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p. 36

AN OVERVIEW OF THE ESSENTIAL DIFFERENCES AND SIMILARITIES OF SYSTEM IDENTIFICATION TECHNIQUES

Raman K. Mehra
Scientific System
500 W. Cummings Pk
Suite 3950
Woburn, MA 01801

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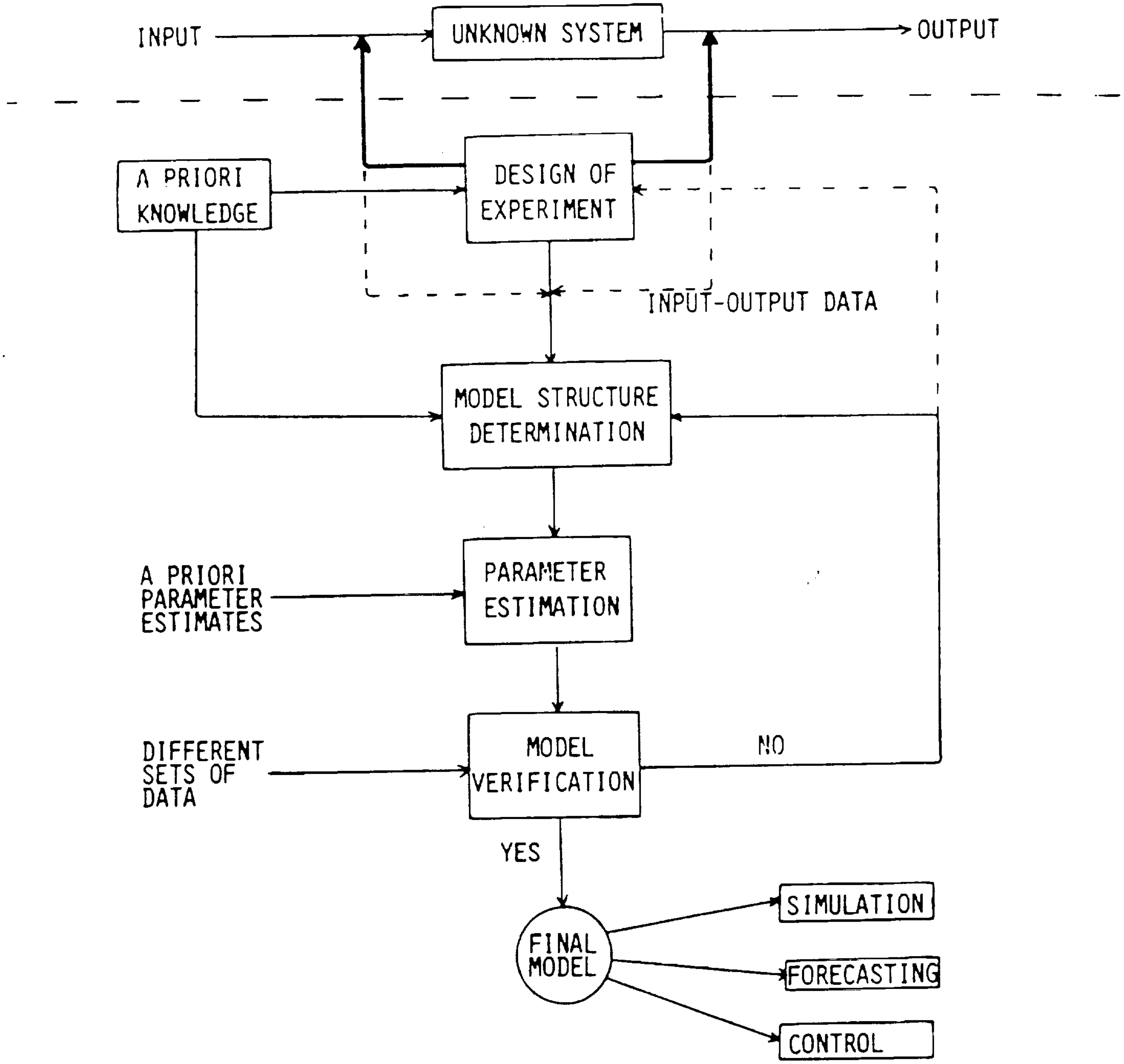
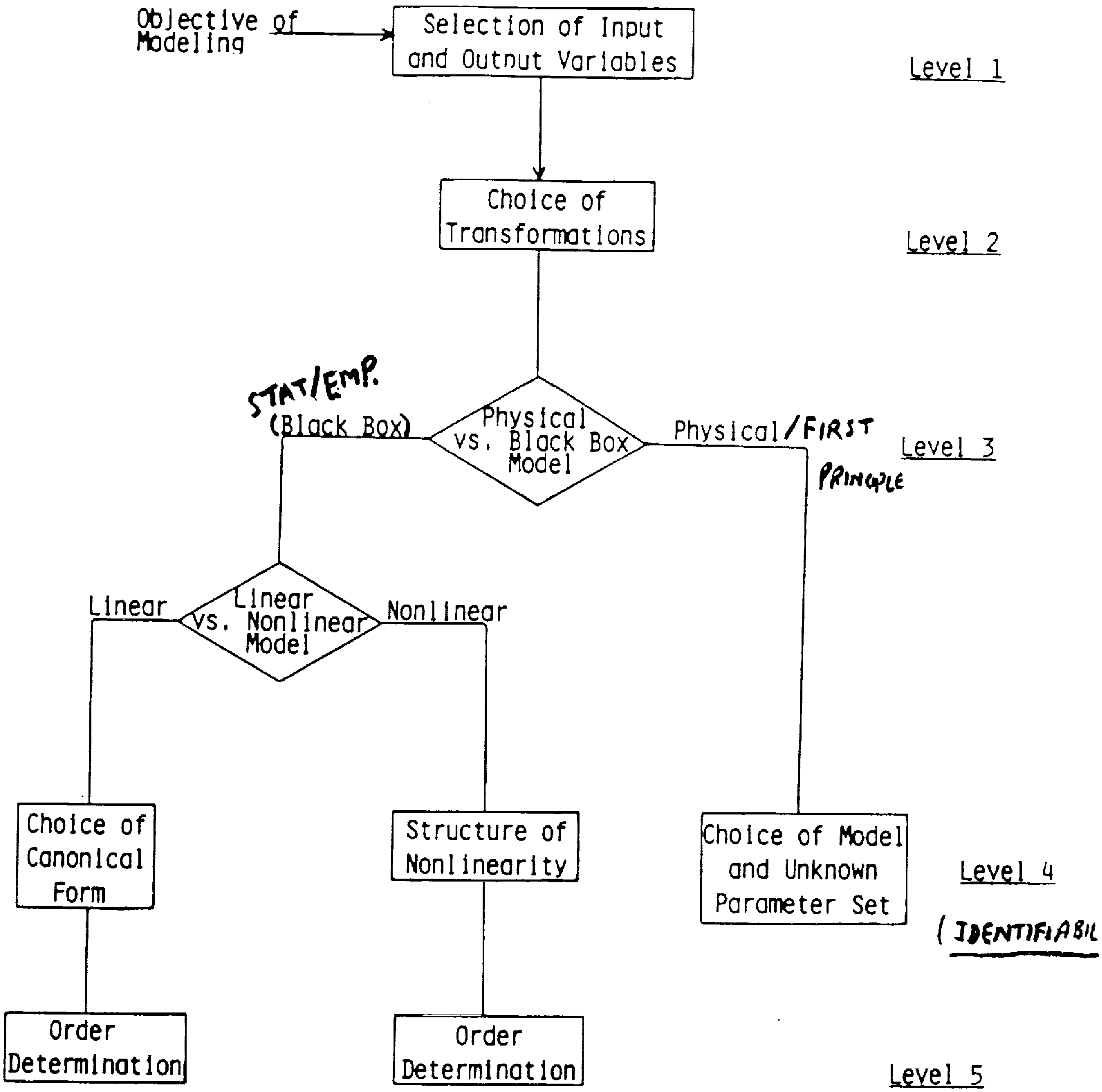
Outline

1. System Identification - Four Basic Steps
2. Bayesian Statistical Decision Theory Framework
3. Maximum Likelihood Estimation and E-M algorithm
4. Similarities and Differences between Indentification Methods
5. Minimal Realizations - Deterministic & Stochastic
6. Structural Mode Identification using Stochastic Realization Algorithm
7. Identification Results
    - Membrane Simulations
    - AL Grid
    - X-29 Flutter Flight Test Data
8. Conclusions



STEPS IN SYSTEM MODELING AND IDENTIFICATION



Hierarchical Levels in Model Structure Determination

2. Bayesian Statistical Decision Theory Framework for System Identification

1. Unknown state of the world denoted by state vector $\{x(t)\}$.

2. A set of models $M_1, M_2, \ldots M_\ell$ with a priori probabilities $p(M_i)$, which describe the evolution of the state $x(t)$.

3. Unknown parameters θ_i associated with Model M_i and prior probibilities $p(\theta_i | M_i)$

4. Vector of observations $\{y\ (t)\}$ related to the state of the system $\{x(t)\}$, according to probability distributions. $p(\{y(t)\} |\ \{x(t\}, M_i)$

5. Loss function $\mathcal{L}\ (M, \hat{M})$ which expresses the loss to the decision maker of choosing model $\hat{M}$ when M is the true model.

Bayesian Solution:

Obtain posterior distributions $p(M_i | \{y(t)\})$ and select M_i which minimizes the expected value of the loss function.

Bayes Rule:

$$p(M_i | \{y(t)\}) = \frac{p(\{y(t)\} | M_i)\ p(M_i)}{p(\{y(t)\})}$$

$$p(\{y(t)\} | M_i\}) = \int_{\otimes} p(\{y(t)\} | M_i, \theta_i)\ p(\theta_i) d\theta_i$$

Likelihood Function:

$$L(M_i, \theta_i) = p(\{y(t)\} | M_i, \theta_i)$$

Maximum Likelihood Estimation

- $\hat{\theta}_{ML}$ maximizes $p(\{y(t)\}|\theta)$ or its logarithm, $LL(\theta) = \log p(\{y(t)\}|\theta)$

- Cramer-Rao Lower Bound:

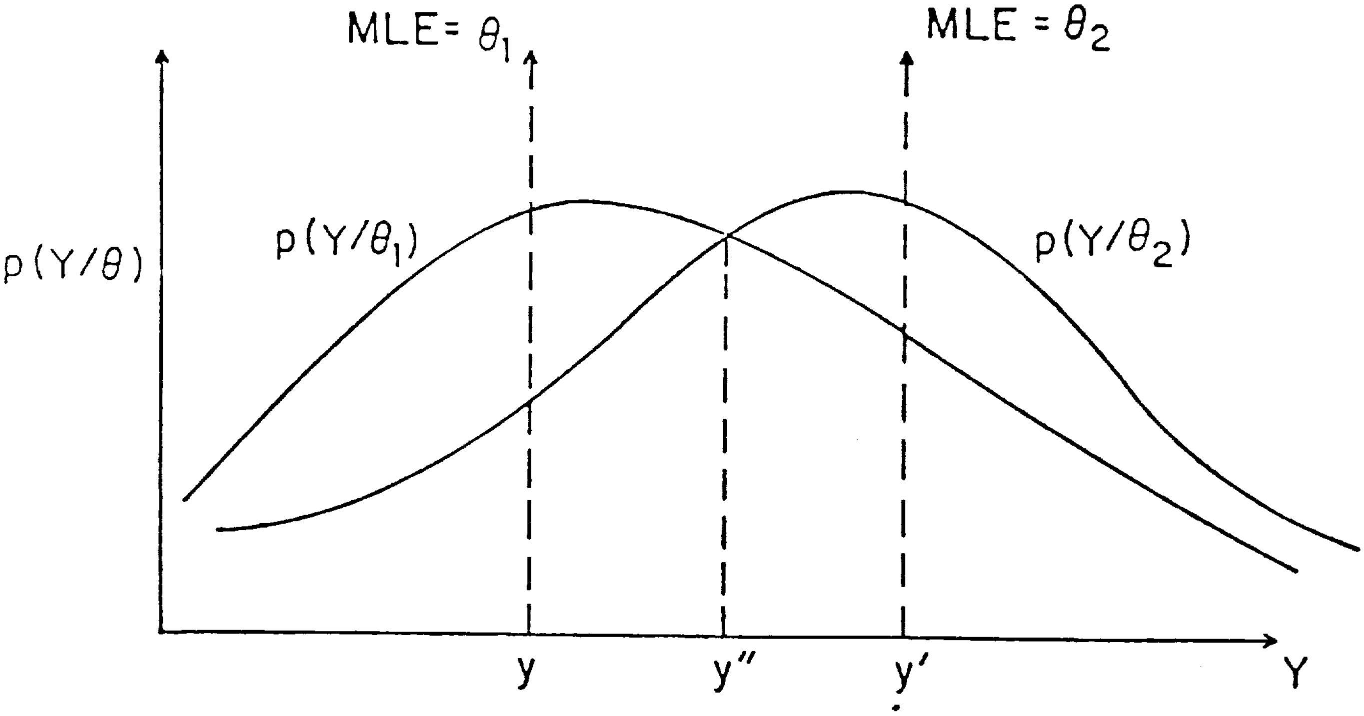
$$\mathrm{cov}\,(\hat{\theta}) \geq \left[E\frac{\partial LL(\theta)}{\partial\theta} \quad \left(\frac{\partial LL(\theta)}{\partial\theta}\right)^T\right]^{-1}$$

  The quantity in brackets is the **Fisher Information Matrix** which is very useful for determining Identifiability and for Input Design.

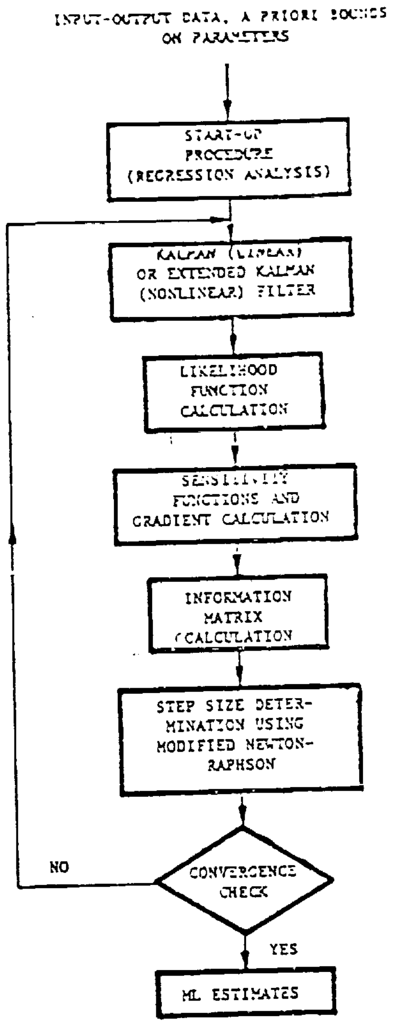
- when $p(\{y(t)\}|\theta)$ is Gaussian and θ effects the conditional mean linearly, $LL(\theta)$ is quadratic in θ and MLE is same as Weighted Least Squares.

- $\hat{\theta}_{ML}$ can be obtained by a sequence of Expectation & Maximization steps (E-M Algorithm), each one of which is simpler than direct maximization of the Likelihood Function.

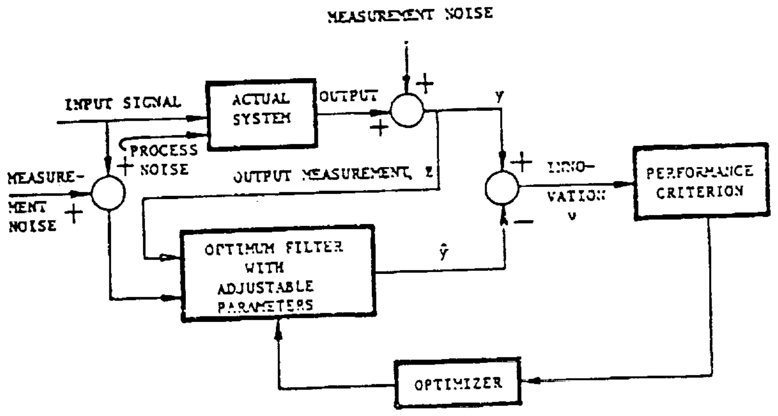
  E-step: Estimate state given parameters
  E-step: Estimate parameters given state statistics.



MAXIMUM LIKELIHOOD ESTIMATION



Steps in Maximum Likelihood Estimation



Block Diagram for Maximum Likelihood

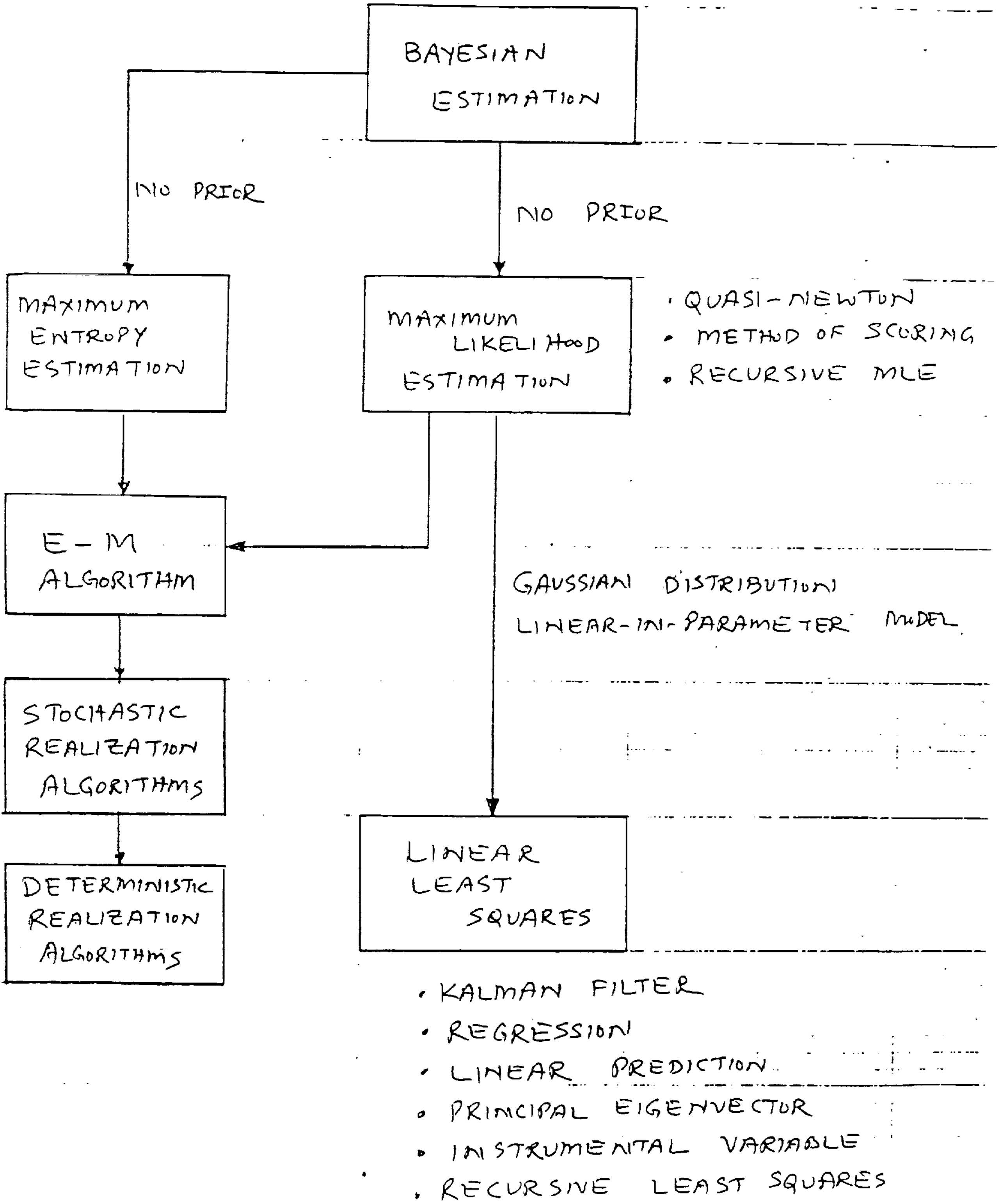


FIG: RELATIONSHIP OF DIFFERENT TECHNIQUES FOR PARAMETER ESTIMATION

Combined Model Structure Determination and Parameter Estimation for Linear Systems

$$x_{t+1} = A\,x_t + B\,u_t + K\,e_t$$

$$y_t = Cx_t + D\,u_t + e_t$$

$$t = 1, \ldots N$$

Stochastic Realization:

Given $\{y_t, u_t\}$, identify system order n and matrices A, B, C, D, K and Cov (e_t).

Deterministic Realization:

Given impulse response parameters $CA^{k-1}B$, identify A, B, C and n

Solutions:

- Deterministic (Ho-Kalman, Balanced Realization, ERA)
- Stochastic (Akaike, Mehra, Aoki, QMARKOV, CVA, SRA)

Stochastic Realization Algorithm

DEFINE:

$p_t = \text{column}\left[y_{t-1}\ u_t, y_{t-2}, u_{t-1}, \ldots \right]$, Past

$f_t = \text{column}\left[y_t, y_{t+1}, y_{t+2}, \ldots \right]$, Future

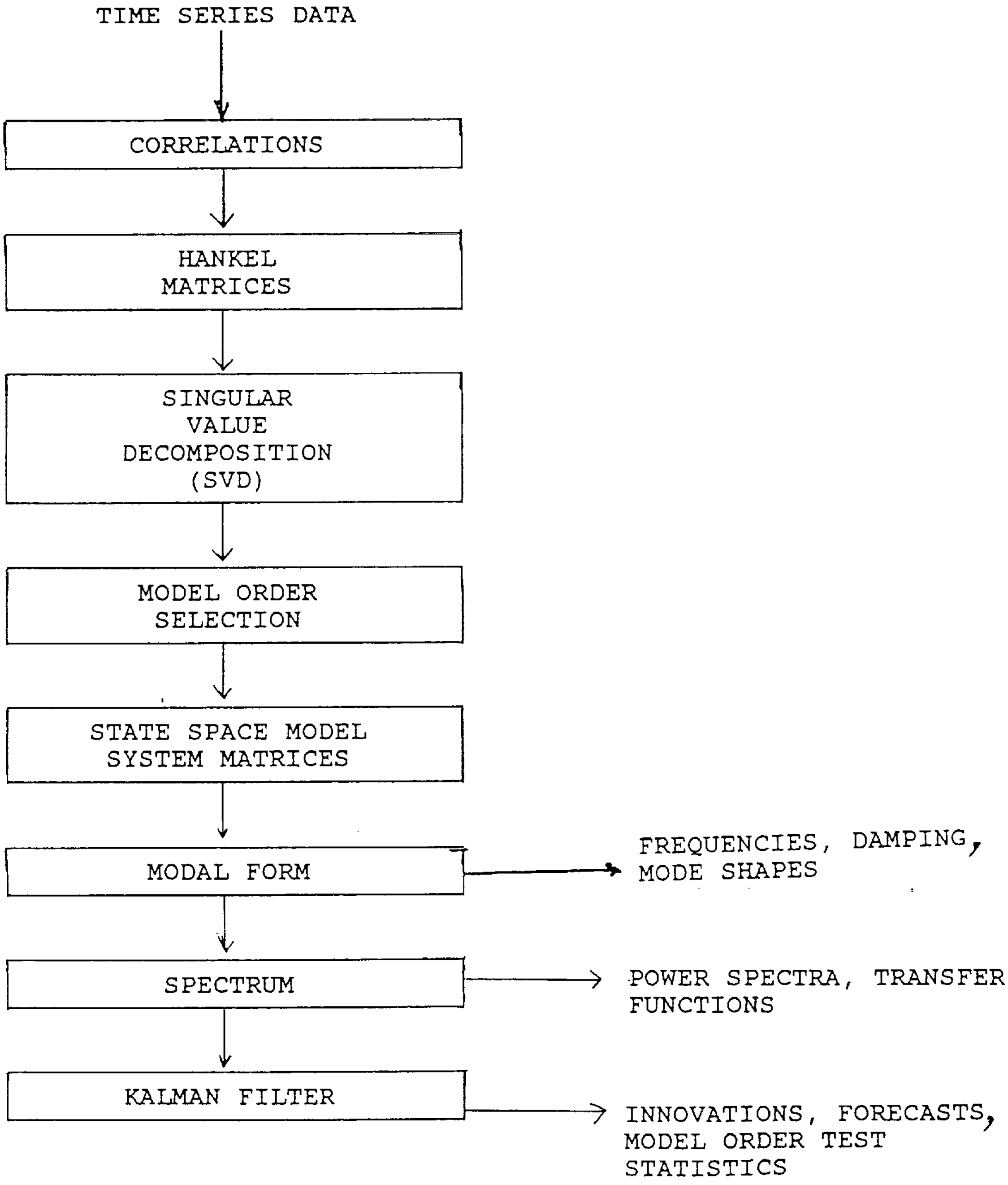
Correlation of Past & Future:

$H = E\,(f_t\, p_t{}')$, Hankel Matrix

Singular Value Decomposition:

$H = U \Sigma V'$

STEPS IN SS-SRA



Identification Results

1. Membrane Simulation:

   - Data provided by Mark Norris, AFAL
   - Two lightly damped modes at 0.5Hz
   - 5 velocity measurements contaminated with different levels of multiplicative noise (1% to 1,000%).
   - SRA results are satisfactory for <u>300% noise!</u>

FREE-FREE MEMBRANE SIMULATION

- DATA GENERATED BY MARK NORRIS (AFAL) WITH NOISE RANGING FROM 0% TO 1000% AND 2 DIFFERENT INITIAL CONDITIONS (VEL 1 AND VEL 2).

- TRUE MODEL:

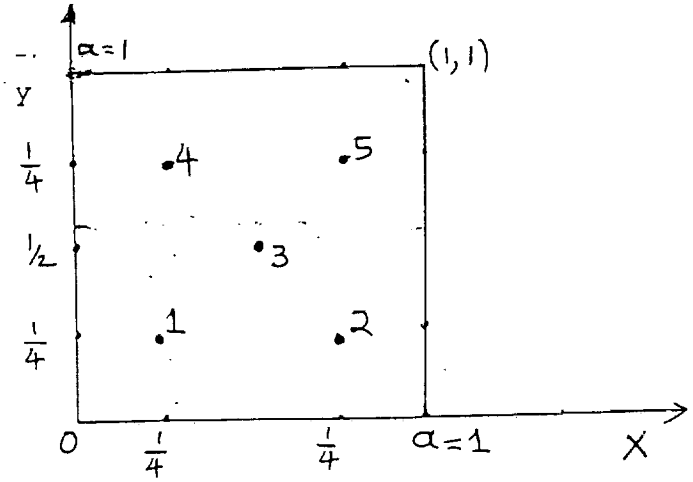
NATURAL FREQUENCIES	DAMPING
0 Hz	0
0.5 Hz	0.01
0.5 Hz	0.01
(REPEATED EIGENVALUE)	

MODE SHAPES:

$$\phi_1(X, Y) = 1$$

$$\phi_2(X, Y) = \cos(\Pi X)$$

$$\phi_3(X, Y) = \cos(\Pi Y)$$



TENSION T=1 N/

DENSITY ρ=1kg/

a=1m

MEMBRANE SENSOR LOCATIONS

IDENTIFICATION RUNS BY SCIENTIFIC SYSTEMS ON

MEMBRANE SIMULATION DATA USING SS-SRA

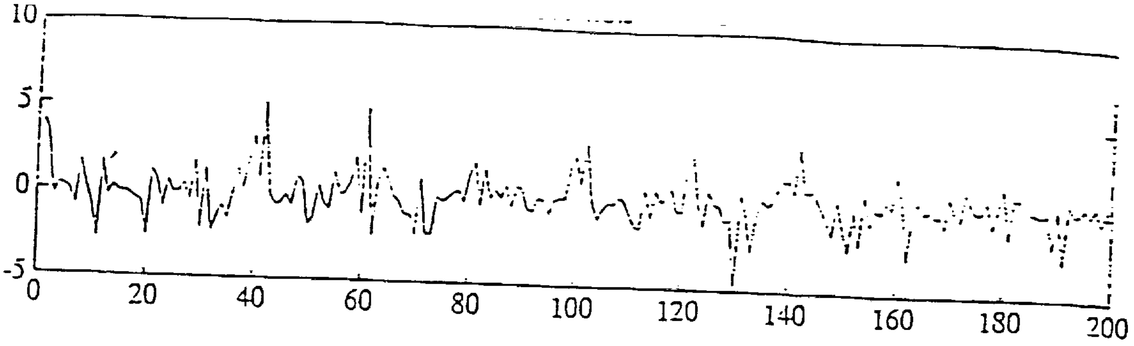
1. IDENTIFICATION DONE USING BOTH FREE AND FORCED RESPONSE DATA WITH 200 POINTS FOR 5 AND 10 OUTPUTS AT 0.1 HZ.

2. FREE RESPONSE RESULTS ARE BETTER DUE TO POOR INPUT EXCITATION FOR THE FORCED RESPONSE DATA

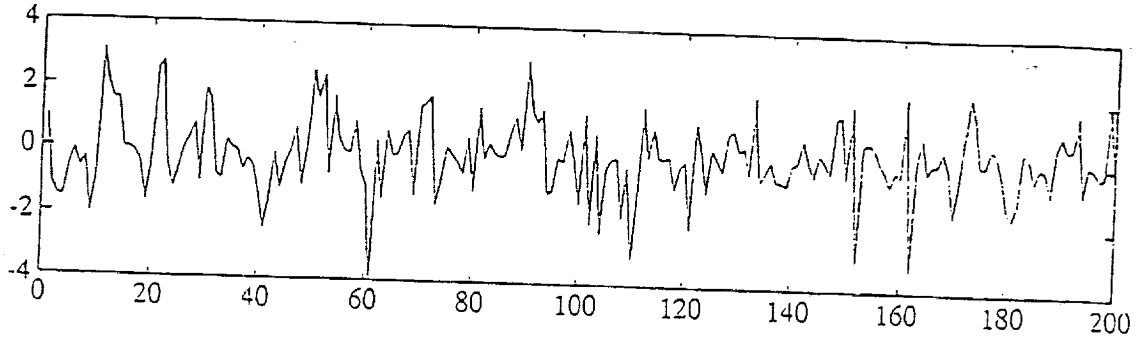
3. FREE RESPONSE RESULTS WITH VEL2 INITIAL CONDITION (IC) ARE BETTER THAN VEL1 IC RESULTS DUE TO BETTER OUTPUT EXCITATION

4. BEST RESULTS ARE OBTAINED BY AVERAGING THE CORRELATIONS FROM VEL1 AND VEL2 OUTPUTS, PARTICULARLY IN THE IDENTIFICATION OF MODE SHAPES. THESE RESULTS ARE SUPERIOR TO RESULTS USING 10 OUTPUTS AND CONCATENATED DATA (400 POINTS).

5. SATISFACTORY RESULTS ARE OBTAINED BY SS-SRA FOR ALL NOISE CASES INCLUDING 300% NOISE, AS OPPOSED TO ERA RESULTS WHICH DEGRADE GREATLY AFTER 50% NOISE. THE ACCURACY OF THE RESULTS IS SURPRISING CONSIDERING THE NOISE IS MULTIPLICATIVE.

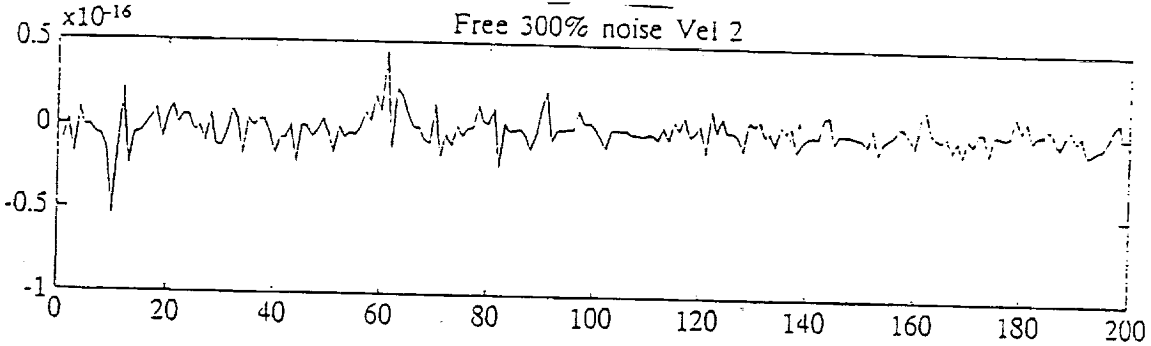


Series 1

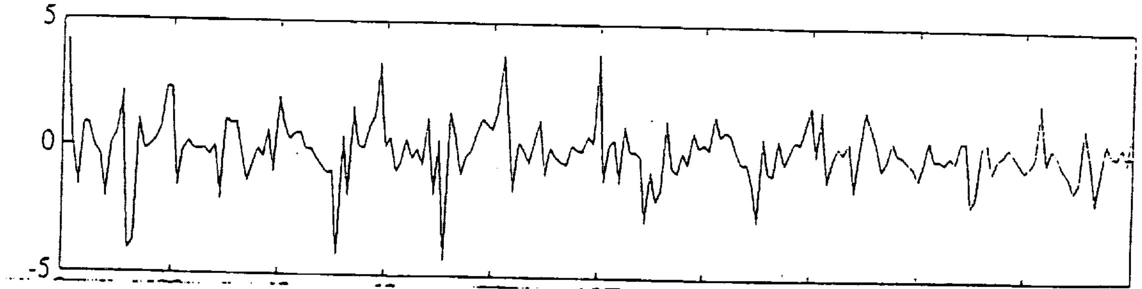


Series 2

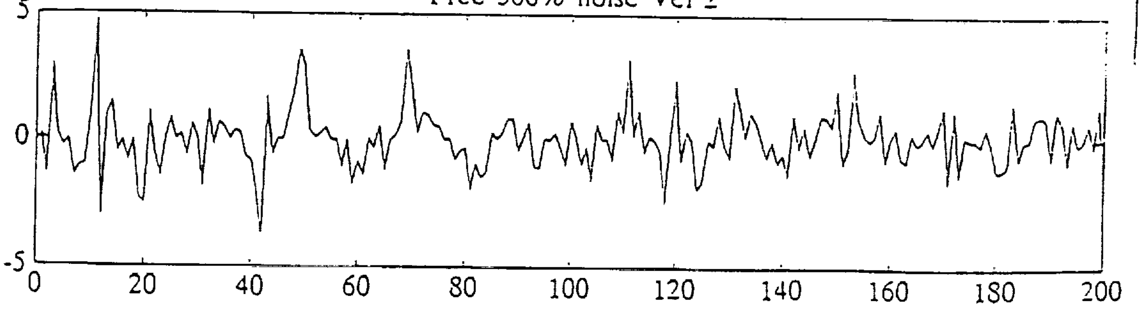
Free 300% noise Vel 2



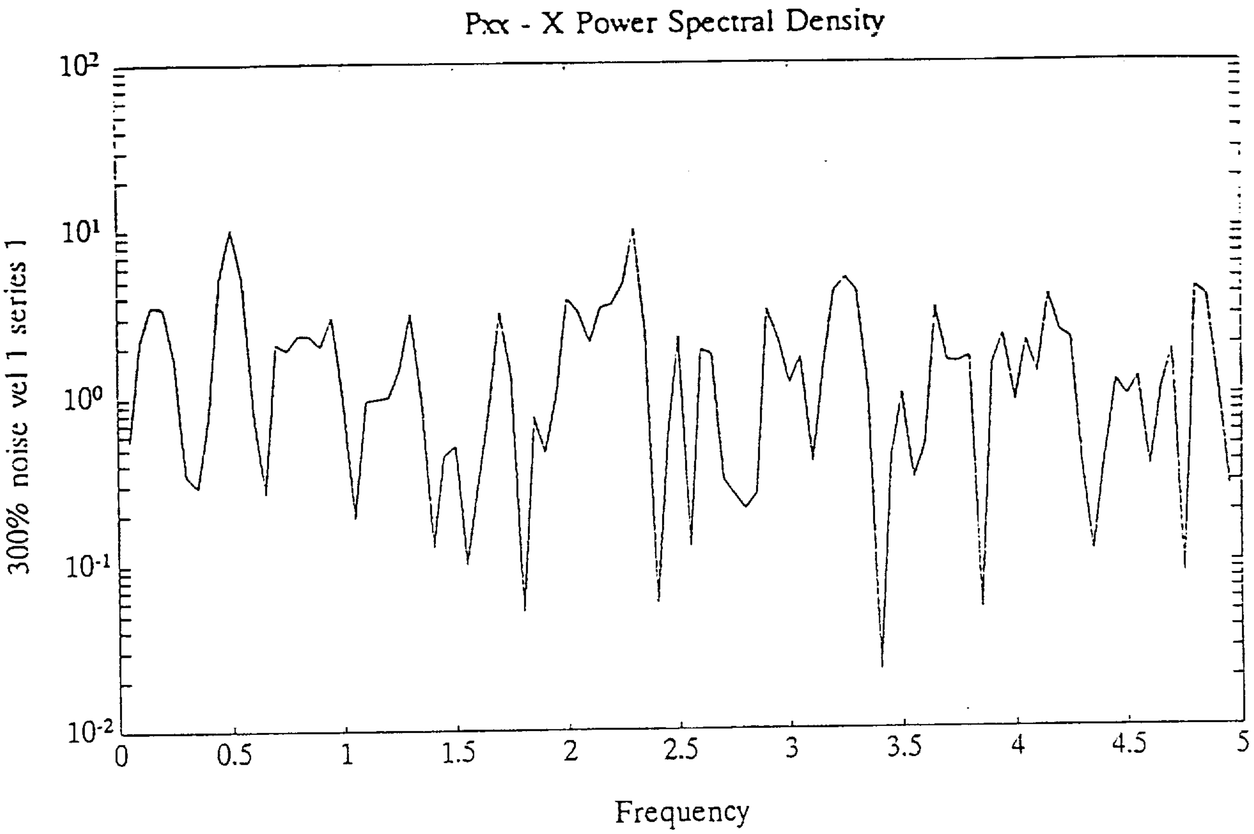
Series 3

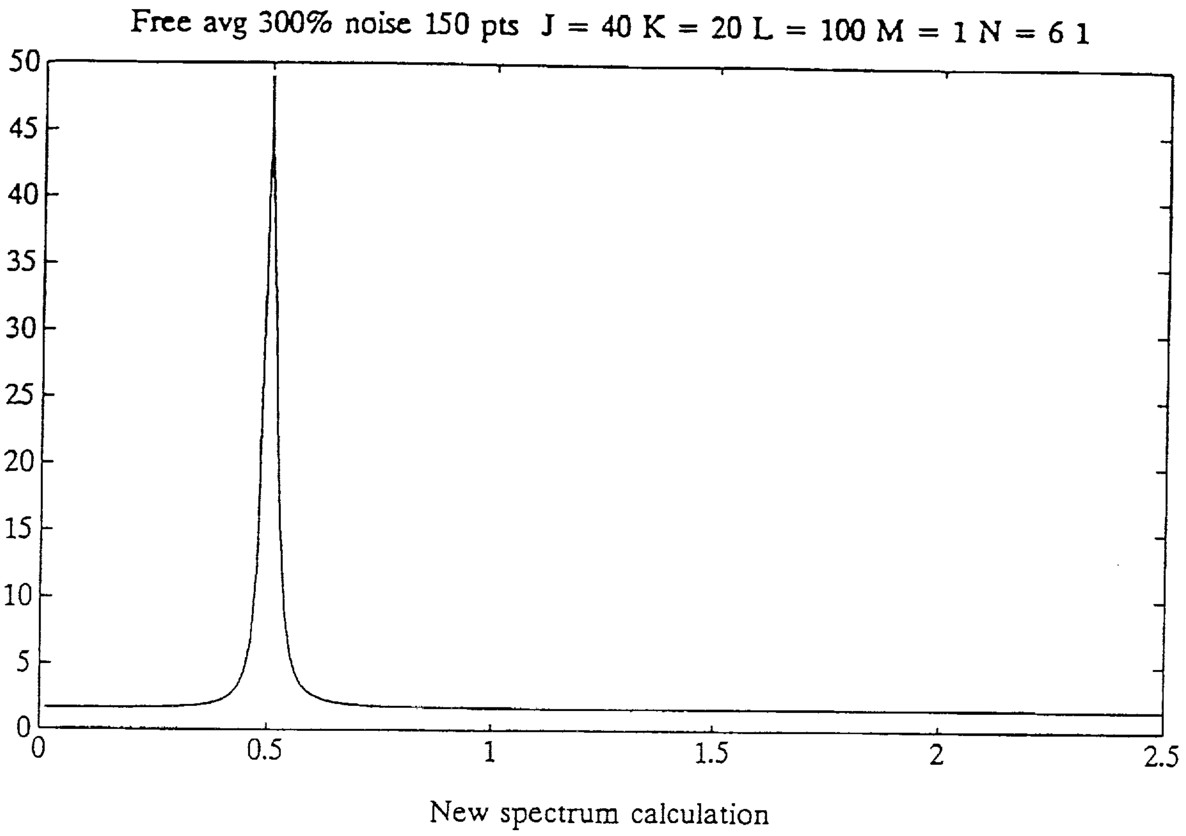


Free 300% noise Vel 2

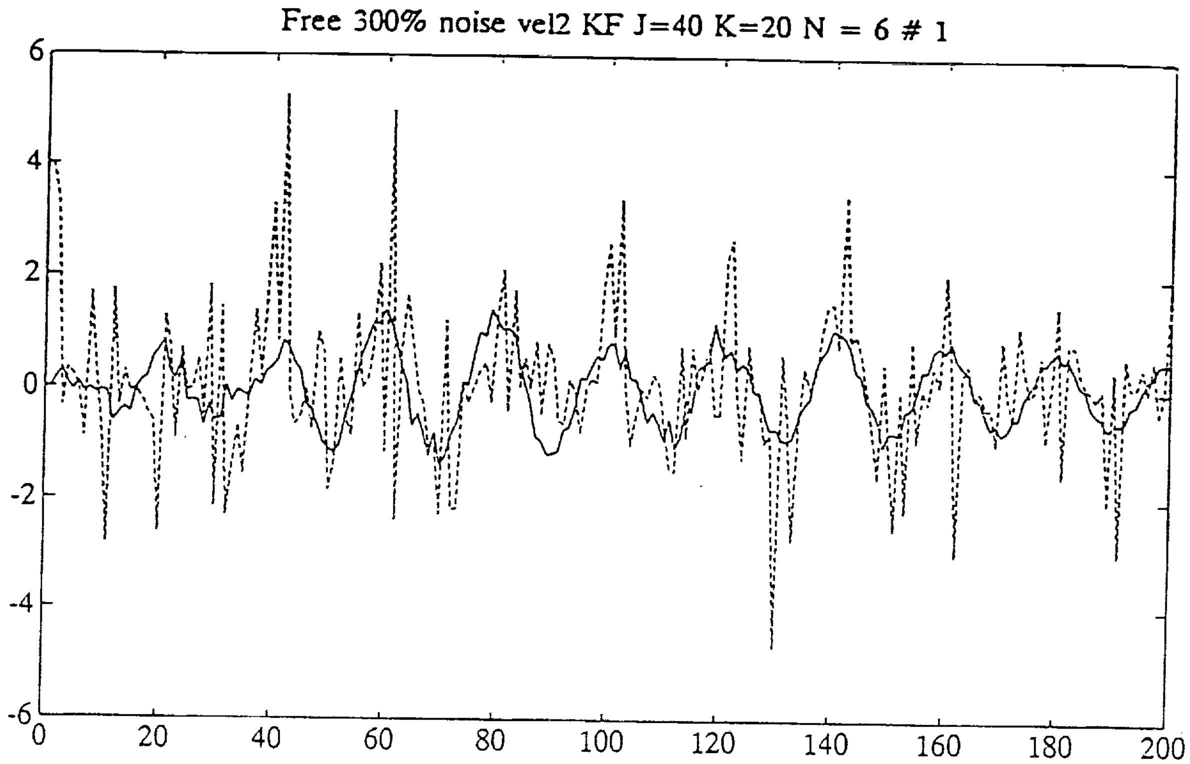


Series 5

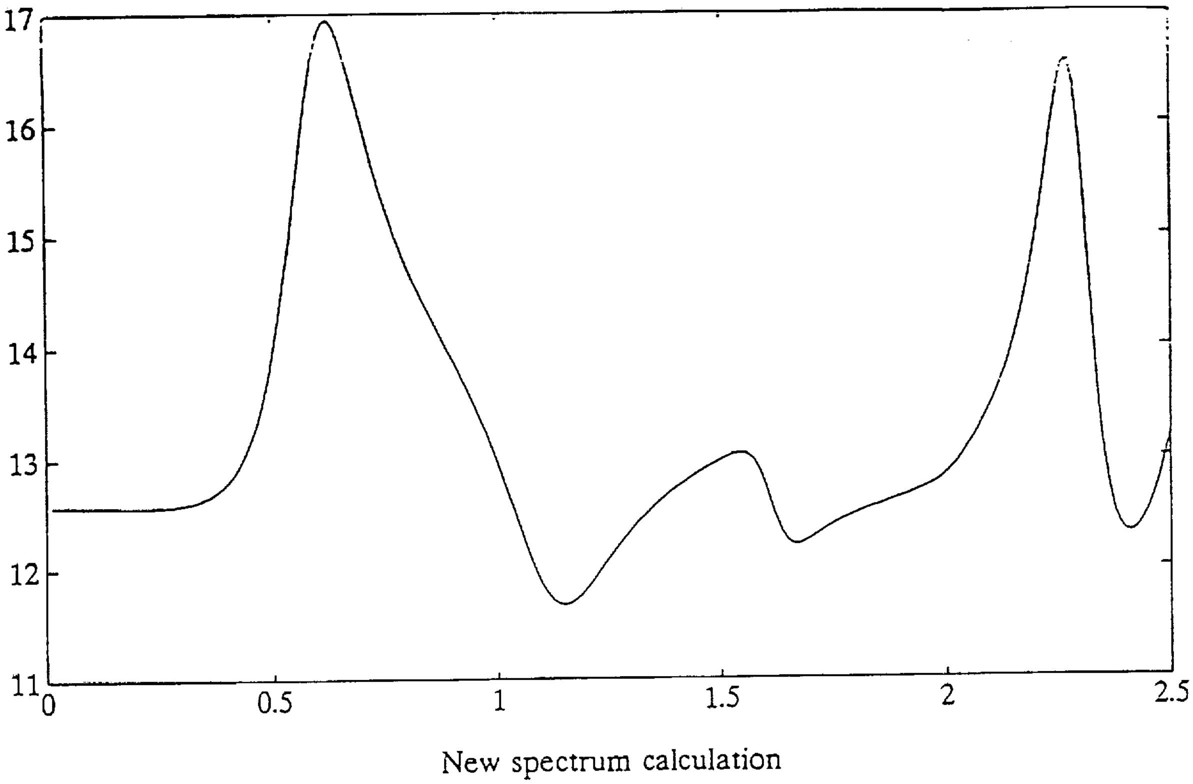

Pxx - X Power Spectral Density
10^2
10^1
10^0
10^-1
10^-2
300% noise vel 1 series 1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency


Free avg 300% noise 150 pts J = 40 K = 20 L = 100 M = 1 N = 6 1
50
45
40
35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
New spectrum calculation

```
ans =

Free avg 300% noise 150 pts  J = 40 K = 20 L = 100 M = 1 N = 6


ans =

Magnitude    Phase      Freq      Damping       Air Force Values


ans =

    0.9808     2.3658     3.7655     0.0082          0          0
    0.9808    -2.3658     3.7655     0.0082     0.5000     0.0100
    0.9920     0.3120     0.4968     0.0256     0.5000     0.0100
    0.9920    -0.3120     0.4968     0.0256          0          0
    0.9899     0.3206     0.5105     0.0318          0          0
    0.9899    -0.3206     0.5105     0.0318          0          0


ans =

      Singular Values


ans =

    1.0000    7.4279
    2.0000    7.4144
    3.0000    2.8451
    4.0000    2.8444
    5.0000    2.2254
    6.0000    2.2090
    7.0000    1.6678
    8.0000    1.6350
    9.0000    1.6062
   10.0000    1.5315
   11.0000    1.5038
   12.0000    1.4938
   13.0000    1.4624
   14.0000    1.4458
   15.0000    1.4098
   16.0000    1.3894


cyy0 =

  Columns 1 through 3

     1.954367688700000e+00    -9.804237999999993e-03     1.222204885150001e-03
    -9.804237999999993e-03     1.533202479950000e+00    -1.735176744450000e-03
     1.222204885150001e-03    -1.735176744450000e-03     4.806341140050000e-04
     1.715561279090000e-01    -1.606591878000000e-01     1.414021973600000e-03
    -4.222937015000000e-01     2.431914045500003e-02     3.260060618799988e-04

  Columns 4 through 5

     1.715561279090000e-01    -4.222937015000000e-01
    -1.606591878000000e-01     2.431914045500003e-02
```


Free 300% noise vel2 KF J=40 K=20 N = 6 # 1
6
4
2
0
-2
-4
-6
0
20
40
60
80
100
120
140
160
180
200

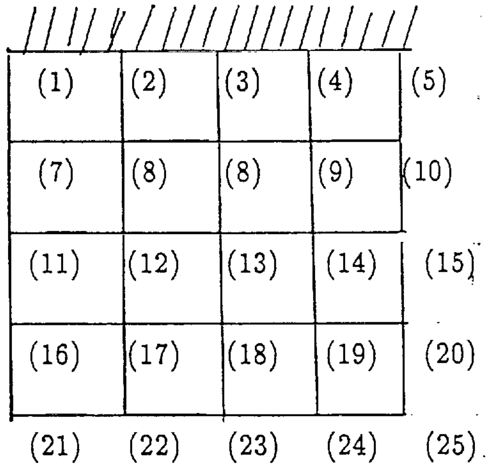
free avg 10Hz 1000% noise J = 40 K = 20 L = 100 M = 1 N = 29 2



AFAL Grid Data

- Data provided by Mark Norris, AF Astronautics Lab
- Three sets of data at 100hz.
  - (i) PMASS Input (2000 pts)
  - (ii) X-TORQUE Input (24,000 pts)
  - (iii) Y-TORQUE Input (24,000 pts)
- SRA identifies all 11 modes in all cases and proivides good estimates of freq., dampings and mode shapes.
- ERA identifies all 11 modes only for combined X and Y Torque inputs
- Damping estimates from SRA are satisfactory with 2000 pts of P-MASS Input.
- SRA gives satisfactory identification results with single output.

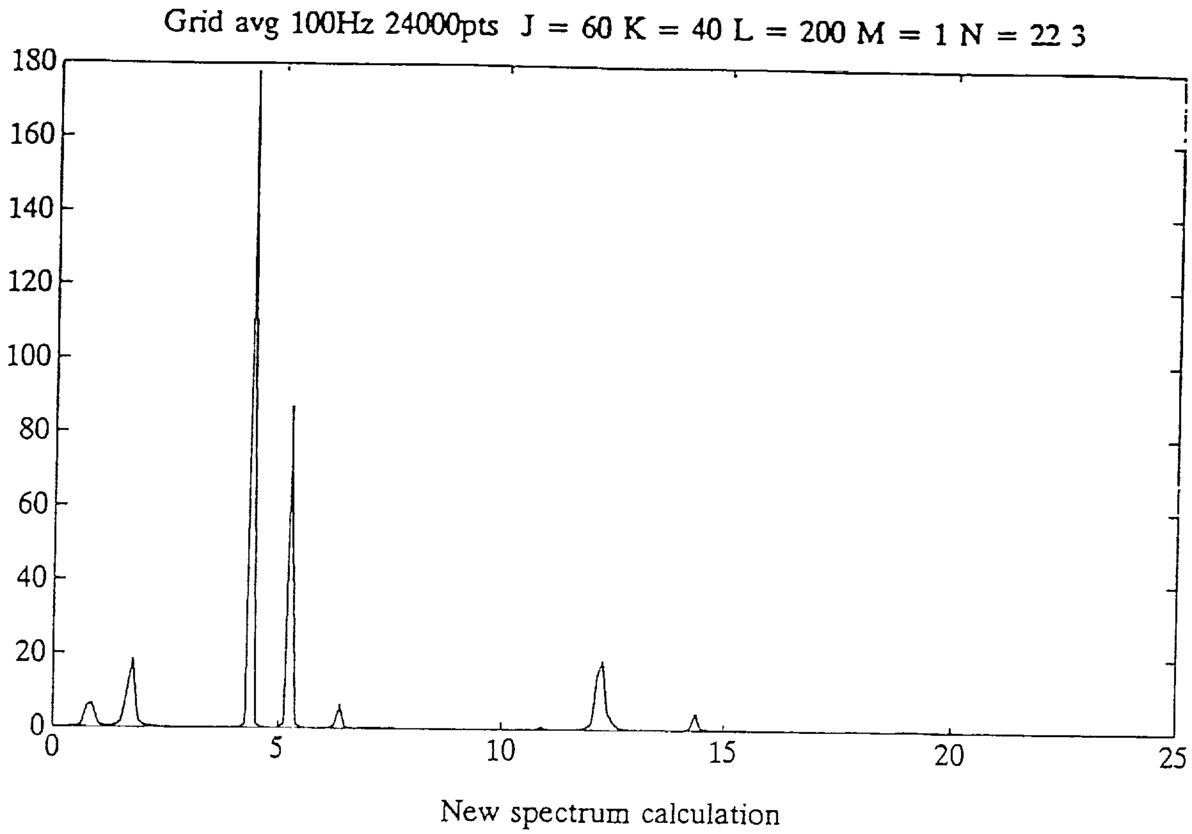
AFAL GRID



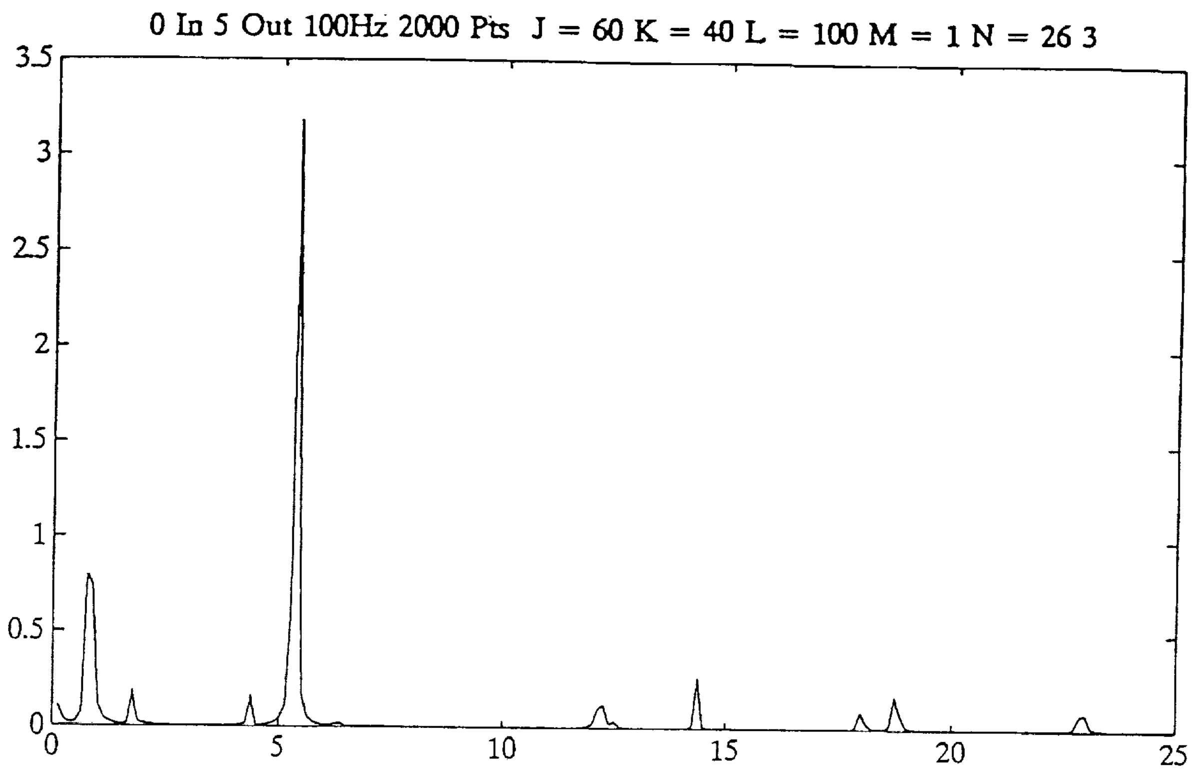
↑

→ X Z - OUT OF PAPER (NORMAL TO GRID)

NODES (1) – (5) HAVE NO DISPLACEMENT AND NO ROTATION.


Grid avg 100Hz 24000pts J = 60 K = 40 L = 200 M = 1 N = 22 3
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
New spectrum calculation

```
ans =

Grid avg 100Hz 24000pts  J = 60 K = 40 L = 200 M = 1 N = 22


ans =

Magnitude    Phase      Freq     Damping      Air Force Values


ans =

    0.9978    1.1919   18.9699    0.0019     0.8100    0.0023
    0.9978   -1.1919   18.9699    0.0019     1.6610    0.0016
    0.9987    1.1350   18.0648    0.0011     4.3550    0.0013
    0.9987   -1.1350   18.0648    0.0011     5.2120    0.0017
    0.9980    0.9013   14.3439    0.0023     6.2910    0.0013
    0.9980   -0.9013   14.3439    0.0023    10.7550    0.0011
    0.9986    0.7822   12.4486    0.0018    12.1530    0.0016
    0.9986   -0.7822   12.4486    0.0018    12.3840    0.0016
    0.9986    0.7659   12.1900    0.0018    14.2540    0.0019
    0.9986   -0.7659   12.1900    0.0018    17.9550    0.0015
    0.9991    0.6813   10.8428    0.0013    18.7720    0.0010
    0.9991   -0.6813   10.8428    0.0013          0         0
    0.9994    0.1066    1.6967    0.0053          0         0
    0.9994   -0.1066    1.6967    0.0053          0         0
    0.9993    0.0512    0.8142    0.0132          0         0
    0.9993   -0.0512    0.8142    0.0132          0         0
    0.9991    0.3986    6.3441    0.0022          0         0
    0.9991   -0.3986    6.3441    0.0022          0         0
    0.9994    0.3292    5.2401    0.0019          0         0
    0.9994   -0.3292    5.2401    0.0019          0         0
    0.9994    0.2750    4.3775    0.0021          0         0
    0.9994   -0.2750    4.3775    0.0021          0         0


ans =

      Singular Values


ans =

    1.0000    7.4654
    2.0000    6.0473
    3.0000    4.9555
    4.0000    3.1561
    5.0000    2.9364
    6.0000    2.8078
    7.0000    2.4539
    8.0000    2.3086
    9.0000    1.0343
   10.0000    0.9902
   11.0000    0.7768
   12.0000    0.7767
   13.0000    0.7255
   14.0000    0.7051
   15.0000    0.1895
   16.0000    0.1849
```

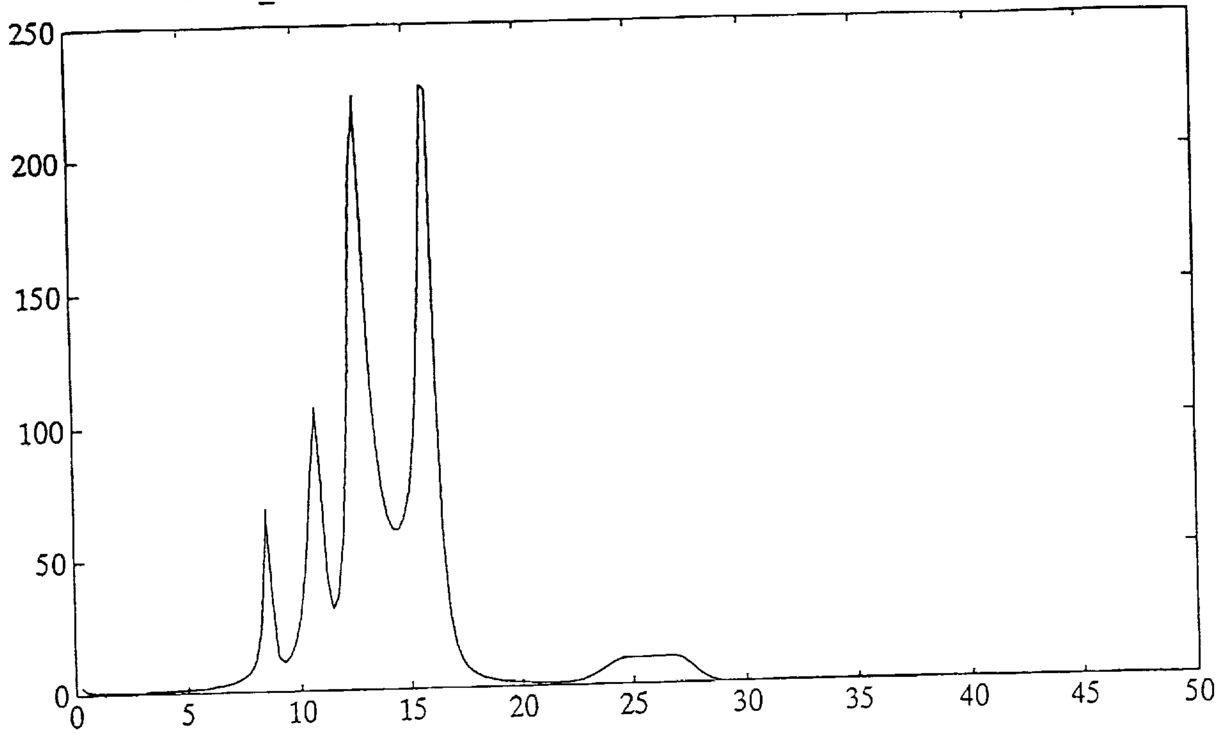

0 In 5 Out 100Hz 2000 Pts J = 60 K = 40 L = 100 M = 1 N = 26 3
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25

```
ans =

0 In 5 Out 100Hz 2000 Pts  J = 40 K = 20 L = 100 M = 1 N = 26


ans =

Magnitude    Phase     Freq     Damping     Air Force Values    Computed Values


ans =

                                                                        .0019
   0.9853    1.4770   23.5089    0.0100    0.8100     0.0023           .0078
   0.9853   -1.4770   23.5089    0.0100    1.6610     0.0016           .0024
   0.0163    3.1416   82.3967    0.7948    4.3550     0.0013           .0023
   0.9984    1.1810   18.7954    0.0013    5.2120     0.0017           .0044
   0.9984   -1.1810   18.7954    0.0013    6.2910     0.0013           .0022
   0.9985    1.1336   18.0411    0.0014   10.7550     0.0011           .0036
   0.9985   -1.1336   18.0411    0.0014   12.1530     0.0016           .0017
   0.9989    0.9029   14.3699    0.0013   12.3840     0.0016           .0013
   0.9989   -0.9029   14.3699    0.0013   14.2540     0.0019           .0014
   0.9986    0.7821   12.4471    0.0017   17.9550     0.0015           .0013
   0.9986   -0.7821   12.4471    0.0017   18.7720     0.0010
   0.9972    0.7675   12.2151    0.0036         0          0
   0.9972   -0.7675   12.2151    0.0036         0          0
   0.9985    0.6840   10.8855    0.0022         0          0
   0.9985   -0.6840   10.8855    0.0022         0          0
   0.9982    0.3981    6.3367    0.0044         0          0
   0.9982   -0.3981    6.3367    0.0044         0          0
   0.9992    0.3355    5.3394    0.0023         0          0
   0.9992   -0.3355    5.3394    0.0023         0          0
   0.9993    0.2752    4.3797    0.0024         0          0
   0.9993   -0.2752    4.3797    0.0024         0          0
   0.9992    0.1078    1.7151    0.0078         0          0
   0.9992   -0.1078    1.7151    0.0078         0          0
   0.9994         0    0.0099    1.0000         0          0
   0.9990    0.0508    0.8082    0.0190         0          0
   0.9990   -0.0508    0.8082    0.0190         0          0


ans =

      Singular Values


ans =

    1.0000    2.5860
    2.0000    2.3951
    3.0000    2.3470
    4.0000    2.0355
    5.0000    1.9648
    6.0000    0.2399
    7.0000    0.2144
    8.0000    0.2021
    9.0000    0.1929
   10.0000    0.1778
   11.0000    0.0959
   12.0000    0.0919
```

X-29 Flutter Flight Test Data

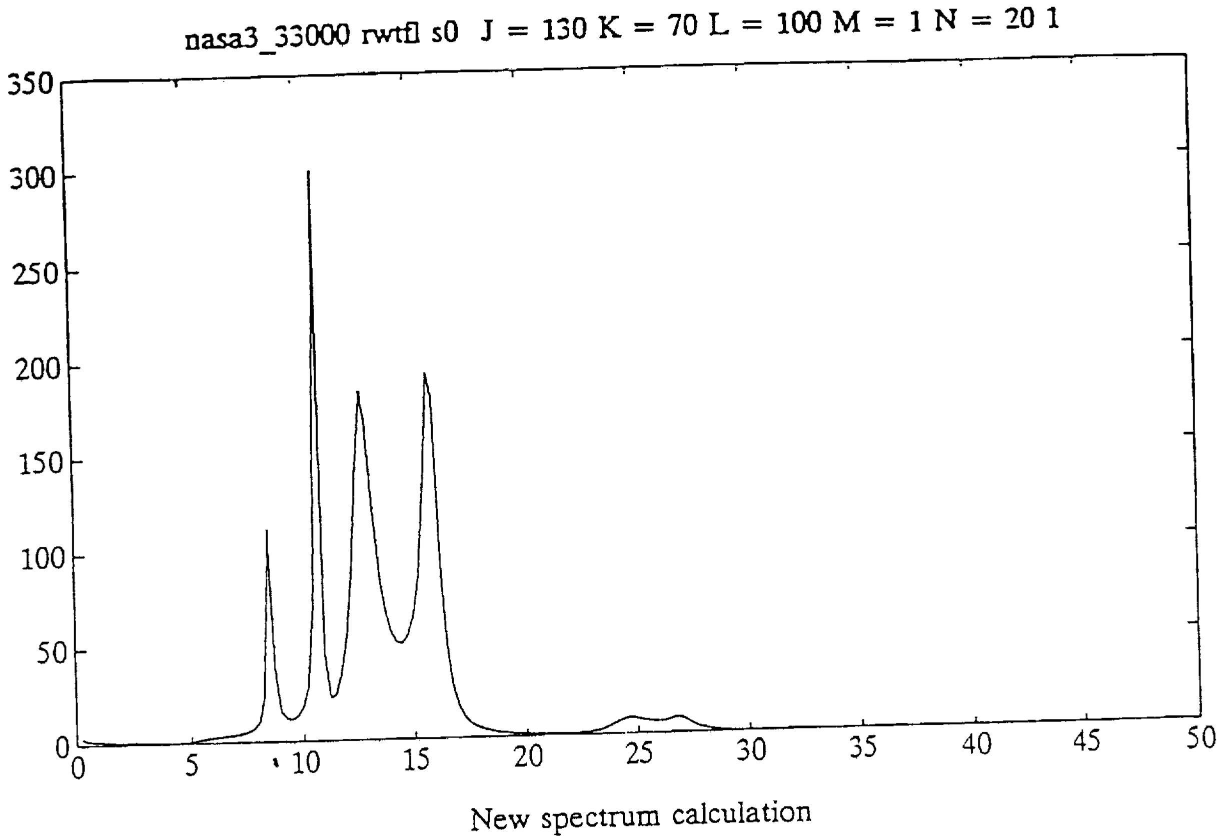
- Data provided by Mike Kehoe, NASA Dryden (33,000 points, 400hz).
- X-29 Flutter modes are excited by natural turbulence
- SRA identifies all the modes from each data channel.
- Damping and frequency estimates in good agreement with NASA values.
- SRA parameter estimates based on 2,000 points are satisfactory.

nasa3_33000 rwtfl s0 J = 130 K = 70 L = 100 M = 1 N = 14 1



New spectrum calculation

```
ans =

nasa3_33000 rwtfl s0  J = 130 K = 70 L = 100 M = 1 N = 14


ans =

Magnitude      Phase       Freq      Damping       Nasa Values


ans =

     0.6474          0    27.6811     1.0000     8.5900          .0182
     0.9845     0.4328    27.5681     0.0361    10.6400          .0370
     0.9845    -0.4328    27.5681     0.0361    12.7900          .0224
     0.9774     0.3760    23.9821     0.0606    15.7200          .0286
     0.9774    -0.3760    23.9821     0.0606          0              0
     1.0000          0     0.0023     1.0000          0              0
     0.9941     0.2502    15.9314     0.0236          0              0
     0.9941    -0.2502    15.9314     0.0236          0              0
     0.9924     0.1979    12.6095     0.0387          0              0
     0.9924    -0.1979    12.6095     0.0387          0              0
     0.9973     0.1348     8.5838     0.0197          0              0
     0.9973    -0.1348     8.5838     0.0197          0              0
     0.9936     0.1694    10.7909     0.0377          0              0
     0.9936    -0.1694    10.7909     0.0377          0              0


ans =

         Singular Values


ans =

     1.0000   60.7314
     2.0000   41.1338
     3.0000   38.8540
     4.0000   23.9111
     5.0000   23.5237
     6.0000    5.4193
     7.0000    5.0483
     8.0000    4.6103
     9.0000    4.4074
    10.0000    3.6840
    11.0000    3.4194
    12.0000    1.2080
    13.0000    1.0580
    14.0000    0.6707
    15.0000    0.6336
    16.0000    0.4258
    17.0000    0.3790
    18.0000    0.3534
    19.0000    0.3174
    20.0000    0.1659
    21.0000    0.1548
    22.0000    0.1426
    23.0000    0.1332
    24.0000    0.1189
```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300
250
200
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100
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0
0
5
10
15
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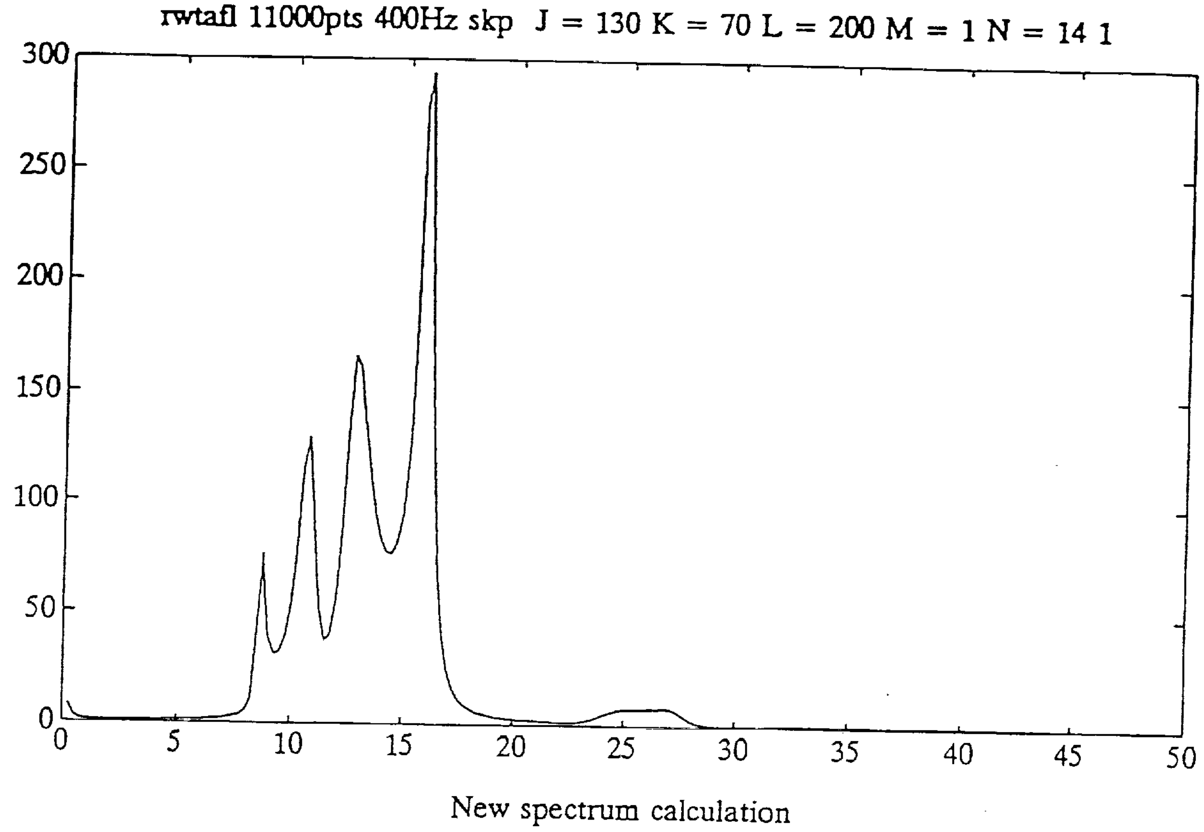
25
30
35
40
45
50
New spectrum calculation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300
250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
45
50
New spectrum calculation

CONCLUSIONS

1. BAYESIAN STATISTICAL DECISION TEHORY PROVIDES A UNIFYING FRAMEWORK FOR SYSTEM IDENTIFICAION TECHNIQUES. FUNDAMENTAL SIMILARITIES ALSO EXIST BETWEEN STRUCTURAL MODE IDENTIFICATION AND EXTRACTION OF SINUSOIDS FROM NOISE. (APPLICATION IN RADAR AND SONAR SIGNAL PROCESSING

2. STOCHASTIC REALIZATION ALGORITHM (SRA) IS HIGHLY ROBUST AGAINST NOISE. IT PROVIDES COMPLETE IDENTIFICATION OF MULTI-INPUT MULTI-OUTPUT SYSTEMS IN A FULLY STOCHASTIC SETTING.

3, SRA OUTPERFORMS ERA UNDER CONDITIONS OF
   - LOW SIGNAL TO NOISE RATIO
   - SHORT DATA LENGHTS
   - POOR INPUT EXCITATION
   - SINGLE OUTPUT MODELING

4. SRA CAN INDENTIFY SYSTEMS WITHOUT INPUT MEASUREMENTS. NO PRIOR KNOWLEDGE OF SYSTEM MODEL STRUCTURE AND ORDER REQUIRED.

5. SRA STATE SPACE MODELS ARE IN NESTED KALMAN FILTER FORM. LOWER ORDER MODELS ARE OBTAINED FROM HIGHER ORDER MODELS BY SIMPLE DELETION OF ROWS AND COLUMNS. NOISE CONVARIANCES ARE IDENTIFIED ALONG WITH SYSTEM MATRICES.

6. MODEL ORDER DETERMINATION IS DONE EASILY USING BREAKS IN SINGULAR VALUES AND AIC (AKAIKE INFORMATION CRITERION).

7. AS SYSTEM ORDER IN INCREASED, FREQUENCIES WITH LARGE SPECTRAL POWER ARE IDENTIFIED FIRST.

8. SEPARATION OF CLOSELY SPACED FREQUENCIES REQUIRES INCREASE IN THE DIMENSION OF THE HANKEL MATRIX, BUT NOT THE SYSTEM DIMENSION.

9. **SURPRISING RESULTS:**
   - o SATISFACTORY FREQUENCY AND DAMPING IDENTIFICATION CAN BE DONE FROM SINGLE OUTPUT DATA SERIES.
   - o FREQUENCY AND DAMPING PARAMETERS IDENTIFIED FOR THE 300% NOISE CASE (MEMBRANE SIMULATION). THE NOISE CAN BE MULTIPLICATIVE!
   - o THE IMPROVEMENT IN IDENTIFICATION RESULTS USING INPUT DATA IS MINOR.

10. IMPORTANCE FOR CONTROL DESIGN:

    ADAPTIVE CONTROL IS FEASIBLE SINCE MODES WITH DISCERNIBLE SPECTRAL POWER CAN BE IDENTIFIED USING SRA. SINCE IT IS NOT POSSIBLE TO DESIGN CONTROLLERS, WHICH ARE ROBUST AGAINST ALL POSSIBLE UNCERTAINTIES, ADAPTIVE CONTROL OF STRUCTURAL MODES USING ON-LINE IDENTIFICATION IS A MORE PRACTICAL APPROACH.