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Component Mode Damping Assignment Techniques

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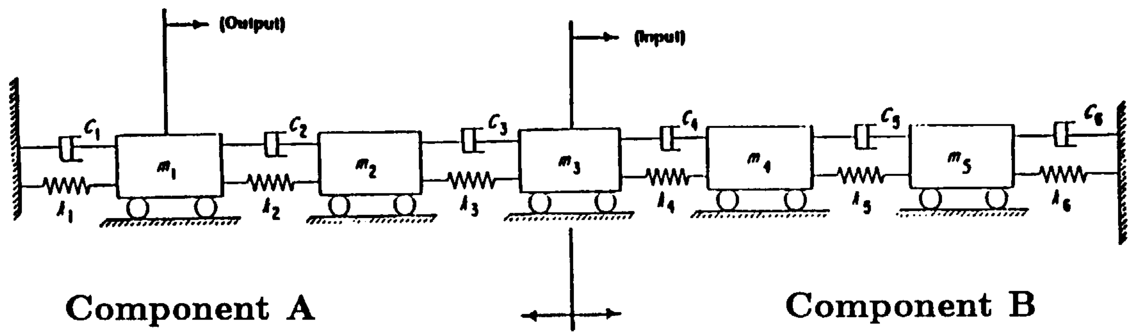
Background

- Multibody Dynamics Simulation Packages (e.g., DISCOS) Assemble Components' Dynamical Characteristics To Produce Dynamical Equations of A System of Interconnected Bodies (e.g., Galileo Spacecraft),
- For Flexible Components, Their Modal Characteristics Are Used As Inputs To The Package. In Particular, The Component Modal Damping Matrices Are Usually Assumed To be Diagonal,
- From Experience Obtained From Working With Large Space Structures, A Uniform Damping Factor of Not More Than 0.25% Can Usually Be Conservatively Assumed For All The System Modes,
- With Assumed Levels of Components' Damping, the Assembled System's Damping Might or Might Not Adhere to This "Rule of Thumb". If It Doesn't, A Time-consuming, Iterative Procedure Must Then Be Used to Adjust The Components' Damping Until It Does.

Objective

- **To Develop Techniques to Determine the Component Modes' Damping Factors Given the System Modes' Damping Factors**



System (5-DOF)	=	**Component A (3-DOF)**	+	**Component B (3-DOF)**	

$\omega_k, \quad k = 1, ..., 5$ $\qquad$ $\omega_{Ai}, \quad i = 1, ..., 3$ $\qquad$ $\omega_{Bj}, \quad j = 1, ..., 3$

$\varsigma_k, \quad k = 1, ..., 5$ $\qquad$ $\varsigma_{Ai}, \quad i = 1, ..., 3$ $\qquad$ $\varsigma_{Bj}, \quad j = 1, ..., 3$

- **Given the System Modes' Damping Factors (ς_k, $k = 1,...,5$), Find the Component Modes' Damping Factors (ς_{Ai}, $i = 1,...,3$ and ς_{Bj}, $j = 1,...,3$).**

The Approach

- To Derive, from First Principles, a Relation Between the System Modal Damping Matrix and the Component Modal Damping Matrices,
- To Formulate and Solve an Optimization Problem that Enforce The Derived Component/System Modal Damping Relation.

Components' Equations of Motion in Physical and Modal Coordinates

$$[M_A][\ddot{x}_A] + [C_A][\dot{x}_A] + [K_A][x_A] = [G_A]u\,,$$

$$[x_A] = [V_A][q_A]\,,$$

$$[I_{n_A}][\ddot{q}_A] + [\bar{C}_A][\dot{q}_A] + [\bar{K}_A][q_A] = [\bar{G}_A]u\,.$$

A Similar Set of Equations May Be Written For Component B.

Remarks

- Viscous Damping is a Simplified Mathematical Representation of a Rather Complex Situation Which Might Include Other Forms of Energy Dissipation (e.g., Hysteresis Damping, etc.).
- We Assume That If The Damping Is Small, These Damping Effects Can Be Grossly Represented By An Equivalent Viscous Term.
- $[\bar{C}_A]$ And $[\bar{C}_B]$ Are in General Non-diagonal, but Are Assumed to be Diagonal. Results Are Also Obtained Without This Assumption.

System's Equations of Motion in Physical and Modal Coordinates

- **Physical Coordinates : Constructed from the Components' EOM Using the Compatibility Relations**

$$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} [P_A] \\ [P_B] \end{bmatrix} [x] \,,$$

$$[M][\ddot{x}] + [C][\dot{x}] + [K][x] = [G]u \,,$$

Where $[M]$ May Be Expressed In Terms of $[M_A]$, $[M_B]$, $[P_A]$, and $[P_B]$. $[C]$ May Be Expressed In Terms of $[C_A]$, $[C_B]$, $[P_A]$, and $[P_B]$, etc.

- **Modal Coordinates**

$$[x] = [V][q] \,,$$

$$[I_n][\ddot{q}] + [\bar{C}][\dot{q}] + [\bar{K}][q] = [\bar{G}]u \,,$$

Where $[V]^T[M][V] = [I_n]$, $[\bar{G}] = [V]^T[G]$, $[\bar{K}] = [V]^T[K][V]$ (= Diag [ω_k^2], k = 1,...,5), And $[\bar{C}] = [V]^T[C][V]$ (Assumed to Be Diagonal).

One Way to Establish a Relation Between the System Generalized Coordinate $[q]$ and Those of the Components $[q_A]$ and $[q_B]$

Note that : $[x_A] = [V_A]\ [q_A] = [P_A]\ [x] = [P_A]\ [V]\ [q]$.

Hence

$$[q_A] = [Q_A][q]\,,$$

$$[q_B] = [Q_B][q]\,.$$

- $[Q_A](n_A \times n) = [V_A]^{-1}[P_A][V]$, $\qquad [Q_B](n_B \times n) = [V_B]^{-1}[P_B][V]$.
- $[V_A]$ and $[V_B]$ are non-singular matrices.
- **Relations Between Components' Modal Matrices and System's Modal Matrices**

$$[Q_A]^T[Q_A] + [Q_B]^T[Q_B] = [I_n]\,,$$

$$[Q_A]^T[\bar{C}_A][Q_A] + [Q_B]^T[\bar{C}_B][Q_B] = [\bar{C}]\,,$$

$$[Q_A]^T[\bar{K}_A][Q_A] + [Q_B]^T[\bar{K}_B][Q_B] = [\bar{K}]\,,$$

$$[Q_A]^T[\bar{G}_A] + [Q_B]^T[\bar{G}_B] = [\bar{G}]\,.$$

Required Relation Between the System's Damping Matrix and the Components' Damping Matrices.

Alternative Way To Express The Component/System Modal Damping Relation

$$\sum_{i=1}^{i=n_A} c_{A\,i}\, R_{A\,i} + \sum_{j=1}^{j=n_B} c_{B\,j}\, R_{B\,j} = [\bar{C}]\,,$$

$$\sum_{i=1}^{i=n_A} \omega_{A\,i}^2\, R_{A\,i} + \sum_{j=1}^{j=n_B} \omega_{B\,j}^2\, R_{B\,j} = [\bar{K}]\,.$$

Remarks :

- R_{Ai} **and** R_{Bj} **Are Determined From** Q_A **and** Q_B **Respectively,**
- R_{Ai} **and** R_{Bj} **May be Interpreted as "Weighting" Matrix that Determines the Contributions of the** i^{th} **Mode of Component A and** j^{th} **Mode of Component B to the System Damping,**
- **Same Weighting Matrices also Determine the Contributions of the Component Modes to the System Stiffness Matrix.**

Optimization Problem

$$\min_{c_{Ai},c_{Bj}} J = \frac{1}{2} \| \bar{C} - \sum_{i=1}^{i=n_A} c_{A\,i} R_{A\,i} - \sum_{j=1}^{j=n_B} c_{B\,j} R_{B\,j} \|_F^2 ,$$

where $\| \bullet \|_F^2$ is the Squared Frobenius Norm of the Matrix Concerned

The Optimality Conditions :

$$\frac{\partial J}{\partial c_{A1}} = \cdots = \frac{\partial J}{\partial c_{B1}} = \cdots = 0 , \qquad \begin{bmatrix} c_{A1} \\ c_{A2} \\ \cdots \\ c_{B1} \\ c_{B2} \\ \cdots \end{bmatrix} = A^{-1} \times B .$$

where A is a $(n_A + n_B) \times (n_A + n_B)$, and B is a $(n_A + n_B) \times 1$ matrices.

$$A = \begin{bmatrix} \| R_{A1} R_{A1} \| & .. & \| R_{A1} R_{An_A} \| & .. & \| R_{A1} R_{Bn_B} \| \\ \| R_{A2} R_{A1} \| & .. & \| R_{A2} R_{An_A} \| & .. & \| R_{A2} R_{Bn_B} \| \\ .. & .. & .. & .. & .. \\ \| R_{Bn_B} R_{A1} \| & .. & \| R_{Bn_B} R_{An_A} \| & .. & \| R_{Bn_B} R_{Bn_B} \| \end{bmatrix} , \qquad B = \begin{bmatrix} \| R_{A1} \bar{C} \| \\ \cdots \\ \| R_{An_A} \bar{C} \| \\ \| R_{B1} \bar{C} \| \\ \cdots \\ \| R_{Bn_B} \bar{C} \| \end{bmatrix} .$$

- $\| X Y \| = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} \times Y_{ij}$
- **From Cauchy's Inequality Theorem, the Determinant of A is Always Greater than Zero Unless the Matrices $R_{A1},\ldots,R_{An_A},\ldots,R_{Bn_B}$ are Linearly Dependent**

Iterative Gradient Solution

$$[c_{A1}]_{k+1} = [c_{A1}]_k - \epsilon[\frac{\partial J}{\partial c_{A1}}]_k \,,$$

$$\cdots$$

$$[c_{B1}]_{k+1} = [c_{B1}]_k - \epsilon[\frac{\partial J}{\partial c_{B1}}]_k \,,$$

$$\cdots$$

- **k = Current Iteration Step**
- **ϵ = Small Positive Constant**

$$[\frac{\partial J}{\partial c_{A1}}]_k = -\sum_{r=1}^{r=n}\sum_{s=1}^{s=n}[R_{A1}]_{rs}([\bar{C}]_{rs} - \sum_{i=1}^{i=n_A}(c_{A\,i})_k\,[R_{A\,i}]_{rs} - \sum_{j=1}^{j=n_B}(c_{B\,j})_k\,[R_{B\,j}]_{rs}) \,,$$

$$\cdots$$

$$[\frac{\partial J}{\partial c_{B1}}]_k = -\sum_{r=1}^{r=n}\sum_{s=1}^{s=n}[R_{B1}]_{rs}([\bar{C}]_{rs} - \sum_{i=1}^{i=n_A}(c_{A\,i})_k\,[R_{A\,i}]_{rs} - \sum_{j=1}^{j=n_B}(c_{B\,j})_k\,[R_{B\,j}]_{rs}) \,.$$

$$\cdots$$

- **The Iteration Stops When the Magnitudes of all the Gradients are Smaller Than a Prescribed Quantity (e.g., 10^{-10}).**

Modifications to the Optimization Problem

- **Weighting Matrix**

$$J_W = \frac{1}{2} \sum_{r=1}^{r=n} \sum_{s=1}^{s=n} W_{rs}^2 \alpha_{rs}^2$$

- **Inequality Constraints**

  - **Results Obtained From the Unconstrained Optimization Problem Might or Might not be "Physically Meaningful." Situations Arise in which, for a Given $[\bar{C}]$, One or More of c_{Ai} ($i = 1,\ldots,n_A$) and c_{Bj} ($j = 1,\ldots,n_B$) Might Be Negative**

  - **To Overcome the Difficulty, the Formulated Optimization Problem May be Modified with the Additions of Inequality Constraints**

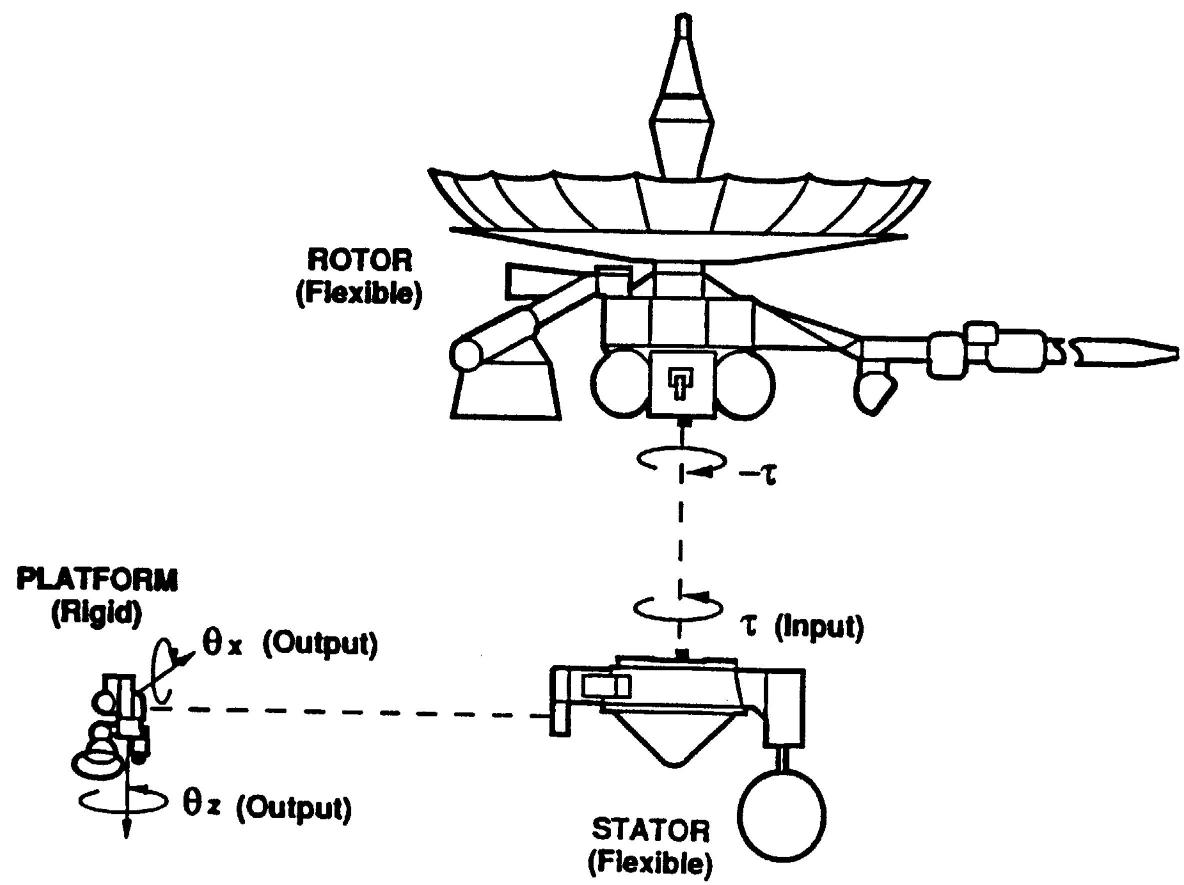
$$c_{Ai} \geq 0\,, \qquad i = 1, \ldots, n_A\,,$$
$$c_{Bj} \geq 0\,, \qquad j = 1, \ldots, n_B\,.$$

  - **No More Analytical (Algebric) Solution**

  - **May Be Solved Iteratively by, e.g., the Gradient Projection Method.**

Example : Galileo Spacecraft Cruise Model

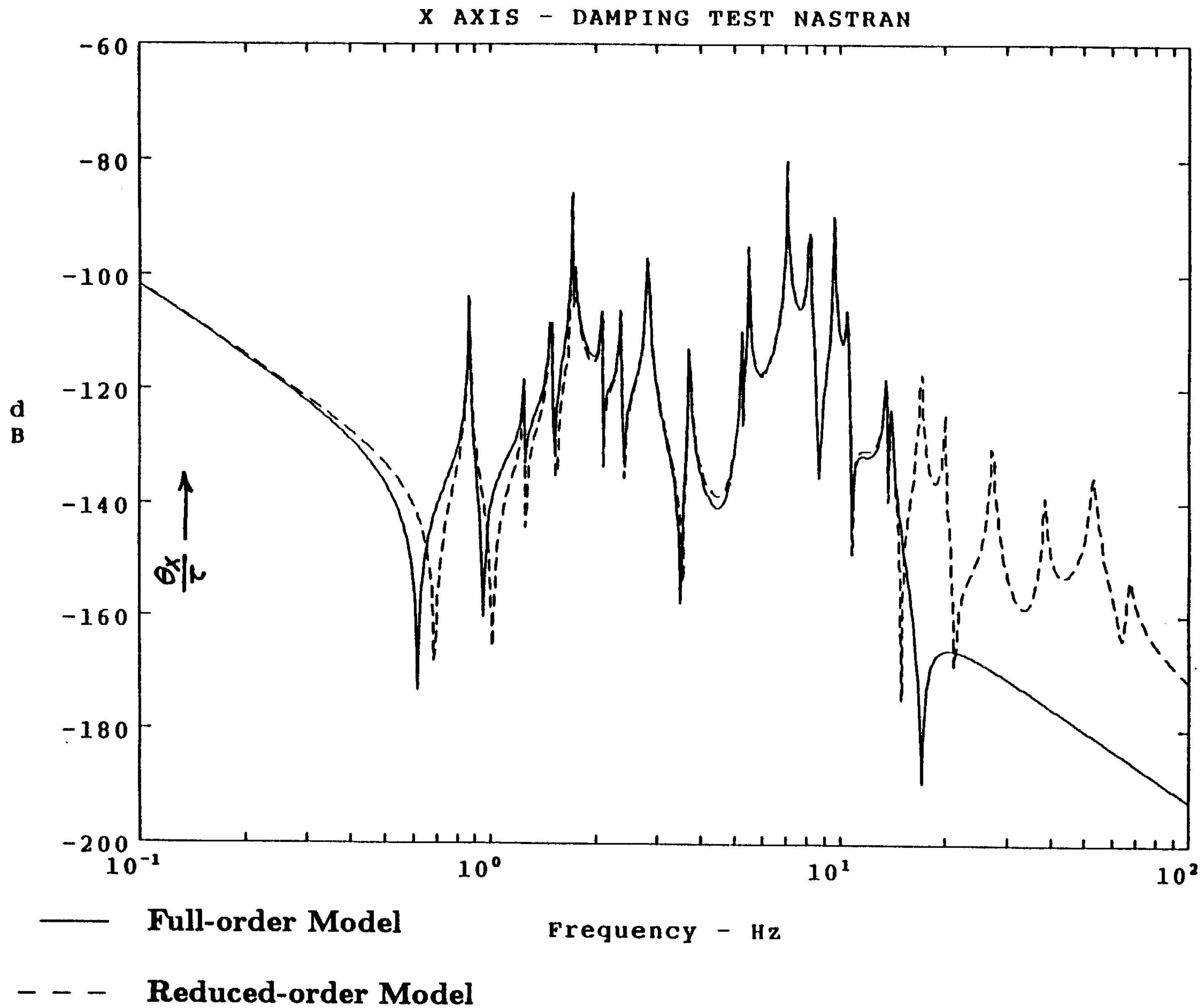
- A Complex High-order Model From Nastran FEM
- 26 Rotor Modes, 19 Stator Modes (After SVD), 6 Scan-platform Modes;
- Assembled System has 8 Rigid-body, 20 Retained, and 11 Extraneous Modes;
- Equal Weighting on All the Retained, and Zero Weighting on All Extraneous Modes.



Damping Ratios of the Reassembled System's Rigid-body, Retained, and Extraneous Modes

Mode	Frequency (Hz)	Rigid-body (%)	Retained (%)	Extraneous (%)
1	0	0		
2	0	0		
3	0	0		
4	0	0		
5	0	0		
6	0	0		
7	0	0		
8	0.0001	0		
9	0.8644		0.2500	
10	1.2355		0.2500	
11	1.4788		0.2500	
12	1.6782			0.5228
13	1.7070		0.2500	
14	1.7346		0.2500	
15	2.0724		0.2500	
16	2.3513		0.2500	
17	2.8152		0.2500	
18	3.7066		0.2500	
19	4.1724		0.2500	
20	5.2314		0.2500	
21	5.2473		0.2500	
22	5.4695		0.2500	
23	6.0124			22.4335
24	7.0593		0.2500	
25	7.1711			41.0165
26	8.1534		0.2500	
27	9.3513			14.5847
28	9.6102		0.2500	
29	10.3556			0.2146
30	10.4210		0.2500	
31	10.5549		0.2500	
32	13.5342		0.2500	
33	13.9894		0.2500	
34	17.0659			0.5842
35	19.9321			0.4905
36	27.2954			0.7231
37	38.4226			1.0406
38	53.0927			1.4643
39	66.6673			1.9455

Bode Plot, Galileo Spacecraft Cruise Model, (X-axis)



Summary

(1) A Relation Between the System Modal Damping Matrix and the Component Modal Damping Matrices is Derived from First Principles;

(2) An Optimization Problem is then Formulated to Select All the Component Modes' Damping Ratios that Best Satisfy the Above Derived Relation;

(3) A Weighting Matrix is Used in the Cost Functional to Stress the Relative Importance of the Diagonal Terms in the Damping Matrix. Inequality Constraints are Also Added to the Optimization Problem to Pick Only Nonnegative Component Modes' Damping Factors;

(4) The Optimization Problem May be Solved Algebraically or Iteratively;

(5) The Proposed Techniques are Successfully Used on a High-order, Finite-element Model of the Galileo Spacecraft.