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STABILITY CONSIDERATIONS for MAGNETIC SUSPENSION SYSTEMS USING ELECTROMAGNETS MOUNTED in a PLANAR ARRAY

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Background

LGMSS model will be suspended by magnetostatic REPULSION

- rarely used before at large gaps
- wind tunnel systems employ attraction or combined attraction/repulsion

Spacing between model and electromagnets is >> model scale, same order as electromagnet scale

- electromagnet fields not affected by presence of model
- model "sees" applied fields/gradients relatively independent of details of electromagnet geometry

Problems :

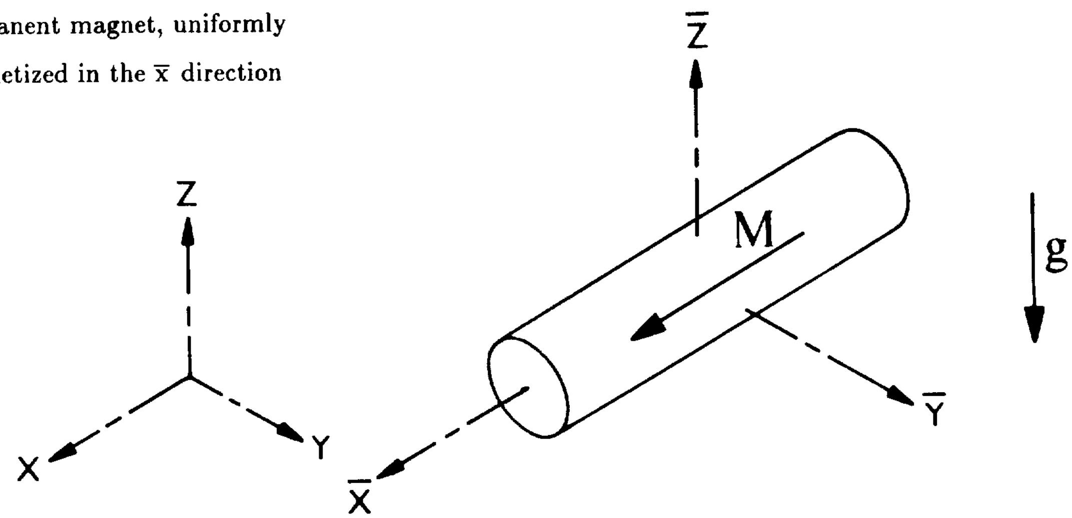
1 - How does model behave in applied fields/gradients?

2 - How to (efficiently) create desired fields/gradients?

The Large Gap Magnetic Suspension System (LGMSS) has been described in the two previous presentations. The analytic approach adopted is similar to that used for many years with wind tunnel Magnetic Suspension and Balance Systems (MSBS), which are also large-gap systems. The motivation for the present study is the concern that the use of a repulsive suspension approach may present new problems of stability and dynamics.

Model

Model is cylindrical
permanent magnet, uniformly
magnetized in the $\overline{x}$ direction



Allow small translations and rotations from datum

Use Euler angles - θ_x (roll), θ_y (pitch), θ_z (yaw) to specifiy orientation

The cylindrical model core (magnetic material contained within the final model envelope) is the originally specified configuration. Axial magnetization is the most natural choice and is the configuration chosen for wind tunnel MSBSs. The key point of interest is the natural behaviour of the model in the quasi-steady applied fields required to suspend the deadweight of the model.

Governing Equations

$$\delta\vec{F} = (\vec{M} \cdot \nabla)\vec{B}\ \delta V \qquad \delta\vec{T} = (\vec{M} \times \vec{B})\ \delta V$$

$\vec{B}$ is applied $\vec{B}$ from electromagnets,

Provided $\vec{M}$, $\vec{B}$ are relatively uniform over magnetic core then :

$$\vec{F} \simeq \mathrm{Vol}\,(\vec{M} \cdot \nabla)\ \vec{B}_{centroid}$$

$$\vec{T} \simeq \mathrm{Vol}\,(\vec{M} \times \vec{B}_{centroid})$$

For the configuration chosen, these approximate and simplified equations are adequate and again correspond to traditional practice with wind tunnel MSBSs [Refs 1,2]

Force and Torque Equations

Expanding force and torque expressions and using small rotations :

$$T_{\bar{y}} = \text{Vol } M_{\bar{x}} \; (-\theta_y B_x - B_z)$$

$$T_{\bar{z}} = \text{Vol } M_{\bar{x}} \; (-\theta_z B_x + B_y)$$

$$F_{\bar{x}} = \text{Vol } M_{\bar{x}} \; (B_{xx} + 2\,\theta_z\, B_{xy} - 2\,\theta_y\, B_{xz})$$

$$F_{\bar{y}} = \text{Vol } M_{\bar{x}} \; (-\theta_z\, B_{xx} + B_{xy} + \theta_z\, B_{yy} - \theta_y\, B_{yz})$$

$$F_{\bar{z}} = \text{Vol } M_{\bar{x}} \; (\theta_y\, B_{xx} + B_{xz} + \theta_z\, B_{yz} - \theta_y\, B_{zz})$$

With small displacements :

$$B_x \simeq \left\{B_x\right\}_o + \left\{B_{xx}\right\}_o x + \left\{B_{xy}\right\}_o y + \left\{B_{xz}\right\}_o z + \text{ etc} \cdots$$

$$B_{xx} \simeq \left\{B_{xx}\right\}_o + \left\{(B_{xx})_x\right\}_o x + \left\{(B_{xx})_y\right\}_o y + \text{ etc} \cdots$$

Following some manipulation, the force and torque equations reduce to this form. Further details can be found in References 3,4. By way of illustration, the $\{\text{Vol } M_{\bar{x}}\ B_z\}$ term in the $T_{\bar{y}}$ equation is considered the "primary" term and arises directly from the expansion of the vector cross-product. In the same equation, the $\{\text{Vol } M_{\bar{x}}\ \theta_y B_x\}$ term indicates a tendancy for the magnetization vector to align itself with an applied field - the "compass needle" effect.

The effect of core translations is incorporated in the evaluation of fields at the model centroid. The subscript "o" implies evaluation at the datum, untranslated origin.

Reduction to State-Space Form

Choose model STATE of $\left\{ \Omega_{\bar{y}} \ \ \Omega_{\bar{z}} \ \ \theta_{\bar{y}} \ \ \theta_{\bar{z}} \ \ V_{\bar{x}} \ \ V_{\bar{y}} \ \ V_{\bar{z}} \ \ \bar{x} \ \ \bar{y} \ \ \bar{z} \right\}$

- no torque about $\bar{x}$ axis - roll degree-of-freedom
- model is initially in equilibrium (determined separately)

Write perturbation equations in STATE-SPACE form -

$$\left\{\delta \dot{X}\right\} = \mathcal{A} \left\{\delta X\right\} + \mathcal{B} \left\{\delta I\right\}$$

Specify "weighting" matrix ($\mathcal{W}$), carry model mass and inertia on leading diagonal

With further manipulation, detailed more fully in References 3,4, the equations of motion can be reduced to State-Space form, where the model state is actually a perturbation from equilibrium. The equilibrium conditions, notably the electromagnet currents, are determined seperately.

"A" and "B" matrices

$$\mathcal{A} = \mathrm{Vol}\ M_{\overline{x}}\ \mathcal{W} \begin{bmatrix} 0 & 0 & -B_x & 0 & 0 & 0 & 0 & -B_{xz} & -B_{yz} & -B_{zz} \\ 0 & 0 & 0 & -B_x & 0 & 0 & 0 & B_{xy} & B_{yy} & B_{yz} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2\,B_{xz} & 2\,B_{xy} & 0 & 0 & 0 & (B_{xx})_x & (B_{xx})_y & (B_{xx})_z \\ 0 & 0 & B_{yz} & (B_{yy}-B_{xx}) & 0 & 0 & 0 & (B_{xy})_x & (B_{xy})_y & (B_{xy})_z \\ 0 & 0 & (B_{xx}-B_{zz}) & B_{yz} & 0 & 0 & 0 & (B_{xz})_x & (B_{xz})_y & (B_{xz})_z \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{B} = \mathrm{Vol}\ M_{\overline{x}}\ \mathcal{W} \begin{bmatrix} -K_{z_1} & \cdots & \\ \vdots & \ddots & \\ & & (10*5) \\ & & \end{bmatrix}$$

The $\mathcal{A}$ matrix determines the system's dynamics. The form shown above is reasonably general. The field and field gradient terms are the equilibrium suspension values. Thus for any desired system configuration and detail design, values can be quickly evaluated and the model dynamics found from the eigenvalues and eigenvectors.

The $\mathcal{B}$ matrix is filled with coefficients specifying the field and field gradient components generated by each electromagnet in turn.

5 - Coil System

$I_o = I_{max} \left\{ -0.7753,\ -0.2417,\ 0.6293,\ 0.6293,\ -0.2417 \right\}$

Coil #	B_x (Tesla)	B_y	B_z	B_{xx} (T/m)	B_{xy} ……
1	0.0216	0	-0.0198	0.0092	0
2	0.0067	0.0206	-0.0198	-0.0269	0.0118
3	-0.0175	0.0127	-0.0198	-0.0046	-0.0191
4	-0.0175	-0.0127	-0.0198	-0.0046	0.0191
5	0.0067	-0.0206	-0.0198	-0.0269	-0.0118

- calculated using OPERA (from VF/GFUN, TOSCA), using cartesian polynomial fitting of field at grid points

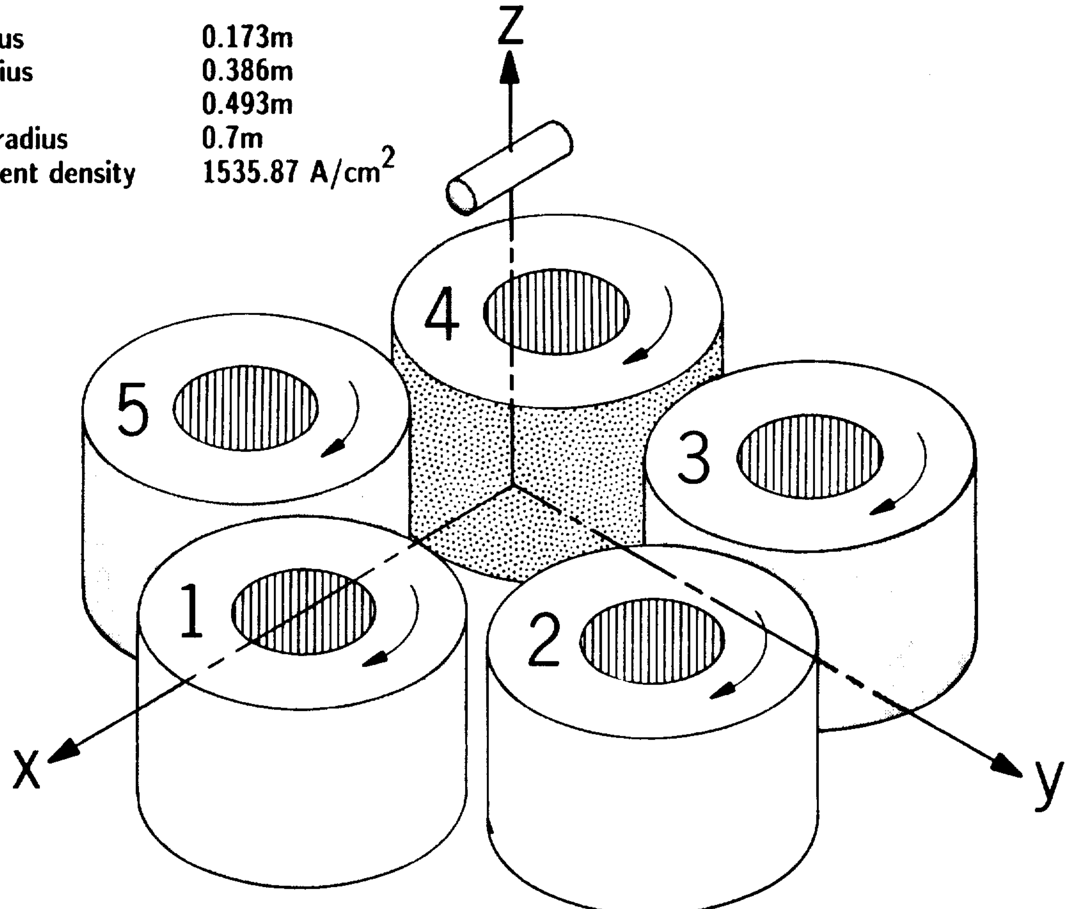
Natural Modes

MODE 1	14.3 rad/s	Unstable divergence	x, θ_y	Axial + pitch
MODE 2	4.6 rad/s	Stable oscilliatory	x, θ_y	Axial + pitch
MODE 3	12.9 rad/s	Unstable divergence	θ_z	Yaw rotation
MODE 4	5.0 rad/s	Stable oscilliatory	z	Vertical motion
MODE 5	2.5 rad/s	Unstable divergence	y	Lateral translation

The electromagnet configuration corresponds to one of the final designs emerging from the Madison Magnetics Incorporated design study (Reference 5). Equilibrium suspension is achieved with the current distribution shown, e.g. 77.53% of maximum design current in Coil number 1, the sign determined by the arbitrary sign convention chosen (see later Figure). The $\mathfrak{B}$ matrix can be constructed directly from the field and field gradient terms indicated in the Table. The coefficients of the $\mathcal{A}$ matrix are found by summation of the products of each coefficient with the relevant current fraction. Field calculations are carried out using an analysis and post-processing package, "OPERA", which uses numerically evaluated integral expressions for field around simple conductor geometries. Once the $\mathcal{A}$ matrix is found, eigenvalues and eigenvectors are found using "PC-MATLAB". The frequency, stability and shape of each mode is of interest. Mode 5 is rather benign (low frequency) and represents the model, in a sense, "falling off" the suspension electromagnets. Stability in suspension height is expected in a repulsive mode suspension and is shown in Mode 4. Mode 3 is the "compass needle" effect, with the model attempting to reverse direction so as to align the magnetization vector with the axial ($B_{\bar{x}}$) field. Mode 1 appears to represent similar behaviour in the orthogonal plane, though coupling into translation is exhibited. Mode 2 is an unexpected result. Unstable translation would have been expected by analogy with Mode 5.

5-Coil System

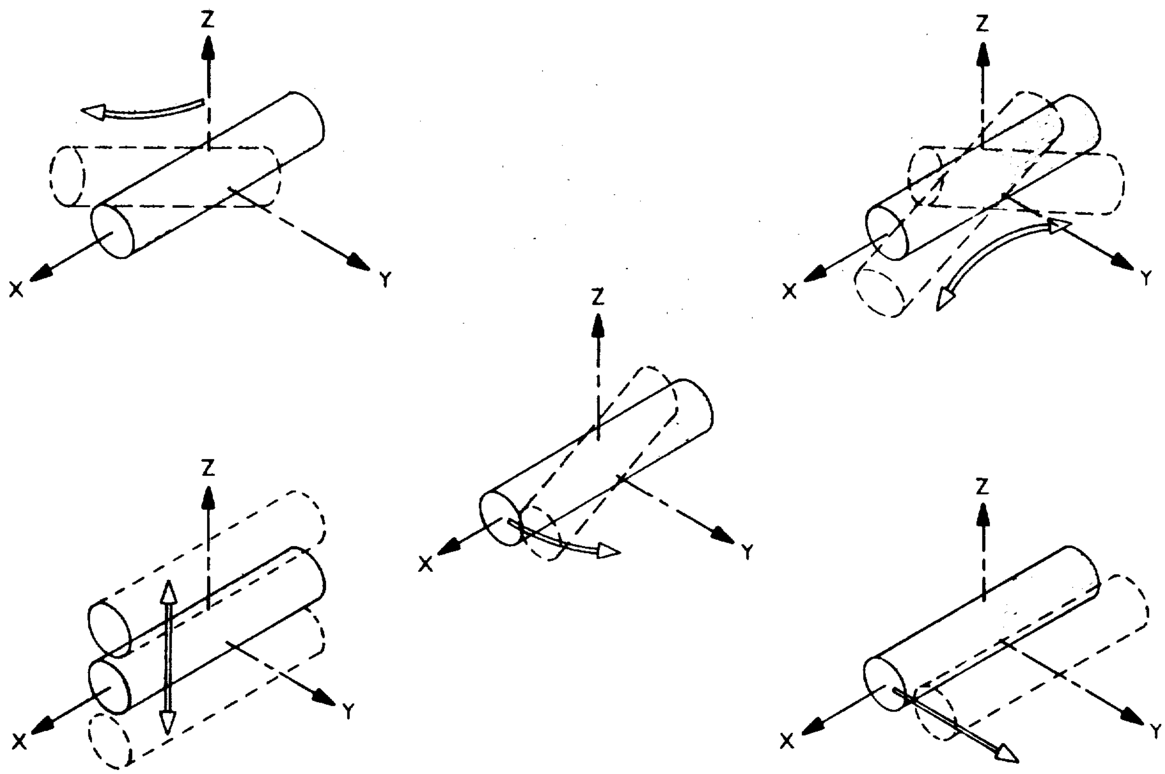
Inner radius	0.173m
Outer radius	0.386m
Depth	0.493m
Location radius	0.7m
Max. current density	1535.87 A/cm^2



Model	23.11 kg, 0.6kg m^2
Permanent magnet	0.1016m dia. * 0.3048m, 1.2 Tesla

The important dimensions of the Madison Magnetics design and the sign convention for positive current direction are shown here. The levitation height, measured from the model axis to the top face of the electromagnets, is 0.9144m (36 inches).

Natural Modes



The three unstable modes are :-

Mode 1 - shown top left; Mode 3 - shown center and Mode 5 - lower right

The two stable, oscilliatory modes are :-

Mode 2 - shown top right and Mode 4 - shown lower left

Origins of Modes ?

Suppose only B_{xz} applied (to support weight of model)

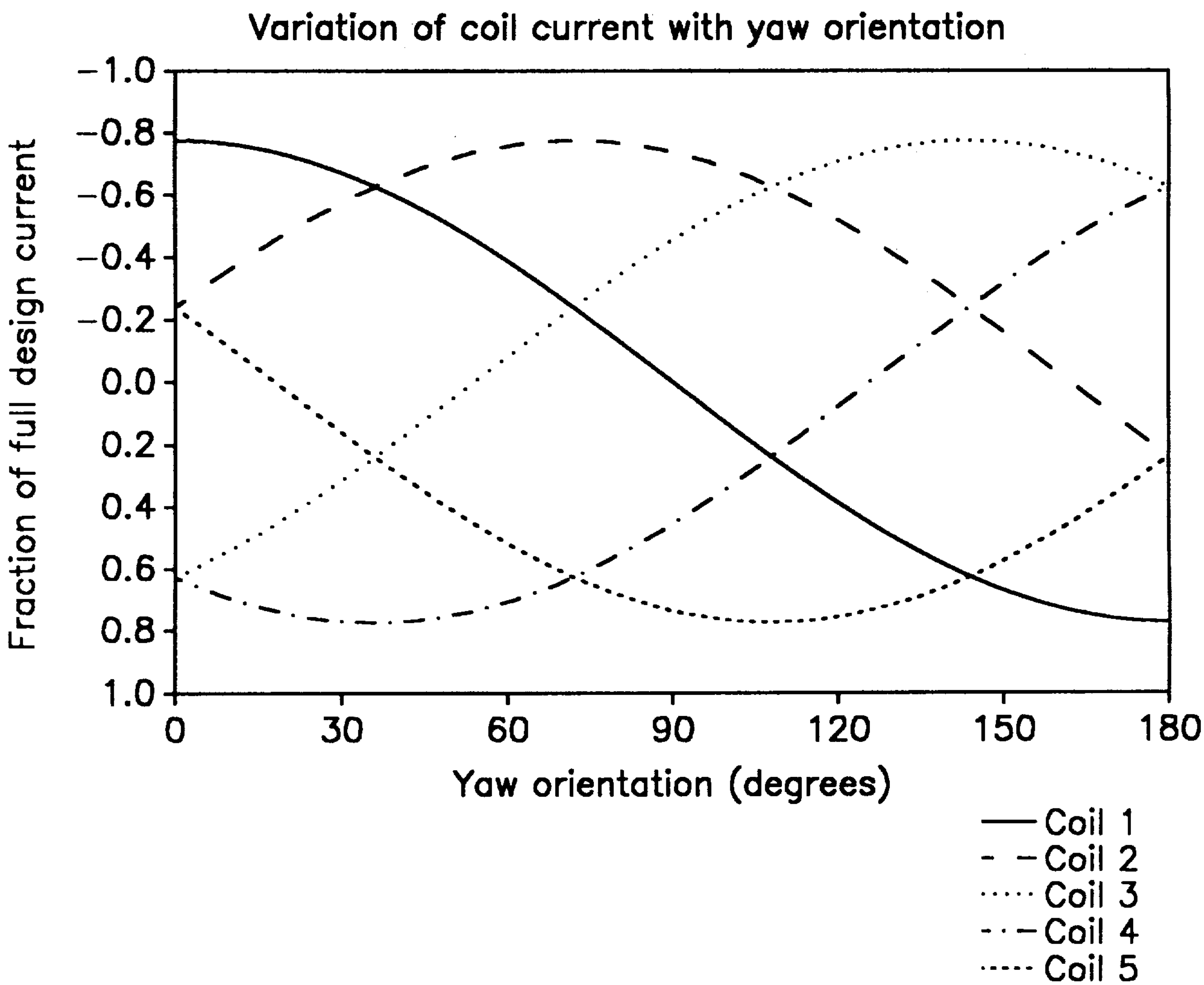
MODE A	9.3 rad/s	Unstable divergence	x, θ_y	Axial + pitch
MODE B	9.3 rad/s	Stable oscilliatory	x, θ_y	Axial + pitch

Adding B_x (typical value)

MODE A	14.3 rad/s	Unstable divergence	x, θ_y	Axial + pitch
MODE B	6.9 rad/s	Stable oscilliatory	x, θ_y	Axial + pitch
MODE C	12.9 rad/s	Unstable divergence	θ_z	Yaw rotation

In order to better understand the origin of Modes 1 and 2, the $\mathcal{A}$ matrix was re-solved with unnecessary field and field gradient terms arbitrarily zeroed. With only the gradient required to generate the lifting force on the model (opposing weight), two modes are found, vaguely approximating Modes 1 and 2 as previously shown. If the axial field is re-applied, still holding all "second-order gradients" (terms of the form $(B_{ij})_k$ zero, the stable oscilliatory mode is moved to a lower frequency, the unstable mode to a higher frequency. The "compass-needle" mode (Mode 3) now appears.

Effect of Rotation in Azimuth (Yaw)



As the model rotates about the vertical axis, electromagnet currents are smoothly redistributed between electromagnets. The current variation in each electromagnet is virtually sinusoidal. It is found that there are no significant changes in the model's modes of motion as the rotation proceeds.

Six Coil System

Try to control B_x ($\Rightarrow 0$).

$$\text{Find } \frac{I_{B_x}}{I_{B_{xz}}} = \begin{bmatrix} 0.933 \\ 1.0 \\ 1.0 \\ 0.933 \\ 1.0 \\ 1.0 \end{bmatrix}$$

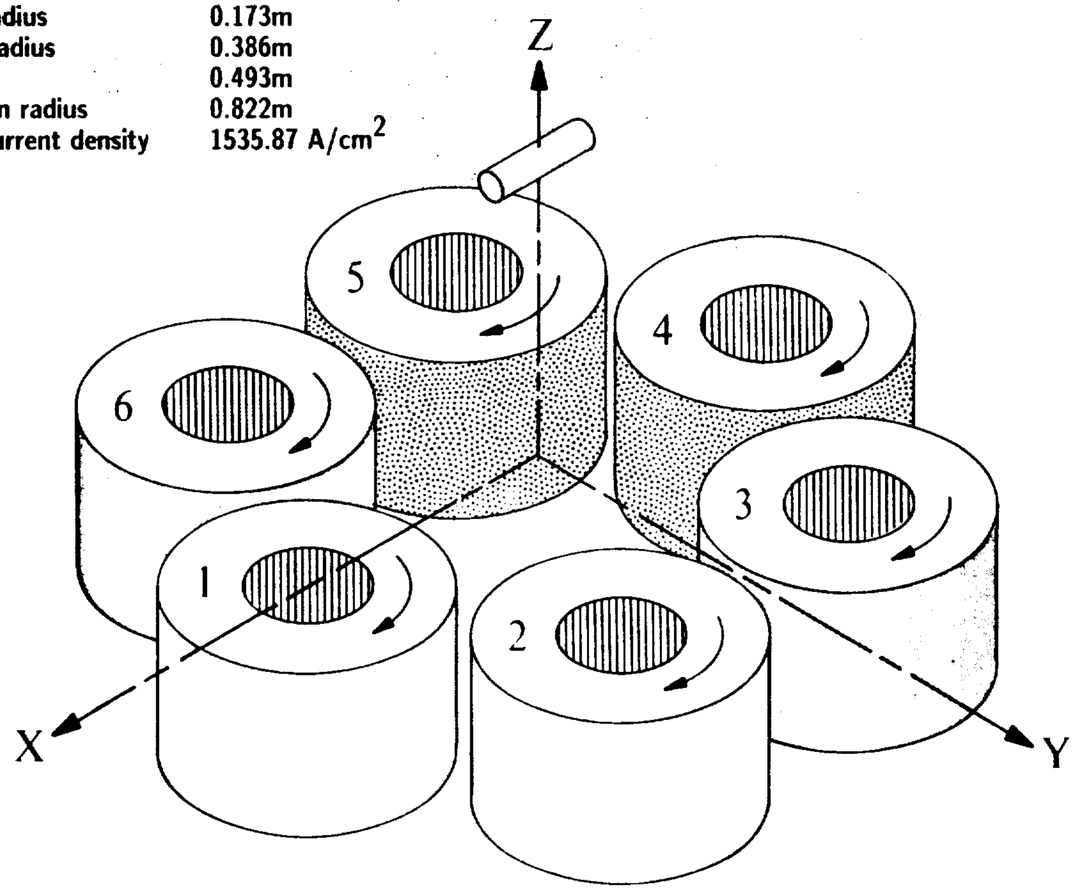
Currents are all large and roughly +/-/+/-/+/- for normal suspension with $B_x = 0$

In an attempt to lower the frequency of the highest frequency unstable mode, an attempt is made to control the value of B_x, preferably forcing it to zero. This is only feasible if an additional electromagnet is added to the configuration. This is done by preserving the same individual electromagnet geometry and spacing between electromagnets, but locating all at a larger radius.

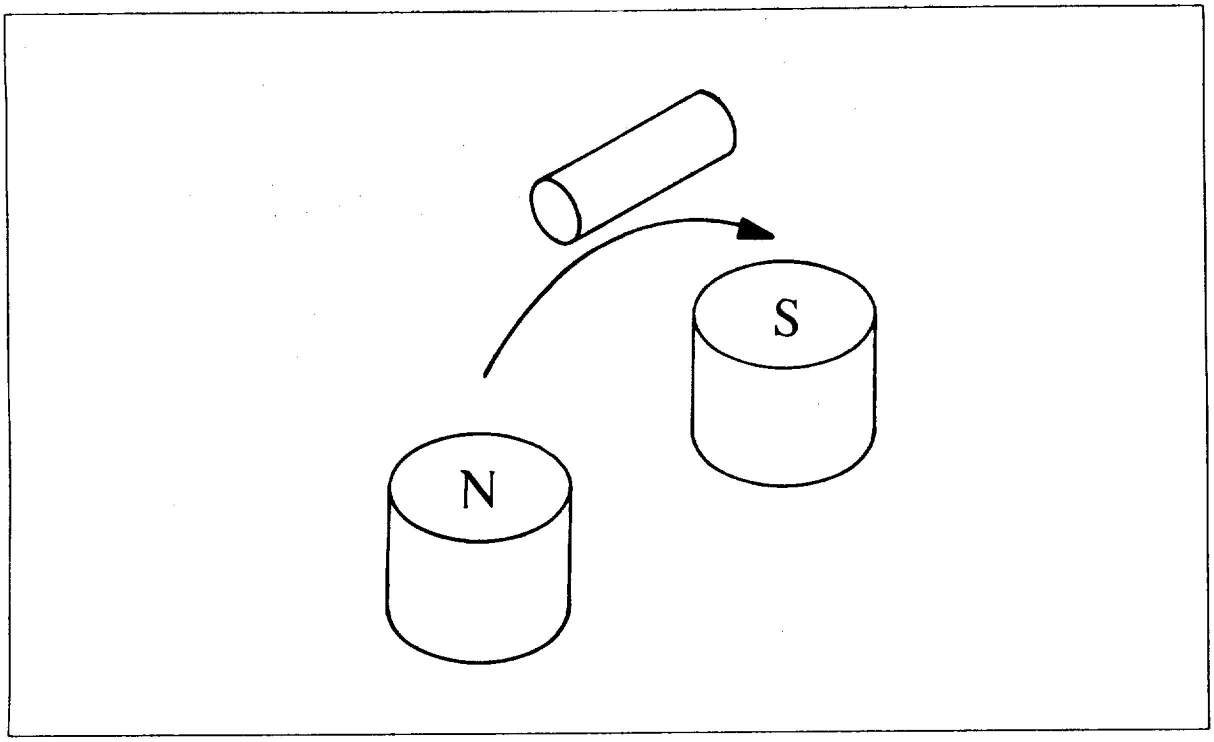
It is quickly found that the resulting configuration is ineffective in controlling B_x. This is due to the fact that the current distribution required to generate the gradient B_{xz} is virtually identical to that required to generate B_x, each field or field gradient component being generated independently of all others in both cases.

6-Coil System

Inner radius	0.173m
Outer radius	0.386m
Depth	0.493m
Location radius	0.822m
Max. current density	1535.87 A/cm^2



Model	23.11 kg, 0.6kg m^2
Permanent magnet	0.1016m dia. * 0.3048m, 1.2 Tesla

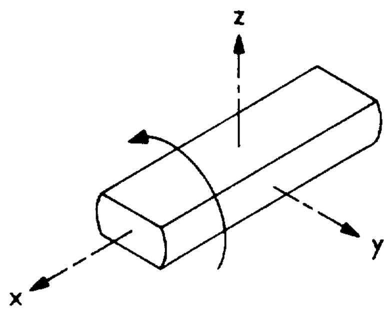


The cause of the problem is easily understood with a North-South pole representation. The model requires a N-S pole pair distributed along the x axis as shown. This inevitably generates an axial field in the direction opposite to the model magnetization.

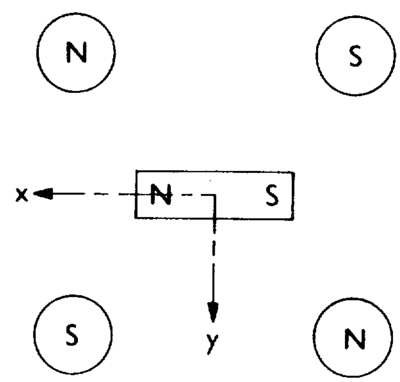
It can be noted that in wind tunnel MSBSs, this problem either does not arise, if electromagnets are located symmetrically above and below the model, or is exploited, where electromagnets are only positioned above the model, creating a useful natural magnetizing field.

Seven / Eight Coil Systems

Attempt to control B_{xz_y} to produce rolling moment (T_x) with non-axisymmetric core.



Problem:



Nevertheless, apparently just possible with 7-coil system

8-coil system has excessive symmetry and will not produce B_{xz_y}

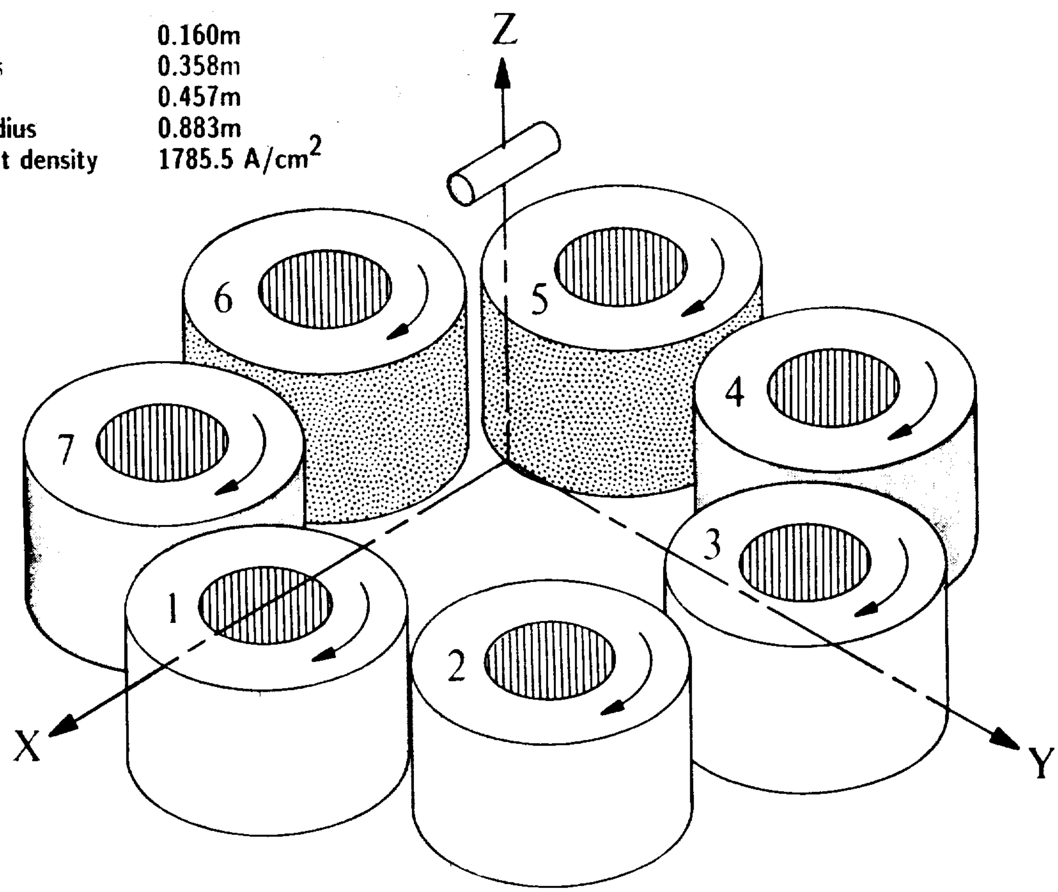
Wind tunnel systems have high level symmetry with coils clustered on all sides of suspended model

Assuming that the axial field cannot be sensibly controlled, attention is turned to the possibility of generating a lateral gradient of the "lift" field gradient, B_{xz_y}, which has been exploited in wind tunnel MSBS work in the past, for generation of rolling moment. If possible, this would provide a means for controlling the 6th degree of freedom, presently presumed to be passively stabilized.

It is found that the system does not work for the 6-electromagnet arrangement, due to the "roll" field being generated by a current distribution identical to that required for the generation of lateral force, F_y. Additional electromagnets are therefore added, in this case requiring a reduction in size of each, along with an increase in the radius of their centers. The total ampere-turns in each electromagnet is held constant. It is found that the problem of inseparability of roll and sideforce again arises with the 8-electromagnet arrangement. With only 7 electromagnets, the symmetry of the arrangement is of a sufficiently low order to permit separation of these two fields, though not very effectively.

7-Coil System

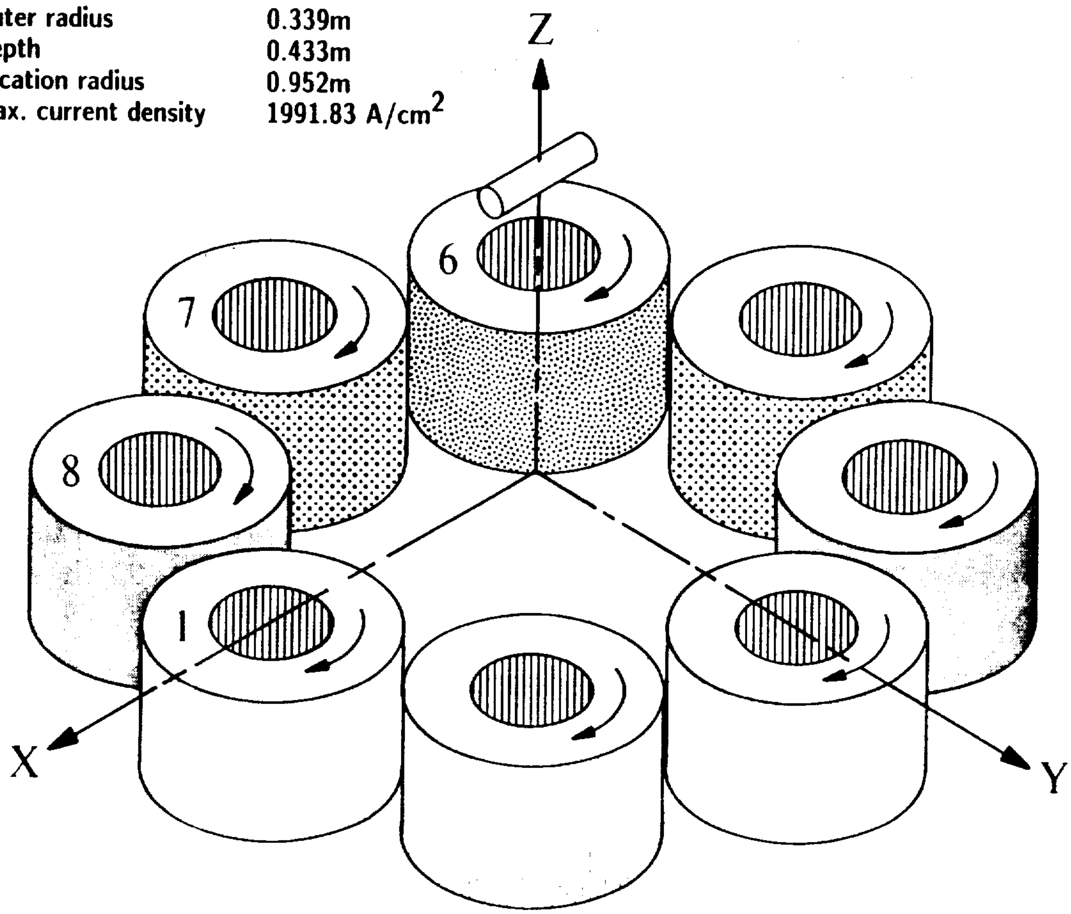
Inner radius	0.160m
Outer radius	0.358m
Depth	0.457m
Location radius	0.883m
Max. current density	1785.5 A/cm^2



Model	23.11 kg, 0.6kg m^2
Permanent magnet	0.1016m dia. * 0.3048m, 1.2 Tesla

8-Coil System

Inner radius	0.152m
Outer radius	0.339m
Depth	0.433m
Location radius	0.952m
Max. current density	1991.83 A/cm^2



Model	23.11 kg, 0.6kg m^2
Permanent magnet	0.1016m dia. * 0.3048m, 1.2 Tesla

Future Work

Incorporate modal analysis into control system simulation and design

- may require coupled axial and pitch degrees-of-freedom
- high frequency unstable modes place burden on power supplies and controller in LGMSS application

Study second-order effects

Study influence of eddy currents

Simulation efforts are underway, extending work reported in References 3,4. To achieve optimum performance, the coupling between the axial translation and pitch degrees-of-freedom needs to be addressed. Existing work with wind tunnel MSBSs has dealt with similar effects by insertion of a "decoupling" matrix into the control loops, such that the controller can be configured as 5 (or 6) parallel and quasi-independent loops, each stabilizing one of the natural degrees-of-freedom. Other approaches are possible.

Conclusions - N-coil ring

Will work for 5 degree-of-freedom control

360 degree azimuth (θ_z) range easily achieved

Large motion capability around datum expected

Have to control all currents in (superconducting) electromagnets

6th degree-of-freedom can be passively stabilized if $\vec{g}$ present

Alternative roll control schemes are available

If $\vec{g}$ present, highest frequency modes are a problem

With $\vec{g}$ absent, modes may not be a problem

These conclusions are based on this study and the results of previous design studies for 5- and 6-electromagnet configurations. At present, design work for the LGMSS is focussing on an alternative configuration, where the axis of magnetization is vertical, parallel with the gravity vector.

Speculation

Change the direction of magnetization ??

With vertical magnetization (parallel to $\vec{g}$), B_{zz} would support weight. Can (must) arrange B_z to provide roll/pitch stability.

Modes would be :

Stable oscilliatory	θ_x	Provided B_z is correct sign
Stable oscilliatory	θ_y	Provided B_z is correct sign
Neutral	θ_z	Depends on system axisymmetry
Unstable divergent	x	Highly dependant
Unstable divergent	y	on electromagnet
???????	z	configuration

It appears possible to achieve a lower value of the frequency of the highest frequency unstable modes, by aligning the magnetization vector with the gravity vector. Further analysis is required of this and other possible configurations.

References

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