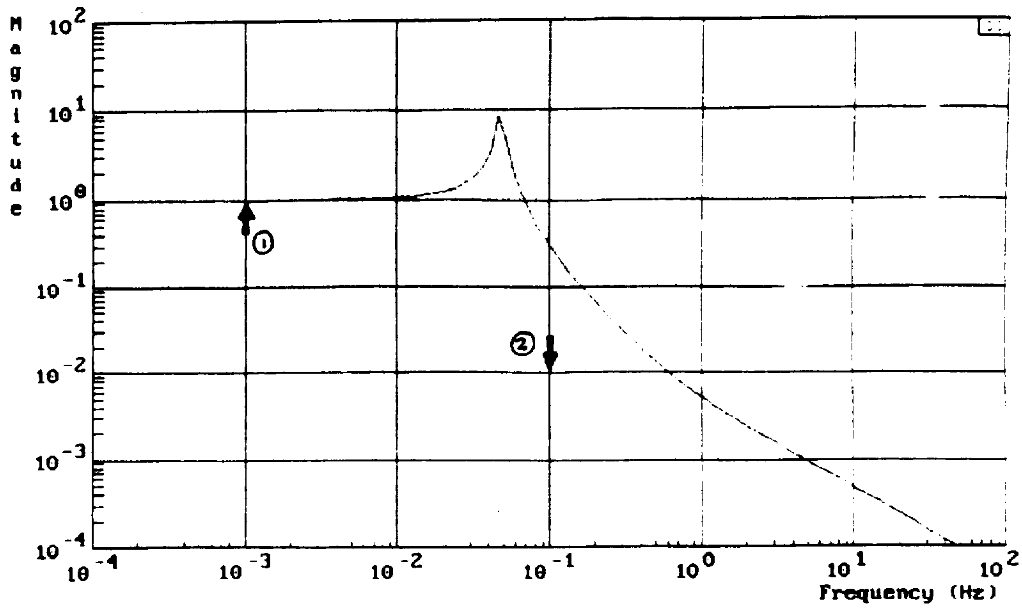

Figure 5: Specifications (1) and (2) and Uncompensated Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$

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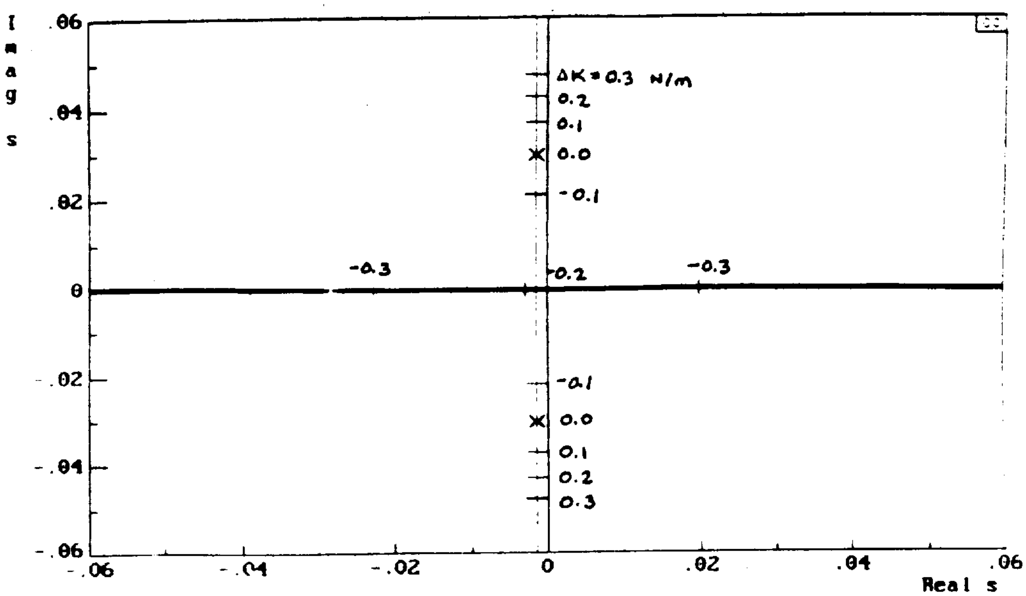


Figure 6: Root Locus for Equivalent Stiffness Design with respect to Umbilical Stiffness Error

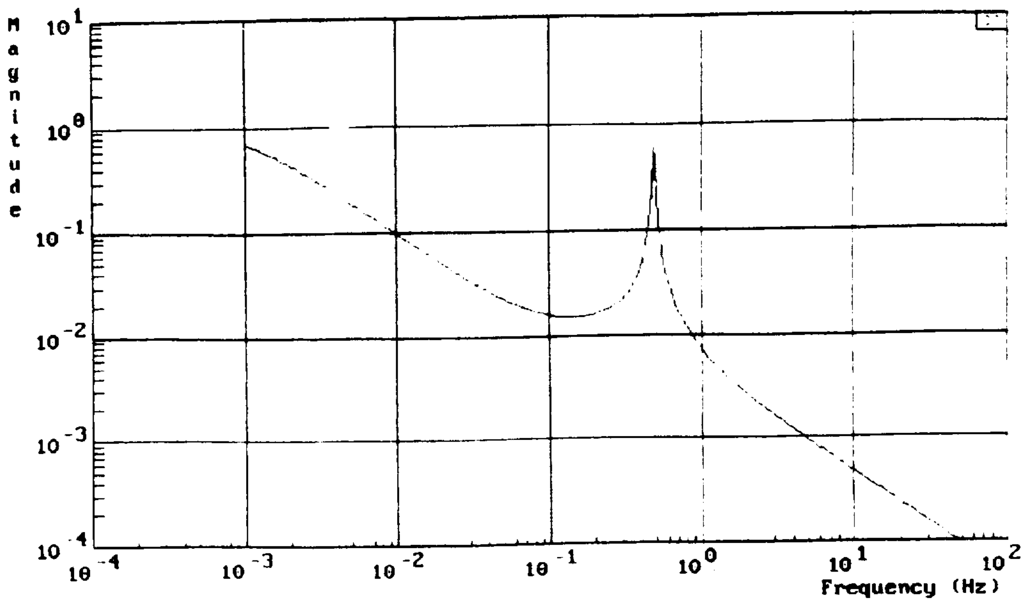


Figure 7: Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$ for Inertial Stiffness with Feedforward Design

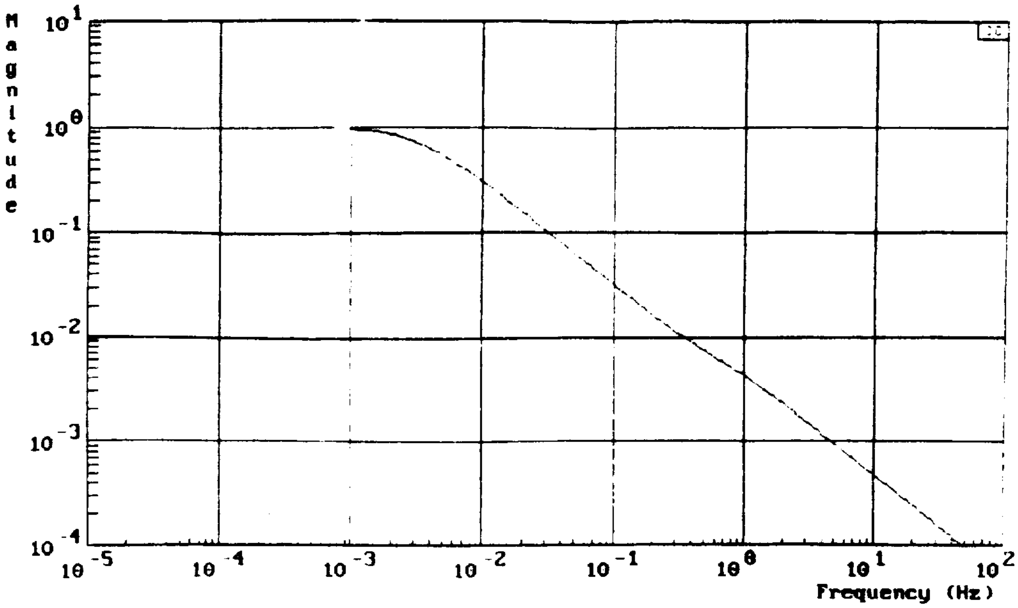


Figure 8: Transmissibility $\ddot{X}(s)/\ddot{Y}(s)$ for Inertial Damping Design

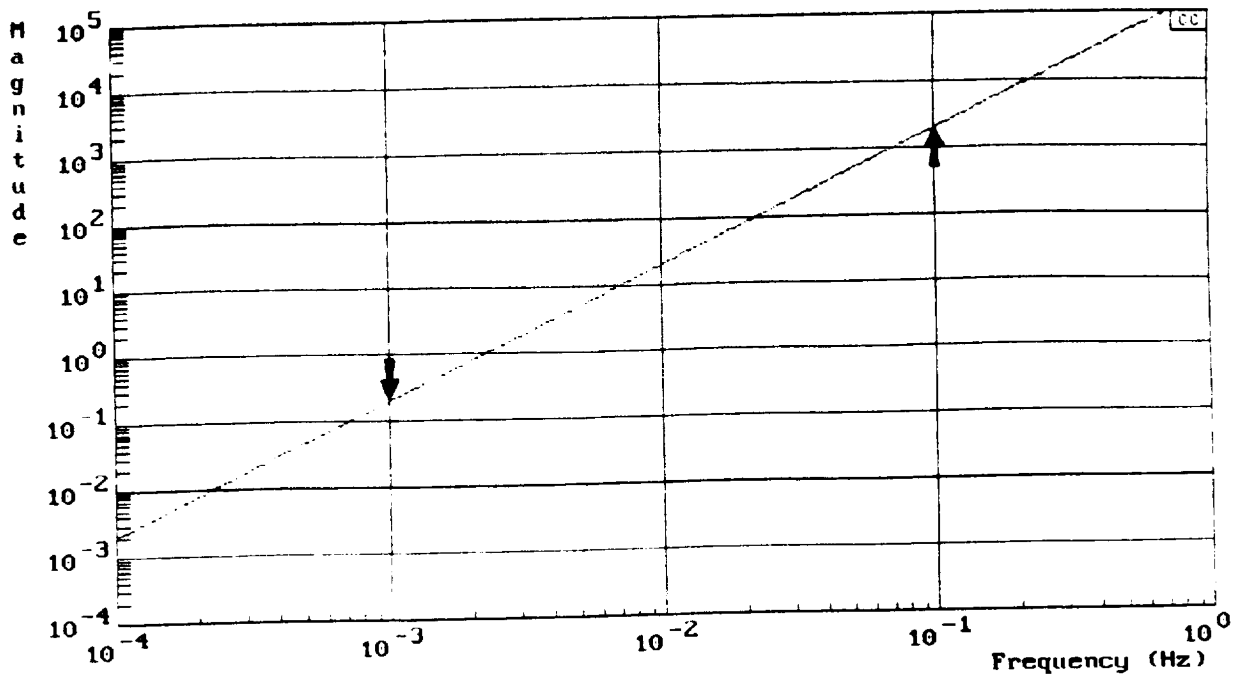


Figure 9: Designs Specifications and $H_p(s)$

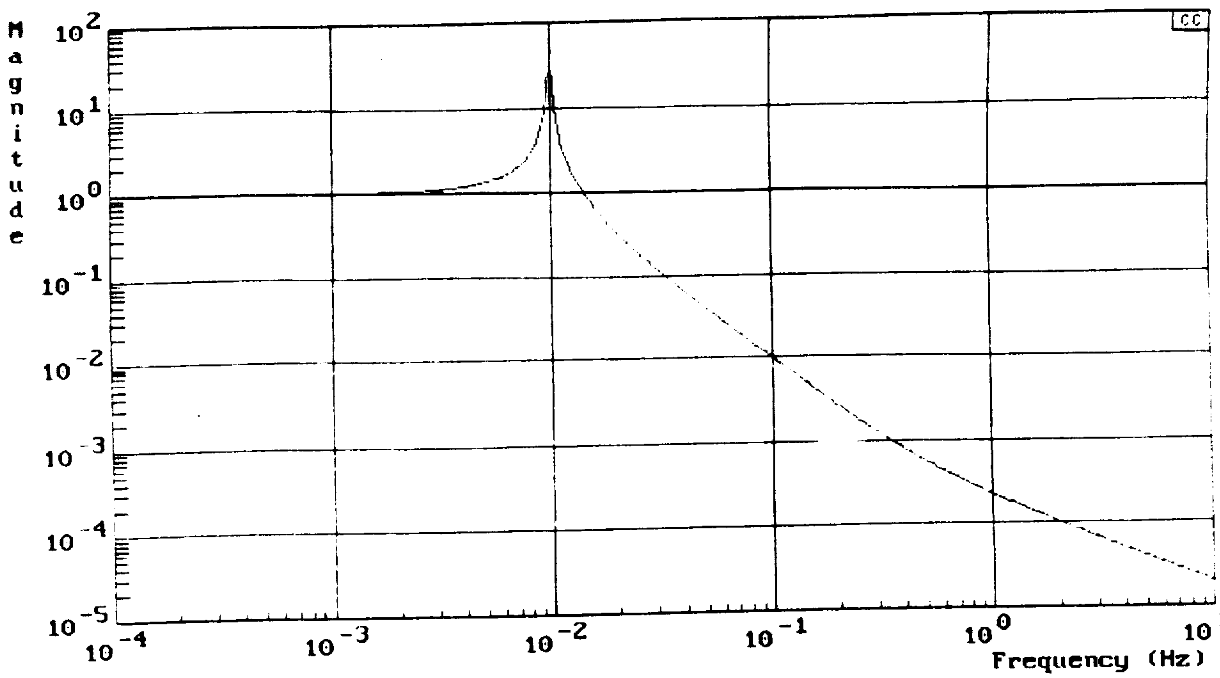


Figure 10: Resultant Transmissibility for Loop Shaped Design

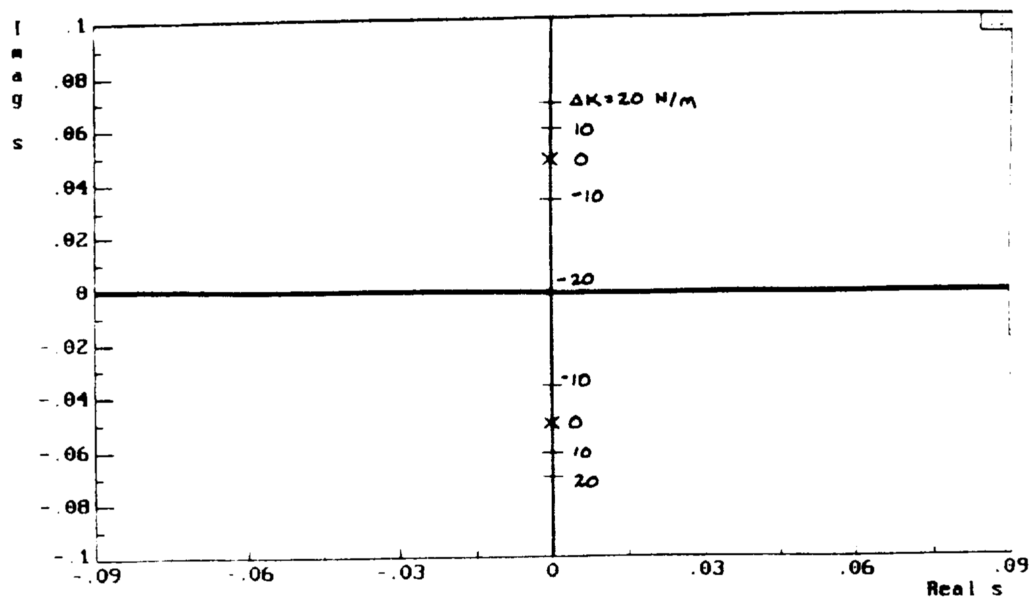


Figure 11: Root Locus of Loop Shaped Design with respect to Umbilical Stiffness Error

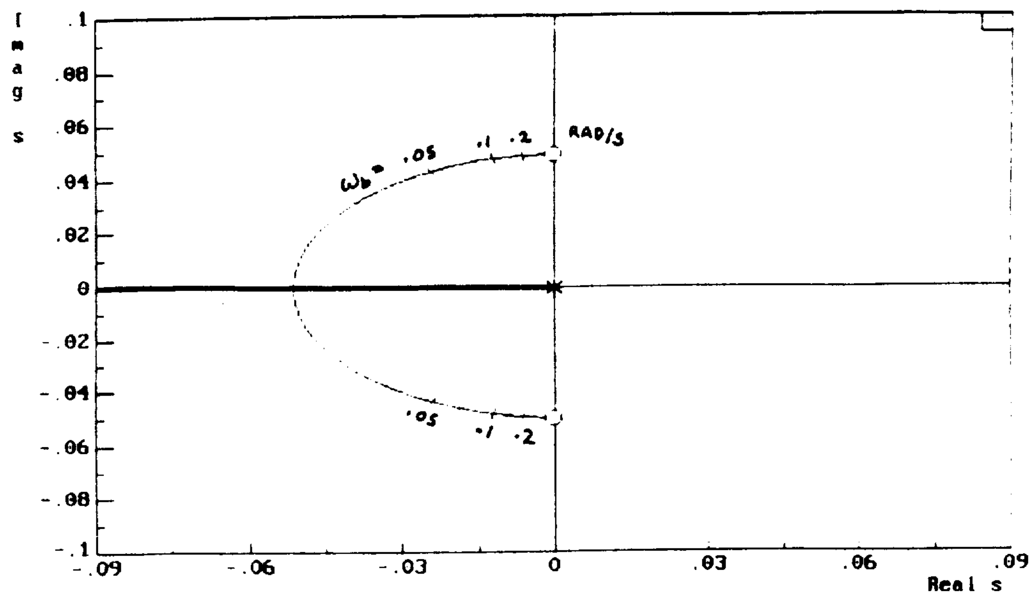


Figure 12: Root Locus of Loop Shaped Design with respect to Actuator Finite Bandwidth. ω_b = actuator pole break frequency

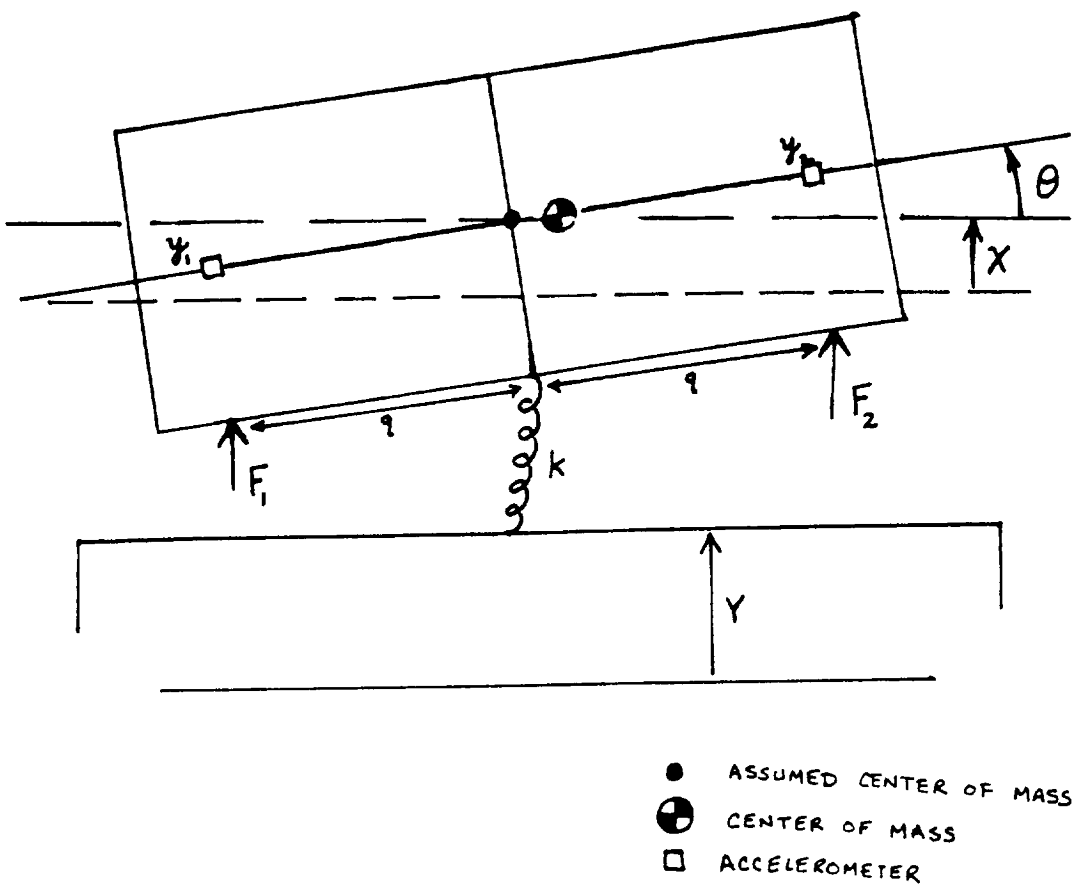


Figure 13: Two–Degree–of–Freedom Active Isolation System

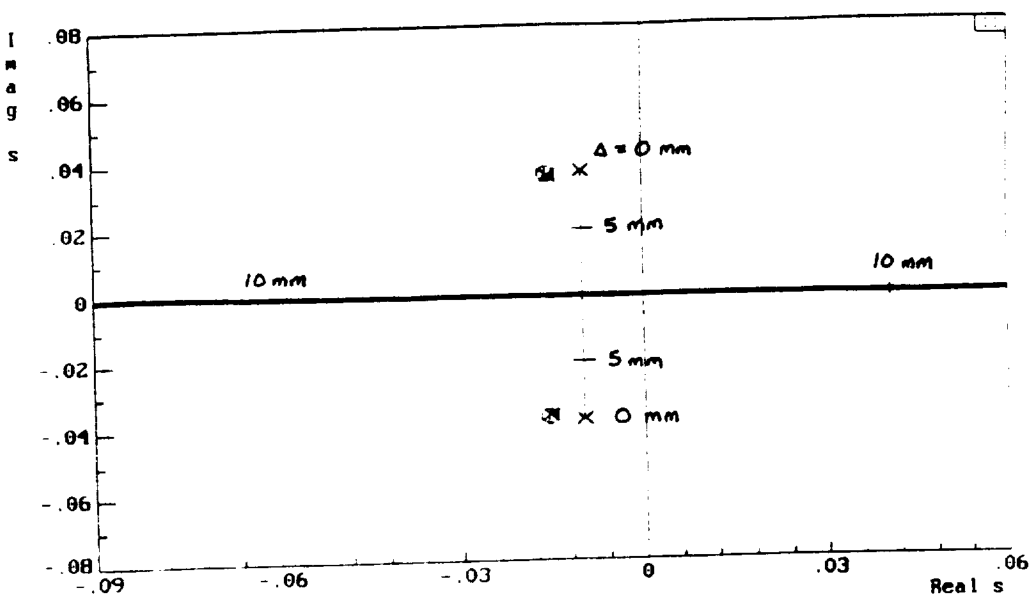


Figure 14: Root Locus of Two–Degree–of–Freedom with respect to Center of Mass Error Δ

CONTROL ISSUES OF MICROGRAVITY VIBRATION ISSOLATION

C.R. Knospe, Research Assistant Professor
R.D. Hampton, Graduate Student

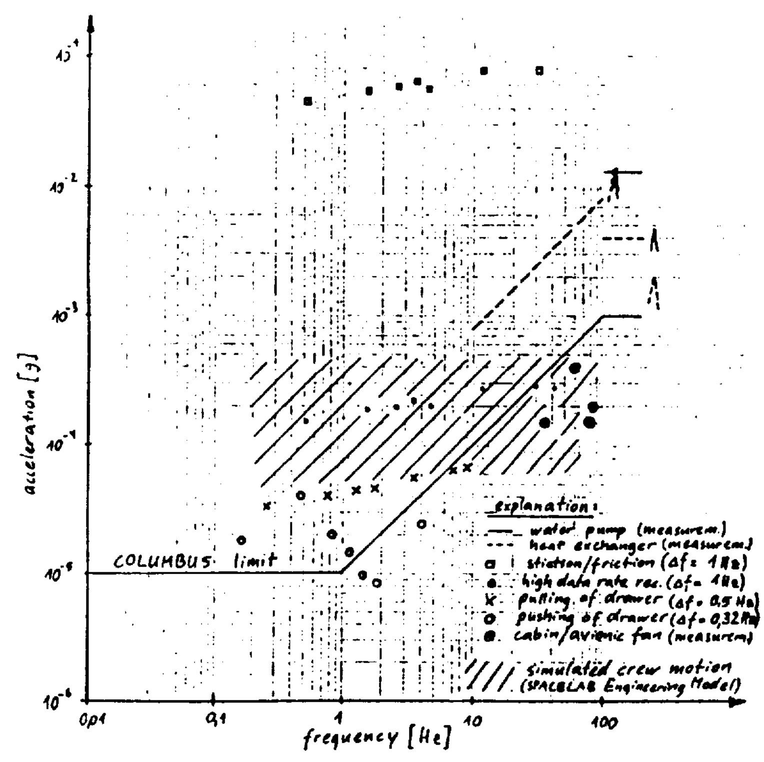
Workshop on Aerospace Applications
of Magnetic Suspension
September 25, 1990

PROBLEM:

ORBITAL ENVIRONMENT CONTAMINATED BY BROADBAND DISTURBANCES

UNACCEPTABLE FOR FLUIDS AND MATERIALS EXPERIMENTS

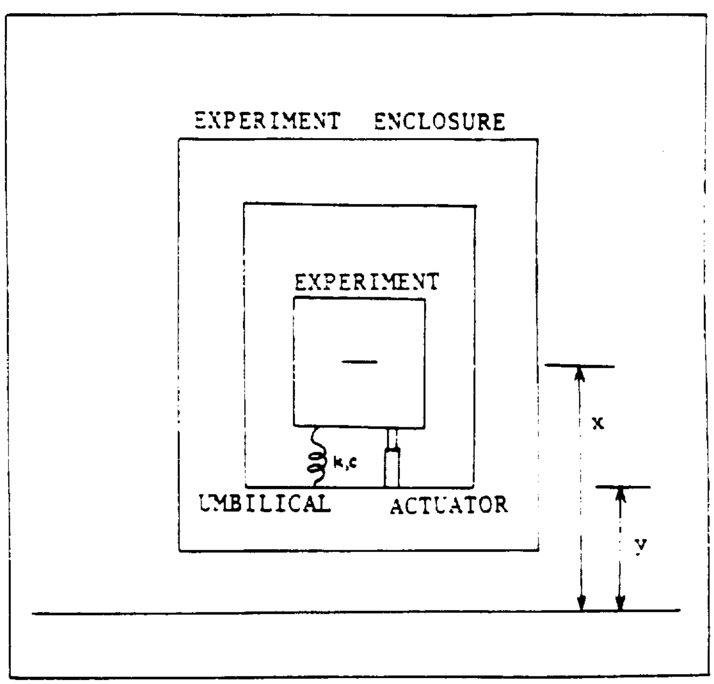
UMBILICAL NEEDED



PERFORMANCE SPECIFICATIONS

- UNITY TRANSMISSIBILITY FROM DC TO 0.001 Hz
- 40 dB ATTENUATION ABOVE 0.1 Hz
- BOTH STABILITY AND PERFORMANCE ROBUSTNESS

Robustness: The ability to withstand ummodelled effects



OUTLINE

(1) CONTROL THEORY REVIEW

(2) PASSIVE ISOLATION ANALOGIES

(3) CONTROLLER DESIGN

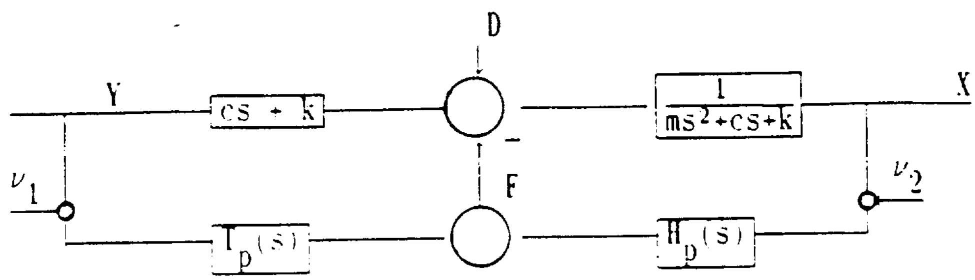
(4) MDOF EXAMPLE

(5) LINEAR QUADRATIC REGULATOR

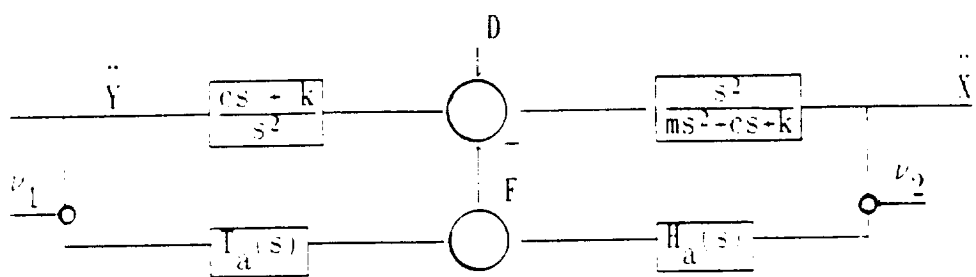
(6) CONCLUSIONS

SYSTEM BLOCK DIAGRAMS

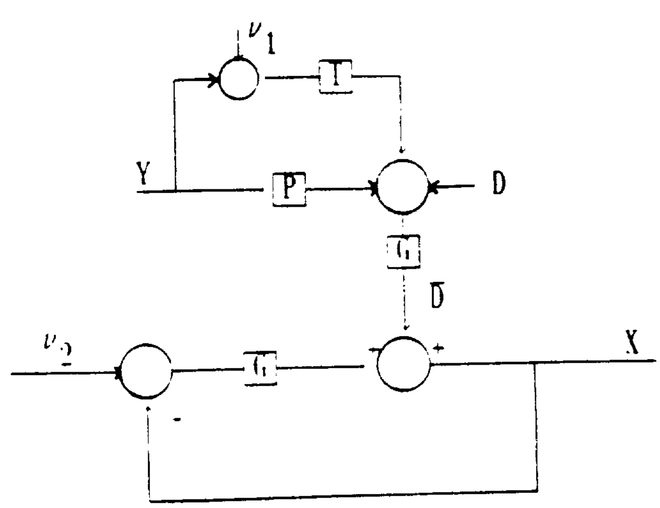
POSITION FORM



ACCELERATION FORM



UNITY FEEDBACK FORM



THE LOOP SHAPING TRADE-OFF

SENSITIVITY FUNCTION

$$S(s) \equiv \frac{X(s)}{\bar{D}(s)} = \frac{1}{1 + GH}$$

COMPLIMENTARY SENSITIVITY FUNCTION

$$C(s) \equiv \frac{X(s)}{V_2(s)} = \frac{GH}{1 + GH}$$

THE TRADE-OFF

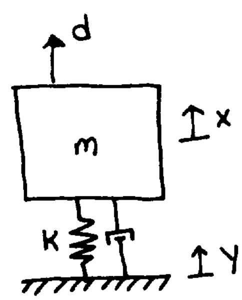
$$S(s) + C(s) \equiv 1$$

LOOP SHAPING: PASSIVE ISOLATION

SPRING-MASS SYSTEM:

$$\frac{\ddot{X}(s)}{\ddot{Y}(s)} = \frac{G}{1 + G}$$

$$\frac{\ddot{X}(s)}{D(s)/m} = \frac{1}{1 + G}$$



$$G(s) = \frac{cs + k}{ms^2}$$

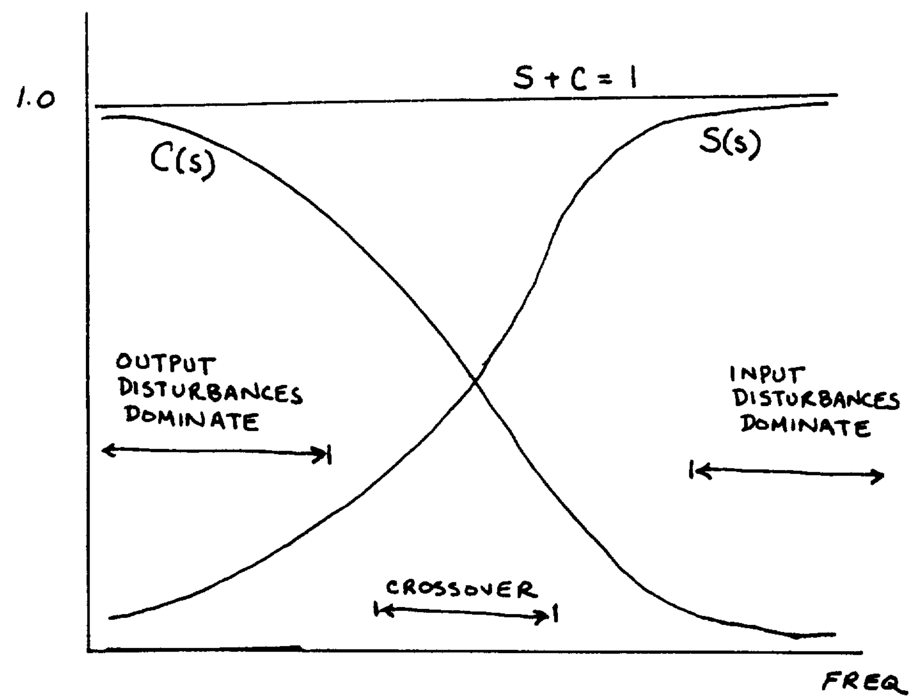
Direct disturbances act as output disturbances while wall accelerations act as input disturbances.

Passive isolation design:

soft mount for base disturbance
stiff mount for direct disturbance

LOOP SHAPING DESIGN

BODE PLOTS:



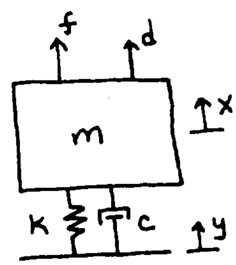
The classical conrol framework displays the trade-offs between input and output disturbance rejection and stability and robustness.

PASSIVE ISOLATION ANALOGIES

THREE ANALOGIES:

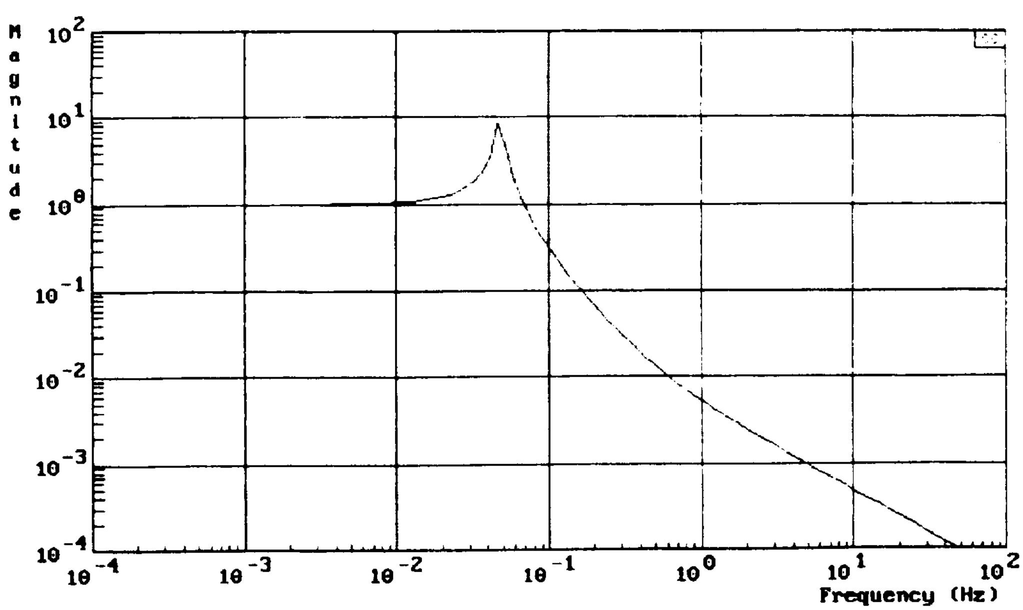
RELATIVE STIFFNESS
INERTIAL STIFFNESS
INERTIAL DAMPING

SYSTEM MODEL:



m=220 kg
k=20 N/m
c=6.63 N s/m

TRANSMISSIBILITY:

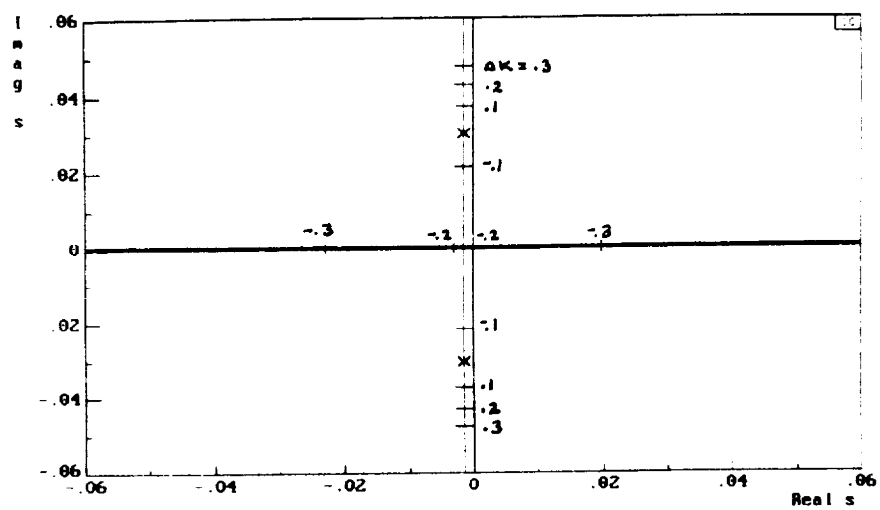


EQUIVALENT STIFFNESS

Lower the natural frequency of the umbilical by adding negative stiffness to the system through the controller

ROBUSTNESS PROBLEM: If the negative stiffness added exceeds that of the umbilical, the system is unstable. But, to lower the natural frequency significantly, the negative stiffness introduced must be nearly that of the umbilical.

Root locus plot:

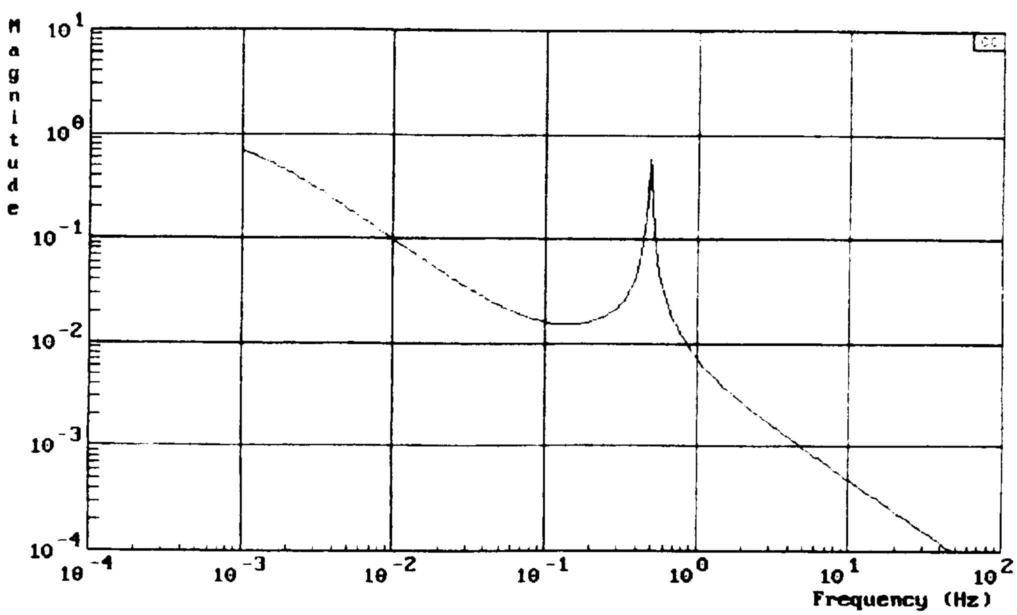


Design has less than 0.1 degree phase margin.

INERTIAL STIFFNESS

Fasten the experiment platform rigigly to an inertial structure through inertial position feedback

PROBLEM: The natural frequency of the system is actually increased; isolation is obtained by lowering the DC gain of the system.Therefore,the resultant system does not have unity transmissibility up to 0.001 Hz. This can be fixed with feedforward.



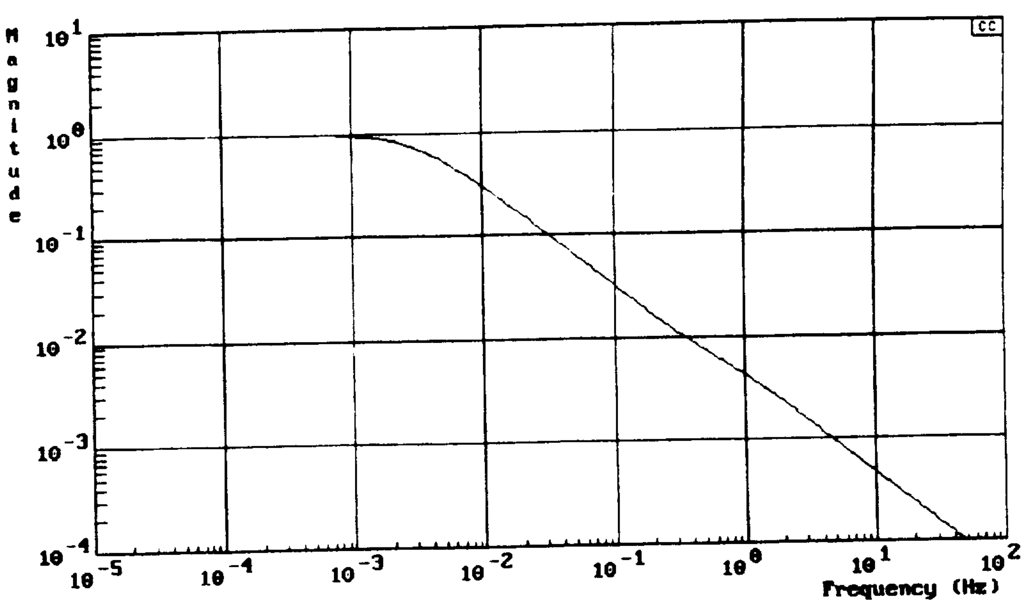
Requires inertial position and velocity measurements.

INERTIAL DAMPING

Prevent experiment movement through fastening the system to inertial space via a damper.This may be accomplished through inertial velocity feedback.

•This method does not compromise the system's DC gain.

PROBLEM: The resulting transmissibility only rolls off at 20 dB/decade. Therefore, the 40 dB at 0.1 Hz and unity transmissibility up to 0.001 Hz design specifications cannot be both achieved.



PASSIVE ISOLATION ANALOGIES:

The passive isolation analogies yield some insight into control system design.

THREE LIMITATIONS AS A DESIGN TOOL:

- Inflexibility to shape the response with simple analogical elements, stiffness and damping.

  This inflexibility can be seen in the inability of the analogical elements to yield an unstable controller.

- Inability to easily and effectively extend to multiple-degree-of-freedom problems.

- Completely ignores the robustness problem inherent to active control system design.

LOOP SHAPING DESIGN

CLOSED LOOP TRANSFER FUNCTION:

$$G_{cl}(s) \equiv \frac{X(s)}{Y(s)} = \frac{G_p P}{1 + G_p H_p}$$

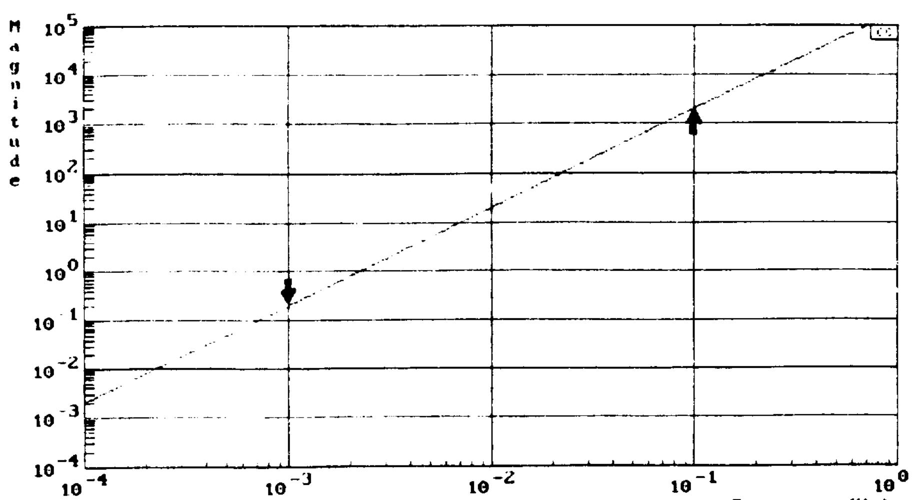
SPECIFICATIONS:

$|G_{cl}| \approx 1.0 \qquad |G_p H_p| < 0.01 \qquad |H_p| < 0.2, \quad \frac{\omega}{2\pi} < 0.001\ \text{Hz}$

$|G_{cl}| < 0.01 \qquad |G_p H_p| > 100 \qquad |H_p| > 2000, \quad \frac{\omega}{2\pi} > 0.1\ \text{Hz}$

Low frequency: Unity transmissibility
High frequency: 40 dB attenuation

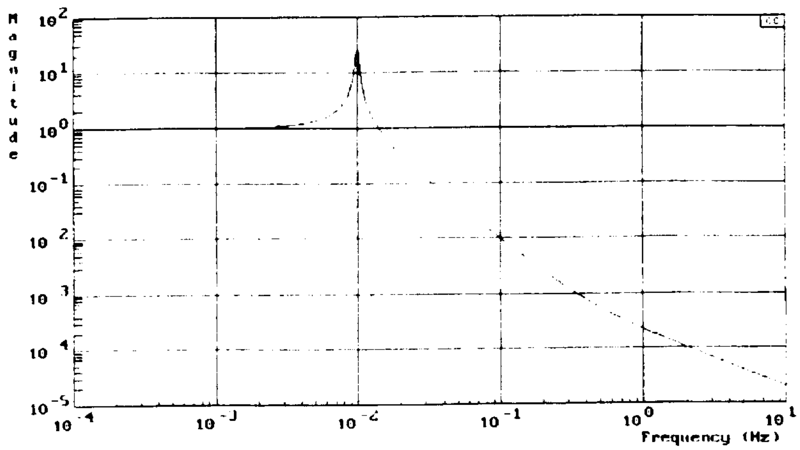
DESIGN SPECIFICATIONS AND FEEDBACK CONTROLLER



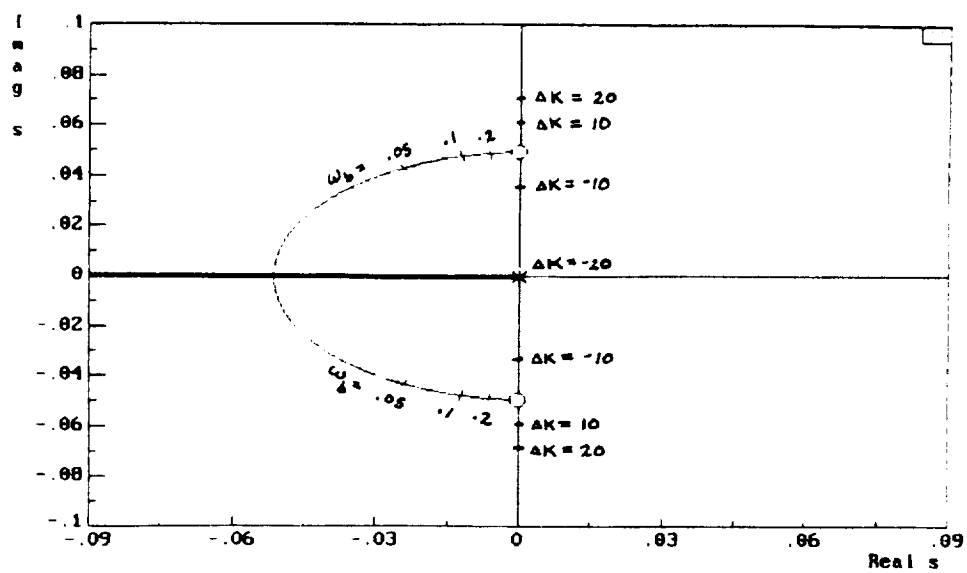
ACCELERATION FEEDBACK

LOOP SHAPING DESIGN

RESULTING TRANSMISSIBILITY:



ROBUSTNESS ANALYSIS THROUGH ROOT LOCUS:

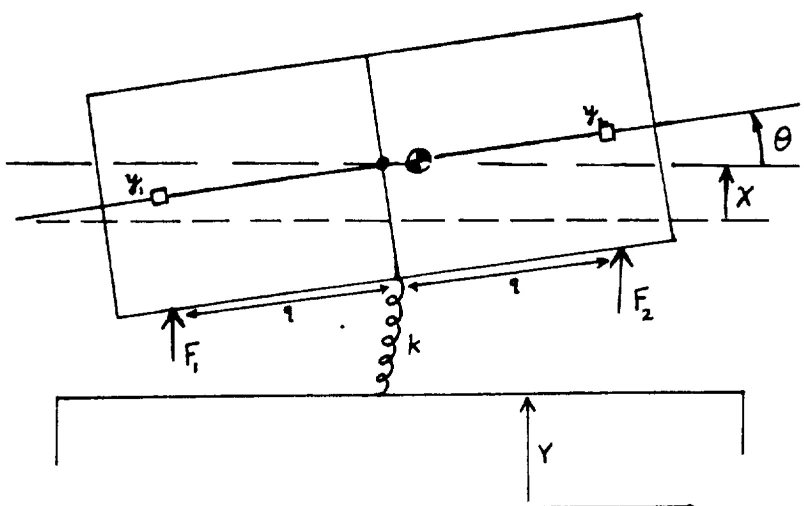


Need to modify this design to

- add damping to remove resonance
- limit control system bandwidth
- add integral term to provide centering

MDOF DESIGN EXAMPLE

EXAMPLE PROBLEM:



METHODOLOGY:

Decoupling system to rotational and translational modes, designing SISO controllers for each mode.

- Rotational mode requires angular position and velocity feedback.
- Translational mode requires translational acceleration and velocity feedback.

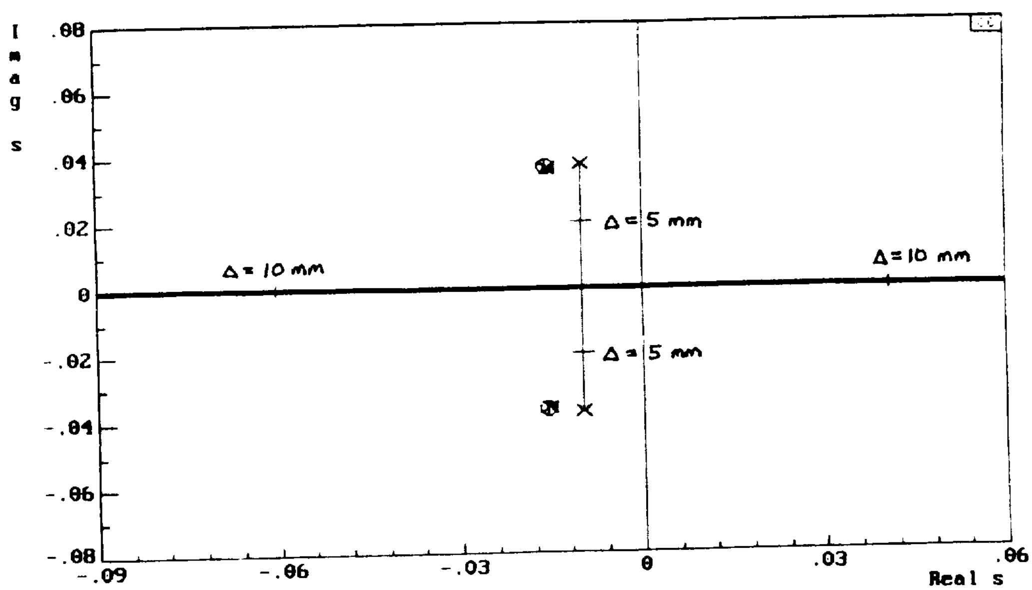
ROBUSTNESS TO CENTER OF MASS UNCERTAINTY

Decoupling is dependent on accurate knowledge of the center of mass location.

Characteristic Equation:

$$\left[(m+a)s^2 + cs + k\right]\left[Is^2 + ns + b\right] - \Delta^2\left[m(as^2 + cs + k)\right] = 0$$

Root Locus with respect to Center of Mass Uncertainty



As little as 6 mm uncertainty can produce instability.

LINEAR QUADRATIC REGULATOR

MIMO control design, since it requires a high degree of coordination, must proceed by a synthesis procedure.

LQR SYNTHESIS:

- Quadratic Norm Performance Function
- State Feedback Controller

PROBLEMS:

- State feedback is feedback of inertial positions and velocities. The resulting system does not have unity transmissibility at DC.
- Ignores the disturbances. Actually treats them as white noise.
- Weak at loop shaping

FIXES:

- Frequency weighted performance functions
- Disturbance accomodation

Both these require the addition of psuedo-states and permit loop shaping via singular value analysis.

CONCLUSIONS

- The active isolation problem should be examined from a control perspective.
- Design proceeds best from the classical control framework of loop shaping.
- Loop shaping results in an acceleration feedback design which increases the effective mass of the isolation platform.
- Decoupling/SISO design procedures for MIMO control problems may result in controllers with poor robustness.
- Linear Quadratic Regulator synthesis for the microgravity problem requires frequency weighted cost functions and disturbance accomodation.

The authors wish to acknowledge G. Brown and C. Grodsinsky for many helpful discussions of this problem. This work was supported in part by NASA Lewis Research Center and the Commonwealth of Virginia. Some of this research was performed at NASA LRC as part of the Summer Faculty Fellowship Program.