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SPATIAL OPERATOR APPROACH TO FLEXIBLE MULTIBODY SYSTEM DYNAMICS AND CONTROL

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SUMMARY

This paper extends to flexible multibody systems the recent [1-6] results of the author on the use of spatially recursive filtering and smoothing techniques to multibody dynamics. The configuration analyzed is that of a mechanical system of flexible bodies joined together by articulated joints. It is established that the composite flexible multibody mass matrix $\mathcal{M}$ can be factored as $\mathcal{M} = (I + \mathcal{K})\mathcal{D}(I + \mathcal{K}^*)$ in which $\mathcal{K}$ is a lower-triangular factor, $\mathcal{D}$ is a diagonal operator, and $\mathcal{K}^*$ is an upper-triangular factor. The operators $(I + \mathcal{K})$ and $\mathcal{D}$ can be constructed by means of a spatially recursive Kalman filter that begins at the tip of the system and proceeds inwardly toward the base. Similarly, the upper- triangular factor $(I + \mathcal{K}^*)$ is constructed by means of a corresponding outward smoothing recursion. The inverse $(I - \mathcal{L}) = (I + \mathcal{K})^{-1}$ of the causal factor $(I + \mathcal{K})$ is also a lower-triangular matrix. This inverse $(I - \mathcal{L})$ and its upper-triangular transpose $(I - \mathcal{L}^*)$ can also be computed by means of filtering and smoothing operations respectively. This means that the inverse $\mathcal{M}^{-1}$ of the mass matrix can be factored as $\mathcal{M}^{-1} = (I - \mathcal{L}^*)D^{-1}(I - \mathcal{L})$. The foregoing factorization results are used to develop spatially recursive algorithms for multibody system inverse and forward dynamics. The algorithms are what is referred to as Order N in the sense that the total number of arithmetic operations increases only linearly with the number of bodies in the system.

1. INTRODUCTION

The problem of flexible multibody system forward dynamics consists of finding the joint angle accelerations and the flexible body accelerations, given the applied moments at the joints and the forces due to the elastic deformation of the flexible bodies in the system. The closely related problem of inverse dynamics is to find the set of joint moments that must be applied in order to achieve a prescribed set of system accelerations. These problems are particularly important in the simulation and control design for systems which are not readily tested in a ground laboratory. Examples of such systems include future space manipulators (referred to as space cranes) to be used for handling and retrieval of free-flying satellites and space platform modules. Flexible dynamics problems are also encountered in multiarm manipulation of such flexible task objects as thermal blankets, hoses, extensible cables, and spring-loaded mechanisms.

2. PROBLEM STATEMENT

Consider a mechanical system consisting of N flexible bodies numbered $1, \cdots, N$ connected together by N joints numbered $1, \cdots, N$ to form a branch-free kinematic chain. The bodies and joints are numbered in an increasing order that goes from the tip of the

system toward the base. Joint k in the sequence connects bodies k and $k+1$. Joint 0 can be selected at any arbitrary point in body 1.

A typical flexible body k is characterized by a finite-element model consisting of a finite number of nodes defined at the spatial locations i. These locations are expressed in a coordinate system attached to the body. The set of all finite-element nodes for body k is denoted by $\Omega(k)$, and the total number of nodes is N_k.

The finite-element model for body k also involves a mass matrix m_k and a stiffness matrix s_k, which are assumed to be obtained from a stand-alone structural dynamics analysis of this body. It is assumed that the flexible body mass and stiffness matrices are time-independent quantities computed in advance. Alternatively, the flexible body mass and stiffness properties are characterized by pre-computed vibrational modes and the corresponding modal frequencies.

A 6-dimensional displacement at node i of body k is denoted by $u_k(i) = [\alpha(i), x(i)]$ in which $\alpha(i)$ is a 3-dimensional rotation and $x(i)$ is a 3-dimensional translation. These nodal displacements are expressed in a local coordinate frame attached to body k. The corresponding velocities and accelerations are respectively $\dot{u}_k(i)$ and $\ddot{u}_k(i)$ and are also expressed in the same local coordinate frame. The displacement field $u_k = [u_k(1), \cdots, u_k(N_k)]$ produces an elastic force field $f_k = [f_k(1), \cdots, f_k(N_k)]$ which can be computed as $f_k = -s_k u_k$ in terms of the stiffness matrix s_k.

The joints labeled are single-degree-of-freedom joints, which allow rotation along the joint axis only. For these joints, $\widehat{h}(k)$ is a unit vector along the axis of rotation; $F(k)$ is the active moment applied about the axis of joint k; $\theta(k)$ is the corresponding joint angle which is positive in the right-hand-sense about $\widehat{h}(k)$. The relative angular velocity and acceleration at joint k are denoted by $\dot{\theta}(k)$ and $\ddot{\theta}(k)$.

The objective in forward dynamics is to outline a recursive method for computation of the joint-angle accelerations $\ddot{\theta}(k)$ and the flexible-body nodal accelerations $\ddot{u}_k(i)$ for i in $\Omega(k)$, given the applied moments $F(k)$ and the elastic forces f_k. The objective in inverse dynamics is to compute the set of forces and moments that must be applied in order to achieve a set of prescribed accelerations.

3. STATE SPACE MODEL

The following state space model [1] for propagation of forces, velocities and accelerations makes it easy to express the recursive dynamics algorithms.

The term spatial force refers to a 6×1 vector $X(i)$ whose first three components are pure moments and whose last three components are pure translational forces. Similarly, the term spatial velocity $V(i)$ describes a 6×1 vector of angular and linear velocities. The spatial accelerations $\lambda(i)$ are obtained by appropriate [1] time differentiation of the spatial velocities $V(i)$. If the argument k is used, the corresponding force $X(k)$, velocity $V(k)$, and acceleration $\lambda(k)$ are defined at a typical joint k. If the argument i is used, the corresponding force $X_k(i)$ velocity $V_k(i)$ and acceleration $\lambda_k(i)$ are defined at node i of the body k finite-element model.

The vector $X^+(k)$ is used to represent the spatial force on the "positive" side of joint k. The $+$ superscript indicates that the corresponding variable is evaluated at a point on body $k+1$ immediately adjacent and on the "positive" side, toward the base, of joint k.

The notation $X^-(k)$ is used to represent the force on the "negative", toward the tip, side of hinge k. Similarly, the notation $V^+(k)$ and $V^-(k)$ is used respectively for the spatial velocity on the positive and negative sides of joint k.

To propagate forces, velocities and accelerations between the two typical spatial locations i and k in the multibody system, define the "transition" matrix [1]

$$\phi(k,i) = \begin{pmatrix} U & \widetilde{L}(k,i) \\ 0 & U \end{pmatrix} \tag{3.1}$$

in which $L(k,i)$ is the vector from point k to point i; and $\widetilde{L}(k,i)$ is the 3×3 matrix equivalent to $L(k,i) \times (\cdot)$. This matrix has the following semi-group properties typically associated with a transition matrix for a discrete linear state space system:

$$\phi(k,m) = \phi(k,i)\phi(i,m); \quad \phi(k,k) = U \tag{3.2}$$

and $\phi^{-1}(k,m) = \phi(m,k)$. This matrix is used in the next section to develop spatial recursions for the kinematics and dynamics of the flexible multibody problem.

4. FLEXIBLE MULTIBODY KINEMATICS

4.1 Kinematics Internal to a Typical Flexible Body

The velocity $V^+(k)$ on the positive side of the joint k and the velocity $V_k(i)$ at nodal point i in body k are related by

$$V_k(i) = \phi^T(k,i)V^+(k) + v_k(i) \quad \text{for all} \quad i \in \Omega(k) \tag{4.1}$$

in which

$$\begin{aligned} v_k(i) &= \phi^T(k,i)H^T(k)\dot{\theta}(k) + \dot{u}_k(i) \qquad i = 1, \cdots, N_{k-1} \\ v_k(i) &= \phi^T(k,i)H^T(k)\dot{\theta}(k) \qquad i = N_k \end{aligned} \tag{4.2}$$

In Eqs. 4.1 and 4.2, $\dot{u}_k(i)$ denotes the relative spatial velocity of the mass element at node i. The set $\Omega(k)$ is the set of all nodes in the finite-element model for body k. Note that the last nodal point N_k in body k is assumed to be rigidly attached to the negative side of joint k. Hence, this point does not undergo an elastic displacement with respect to the joint k. This is reflected in Eq. (4.2).

4.2 Recursive Kinematics for Flexible Multibody System

The sequence of velocities $V^+(k)$ satisfies

LOOP $k = N-1, \cdots, 1$

$$V^+(k) = \phi^T(k+1,k)V^+(k+1) + C^T(k+1,k)v_{k+1}(1) \tag{4.3}$$

END LOOP;

with the terminal condition $V^+(N+1) = 0$. By definition, $v_{k+1}(1)$ is the first 6-dimensional component of the relative velocity vector v_{k+1}, i. e., $v_k = [v_k(1), \cdots, v_k(N)]$. Note also

that $v_{N+1} = 0$. The $6\times 6N$ matrix $C^T(k+1,k)$ is defined as $C^T(k+1,k) = [\phi^T(1,k),\cdots,0]$. Outward integration of this iterative equation leads to

$$V^+(k) = \sum_{j=k}^{N-1} \phi^T(j,k)C^T(j+1,j)v_{j+1}$$

4.3 Recursive Kinematics Using Spatial Operators

To express the kinematic relationships in Eqs. (4.1)-(4.3) in terms of an equivalent spatial operator [4] notation, define first the spatial operators Φ, h, C, B, and H as

$$\Phi = \begin{pmatrix} I & 0 & \cdots & 0 \\ \phi(2,1) & I & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \phi(N,1) & \phi(N,2) & \cdots & I \end{pmatrix} \qquad h = [I, 0]$$

$$C = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ C(2,1) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & \cdots & C(N,N-1) & 0 \end{pmatrix}$$

$$B = diag[B(1),\cdots,B(N)] \quad H = diag[H(1),\cdots,H(N)]$$

in which the spatial operators $B(k)$ are defined as

$$B(k) = [\phi(k,1),\phi(k,2),\cdots,\phi(k,N_k)]$$

Based on this notation, the kinematic relationships in Eqs (4.1)-(4.3) can be expressed as

$$V = B^*V^+ + v \tag{4.4}$$

$$v = B^*H^*\dot{\theta} + h^*\dot{u}; \qquad v = \mathcal{H}^*\dot{X} \tag{4.5}$$

$$V^+ = \Phi^*C^*v \tag{4.6}$$

with $V = [V_1,\cdots,V_N]$, $v = [v_1,\cdots,v_N]$, $\mathcal{H}^* = [h^*, B^*H^*]$, $\dot{X} = [\dot{u},\dot{\theta}]$, $u = [u_1,\cdots,u_N]$ and $\theta = [\theta_1,\cdots,\theta_N]$. Combination of (4.4) and (4.6) leads to

$$V = (I + B^*\Phi^*C^*)v \tag{4.7}$$

While the kinematic relationships in (4.4)-(4.7) apply to spatial velocities, similar relationships can be derived for the corresponding accelerations by appropriate [1] time differentiation.

5. INVERSE DYNAMICS

RESULT 5.1. *The total kinetic energy in the multibody system can be computed in term of the composite velocity vector* $\dot{X} = [\dot{u}, \dot{\theta}]$ *as*

$$K.E. = (1/2)\dot{X}^* \mathcal{M} \dot{X}$$

in which the system mass matrix $\mathcal{M}$ *is*

$$\mathcal{M} = \mathcal{H}(I + C\Phi B)m(I + B^*\Phi^* C^*)\mathcal{H}^* \tag{5.1}$$

Eqs. (4.4)-(4.7) and (5.1) lead to a recursive inverse dynamics solution consisting of an outward sweep in which a sequence of system accelerations are computed. These accelerations are then multiplied by the appropriate blocks m_k of the mass matrix m in (5.1). Then, an inward recursion is performed to compute the required applied moments. Because of the factorization $\mathcal{M} = \mathcal{H}(I + C\Phi B)m(I + B^*\Phi^* C^*)\mathcal{H}^*$ of the mass matrix $\mathcal{M}$, these two recursions are equivalent to multiplication of the system accelerations $\ddot{X}$ by the composite mass matrix $\mathcal{M}$.

6. MULTIRIGID SYSTEM: A SPECIAL CASE

If the flexible bodies in the system are rigidized, by setting the nodal point velocities to zero, then the flexible body mass matrix of Sec. (5) becomes the multirigid body mass matrix analyzed by the author in [1-4].

Multirigid Body Mass Matrix. *The multirigid body mass matrix*

$$\mathcal{M} = H\Phi M \Phi^* H^* \tag{6.1}$$

in which

$$M = BmB^* = diag[M(1), \cdots, M(N)] \tag{6.2}$$

can be obtained from the flexible body mass matrix by setting the elastic state-to-output operator h *to zero. The diagonal block* $M(k)$ *in Eq. (6.2) is the rigid spatial mass matrix of the rigidized body* k *about joint* k.

Recursive Evaluation of the Multirigid Body Mass Matrix

The elements $m_R(k,j)$ of the mass matrix in (6.1) can be computed by

$$R(0) = 0$$

LOOP $k = 1, \cdots, N$

$$R(k) = \phi(k, k-1)R(k-1)\phi^T(k, k-1) + M(k)$$

$$m_R(k,k) = H(k)R(k)H^T(k)$$

$$x(k) = r(k)H^T(k)$$

Loop $i = k+1, \cdots, N$

$$x(i) = \phi(i, i-1)x(i-1)$$

$$m_R(i,k) = H(i)x(i)$$

End i **Loop**;

END k LOOP;

7. FLEXIBLE SYSTEM MASS MATRIX

The goal of this section is to arrive at a spatially recursive algorithm that computes the flexible multibody system mass matrix by means of an inward recursion from the tip to the base. The approach used to do this is to first establish the identity in (7.1) below.

Result 7.1. *The matrix* $(I+C\Phi B)m(I+B^*\Phi^*C^*)$ *in the flexible multibody system mass matrix* $\mathcal{M}$ *can be expressed as*

$$(I + C\Phi B)m(I + B^*\Phi^*C^*) = r + C\Phi Br + rB^*\Phi^*C^* \tag{7.1}$$

in which $r = m + CRC^*$. *Furthermore, the diagonal matrix* $r = diag[r(1), \cdots, r(N)]$ *is a block-diagonal matrix whose diagonal blocks* $r(k)$ *are given by*

$$r(k) = m(k) + C(k, k-1)R(k-1)C^T(k, k-1) \tag{7.2}$$

Inward Recursion for the Flexible Mass Matrix

$$R(0) = 0$$

LOOP $k = 1, \cdots, N$;

$$r(k) = m(k) + C(k, k-1)R(k-1)C^T(k, k-1)$$

$$\mathcal{M}(k,k) = \mathcal{H}(k)r(k)\mathcal{H}^T(k)$$

$$x(k) = r(k)\mathcal{H}^T(k)$$

Loop $i = k+1, \cdots, N$;

$$x(i) = \phi(i, i-1)x(i-1)$$

$$\mathcal{M}(i,k) = \mathcal{H}(i)x(i)$$

End i **Loop**;

END k LOOP;

Spatial Operator Notation

In spatial operator notation, the above recursions for the diagonal blocks of the mass matrix become

$$r = CRC^* + m; \quad R = BrB^*; \quad r = CBrB^*C^* + m$$

The above results are an extension to flexible multibody systems of the results obtained earlier by the author [1-4] for multirigid body systems.

8. INNOVATIONS FACTORIZATION OF THE MASS MATRIX AND ITS INVERSE

The flexible multibody system mass matrix $\mathcal{M}$ can be factored as

$$\mathcal{M} = (I + \mathcal{K})\mathcal{D}(I + \mathcal{K}^*) \tag{8.1}$$

in which the causal operator $\mathcal{K}$ and the diagonal operator $\mathcal{D}$ are

$$\mathcal{K} = \begin{pmatrix} hC\Phi Bg & hC\Phi G \\ H\Phi Bg & H\phi G \end{pmatrix}; \quad \mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}$$

The Kalman gain operators g and G are defined in terms of the following Riccati-like equations

$$p = C(P - GDG^*)C^* + m \tag{8.2a}$$

$$P = B(p - gdg^*)B^* \tag{8.2b}$$

$$G = PH^*D^{-1}; \qquad g = ph^*d^{-1}$$

$$D = HPH^*; \qquad d = hph^*$$

Inverse of the Causal Factor $(I + \mathcal{K})$

$$(I + \mathcal{K})^{-1} = I - \mathcal{L} \tag{8.3}$$

in which $\mathcal{L}$ is the causal operator

$$\mathcal{L} = \begin{pmatrix} h\widetilde{C}\Psi Bg & h\widetilde{C}\psi G \\ H\Psi Bg & H\psi G \end{pmatrix}$$

Some of the spatial operators used in this result are defined as

$$\widetilde{B}(k) = B(k)[I - g(k)h(k)]; \qquad \widetilde{B} = diag[\widetilde{B}(1), \cdots, \widetilde{B}(N)]$$

$$\widetilde{C}(k, k-1) = C(k, k-1)[I - G(k-1)H(k-1)]$$

$$\widetilde{C} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \widetilde{C}(2,1) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \widetilde{C}(N, N-1) & 0 \end{pmatrix}$$

$$\psi(k, i) = \widetilde{B}(k)\widetilde{C}(k, k-1) \cdots \widetilde{B}(i+1)C(i+1, i)$$

$$\psi = \sum_{i=1}^{k-1} \psi(k, i); \qquad \Psi = \sum_{i=1}^{k} \psi(k, i)[I - G(i)H(i)]$$

This states that the causal factor $I + \mathcal{K}$ is causally invertible. Furthermore it states that the inverse $I - \mathcal{L}$ can be computed by means of a spatially recursive Kalman filter. This Kalman filter will be described in more detail in the following section. Here, the immediate objective is to obtain the following factorization for the inverse $\mathcal{M}^{-1}$ of the flexible multibody system mass matrix $\mathcal{M}$. bigskip

Innovations Factorization of the Mass Matrix Inverse.

$$\mathcal{M}^{-1} = (I - \mathcal{L}^*)\mathcal{D}^{-1}(I - \mathcal{L}) \tag{8.4}$$

This result states that the inverse of the mass matrix is the product of an anticausal factor, a diagonal operator, and a causal factor. Recall that the equations of motion for the flexible multibody system, disregarding without loss of generality the effects of velocity dependent coriolis and gyroscopic terms, are:

$$\mathcal{M}\ddot{X} = \mathcal{F}; \qquad \ddot{X} = \mathcal{M}^{-1}\mathcal{F} \tag{8.5}$$

in which $\mathcal{F} = [f, F]$ is a composite vector made up of the elastic forces f and the applied joint moments F.

Use of (8.4) in (8.5) leads to

$$\ddot{X} = (I - \mathcal{L}^*)\mathcal{D}^{-1}(I - \mathcal{L})\mathcal{F}$$

This equation states that the known forces $\mathcal{F}$ must be operated upon by a two-stage filtering and smoothing process in order to obtain the system accelerations $\ddot{X}$. The first operation involves the causal factor $(I - \mathcal{L})$ which can be mechanized by a spatially recursive Kalman filter. The result of the first stage is an innovations process defined as $(I - \mathcal{L})\mathcal{F}$ and a residual acceleration process defined as $\mathcal{D}^{-1}(I - \mathcal{L})\mathcal{F}$. This residual process is operated upon by an outward smoothing computation represented by the anticausal factor $(I - \mathcal{L}^*)$ to obtain the system accelerations $\ddot{X}$. These filtering and smoothing operations are described more completely in the following section.

9. RECURSIVE FORWARD DYNAMICS

Riccati Equation for Articulated Inertias

$$P^+(0) = 0 \tag{9.1}$$

LOOP $k = 1, \cdots, N$;

$$p_k = m_k + C_{k,k-1}P^+(k-1)C^T_{k,k-1} \tag{9.2}$$

$$d_k = h_k p_k^- h_k^T \tag{9.3}$$

$$g_k = p_k^- h_k^T d_k^{-1} \tag{9.4}$$

$$p_k^+ = (I - g_k h_k)p_k^- \tag{9.5}$$

$$P^-(k) = \sum_{i \in \Omega(k)} \sum_{j \in \Omega(k)} \phi(k,i)p_k^+(i,j)\phi^T(k,j) \tag{9.6}$$

$$D(k) = H(k)P^-(k)H^T(k) \tag{9.7}$$

$$G(k) = P^-(k)H^T(k)D^{-1}(k) \tag{9.8}$$

$$P^+(k) = [I - G(k)H(k)]P^-(k) \tag{9.9}$$

END LOOP;

This discrete-step Riccati-like equation computes a sequence of rigid Kalman gains $G(k)$ defined at every joint angle k and a corresponding sequence of flexible body Kalman gains $g_k = g_k(i,j)$ defined for every pair of nodes i,j in body k. This inward recursion is performed simultaneously with a filtering algorithm described below.

INWARD FILTERING: SPATIAL FORCES

$$Z^+(0) = 0 \tag{9.10}$$

LOOP $k = 1, \cdots, N$;

$$z^-(k) = C_{k,k-1}Z^+(k-1) \tag{9.11}$$

$$e_k^- = f_k - h_k z_k \tag{9.12}$$

$$e_k^+ = d_k^{-1} e_k^- \tag{9.13}$$

$$z_k^+ = z_k^- + g_k e_k^- \tag{9.14}$$

$$Z^-(k) = \sum_{i \in \Omega(k)} \phi(k,i) z_k^+(i) \tag{9.15}$$

$$E^-(k) = F(k) - H(k)Z^-(k) \tag{9.16}$$

$$E^+(k) = E^-(k)/D(k) \tag{9.17}$$

$$Z^+(k) = Z^-(k) + G(k)E^-(k) \tag{9.18}$$

END LOOP;

The result of this filtering stage is a sequence of residuals $E^+(k)$ defined at every joint k and a sequence of flexible body residuals $e_k^+(i)$ defined at every nodal element i of every flexible body k.

OUTWARD SMOOTHING: SPATIAL ACCELERATIONS

$$A^+(N) = 0 \tag{9.19}$$

LOOP $k = N, \cdots, 1$;

$$\ddot{\theta}(k) = E^+(k) - G^T(k)A^+(k) \tag{9.20}$$

$$A^-(k) = A^+(k) + H^T(k)\ddot{\theta}(k) \tag{9.21}$$

$$a_k(N_k) = \phi^T(k, N_k) \tag{9.22}$$

$$a_k^+(i) = \phi^T(k,i)a_k(N_k) \tag{9.23}$$

$$\ddot{u}_k(i) = e_k^+(i) - \sum_{j=1}^{N_k-1} g_k^T(j,i)a_k^+(j) \tag{9.24}$$

$$a_k^-(i) = a_k^+(i) + \ddot{u}_k(i) \tag{9.25}$$

$$A^+(k-1) = C^T(k,k-1)a_k^- \tag{9.26}$$

END LOOP;

The result of this smoothing stage is a sequence of joint angle accelerations $\ddot{\theta}(k)$ and flexible body accelerations $\ddot{u}_k(i)$.

PHYSICAL INTERPRETATION

The forward dynamics problem is solved by a spatially recursive Kalman filtering process which begins at the tip of the system and proceeds inwardly toward the base. This filtering algorithm computes: (1) a sequence of spatial forces $z(k)$ at the flexible bodies and $Z(k)$ at the joints; (2) a sequence of residuals $e^+(k)$ and $E^+(k)$; and (3) a sequence of Kalman gains g_k at the flexible bodies and $G(k)$ at the joints. The filtering stage uses as an input the elastic forces $f(k)$ at body k and the applied joint moments $F(k)$ at joint k. The residuals and the Kalman gains are stored for subsequent processing by an outward smoothing stage.

The smoothing stage is an outwardly recursive process which begins at the base of the system and proceeds from body to body toward the tip. The smoother computes a sequence of spatial accelerations a_k at the flexible bodies and $A(k)$ at the joints. The smoother also computes a sequence of relative elastic accelerations $\ddot{u}_k$ at the flexible bodies and joint angle accelerations $\ddot{\theta}(k)$ at the joints.

Riccati Equation

One of the central features of the inward filtering algorithm is the spatial Riccati equation in Eqs. (9.1)-(9.9) which accumulates the outboard spatial inertia as the recursive computations are performed.

This Riccati equation begins at the tip of the system with the initial condition $P^+(0) = 0$ in Eq. (9.1). This initial condition means physically that there is no spatial inertia outboard of this fictitious joint.

Eq. (9.2) is used to add to the body k free-free mass matrix m_k the spatial inertia of a fictitious articulated rigid body which is equivalent to collection of bodies outboard of joint $k-1$. This equivalent inertia is transferred from the joint $k-1$ to the attachment nodal point 1 in body k by the transition operator $C(k,k-1)$.

Eq. (9.3) computes the flexible body k articulated mass matrix. This matrix can be viewed as a reduced-order body k mass matrix. The order reduction occurs because the operator $h(k)$ has the effect of constraining the last nodal point N_k in the finite-element model of body k.

The Kalman gain $g(k)$ in Eq. (9.4) is a $6N \times 6(N-1)$ matrix which is used to compute the projection operator $[I - g(k)h(k)]$. This projection operator, when multiplied in (9.5) by the matrix $p^-(k)$ leads to the updated matrix $p^+(k)$ which has a null space of dimension $N-6$ in the direction of the operator $h(k)$. Eqs. (9.6) transfers the flexible articulated body inertia to an equivalent rigid body mass matrix at joint k.

Eqs. (9.7)-(9.9) are identical to the computations involved in crossing joint k in the multirigid body forward dynamics algorithm in [1]. They involve the following computations: (1) evaluation of the scalar articulated inertia $D(k)$ about joint k. This inertia is the inertia about joint k of the composite body outboard of joint k, with all of the degrees of freedom outboard of joint k being unlocked; (2) computation of the Kalman gain $G(k)$ in Eq. (9.8) to determine the projection operator $[I - G(k)H(k)]$ in Eq. (9.9). When this operator pre- multiplies the rigid-body spatial inertia $P^-(k)$, the updated spatial articulated $P^+(k)$ results. The spatial inertia $P^+(k)$ is that of a fictitious body which has no inertia along the joint k axis.

After crossing the joint k in Eq. (9.9), the algorithm lets $k \to k+1$ and returns to Eq. (9.2). The process of crossing a flexible body and a joint has been completed.

Filtering

The filtering algorithm in Eqs. (9.10)-(9.18) is a spatially recursive Kalman filter based on an inward sequence which is performed together with the Riccati equation just described. The Kalman filter begins at the tip of the system, the fictitious point "0", with the initial condition $Z^+(0) = 0$ in Eq. (9.10), which indicates that there are no external applied forces at this point. This begins a recursive process which takes as inputs the sequence of elastic forces $f(k)$ and the sequence of applied joint moments $F(k)$. The outputs of this process are a sequence of flexible-body residuals $e^+(k)$ and joint axis residuals $E^+(k)$.

Eqs. (9.11)-(9.13) compute the flexible-body residual $e^+(k)$ at body k. First, Eq. (9.11) determines the spatial force $z^-(k)$ that exists in body k due to the previously determined force $Z^+(k-1)$ at joint $k-1$ reflecting the presence of all of the bodies outboard of this joint. In Eq. (9.11), this force is multiplied by the operator $h(k)$ to obtain the predicted output force $h(k)z^-(k)$. The flexible body innovations $e^-(k)$ in Eq. (9.12) can be viewed as an "error" quantity equal to the difference between the actual force $f(k)$ due to the body stiffness and the predicted force $h(k)z^-(k)$ due to the preceding bodies $1, \cdots, k-1$ outboard of joint $k-1$. The residual acceleration process $e^+(k)$ is computed from $e^-(k)$ by dividing by the articulated spatial mass matrix $d(k)$ which emerges from the Riccati equation. This division is indicated in Eq. (9.13). The flexible body residual $e^+(k)$ has a very interesting physical interpretation. It corresponds to the inertial acceleration that the finite-element nodes in body k would undergo, if the "future" degrees of freedom were locked.

The computation in Eq. (9.14), which determines the updated spatial force distribution $z^+(k)$ in body k, has the effect of unlocking the 6 degrees of freedom associated with the all of the nodal points in body k except the last one.

Eq. (9.15) sums the spatial force estimates $z^+(k)$ at the nodal points in body k and transfers them to joint k. The result of this summation is the 6-dimensional spatial force $Z^-(k)$. This force reflects at joint k the effect of all of the preceding bodies. The next

steps, conducted in Eqs. (9.16)- (9.18), cross or unlock joint k. These steps are identical to that used in the multirigid body algorithms [1] and result in the updated spatial force $Z^+(k)$ on the positive, inward toward the base, of joint k.

At this juncture, the filtering algorithm lets $k \to k+1$ and returns to Eq. (9.11) to start the computations necessary to cross the next body.

Smoothing

The smoothing process in Eqs. (9.19)-(9.26) is an outward recursion which starts at the base of the system and proceeds outwardly to its tip. The smoothing process produces a sequence of rigid-body spatial accelerations $A(k)$ at the joints and of flexible-body accelerations $a(k)$ at the nodal points of the flexible bodies. It also produces the relative accelerations $\dot{u}(k)$ at the flexible bodies and the joint-angle accelerations $\ddot{\theta}(k)$ at the joints. The smoother uses as inputs the sequences of residual accelerations $e^+(k)$ and $E^+(k)$. It also uses the Kalman gain sequences $g(k)$ at the flexible bodies and $G(k)$ at the joints.

The outward smoothing sequence begins with the terminal condition $A^+(N) = 0$, which corresponds to the assumption that the base of the system is immobile. Eqs. (9.20) and (9.21) can be viewed as specifying the computations necessary to cross joint k in the outward direction.

Eq. (9.22) computes the spatial acceleration $a_k(N_k)$ of the attachment point N_k. Eq. (9.23) computes the spatial accelerations $a_k^+(i)$ at the internal finite-element nodes of the flexible body k. The "+" indicates that the corresponding acceleration is that of a rigid body frame attached to a rigidized flexible body obtained by setting the elastic displacements to zero. The elastic displacement accelerations at the finite-element nodes are computed by Eq. (9.24). Eq. (9.25) then computes the total inertial accelerations of the finite- element nodes in body k. The spatial acceleration of the first node, also referred as an attachment node, is then propagated by Eq. (9.26) to the positive side of joint $k-1$. At this stage, the algorithm lets $k \to k-1$ and returns to Eq. (9.20) to begin the computations associated with the next body $k-1$.

Modal Expansions

The above algorithm has been expressed in terms of nodal coordinates to model the flexibility of each of the flexible bodies in the system. In many cases, it is more convenient to use what are typically referred to as modal coordinates. A modal model for a flexible body is obtained by doing a modal or eigenfunction analysis of the finite-element model for the same body. Use of these expansions leads to a spatially recursive forward dynamics algorithm that is almost identical in form to that of (9.1)-(9.26) above, but in which the quantities (displacements, velocities, accelerations, forces, and mass) involved are interpreted in terms of modal coordinates as opposed to the nodal coordinates used in (9.1)-(9.26).

10. CONCLUDING REMARKS

The inverse and forward dynamics problems for flexible multibody systems have been solved using the techniques of spatially recursive Kalman filtering and smoothing. These algorithms are easily developed using a set of identities associated with mass matrix factorization and inversion. These identities are easily derived using the spatial operator algebra

developed by the author. Current work is aimed at computational experiments with the described algorithms and at modeling for control design of limber manipulator systems. It is also aimed at handling and manipulation of flexible objects.

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