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Structural Representation for Analysis of a Controlled Structure

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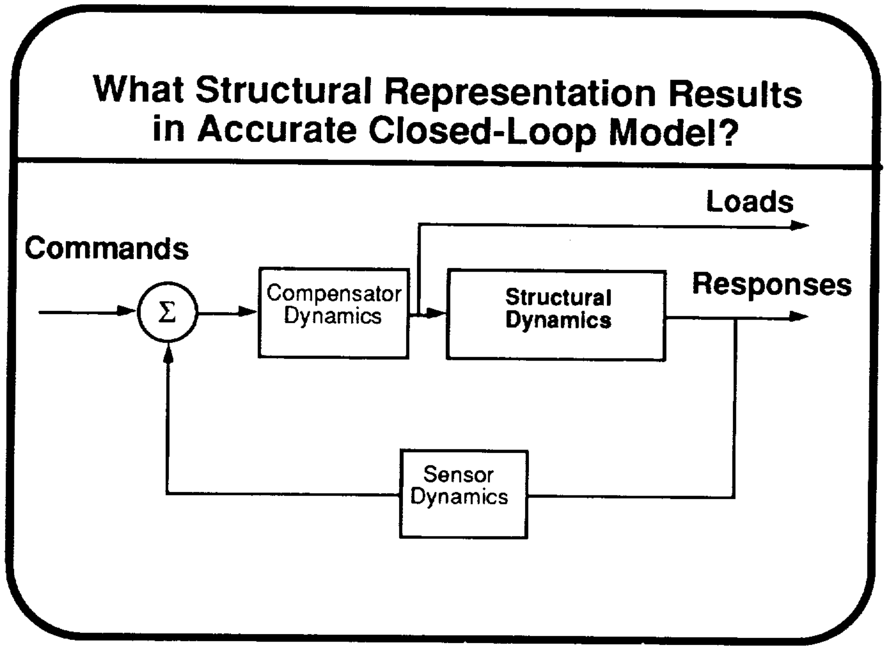


The purpose of this study is to determine what reduced order structural representation is most appropriate for coupling with a control system. The goal is to choose a reduced order structural model which retains as closely as possible the characteristics of the closed-loop model with a full order structural representation. By characteristics of the closed-loop model we mean the closed-loop eigenvalues and the closed-loop transfer functions from commands to loads and from commands to responses. This process does not address the accuracy of the full-order model (usually a finite element model) but only the loss of accuracy associated with reducing the model. For the purposes of this study we will limit ourselves to collocated sensors and actuators. The choice of a structural representation for non-collocated sensors and actuators is not so clear.

What Do We Mean by Accurate Closed-Loop Model?

(1) Accurate Closed-Loop Frequencies:

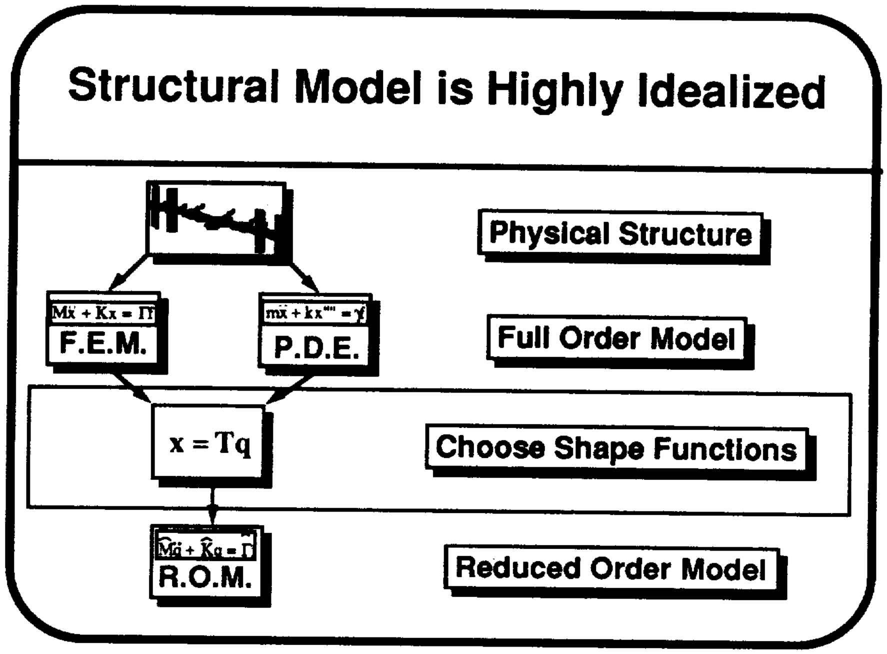
$$\varepsilon = \frac{|\lambda_{approx.} - \lambda_{exact}|}{|\lambda_{exact}|}$$

(2) Accurate Closed-Loop Frequency Response:

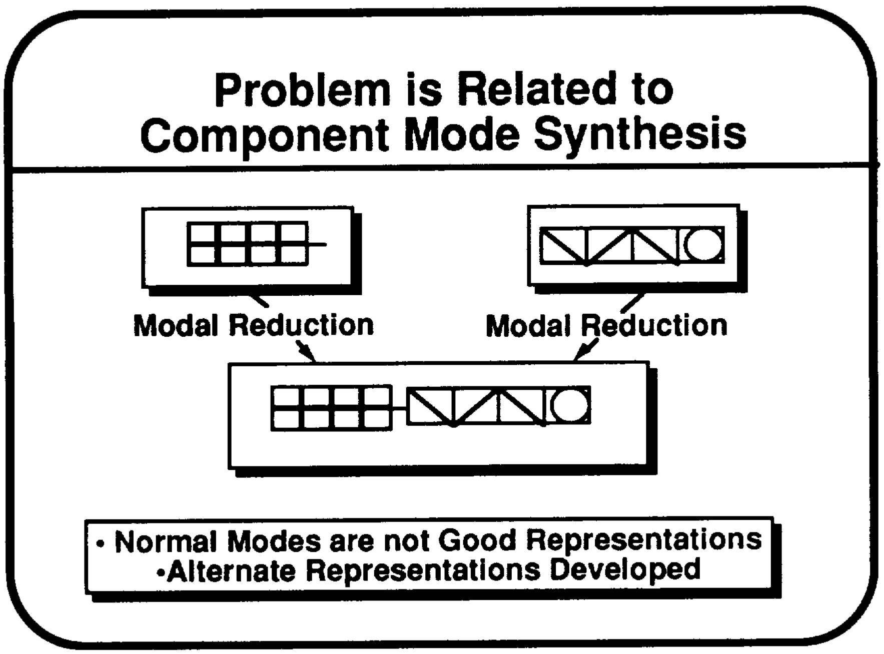
$$\varepsilon(j\omega) = \frac{|G(j\omega)_{approx.} - G(j\omega)_{exact}|}{|G(j\omega)_{exact}|}$$

- **From Commands to Responses**
- **From Commands to Input Loads**

More specifically we will define errors as follows: For each closed-loop frequency we will define a relative error as the distance from the closed-loop pole based on the reduced model to the closed-loop pole based on the full order model, divided by the magnitude of the closed-loop pole based on the full-order model. For the transfer functions from commands to input loads and from commands to responses we will use the same measure to define a relative error as a function of frequency. The reduced order model is said to closely represent the closed-loop model when these error measures are small.



As mentioned before we are only addressing one aspect of the accuracy issue. The overall modeling process starts with a physical structure or drawings of a physical structure and proceeds to develop what we will call a full-order model. Whether this model is based on finite elements or partial differential equations, a number of assumptions were made in its derivation. For the purposes of this study, we will assume that these assumptions are valid to the extent that the full-order model accurately represents modal frequencies in the control bandwidth and also the static deflection due static loads applied at the controller interface locations. The full order model, whether it is represented by a finite element model or partial differential equations, is almost certainly too large for practical control system analysis. The model is reduced by choosing a small number of shape functions (often normal modes). We are addressing the choice of these shape functions, such that accuracy of the closed-loop model is retained with as few functions as possible.



A very similar problem is that of component mode synthesis (CMS). In CMS the goal is to represent a number of substructures by reduced order models based on shape functions such that when these substructures are coupled, the modal frequencies of the coupled model will be as accurate as possible. In the case of control-structure interaction (CSI) we are simply replacing one the substructures by a control system.

Researchers in CMS have demonstrated for more than 20 years that the use of normal modes to represent the substructures can result in large inaccuracies of the coupled model. A number of alternate substructure representations have been developed that result in much more accurate coupled models.

Alternate Representations are Statically Exact

- **Residual Flexibility adds Static Contribution of Neglected Modes**
- **Craig-Bampton Representation adds Static Solution to Cantilevered Modes***
- **Lanczos Vectors are Based on Static Solutions**

***Implemented in NASTRAN as standard method**

Two methods for representing substructures in CMS have emerged as standards. These are normal modes with addition of residual flexibility and a Craig-Bampton representation. The use Lanczos vectors rather than modes has also been suggested.

The residual flexibility method adds static flexibility that is not represented by the retained normal modes. This flexibility can either be represented as a purely static flexibility at the interface or by a high frequency subsystem which contributes quasi-statically at the interface. The residual flexibility subsystem is uncoupled (orthogonal to) the retained normal modes.

The Craig-Bampton method combines a static reduction (Guyan reduction) to the interface degrees of freedom with a set of normal modes calculated with the interface degrees of freedom held fixed.

Lanczos vectors are generated by a series of static solutions and do not require the solution of an eigenvalue problem. The resulting mass and stiffness matrices are tri-diagonal rather than diagonal.

Craig-Bampton Representation Standard in Structural Dynamics

$$\begin{bmatrix} M_{BB} & M_{IB} \\ M_{IB}^T & I \end{bmatrix} \begin{Bmatrix} \ddot{x}_B \\ \ddot{x}_I \end{Bmatrix} + \begin{bmatrix} K_{BB} & 0 \\ 0 & \Omega^2 \end{bmatrix} \begin{Bmatrix} x_B \\ x_I \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

- **Static shapes based on unit deflections of exterior DOF**
- **Guyan reduction to exterior DOF**
- **Static shapes = rigid body modes for rigid body control**
- **Relative not absolute DOF fixed at joints**
- **Results in accurate system models**

The Craig-Bampton method is the most popular method used in CMS. It is conceptually very simple, it is accurate and it is implemented as the standard representation used by MSC/NASTRAN's superelement capability. Two sets of shape vectors are used. The first are static shapes based on unit deflections at the interface. These are the same shape functions used in the Guyan reduction process. The second are normal modes calculated with the interface fixed. The Craig-Bampton representation has the form illustrated above. While it is not diagonal, it can be diagonalized. In this case it is very similar to the residual flexibility representation, with a number of normal modes combined with a set of high frequency modes representing static flexibility at the interface.

When the Craig-Bampton representation is used in CSI for systems with joints, the relative rotation rather than the absolute rotation at the joint can be held fixed during the calculation of "component" modes.

Why Do We Persist in Using Normal Modes?

- Tradition
- Obtainable from any Structural Dynamics Routine
- Physical Interpretation
- Approximately Balanced (in sense of Moore)
- We Don't Understand Damping (modal damping)
- Small Amount of Data to Transfer
- Uncoupled Equations of Motion

Given that normal modes are known to generate poor solutions in CMS problems, and given that alternate representations are better, why are normal modes used so pervasively in CSI? There are a number of reasons why normal modes are convenient. They are standard output from any structural dynamics routine, and are certainly more "standard" than the alternate representations used in CMS. They have a physical interpretation. For lightly damped systems with sufficiently separated frequencies they are approximately balanced and cost decoupled, suggesting that they are natural coordinates to use for model reduction. Modal damping is simple to define. The amount of data transferred is limited to modal frequencies and mode shape coefficients. And finally, the resulting equations of motion are uncoupled, resulting in very fast simulations.

Alternate Representations can be Diagonalized

$$\boxed{\tilde{M}\ddot{q} + \tilde{K}q = \tilde{\Gamma}f} \longrightarrow \boxed{q = E\eta} \longrightarrow \boxed{\ddot{\eta} + \Omega^2\eta = E^T\tilde{\Gamma}f}$$

Alternate Reduced Order Model — **Eigenvalue Solution** — **Diagonalized Representation**

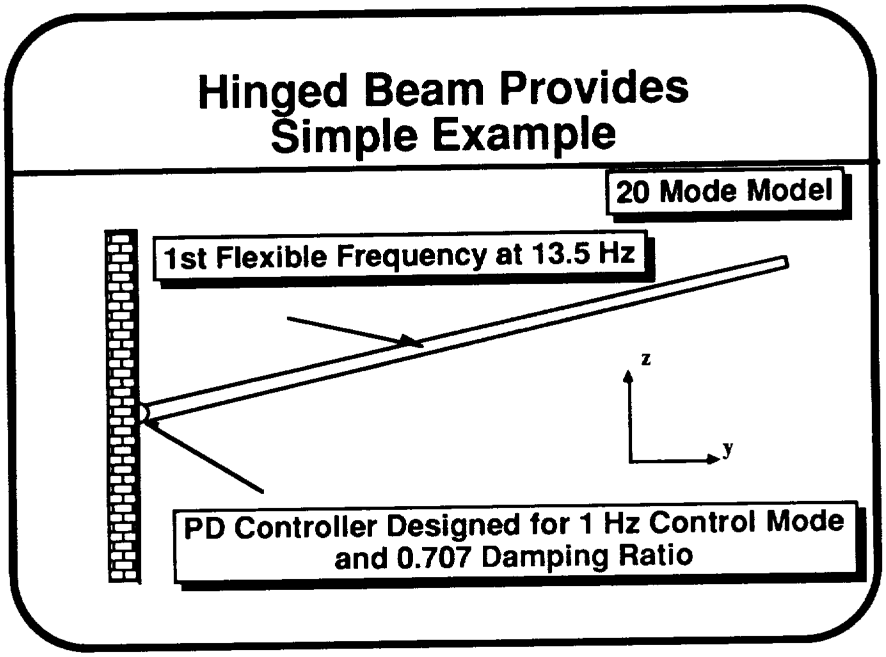
- **Eigenvalue Problem is Small (size of reduced order model)**
- **Diagonalized Representation has Many Advantages of Normal Mode Representation**

Many of the advantages of the normal modes representation are shared by the Craig-Bampton representation, if the reduced order model is diagonalized. The diagonalization involves an eigenvalue solution on the reduced order model, which is typically very fast. The resulting "modes" will include some low frequency normal modes along with some high frequency residual modes which contribute quasi-statically in the low frequency range. The coordinates are now balanced and cost-decoupled with respect to the reduced order model, though not necessarily with respect to the full order model. The amount of information to be transferred is again limited to frequencies and mode shape coefficients and the equations of motion are uncoupled. The high frequency modes may need to be treated carefully during a transient simulation, though extra damping can be added without affecting their contribution in the frequency range of interest.

Summary of Structural Representations

- Goal is Accurate Closed-Loop Model
  - Accurate Closed-Loop Frequencies
  - Accurate Closed-Loop Transfer Functions
- Choice of Shape Functions can be Motivated by CMS
- Alternate Representations are Statically Exact
- Alternate Representations can be Diagonalized

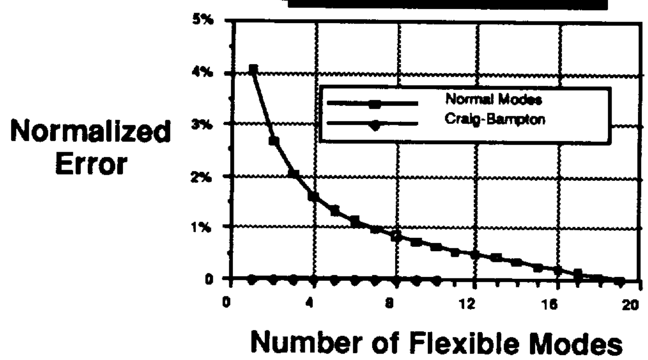
In summary, the goal of this study is to select a minimal number of shape functions that accurately represent the closed-loop model. While normal modes are often used, alternate representations developed in the field of CMS can also be applied. These alternate representations are statically exact at the interface points and can be diagonalized to recover some of the advantages of normal modes. Following, we will show two examples which demonstrate that the Craig-Bampton representation does in fact result in significantly more accurate closed-loop models than the normal modes representation.



One might expect that as a control system became stiffer with respect to the structure, a set of fixed interface modes would be more appropriate, while for a soft control system the free interface modes would be more appropriate. It is certainly true, that as the control system gets stiffer, the inaccuracies associated with the use of free interface modes become larger, however, this simple example shows that the errors can be large even when the control system is significantly softer than the structure. We have chosen PD control gains to give a rigid body frequency of 1 Hz and a damping ratio of 70.7%. The actual frequency and damping ratio will differ due to the flexibility of the beam. The "full-order" model is the finite element model with 20 degrees of freedom.

Craig-Bampton Modes Result in More Accurate Control Frequency

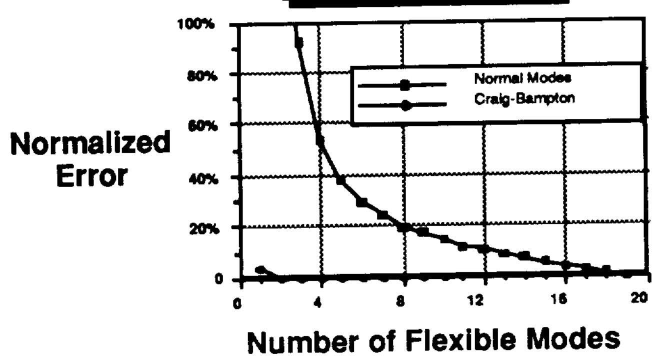
$\omega = 1.05$ Hz, $\zeta = 0.662$



The actual control frequency based on the full 20 degree of freedom model is 1.05 Hz with a damping ratio of 66.2%. Using even one Craig-Bampton mode results in an exact representation of the frequency, while seven normal modes are required to reduce the error to less than 1%.

Craig-Bampton Modes Result in More Accurate 1st Flexible Frequency

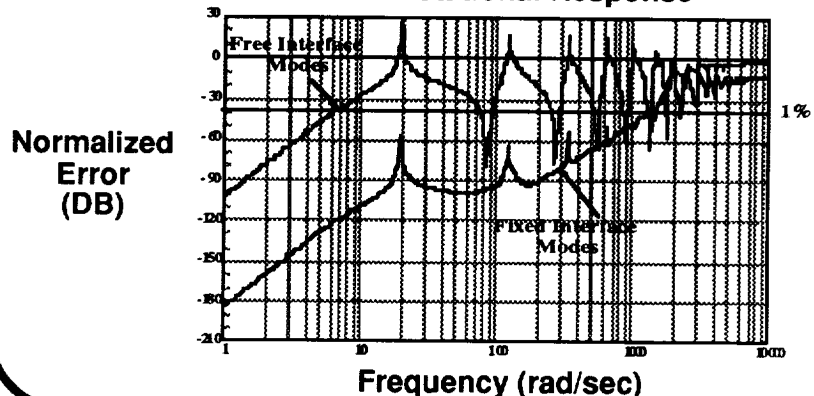
ω= 7.54 Hz, ζ = 1.0



The first closed-loop flexible mode is at 7.54 Hz and is critically damped. In this case two Craig-Bampton modes represent the closed-loop mode exactly, while thirteen normal modes are required to reduce the error to less than 10%. In this case the error in the closed-loop frequency using normal modes is drastic.

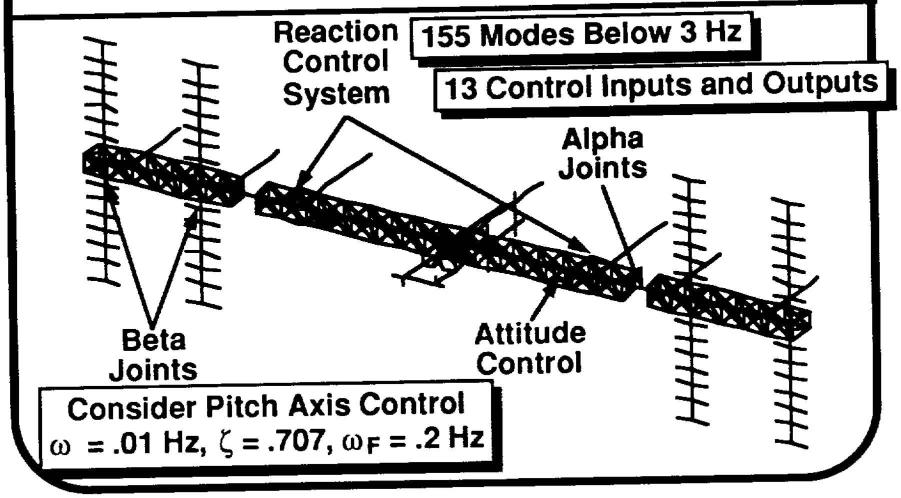
Craig-Bampton Modes Result in More Accurate Transfer Function

Closed-Loop Transfer Function from a Rotational Command to a Rotational Response



Examining the frequency response from a rotational command to a rotational response tells the same story. Using Craig-Bampton modes, the error is less than 1% up to a frequency of 1000 rad/sec, while using normal modes, the error exceeds 1% at just over 6 rad/sec.

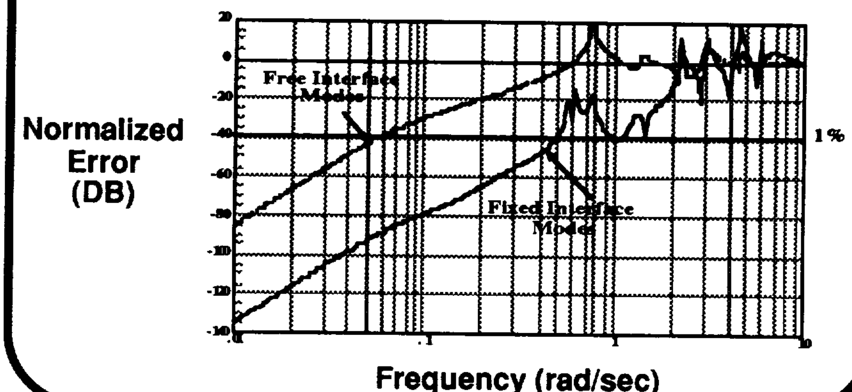
SPACE STATION MODEL IS MORE REALISTIC



The methods presented here were developed for Space Station Freedom, which is a complex structure with very high modal density. Examining pitch control of the Space Station provides a more realistic example. In this case the first significant flexible mode interacting with the control system is near 0.2 Hz. The control gains are chosen to provide a rigid body control frequency of 0.01 Hz and a damping ratio of 70.7%. Once more, this is a system where the control frequency is more than an order of magnitude below the first flexible frequency. In this case it is not possible to calculate the true "full-order" closed-loop model because the finite element model has over 1000 degrees of freedom, so the exact model is one based on 155 Craig-Bampton modes. The two reduced order models are each based on 41 flexible modes. The first based on 41 Craig-Bampton modes, and the second based on 41 normal modes.

Craig-Bampton Modes Result in More Accurate T.F. to Response

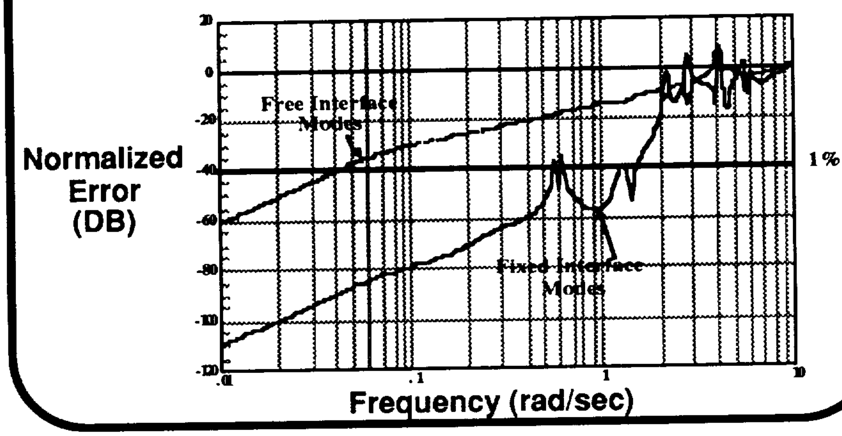
Transfer Function from Pitch Command to Pitch Response



The error in the transfer function from a rotational command to a rotation about the pitch axis again illustrates the improved accuracy associated with the Craig-Bampton representation. In this case the reduced order model based on 41 Craig-Bampton modes is accurate (less than 1% error) up to a frequency of 0.5 rad/sec, while the reduced order model based on 41 normal modes is only accurate up to a frequency of 0.05 rad/sec.

Craig-Bampton Modes Result in More Accurate T.F. to Torque

Transfer Function from Pitch Command to Pitch Torque



Results for the transfer function from rotational commands to torques applied by the controller to the structure suggest similar conclusions. Again the model based on Craig-Bampton modes is significantly more accurate than the model based on normal modes.

CONCLUSIONS

- Alternate Modal Representations are Available and Easy to Implement
- Key Difference is Exact Static Representation
- Alternate Modal Representations can be Diagonalized
- Alternate Modal Representations Result in More Accurate Closed-Loop Models with Collocated Sensors and Actuators
- Non-Collated Sensors and Actuators Less Clear

In conclusion, the use of an alternate structural representation, such as the Craig-Bampton representation, can result in much more accurate results than are obtained when using a truncated set of normal modes. The key difference between the alternate representations and the normal mode representation is the incorporation of a static solution. The alternate representations do not necessarily generate diagonal mass and stiffness matrices, but they can be diagonalized at a minimal effort in order to capture some of the advantages of normal modes.

All the results presented here are based on collocated sensors and actuators. The issue with non-collocated sensors and actuators is somewhat different since it is not clear which points should be held fixed during the modal solution. This is an issue that still needs to be resolved.