FINITE ELEMENT MODELING OF TRUSS STRUCTURES WITH FREQUENCY-DEPENDENT MATERIAL DAMPING

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ABSTRACT

In advanced engineering systems such as large space structures (LSS) or robots, the combination of severe disturbances, stringent requirements, and structural design constraints can result in structures which exhibit significant flexibility. The design of stable, fast-responding structural control systems benefits from accurate knowledge of structural dynamic behavior, including the magnitudes and mechanisms of inherent damping.

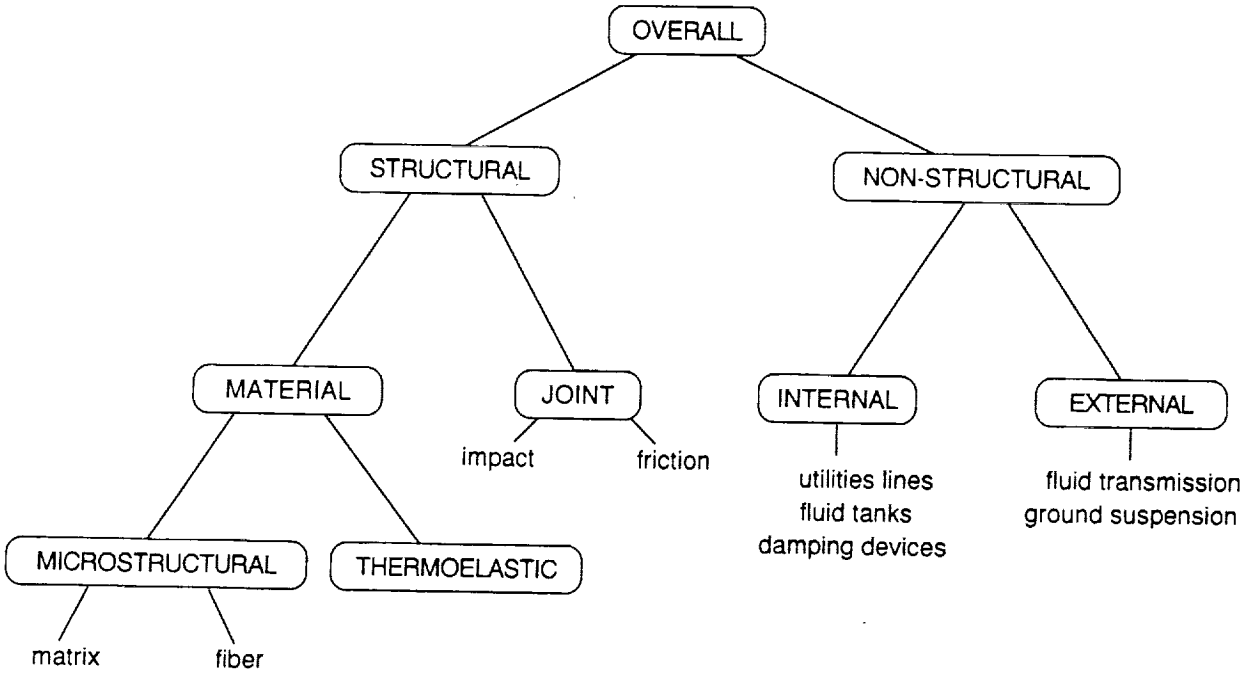
Material damping is likely to be an important, perhaps dominant, contributor to damping in "monolithic" structures and to on-orbit damping in precision spacecraft. The damping of most engineering materials exhibits a strong dependence on frequency.

A physically-motivated modelling technique for structural dynamic analysis that accomodates frequency-dependent material damping has been developed. Key features of the technique are the introduction of augmenting thermodynamic fields (ATF) to interact with the usual mechanical displacement field, and the treatment of the resulting coupled governing equations using finite element analysis methods. The ATF method is fully compatible with current structural finite element analysis techniques.

The method is demonstrated in the dynamic analysis of a 10-bay planar truss structure, a structure representative of those contemplated for use in future space systems. Analytical results from modal analyses of an ATF-damped and an undamped truss structure agree well in terms of modal frequencies, but the ATF analysis also yields modal damping and complex mode shapes. The undamped frequencies are lower by as much as five percent in higher modes because the relaxed (static) modulus value is used, as is usual practice. In addition to preserving the characteristic frequency dependence of material *damping*, the ATF method reflects the frequency dependence of material *modulus* as well.

With the continued development of better analytical tools such as this ATF method, damping will be modelled more accurately in the design of engineering systems and may ultimately become more accessible to design specification.

SOURCES OF DAMPING



MATERIAL DAMPING WILL BE IMPORTANT IN PRECISION SPACE SYSTEMS

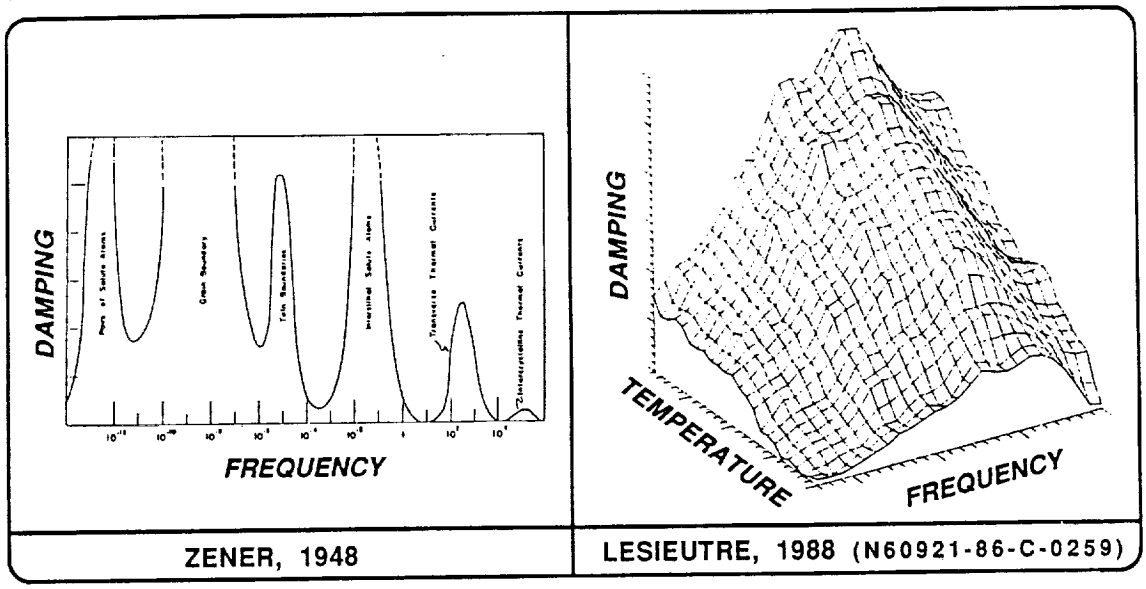
In advanced engineering systems such as large space structures (LSS) or robots, the combination of severe disturbances, stringent requirements, and structural design constraints can result in structures which exhibit significant flexibility. Passive and active damping of these structures is important for several reasons. In terms of performance, higher damping can reduce steady-state vibration levels and can reduce the time needed for transient vibrations to settle. Inherent passive damping can reduce the magnitude of control needed, and can reduce control system complexity. Passive damping can also strongly couple vibration modes which are closely-spaced in frequency and computed assuming no damping. Most importantly, however, the design of stable, fast-responding control systems benefits from accurate knowledge of structural dynamic behavior, which depends on the magnitudes and mechanisms of inherent damping.

Many sources of inherent damping exist for a given aerospace vehicle. A potential classification scheme for such sources is shown above. In general, damping is not well-quantified in the design process, which results in significant design decisions being made on the basis of qualititative information. There is considerable room for improvement in the analysis of damping in aerospace systems. For example, a damping design budget might allocate contributions to total damping required from individual damping sources.

A case can be made for the importance of material damping in at least some precision space vehicles. In common built-up structures which operate in the atmosphere, air damping and joint damping typically dominate system damping. However, air damping is clearly eliminated in space, and the effects of joint damping will be reduced because of requirements for precision ("tight" joints) and typically low vibration levels (friction "lockup"). Material damping is thus likely to be an important, perhaps dominant, contributor to damping in "monolithic" structures and to on-orbit damping in precision spacecraft.

MATERIAL DAMPING IS COMPLEX

- **DEPENDS ON FREQUENCY, TEMPERATURE, DEFORMATION, AMPLITUDE, GEOMETRY**



ZENER, 1948 | LESIEUTRE, 1988 (N60921-86-C-0259)

- **ATF METHOD ADDRESSES FREQUENCY-DEPENDENCE**

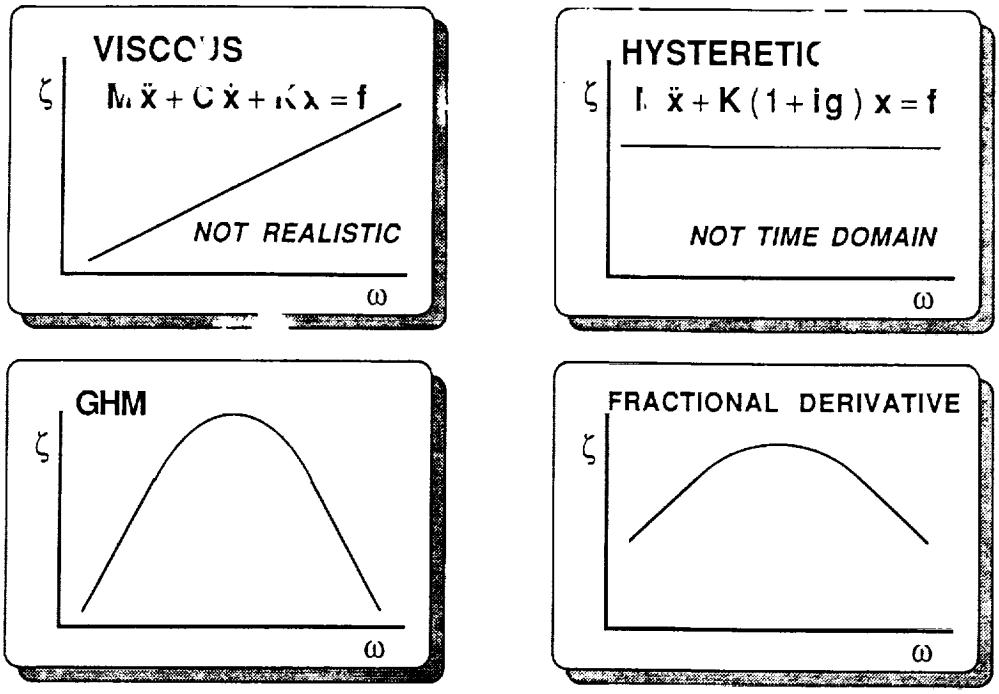
Material damping is generally a complex function of frequency, temperature, type of deformation, amplitude, and structural geometry.

The figure on the left is adapted from the frontispiece of the pioneering text, Elasticity and Anelasticity of Metals and illustrates the typical frequency-dependence of material damping. Note that there are a number of discrete damping peaks, and that a physical (possibly atomic) process is associated with each peak (*e.g.*, "transverse thermal currents").

The figure on the right shows some data recently obtained for aluminum in a flexural vibration test. The frequency dependence of damping is apparent. Similar data were obtained for graphite/aluminum composite materials, although weaker frequency dependence was observed.

Current popular treatments of damping in structural dynamics are generally not physically-motivated and do not preserve the fundamental frequency-dependence of material damping. This observation provided the motivation for the subject work.

DAMPING MODELS AND RELATED WORK



Several methods are currently used to incorporate the effects of material damping into structural models. These methods include viscous damping, frequency-dependent viscous damping, complex modulus, hysteretic damping, structural damping, viscoelasticity, hereditary integrals, and modal damping. Modal damping is probably the most widely used in structural control design applications. Each has some utility, but each suffers from one flaw or another. For example, a one-dimensional structure made from a single material with viscous damping would exhibit damping that increases monotonically with frequency—such behavior is not observed in practice. Although some damping models, such as viscoelasticity, have the potential for better accuracy than more widely-used methods, they are not commonly used in the engineering community—perhaps because of the lack of physical motivation, difficulty of use, or lack of data.

Other researchers have noted the inadeqacy of current damping modeling techniques and have addressed the development of improved methods.

Golla, Hughes, and McTavish (GHM) of the University of Toronto have developed a time-domain finite element formulation of viscoelastic material damping. Their work was guided by the observation that experimental results, often recorded in the frequency domain, are of little direct use in time-domain models. Their results resemble those reported here in an important way—in the introduction of additional "dissipation coordinates." It might not be surprising that additional degrees of freedom would be required to model material behavior with increased accuracy. However, the GHM approach is fundamentally a mathematical one, developing time-domain realizations from frequency-domain models—no attempt is made to provide a physical interpretation of the dissipation coordinates as thermodynamic field variables with a direct relationship to microstructural features of real materials. The GHM technique has been successfully used to fit a portion of an experimentally-determined curve of damping versus frequency, and standard analysis tools can be used to solve the resulting equations.

Bagley and Torvik of the Air Force Institute of Technology have also developed a relevant model of material damping. The core of their concept is the use of fractional time derivatives in material constitutive equations. Their development was motivated by the observation that the frequency dependence observed in real materials is often weaker than the dependence predicted by first-order viscoelastic models. With four and five parameter models, they have been able to accurately represent the elastic and dissipative behavior of over 100 materials over frequency ranges as broad as 8 decades. The application of the general fractional derivative approach to time-domain analysis is an area of continuing research.

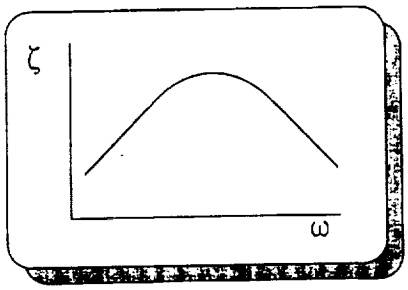
BASIS FOR ATF

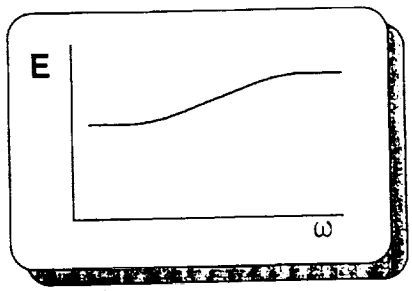
- "INTERNAL STATE VARIABLES" OF MATERIALS SCIENCE MOTIVATE INTRODUCTION OF "AUGMENTING THERMODYNAMIC FIELDS" (ATF)

CONSTITUTIVE EQUATION $\sigma = E_u u' - \delta \xi$

ATF DYNAMICS $\dot{\xi} + B\xi = \left(\frac{B\delta}{\alpha}\right) u'$

- FREQUENCY-DEPENDENT DAMPING AND MODULUS





$$\zeta = \frac{\delta^2}{4E\alpha}\left(\frac{2(\omega/B)}{1+(\omega/B)^2}\right)$$

$$E = E_u\left(1-\frac{\delta^2}{E_u\alpha}\left(\frac{1}{1+(\omega/B)^2}\right)\right)$$

(SMALL DAMPING)

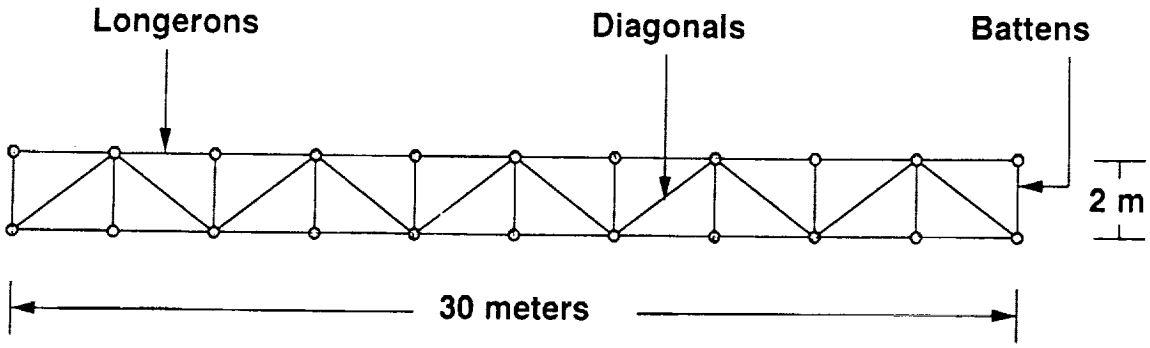
Structural dynamicists are the unintended beneficiaries of a sizable literature on material damping. For many years, crystallographers and metallurgists have used "internal friction" as a probe into the underlying structure of materials. By measuring damping as a function of frequency, temperature, load type, and amplitude, they can determine the mobility and activation energies of various microstructural features of materials. These researchers have identified a multitude of internal variables and relaxation mechanisms which range in geometrical scale from crystal lattice dimensions on up to structural dimensions and, in temporal scale, over a similarly broad range.

The "internal variables" of materials science suggest the introduction of augmenting thermodynamic fields (ATF) to interact with the usual displacement field of continuum structural dynamics. In the one-dimensional case where stress is coupled to a single augmenting thermodynamic field (ATF), and where the ATF dynamics are governed by a first-order relaxation equation, the effective material modulus increases with frequency to an asymptotic value of E_u, and a damping peak is observed. This is in accord with the observed behavior of many engineering materials.

The ATF modeling approach essentially implements a continuum version of the "standard anelastic solid." Weaker frequency dependence, such as that observed in high damping viscoelastic polymers, can be addressed through the introduction of multiple ATFs.

STRAWMAN PROBLEM

- MODAL ANALYSIS OF ATF-DAMPED TRUSS
- ELEMENTS WITH DIFFERENT ELASTIC AND DISSIPATIVE PROP'S



- COMPARE TO RESULTS OF CONVENTIONAL UNDAMPED ANALYSIS

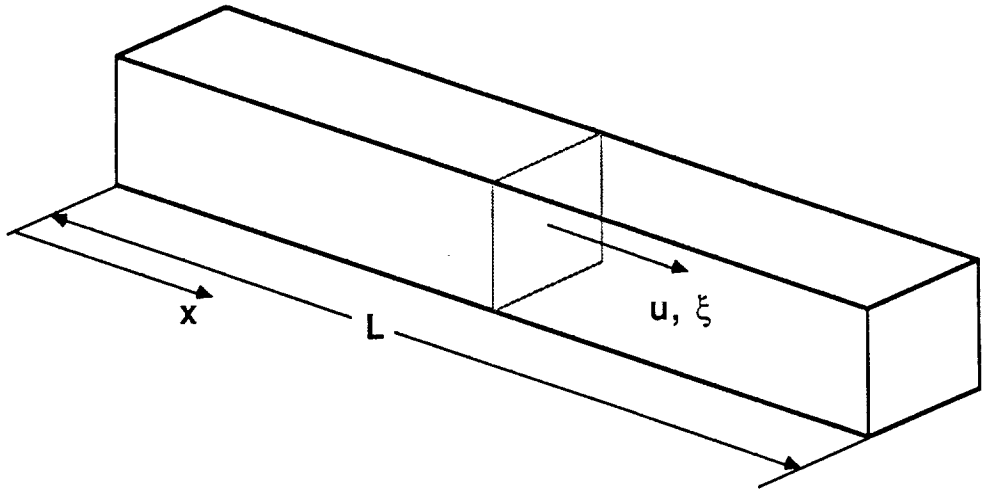
The ATF modeling approach is illustrated through application to the dynamic analysis of a large planar space truss, shown above. Such a structure resembles those proposed for many future space missions. It is an efficient beam-type structure built from 3 basic structural elements: longerons, which are parallel to the beam axis; diagonals, which bisect each rectangular bay; and battens, which are oriented transverse to the beam axis. The total length of the structure is 30 meters and the truss depth is 2 meters.

Members of the truss of interest are connected through frictionless pins so that no bending moments are transmitted through the joints, and so that no damping is introduced by the joints. Joint mass and other non-structural masses are ignored. Finally, each kind of structural element is assumed to be made of a different material having different elastic, inertial, and dissipative properties. For this problem, it is assumed that a single augmenting thermodynamic field is sufficient to characterize the dissipative properties of each material in the frequency range of interest.

Conventional damping analysis, if damping were considered at all, would likely employ the modal strain energy (MSE) method to estimate "modal damping." This is an iterative process, requiring analysis of the undamped structure to determine the frequencies and mode shapes of the undamped modes. The mode shapes are used to determine the distribution of strain energy over the structure, while the frequencies are used to determine frequency-dependent elastic and dissipative properties. Modal damping is then estimated for each mode and, roughly speaking, is numerically equal to the sum of material damping ratios weighted by the fraction of the strain energy stored in each material. The MSE method can lead to serious errors when modes are closely spaced in frequency and when the damped mode shapes are much different than the undamped mode shapes.

The vibration modes and frequencies of this ATF-damped truss are sought for comparison to those of the undamped case. A one-dimensional ATF-damped finite element appropriate for modeling the dynamic behavior of a single truss strut is developed first, then used in the analysis of the overall structure.

FINITE-LENGTH ROD EXECUTING LONGITUDINAL VIBRATION



COUPLED FIELDS: MECHANICAL DISPLACEMENT, u(x)

AUGMENTING THERMODYNAMIC, ξ (x)

ENDS MECHANICALLY UNCONSTRAINED

Consider the case of one-dimensional motion, corresponding to longitudinal vibration of a thin rod of length L. The mechanical displacement along the rod is denoted by u(x) (strain $\epsilon(x)=u'(x)$), and the rod has uniform mass density ρ and *unrelaxed* or *dynamic* modulus of elasticity E. A single augmenting thermodynamic field, $\xi(x)$, is introduced.

The material property δ describes the strength of the coupling of the two dependent fields, u and ξ. In the absence of coupling of the two fields, increments of stress and strain are proportional, with E the relating factor. Analogously, α is the material property that relates changes in A, the thermodynamic conjugate of ξ, to those in ξ. B is the inverse of the time constant for uncoupled relaxation of the augmenting field.

GOVERNING EQUATIONS

PDEs

PRIMARY FORM

$$\rho\ddot{u} - Eu'' = -\delta\xi'$$

$$\dot{\xi} + B\xi = \left(\frac{B\delta}{\alpha}\right)u'$$

ALTERNATE FORM

$$\rho\ddot{u} - Eu'' = -\delta\gamma$$

$$\dot{\gamma} + B\gamma = \left(\frac{B\delta}{\alpha}\right)u''$$

BCs

At ends ($x=0$ and $x=L$)

$$u(x)=u_0 \quad \text{or} \quad \sigma(x)=\sigma_0$$

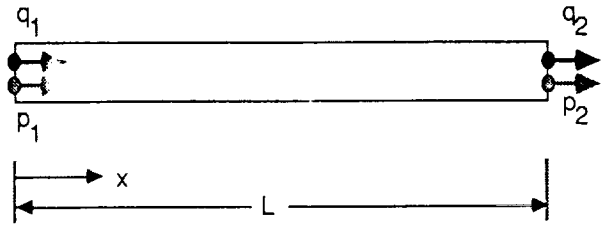
ATF are internal variables (no BC)

The equation of evolution for the mechanical displacement field, u(x), is developed from consideration of momentum balance (zero body forces are assumed). The equation of evolution for the augmenting thermodynamic field, ξ, is found through the use of a basic assumption of irreversible thermodynamics, namely that the rate of change of ξ is proportional to its deviation from an equilibrium value. This results in a first-order differential equation, a "relaxation" equation. The result is a set of two coupled partial differential equations in u and ξ.

The augmenting thermodynamic field is essentially an *internal* field, *i.e.*, there are no explicit boundary conditions that it alone must satisfy. However, the mechanical displacement field must satisfy either displacement ("geometric") or stress ("natural") boundary conditions at each end of the rod, as is the case in undamped structural dynamics. Note that the stress boundary condition involves the augmenting field, ξ.

An alternative formulation of this one-dimensional case may be considered. For example, the preceding primary equations can be expressed in terms of γ, the gradient of the ξ-field. Such a formulation contains only even spatial derivatives, and leads to some benefits in numerical solution, such as symmetric element submatrices and better convergence in terms of damping versus frequency.

FINITE ELEMENT TREATMENT



$$u(x,t) = \zeta^T(x)\, D \begin{Bmatrix} u_1(t) \\ u_2(t) \end{Bmatrix}$$

$$\gamma(x,t) = \Omega^T(x)\, T \begin{Bmatrix} \gamma_1(t) \\ \gamma_2(t) \end{Bmatrix}$$

METHOD OF WEIGHTED RESIDUALS

$$\rho\ddot{u} - Eu'' = -\delta\gamma$$

$$\dot{\gamma} + B\gamma = \left(\frac{B\delta}{\alpha}\right)u''$$

SINGLE ELEMENT

$$\int_0^L \left[\rho D^T\theta\theta^T D\{\ddot{u}\} + ED^T\theta'\theta'^T D\{u\} + \delta D^T\theta\Omega^T T\{\gamma\}\right] dx = 0$$

ALTERNATE FORMS POSSIBLE

MATRIX NOTATION

$$M\{\ddot{u}\} + K\{u\} + F\{\gamma\} = 0$$

The method of weighted residuals (MWR) is used to develop element matrices. The u-γ formulation of the equations has been found to be superior to the u-ξ formulation, and is shown above. Integration by parts is employed, changing the continuity required of the approximating and weighting functions.

The same functions used to approximate the behavior of the dependent fields in the spatial region bounded by the element are used as weighting functions—when there is only one dependent field, this is known as Galerkin's method.

The resulting sets of equations may be written compactly in matrix form, as shown.

In this treatment, both dependent fields are approximated with linear interpolation functions. The corresponding element and the nodal values for the two dependent fields, u and γ are shown in the figure. Anticipating solution of a first-order matrix eigenvalue problem, and to facilitate global matrix assembly, the elemental degrees of freedom are ordered as follows.

$$x = [\dot{q}_1, q_1, p_1, \dot{q}_2, q_2, p_2]^T$$

MATRIX EQUATIONS

UNDAMPED

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}$$

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} 0 & K \\ -I & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ u \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$ FIRST-ORDER FORM

ATF

$$\begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & C \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \dot{u} \\ \dot{\gamma} \end{Bmatrix} + \begin{bmatrix} 0 & K & F \\ -I & 0 & 0 \\ 0 & B & H \end{bmatrix} \begin{Bmatrix} \dot{u} \\ u \\ \gamma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

The structure of the undamped matrix equations of motion are shown in both second-order and first-order form. The structure of the ATF-damped matrix equations is shown for comparision. The ATF equations are more complex, describing the dynamics of the augmenting field as well as the coupling of the two fields.

The "augmented mass matrix" is generally symmetric and positive definite, while the "augmented stiffness matrix" is neither. Certain submatrices, **K** and **H** in particular, are generally symmetric and positive semidefinite, while the properties of the coupling submatrices, **B** anf **F**, depend on the specifics of the numerical approximation scheme employed.

ELEMENT MATRICES

UNDAMPED

$$\begin{bmatrix} \left(\frac{\rho AL}{3}\right) & \left(\frac{\rho AL}{6}\right) \\ \left(\frac{\rho AL}{6}\right) & \left(\frac{\rho AL}{3}\right) \end{bmatrix} \qquad \begin{bmatrix} \left(\frac{EA}{L}\right) & -\left(\frac{EA}{L}\right) \\ -\left(\frac{EA}{L}\right) & \left(\frac{EA}{L}\right) \end{bmatrix}$$

ATF (FIRST ORDER)

$$\begin{bmatrix} \left(\frac{\rho AL}{3}\right) & 0 & 0 & \left(\frac{\rho AL}{6}\right) & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{AL}{3}\right) & 0 & 0 & \left(\frac{AL}{6}\right) \\ \left(\frac{\rho AL}{6}\right) & 0 & 0 & \left(\frac{\rho AL}{3}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \left(\frac{AL}{6}\right) & 0 & 0 & \left(\frac{AL}{3}\right) \end{bmatrix} \qquad \begin{bmatrix} 0 & \left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{3}\right) & 0 & -\left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{6}\right) \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) & 0 & -\left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) \\ 0 & -\left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{6}\right) & 0 & \left(\frac{EA}{L}\right) & \left(\frac{\delta AL}{3}\right) \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -\left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) & 0 & \left(\frac{B\delta A}{\alpha L}\right) & \left(\frac{BAL}{3}\right) \end{bmatrix}$$

For future reference and comparison, details of the elemental mass and stiffness matrices for the undamped and ATF-damped cases are shown. The matrices for the undamped case are shown in second-order form, while the ATF-damped matrices are shown in first-order form. Note that the elements of the undamped matrices also appear in the damped element matrices.

Note that the material modulus, E, is different in the two cases shown. In the undamped case, it is the static or "relaxed" modulus normally used in structural analysis. In the damped case, it is the "unrelaxed" or high-frequency asymptotic modulus. The unrelaxed modulus is always greater than the relaxed value.

FREE VIBRATION PROBLEM

- FREE LONGITUDINAL VIBRATION OF ROD

$$-\begin{bmatrix} 0 & K & F \\ -I & 0 & 0 \\ 0 & B & H \end{bmatrix}\begin{Bmatrix} \dot{u} \\ u \\ \gamma \end{Bmatrix} = \lambda \begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & C \end{bmatrix}\begin{Bmatrix} \dot{u} \\ u \\ \gamma \end{Bmatrix}$$

$$\mathbf{Ax} = \lambda \mathbf{Bx}$$

- GENERAL, UNSYMMETRIC EIGENVALUE PROBLEM
- COMPLEX MODES

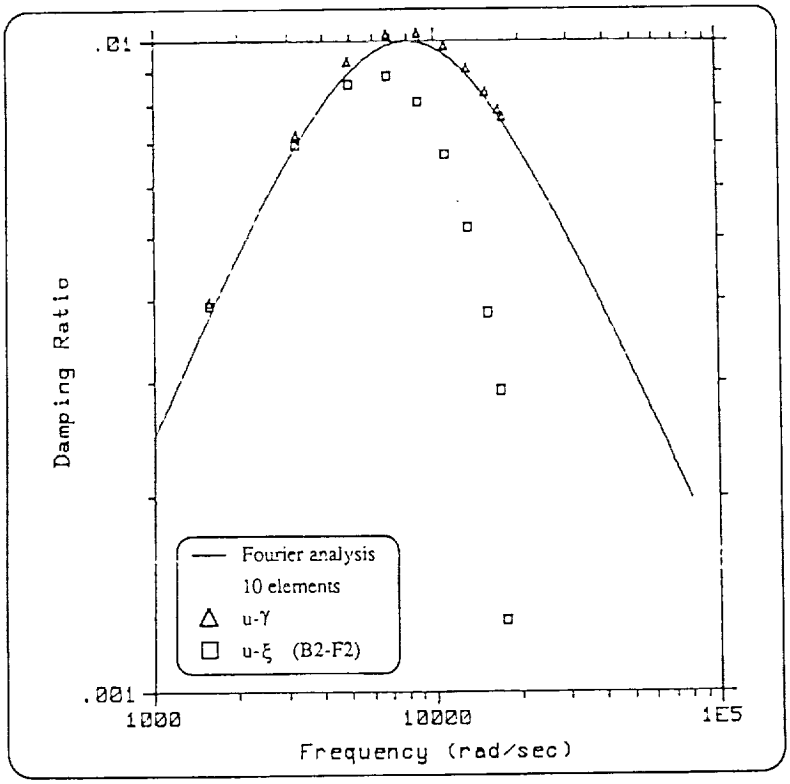
In order to evaluate the performance of this formulation of an ATF-damped rod element, a specific problem was addressed, namely the determination of the natural modes of vibration of a free-free rod. Accordingly, no geometric boundary conditions were enforced.

The elastic properties used correspond roughly to those of aluminum in SI units. The ATF and coupling properties were chosen to yield a peak damping ratio of 0.01 at the frequency of the 5th mode. Note that numerical values for α and δ cannot be uniquely specified in this approach.

The matrix equations of motion were formulated and a general unsymmetric eigenvalue problem solved to yield complex eigenvalues, λ, and mode shapes, x. Global system matrices are assembled from element matrices and geometric boundary conditions are enforced in the usual manner of structural finite element analysis. The damping ratio, ζ, for each mode was determined as the ratio of the negative of the real part of the eigenvalue to the total magnitude and plotted against the magnitude.

Note that the spectrum of eigenvalues contains "vibration modes," "relaxation modes," and "rigid-body modes." In the complex plane, the damped vibration modes lay near the imaginary axis, slightly in the LHP with negative real parts; the relaxation modes lie on the negative real axis. These relaxation modes are characteristic of the response of the γ field.

RESULTS OF TRUSS ELEMENT MODAL ANALYSIS

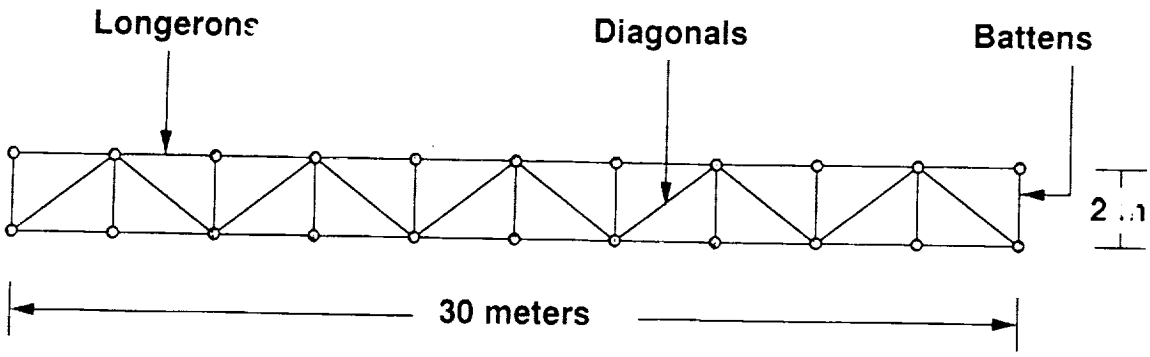


u- γ EQ'NS PROVIDE BETTER DAMPING CONVERGENCE

Typical numerical results for modal damping versus frequency obtained using this approach are shown above. The solid line shows the results expected on the basis of approximate Fourier analysis of the governing equations, while the symbols show the results of ATF-damped modal analysis using 10 elements. The square symbols indicate results obtained using the primary form of the governing equations, while the triangular symbols show the results of using the preferred, alternate form of the equations.

Preservation of the characteristic variation of material damping with frequency is apparent in the results. As previously noted, conventional damping modeling techniques are incapable of producing such results. The frequencies predicted using this method appear to converge from above, as is the case with undamped elements. The finite element implemention based on the alternate form of the governing equations exhibits the proper variation of damping with frequency at all frequencies, while the implementation based on the primary equations "rolls off" too rapidly at high frequencies. Predicted frequencies and damping from both implementations improve with increasing numbers of elements.

LARGE SPACE TRUSS MODAL ANALYSIS

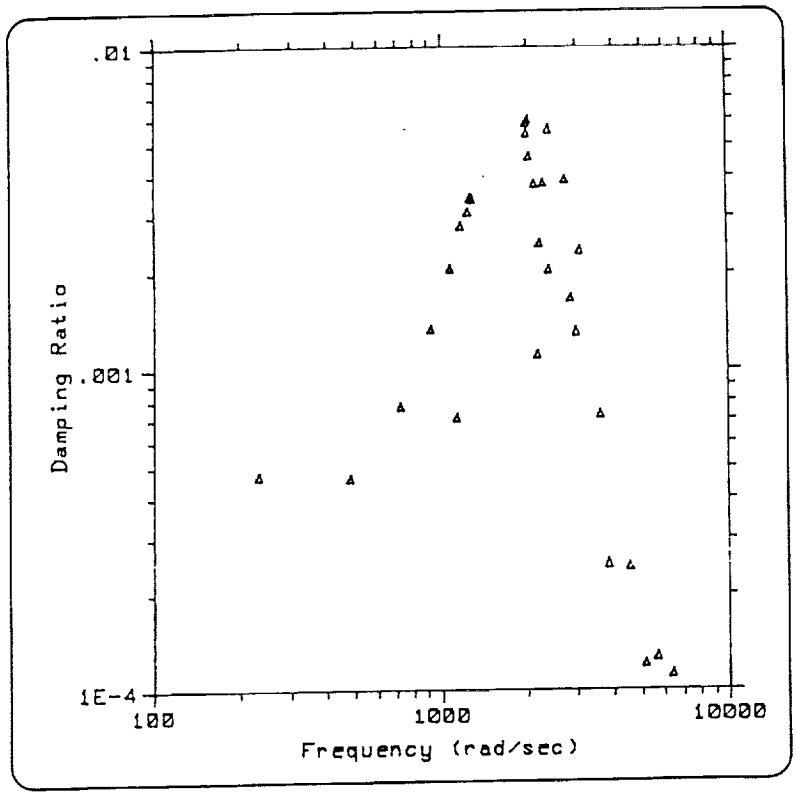


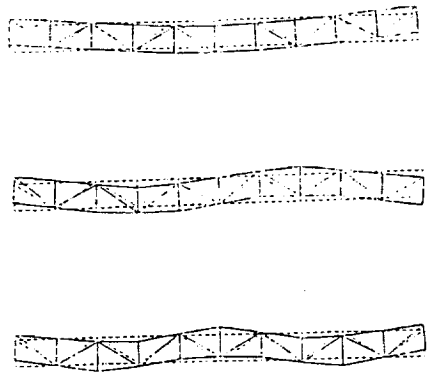
	LONGERON	DIAGONAL	BATTEN
CS area (m2)	31 e-5	19 e-5	6.3 e-5
Modulus (unrelaxed) (Pa)	36.72 e10	18.72 e10	8.4 e10
Density (kg/m3)	2200	1600	2700
Peak damping ratio	0.005	0.01	0.05
Frequency of peak (r/s)	200	2000	8000

The utility of this ATF modeling method is demonstrated in a modal analysis of the previously-described strawman 10-bay, 30-meter planar truss structure. The damped rod elements just described are modified to include the kinetic energy of transverse motion, and to address requirements for interelement continuity of the augmenting fields.

The table above summarizes the key properties of the truss member materials. In particular, the peak damping ratios for each material are different, and are found at different frequencies.

RESULTS OF TRUSS MODAL ANALYSIS





SOME
UNDAMPED
MODE SHAPES

The figure above shows the results of the truss modal analysis in terms of damping versus frequency. Where conventional analysis using the MSE method would have required considerable effort to generate multiple modal damping values, the ATF method delivers them in a single, standard modal analysis. In addition, it delivers more accurate complex modes.

Note that the damping in the lowest bending modes increases quite rapidly with frequency. This is due to two unrelated factors: First, in this kind of truss structure, the fraction of strain energy stored in the diagonal members increases with mode number—this is analagous to the effects of transverse shear in an isotropic beam. Second, in this case, the damping of the material from which the diagonal members are made is increasing at frequencies below 2000 rad/sec. Also note that the 6th flexible mode is an extensional mode, and has considerably lower damping than neighboring bending modes.

RESULTS — TRUSS MODAL ANALYSIS

MODE NO.	UNDAMPED FREQUENCY (R/S)	DAMPED FREQUENCY (R/S)	DAMPING RATIO (10e-3)
4	228.6	231.4	0.47
5	472.2	480.8	0.46
6	701.6	719.3	0.77
7	887.2	917.5	1.32
8	1024.	1069.	2.04
9	1102.	1127.	0.70
10	1109.	1168.	2.77

For comparison, an undamped analysis was performed. The table summarizes the frequency and damping results. The ATF and undamped results agree well in terms of frequency, differing by about one percent for the first mode. The undamped frequencies are lower by as much as five percent in the tenth mode because the relaxed (static) modulus value was used for that analysis, as is usual practice.

SUMMARY

ATF METHOD *AUGMENTING THERMODYNAMIC FIELDS*

- PHYSICALLY-MOTIVATED, PRESERVES CHARACTERISTIC VARIATION OF MATERIAL DAMPING WITH FREQUENCY
- FULLY COMPATIBLE WITH CURRENT FE TECHNIQUES
- MIX DAMPED, UNDAMPED ELEMENTS
- MULTIPLE FIELDS TO FIT DATA

FUTURE WORK

- DAMPING BUDGET
- CONTROLS
- MSE COMPARISON
- OTHER ELEMENTS
- OPTIMIZATION
- IDENTIFICATION
- MATH ASPECTS

A physically-motivated material damping model fully compatible with current computational structural analysis methods has been developed. Termed the Augmenting Thermodynamic Fields (ATF) method, its key feature is the introduction of additional fields to interact with the usual displacement field of continuum structural dynamics. ATF-damped rod elements were developed and provided the basis for a modal analysis of a large space truss structure.

This method preserves the characteristic frequency-dependence of material damping. The results for a single augmenting field per material are readily extended to multiple fields. In addition, the method readily accommodates a combination of damped and undamped elements. Also note that an increase in the accuracy of a structural dynamic model comes with a cost of dimensionality—additional coordinates are required to represent additional aspects of material behavior, *viz*, damping.

With the continued development of analytical tools such as this ATF method, damping will be better treated in the design of engineering systems and may ultimately become more accessible to design specification.

BIBLIOGRAPHY

Ashley, H., "On Passive Damping Mechanisms in Large Space Structures," AIAA Paper 82-0639, 1982.

Bert, C.W., "Material Damping: An Introductory Review of Mathematical Models, Measures, and Experimental Techniques," *Journal of Sound and Vibration*, Vol. 29, No. 2, 1973, pp. 129-153.

Golla, D.F., and Hughes, P.C., "Dynamics of Viscoelastic Structures - A Time-Domain, Finite Element Formulation," *Journal of Applied Mechanics*, Vol. 52, December, 1985, pp. 897-906.

Lesieutre, G.A., and Mingori, D.L., "Finite Element Modeling of Frequency-Dependent Material Damping using Augmenting Thermodynamic Fields," to appear in *Journal of Guidance, Control and Dynamics*, 1990.

Nowick, A.S., and Berry, B.S., *Anelastic Relaxation in Crystalline Solids*, Academic Press, 1972.

Torvik, P.J., and Bagley, D.L., "Fractional Derivatives in the Description of Damping Materials and Phenomena," at the *11th ASME Biennial Conference on Mechanical Vibration and Noise*, Boston, Sept., 1987.

Zener, C.M., *Elasticity and Anelasticity of Metals*, University of Chicago Press, 1948.