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Second Order Modeling of Boundary-Free Turbulent Shear Flows

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Abstract

This paper presents a set of realizable second order models for boundary free turbulent flows. The constraints on second order models based on the realizability principle are re-examined. The rapid terms in the pressure correlations for both the Reynolds stress and the passive scalar flux equations are constructed to exactly satisfy the joint realizability. All other model terms (return-to-isotropy, third moments and terms in the dissipation equations) already satisfy realizability (Lumley 1978, Shih and Lumley 1986). To correct the spreading rate of the axisymmetric jet, an extra term is added to the dissipation equation which accounts for the effect of mean vortex stretching on dissipation. The test flows used in this study are the mixing shear layer, plane jet, axisymmetric jet and plane wake. The numerical solutions show that the new unified model equations (with unchanged model constants) predict all these flows reasonably as the results compare well with the measurements. We expect that these model equations would be suitable for more complex and critical flows.

I. Introduction

The second order closure scheme has been studied for over two decades now and it is playing an increasingly important role in the computation of turbulent flows, for example, in atmospheric turbulence and turbulent combustion. So far this method has achieved success in predicting many different flows (Launder et al[1], Lumley et al[2], Shih and Lumley[3], Shih[4], Chen[5] and Ettestad[6]). However, as Schumann[7] and Lumley[8] have pointed out, many of the second order model equations are not realizable because the models for the pressure correlations in the second moment equations do not satisfy the realizability constraints. They also pointed out that this may cause severe numerical difficulties and produce unphysical results during a numerical computation; such as giving negative turbulent energy or producing correlation coefficients larger than unity in certain critical situations. Hence, there is a strong need for improving second order model equations for both theoretical reasons and practical needs. In this paper we follow Shih and Lumley[9] to directly impose the realizability principle (including joint realizability between the velocity and passive scalar) in the model development and obtain a set of model forms for the rapid part of the pressure correlations. In addition, an extra term is added to the dissipation equation which reflects the effect of the mean vortex stretching on the dissipation.

In order to discuss various model terms appearing in the second moment equations, let us write down the exact equations for the mean quantities and various second moments for incompressible flows (including passive scalar):

$$U_{i,i} = 0 \tag{1}$$

$$D_{,t}U_i = -\frac{1}{\rho}P_{,i} - (\overline{u_i u_j})_{,j} + \nu U_{i,jj} \tag{2}$$

$$D_{,t}F = -(\overline{f u_j})_{,j} + \gamma F_{,jj} \tag{3}$$

$$\begin{aligned} D_{,t}\overline{u_i u_j} = &- \left[\overline{u_i u_j u_k} + \frac{1}{\rho}(\overline{p^{(2)} u_i}\delta_{jk} + \overline{p^{(2)} u_j}\delta_{ik})\right]_{,k} \\ &- \overline{u_i u_k}U_{j,k} - \overline{u_j u_k}U_{i,k} - \frac{1}{\rho}\overline{(p^{(1)}_{,i} u_j + p^{(1)}_{,j} u_i)} \\ &+ \frac{1}{\rho}\overline{(p^{(2)} u_{i,j} + p^{(2)} u_{j,i})} - 2\nu\overline{u_{i,k}u_{j,k}} \end{aligned} \tag{4}$$

$$\begin{aligned} D_{,t}\overline{f u_i} = &- \left[\overline{f u_i u_j} + \frac{1}{\rho}\overline{p^{(2)} f}\delta_{ij}\right]_{,j} \\ &- \overline{u_i u_j}F_{,j} - \overline{f u_j}U_{i,j} - \frac{1}{\rho}\overline{p^{(1)}_{,i} f} \\ &+ \frac{1}{\rho}\overline{p^{(2)} f_{,i}} - (\nu + \gamma)\overline{f_{,j}u_{i,j}} \end{aligned} \tag{5}$$

$$D_{,t}\overline{f^2} = -(\overline{f^2 u_j})_{,j} - 2\overline{f u_j}F_{,j} - 2\gamma\overline{f_{,j}f_{,j}} \tag{6}$$

where U_i, F and P are the mean velocity, mean scalar and pressure, and u_i, f and p are their corresponding fluctuating quantities. Here, we have employed the summation convention on the indices and the following notations: $(\ )_{,i} = \frac{\partial}{\partial x_i}$, $D_{,t}(\ ) = \frac{\partial}{\partial t} + U_k(\ )_{,k}$. For the pressure fluctuation, the following Poisson equation must be satisfied:

$$-\frac{1}{\rho}p_{,jj} = 2U_{i,j}u_{j,i} + u_{i,j}u_{j,i} - \overline{u_{i,j}u_{j,i}} \tag{7}$$

Based on the linearity of p, we have split the pressure fluctuation into two parts: $p^{(1)}$ and $p^{(2)}$, called the rapid and slow pressures respectively:

$$-\frac{1}{\rho}p^{(1)}_{,jj} = 2U_{i,j}u_{j,i} \tag{8}$$

$$-\frac{1}{\rho}p^{(2)}_{,jj} = u_{i,j}u_{j,i} \tag{9}$$

Note that the pressure gradient correlation terms involving $p^{(2)}$ (slow pressure) in the Eqs. (4) and (5) have been seperated into a pressure transport and a pressure strain (or pressure scalar gradient) correlation.

For homogeneous turbulence, using the solution of the Poisson equation (8), we may write

$$-\frac{1}{\rho}\overline{p^{(1)}_{,j}u_i} = -2U_{p,q}\frac{1}{4\pi}\int_V \overline{[u_q(r)u_i(r')]}_{,pj}\frac{dv}{|r-r'|} \tag{10}$$

$$-\frac{1}{\rho}\overline{p^{(1)}_{,i}f} = -2U_{j,k}\frac{1}{4\pi}\int_V \overline{[u_k(r)f(r')]}_{,ij}\frac{dv}{|r-r'|} \tag{11}$$

Equations (10) and (11) may give us a hint on how to construct models for the rapid pressure terms.

For convenience, one often defines $\bar{\epsilon} = \nu\overline{u_{i,k}u_{i,k}}$, $\bar{\epsilon}_f = \gamma\overline{f_{,k}f_{,k}}$. They represent the mechanical and scalar dissipation rates respectively and must be modeled. Their transport equations may be written as follows

$$D_{,t}\bar{\epsilon} = -(\overline{\epsilon u_j})_{,j} - \frac{\bar{\epsilon}^2}{q^2}\Psi \tag{12}$$

$$D_{,t}\bar{\epsilon}_f = -(\overline{\epsilon_f u_j})_{,j} - \frac{\bar{\epsilon}_f{}^2}{\overline{f^2}}\Psi_f \tag{13}$$

Here, Ψ and Ψ_f contain all the constructive and destructive terms of the dissipations (see section II), and $q^2 = \overline{u_i u_i}$.

Lumley[8] suggested that the slow pressure strain (or pressure scalar gradient) correlation term and the viscous dissipation term in the second moment equations can be combined and modeled together, because they are both related to only purly turbulent quantities. Therefore, we write

$$-\Phi_{ij}\bar{\epsilon} = \frac{1}{\rho}\overline{p^{(2)}(u_{i,j}+u_{j,i})} - 2\nu\overline{u_{i,k}u_{j,k}} + \frac{2}{3}\bar{\epsilon}\delta_{ij} \tag{14}$$

$$-\Phi_i\frac{\bar{\epsilon}}{q^2} = \frac{1}{\rho}\overline{p^{(2)}f_{,i}} - (\nu+\gamma)\overline{f_{,j}u_{i,j}} \tag{15}$$

For the rapid terms, Eqs.(10) and (11), we need to model two integrals:

$$I_{pjqi} = -\frac{1}{4\pi}\int_V \overline{[u_q(r)u_i(r')]}_{,pj}\frac{dv}{|r-r'|} \tag{16}$$

$$I_{ijk} = -\frac{1}{4\pi}\int_V \overline{[u_k(r)f(r')]}_{,ij}\frac{dv}{|r-r'|} \tag{17}$$

In the next section, we will discuss the realizability principle and use it to construct the models for various unknown correlations in the second moment equations. In section

III, we test these models in the boundary-free shear flows: mixing shear layer, plane jet and axisymmetric jet, and plane wake. The calculations show that the present models (with unchanged model constants) predict all these flows reasonably as the results show good agreement with the measurements.

II Second Order Closure

Realizability Principle

The concept of realizability was first introduced by Schumann[7] and Lumley[8]. The basic idea is that any non-negative turbulent quantities (say, turbulent energy components, intensity of scalar fluctuations, etc.) must remain positive during the evolution of turbulence, and Schwarz' inequality between any turbulence quantities (say, between fluctuating velocities and scalars) must be satisfied at all the time. The exact turbulent equations derived from the Navier-Stokes equation, for example, Reynolds stress and scalar flux equations, possess these physical and mathematical properties, i.e. the solution of the exact turbulent equations satisfies realizability. However, modeled turbulence equations, obtained by various approximations from the Navier-Stokes equation, often violate realizability and produce unphysical results. In fact, so far none of the second order closure models, with the exceptions of Shih and Lumley[9] and Ristorcelli[10], satisfy complete realizability (which includes joint realizability between velocity and scalar). In this section, we will discuss turbulence models based on realizability. To do that, we define a correlation tensor $D_{ij} = \overline{f^2}\,\overline{u_i u_j} - \overline{fu_i}\ \overline{fu_j}$ which consists of the Reynolds stress and scalar flux. Lumley[8], Shih and Lumley[9] argued that the realizability principle stated above is equivalent to that of non-negativity of the eigenvalues of both the Reynolds stress tensor $R_{ij} = \overline{u_i u_j}$ and the correlation tensor D_{ij}. That is, those eigenvalues must remain positive during the evolution of turbulence. The simplest way to ensure this realizability is to require that the first derivative of the eigenvalues should vanish and the second derivative should remain positive if the eigenvalues vanish, see Figure 1. If we designate the eigenvalues of the Reynolds stress tensor and the correlation tensor by $R_{\alpha\alpha}$ and $D_{\alpha\alpha}$ respectively (no summation convention for Greek indices) we may write these realizability conditions as

$$D_{,t}R_{\alpha\alpha} \rightarrow 0 \quad if \quad R_{\alpha\alpha} \rightarrow 0 \tag{18}$$

$$D_{,t}D_{\alpha\alpha} \rightarrow 0 \quad if \quad D_{\alpha\alpha} \rightarrow 0 \tag{19}$$

$$D_{,tt}R_{\alpha\alpha} \geq 0 \quad if \quad R_{\alpha\alpha} \rightarrow 0 \tag{20}$$

$$D_{,tt}D_{\alpha\alpha} \geq 0 \quad if \quad D_{\alpha\alpha} \rightarrow 0 \tag{21}$$

Eqs.(18) and (19) are the necessary conditions for realizability, and Eqs.(20) and (21) together with (18) and (19) will provide the necessary and sufficient conditions for realizability. For more details, see Lumley[11] and Shih et al[12].

To impose the realizability conditions on various model terms in the equations for the second moments, we need the equations for the eigenvalues of R_{ij} and D_{ij}. In other words, we need the equations for R_{ij} and D_{ij} in the principle axes of R_{ij} and D_{ij}. In the principle axes of R_{ij}, the Reynolds stress equation (4) becomes

$$D_{,t}\overline{u_\alpha^2} = -\left[\overline{u_\alpha^2 u_k} + 2\frac{1}{\rho}\overline{p^{(2)}u_\alpha}\delta_{\alpha k}\right]_{,k} - 2\overline{u_\alpha^2}U_{\alpha,\alpha} + 4U_{p,q}I_{p\alpha q\alpha} - (\Phi_{\alpha\alpha} + \frac{2}{3})\overline{\epsilon} \quad (22)$$

where $\overline{u_\alpha^2}$ are the eigenvalues of R_{ij}. Now, if we impose the realizability condition (18) on Eq.(22), we may obtain a set of constraints for the model terms in the Reynolds stress equations:

$$U_{p,q}I_{p\alpha q\alpha} \rightarrow 0 \quad if \quad \overline{u_\alpha^2} \rightarrow 0 \quad (23)$$

$$(\Phi_{\alpha\alpha} + \frac{2}{3})\overline{\epsilon} \rightarrow 0 \quad if \quad \overline{u_\alpha^2} \rightarrow 0 \quad (24)$$

$$(\overline{u_\alpha^2 u_k} + 2\frac{1}{\rho}\overline{p^{(2)}u_\alpha}\delta_{\alpha k})_{,k} \rightarrow 0 \quad if \quad \overline{u_\alpha^2} \rightarrow 0 \quad (25)$$

Similarly, we may write an equation for D_{ij} in its principle axes and impose the realizability condition (19) to obtain the following constraints on the model terms appearing in both the Reynolds stress and the scalar flux equations:

$$U_{p,q}(\overline{f^2}I_{p\alpha q\alpha} - \overline{fu_\alpha}I_{\alpha pq}) \rightarrow 0 \quad if \quad D_{\alpha\alpha} \rightarrow 0 \quad (26)$$

$$2\frac{\overline{\epsilon}}{q^2}\overline{fu_\alpha}\Phi_\alpha - \overline{f^2}(\Phi_{\alpha\alpha} + \frac{2}{3})\overline{\epsilon} - 2\overline{\epsilon_f}\overline{u_\alpha^2} \rightarrow 0 \quad if \quad D_{\alpha\alpha} \rightarrow 0 \quad (27)$$

$$2\overline{fu_\alpha}(\overline{fu_\alpha u_k} + \frac{1}{\rho}\overline{p^{(2)}f}\delta_{\alpha k})_{,k} - (\overline{f^2 u_\alpha^2 u_k} + \frac{1}{\rho}\overline{p^{(2)}u_\alpha}\delta_{\alpha k})_{,k} - \overline{u_\alpha^2}(\overline{f^2 u_k})_{,k} \rightarrow 0 \quad if \quad D_{\alpha\alpha} \rightarrow 0 \quad (28)$$

The constraints (23-28) on each model term in the Reynolds stress and the scalar flux equations will ensure that the model equations satisfy realizability. The models proposed by Lumley[8] for the third moments and the slow term Φ_{ij}, and the model Φ_i proposed by Shih and Lum- ley[3] already satisfy the abovementioned constraints. What we need here are the models for the rapid terms: I_{pjqi} and I_{ijk}. These terms are usually very important terms in the Reynolds stress and flux equations. Unfortunately, many existing models do not satisfy the conditions (23-28), and therefore, may produce unphysical results.

Models of the Rapid Terms

In the past it has been customary to express I_{pjqi} as a simple linear function of b_{ij} and I_{ijk} as a simple function of $\overline{fu_i}$. We find that it is impossible for these forms to satisfy realizability. The most general forms were first proposed by Shih and Lumley[9] (also see Shih et al[12]). Here, we adopt simpler forms for I_{pjqi} and I_{ijk} which are capable of satisfying realizability:

$$
\begin{aligned}
\frac{I_{pjqi}}{q^2} = {} & \alpha_1 \delta_{qi}\delta_{pj} + \alpha_2(\delta_{pq}\delta_{ij} + \delta_{qj}\delta_{pi}) \\
& + \alpha_3 \delta_{qi} b_{pj} + \alpha_4 \delta_{pj} b_{qi} \\
& + \alpha_5(\delta_{pq} b_{ij} + \delta_{ij} b_{pq} + \delta_{jq} b_{pi} + \delta_{pi} b_{qj}) \\
& + \alpha_6 \delta_{qi} b^2_{pj} + \alpha_7 \delta_{pj} b^2_{qi} \\
& + \alpha_8(\delta_{pq} b^2_{ij} + \delta_{ij} b^2_{pq} + \delta_{jq} b^2_{pi} + \delta_{pi} b^2_{qj}) \\
& + \alpha_9 b_{qi} b_{pj} + \alpha_{10}(b_{pq} b_{ij} + b_{qj} b_{pi}) \\
& + \alpha_{11} b_{qi} b^2_{pj} + \alpha_{12} b_{pj} b^2_{qi} \\
& + \alpha_{13}(b_{pq} b^2_{ij} + b_{ij} b^2_{pq} + b_{qj} b^2_{pi} + b_{pi} b^2_{qj})
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
I_{ikj} = {} & \beta_1 \delta_{ik} \overline{\theta u_j} + \beta_2(\delta_{ij}\overline{\theta u_k} + \delta_{jk}\overline{\theta u_i}) + \beta_3 b_{ik}\overline{\theta u_j} \\
& + \beta_4(b_{ij}\overline{\theta u_k} + b_{jk}\overline{\theta u_i}) + \beta_5(\delta_{ij} b_{kp} + \delta_{kj} b_{ip})\overline{\theta u_p} \\
& + \beta_6 \delta_{ik} b_{jp}\overline{\theta u_p} + \beta_7 b_{ik} b_{jp}\overline{\theta u_p} \\
& + \beta_8(b_{ij} b_{kp} + b_{kj} b_{ip})\overline{\theta u_p} + \beta_9 b^2_{ik}\overline{\theta u_j} \\
& + \beta_{10}(b^2_{ij}\overline{\theta u_k} + b^2_{jk}\overline{\theta u_i}) + \beta_{11}\delta_{ik} b^2_{jp}\overline{\theta u_p} \\
& + \beta_{12}(\delta_{ij} b^2_{kp} + \delta_{jk} b^2_{ip})\overline{\theta u_p} + \beta_{13} b^2_{ik} b_{jp}\overline{\theta u_p} \\
& + \beta_{14} b_{ik} b^2_{jp}\overline{\theta u_p} + \beta_{15}(b^2_{ij} b_{kp} + b^2_{kj} b_{ip})\overline{\theta u_p}
\end{aligned} \tag{30}
$$

where $b_{ij} = \frac{\overline{u_i u_j}}{q^2} - \frac{1}{3}\delta_{ij}$ is called the anisotropy tensor of the Reynolds stress. The coefficients α_i and β_i in Eqs. (29) and (30) are, in general, functions of the invariants of b_{ij} and D_{ij}. However, for passive scalar turbulence, α_i should be only a function of the invariants of b_{ij}. These coefficients need to be determined. To achieve this, we recall the definitions of I_{pjqi} and I_{ijk}, i.e., Eqs.(16) and (17), and find that they have the following properties:

$$I_{pjqi} = I_{jpqi}, I_{pjqi} = I_{pjiq}, I_{ijk} = I_{jik} \tag{31}$$

$$I_{ppqi} = \overline{u_q u_i}, I_{pkqk} = 0, I_{ppk} = \overline{f u_k}, I_{ikk} = 0 \tag{32}$$

We notice that Eqs.(29) and (30) already satisfy the condition imposed by Eq. (31). If we use the condition (32) and the realizability constraints (23) and (26), we may determine the limiting values of all the coefficients. The final expressions are surprisingly simple:

$$
\begin{aligned}
\frac{I_{pjqi}}{q^2} = {} & \frac{1}{30}(4\delta_{pj}\delta_{qi} - \delta_{pq}\delta_{ij} - \delta_{qj}\delta_{pi}) \\
& - \frac{1}{3}(\delta_{qi} b_{pj} - \delta_{pj} b_{qi}) + a_1(\delta_{pq} b_{ij} + \delta_{ij} b_{pq} + \delta_{qj} b_{pi} \\
& + \delta_{pi} b_{qj} - \frac{11}{3}\delta_{qi} b_{pj} - \frac{4}{3}\delta_{pj} b_{qi})
\end{aligned}
$$

$$+ a_2(2\delta_{pj}b^2_{qi} - 3b_{pq}b_{ij} - 3b_{qj}b_{pi} + b_{qi}b_{pj}) \tag{33}$$

$$\begin{aligned} I_{ijk} &= \frac{2}{5}\delta_{ij}\overline{\theta u_k} - \frac{1}{10}(\delta_{ik}\overline{\theta u_j} + \delta_{jk}\overline{\theta u_i}) \\ &+ C_{D1}b_{ij}\overline{\theta u_k} \\ &+ C_{D2}(b_{ik}\overline{\theta u_j} + b_{jk}\overline{\theta u_i}) + C_{D3}\delta_{ij}b_{kl}\overline{\theta u_l}, \end{aligned} \tag{34}$$

where the limiting values of the coefficients are

$$\begin{aligned} &a_1 = -\frac{1}{10},\ a_2 = \frac{1}{10}, \\ &C_{D1} = \frac{1}{10},\ C_{D2} = -\frac{3}{10},\ C_{D3} = \frac{1}{5}. \end{aligned} \tag{35}$$

The last line in (33) and the last two lines in (34) represent the non-linear contribution and if neglected, the linear models used by various other workers will be recovered. It is important to note that the above values of the coefficients a_1, a_2, C_{D1}, C_{D2}, and C_{D3} are their limiting values at the realizability limit, i.e. when $\overline{u_\alpha u_\alpha}$, $D_{\alpha\alpha} \to 0$. For general turbulent flows $\overline{u_\alpha u_\alpha}$ and $D_{\alpha\alpha}$ are not zero and hence the values of the coefficients may deviate from their limiting values. They are, in general, functions of the invariants II and III for a_1 and a_2, and the invariants formed from b_{ij} and $\overline{fu_i}$ for C_{Di}. Some guidance can be obtained by inspecting the following two useful parameters (see Lumley [8] and Shih and Lumley [3]):

$$F = 1 + 27III + 9II \tag{36}$$

$$F_d = 9\ d^3_{ii} - \frac{27}{2}d^2_{ii} + \frac{9}{2}, \tag{37}$$

where

$$II = -\frac{1}{2}b^2_{ii},\quad III = \frac{1}{3}b^3_{ii}$$

$$d_{ij} = \frac{\overline{f^2}\ \overline{u_iu_j} - \overline{fu_i}\ \overline{fu_j}}{\overline{f^2}\ \overline{u_iu_i} - \overline{fu_i}\ \overline{fu_i}}.$$

It can be shown that both F and F_d lie between 0 and 1, and particularly

$$F \to 0 \quad when \quad \overline{u_\alpha u_\alpha} \to 0$$

$$F_d \to 0 \quad when \quad D_{\alpha\alpha} \to 0$$

By using this information, it is convenient to write

$$a_1 = -\frac{1}{10}(1 + AF^\alpha) \tag{38}$$

$$a_2 = \frac{1}{10}(1 + BF^\alpha) \tag{39}$$

$$C_{D1} = \frac{1}{10} + C_1F^\alpha_d \tag{40}$$

$$C_{D2} = \frac{-3}{10} + C_2F^\alpha_d \tag{41}$$

$$C_{D3} = \frac{1}{5} + C_3F^\alpha_d \tag{42}$$

where A, B, C_1, C_2 and C_3 are adjustable constants but α is not as arbitrary as it might seem at first look. The conditions (20) and (21) suggest $\alpha = \frac{1}{2}$ (see Lumley[11] or Shih and Lumley [3]). In the limiting case these coefficients reach their limiting values. Shih et al. [13] took $A = 0.8$ and $B = 0.0$ which fit the DNS data quite well. Shih et al. [14][15] set $C_1 = C_2 = C_3 = 0$ in their computations.

Models of Other Terms

The focus of this paper is on the calculation of the velocity field in the boundary-free flows. Therefore, here we list only the related models for the pressure transport, the third moments, and the return-to-isotropy terms in the second moment equations and the models for the dissipation equation. All these models were proposed by Lumley[8], and they satisfy the realizability conditions discussed in the section II.

Pressure transport term:

$$-\frac{1}{\rho}\overline{p^{[2]}u_i} = C\overline{q^2 u_i} \tag{43}$$

where C is a constant, and Lumley[8] suggested $C = 0.2$.

Third moments:

$$\begin{aligned}\overline{u_i u_j u_k} = &-\frac{1}{3\beta}\frac{q^2}{\overline{\epsilon}}\big[\overline{u_k u_p}(\overline{u_i u_j})_{,p} \\ &+ \overline{u_j u_p}(\overline{u_i u_k})_{,p} + \overline{u_i u_p}(\overline{u_j u_k})_{,p}\big] \\ &+ \frac{\beta - 2}{9\beta}\big[\delta_{ij}\overline{q^2 u_k} + \delta_{ik}\overline{q^2 u_j} + \delta_{jk}\overline{q^2 u_i}\big]\end{aligned} \tag{44}$$

$$\overline{q^2 u_k} = -\frac{3}{4\beta + 10}\frac{q^2}{\overline{\epsilon}}\big[\overline{u_k u_p} q^2_{,p} + 2\overline{u_p u_q}\ \overline{u_k u_q}_{,p}\big] \tag{45}$$

Return-to-isotropy term:

$$\Phi_{ij} = \beta b_{ij} \tag{46}$$

$$\begin{aligned}\beta = &\ 2 + \exp\left(-\frac{7.77}{Re^{1/2}}\right)\Big\{\frac{72}{Re^{1/2}} + 80.1\ln\big[1 \\ &+ 62.4(-II + 2.3III)\big]\Big\}\left(\frac{1}{9} + 3III + II\right)\end{aligned} \tag{47}$$

Model terms in the dissipation equation:

$$\overline{\epsilon u_k} = -\frac{9q^2/\overline{\epsilon}}{5(4\beta + 10)}\overline{\epsilon}_{,p}\Big[\overline{u_k u_p} + 2\frac{\overline{u_k u_q}\ \overline{u_q u_p}}{q^2}\Big] \tag{48}$$

$$\Psi = \psi_0 + \psi_1\frac{q^2}{\epsilon}b_{ij}U_{i,j} \tag{49}$$

$$\begin{aligned}\psi_0 = &\ \frac{14}{5} + 0.98\Big[\exp\left(\frac{2.83}{Re^{1/2}}\right)\Big]\big[1 \\ &- 0.33\ln(1 - 55II)\big] + \psi_{cor}\end{aligned} \tag{50}$$

where $\psi_1 = 2.4$ is a model constant. The term ψ_{cor} is similar to that proposed by Pope[16], which represents the effect of mean vortex stretching on the dissipation:

$$\psi_{cor} = 1.25(1-F)^{0.1}\left(\frac{q^2}{4\bar{\epsilon}}\right)^3 (U_{i,j} - U_{j,i})(U_{j,k} - U_{k,j})(U_{k,i} + U_{i,k}) \tag{51}$$

For isotropic turbulence ($F = 1$), ψ_{cor} becomes zero. For planar flows, ψ_{cor} is also zero because there is no mean vortex stretching. With this extra term in the dissipation rate equation, the spreading rates are predicted very well for both the planar and the round jets.

III Boundary-Free Shear flows

This section presents the results of calculations for some boundary free turbulent shear flows including the two dimensional mixing shear layer, planar jet, axisymmetric jet, and two-dimensional wake. The numerical solutions were obtained by simultaneously solving the set of equations for the mean momentum U_i, Reynolds stress $\overline{u_i u_j}$ and dissipation $\bar{\epsilon}$. All the model constants in the equations remain the same for all the test flows. For these thin shear layer flows, we have adopted the boundary layer approximations, and, therefore, the modeled equations are parabolic (we have kept the viscous diffusion terms in the modeled equations). The numerical scheme is based on the method of Patankar and Spalding[17],[18].

The boundary conditions imposed on these flows are the following: for the mixing layer, the upper and lower free stream velocities have specified values, the derivatives with respect to the transverse direction of all other variables at the upper and lower boundaries have been set to zero. For the jets, the free stream velocity is zero, and the turbulent shear stress is set to zero at the center line of the jets. The transverse derivatives of all other variables at the boundaries are zero (including the mean velocity at the center line). For the two-dimensional wake, the free stream velocity has a specified value. All the other boundary conditions are the same as for the jets.

The initial profiles of all the quantities are arbitrary smooth profiles. The calculations show that all the solutions had reached self-preservation. All the figures presented here are from the far field solutions.

Figures 2, 3 and 4 show the profiles of the mean velocity, turbulent shear stress and energy components for the mixing layer. The experimental data were taken from Bradshaw et al[19], Castro[20] and Gutmark & Wygnanski[21]. The computed mean velocity is in very good agreement with the measurements. The shear stress profile is also satisfactory. The experimental data of the energy components possess scatter but the model shows reasonable agreement with the experiments. The spreading rate (defined as dh/dx, h being the lateral distance between the positions where the velocity is 90% and 10% of the free stream) is calculated to be 0.13, which is also within the experimental scatter.

Figures 5, 6, 7, 8 and 9 show the profiles of the mean velocity, shear stress and energy components for the planar jet. The measurements were taken from Bradbury[22], Heskestad[23], and Gutmark & Wygnanski[21]. The prediction of the mean velocity is in good agreement with the measurements except in the region $\frac{Y}{(X-X_0)} > 0.1$, where the model gives a slight underestimation. The calculations of the shear stress, streamwise and transverse energy components are within the scatter of the experimental data. However, the lateral energy component is apparently overestimated with respect to the measurements. It should be pointed out that this set of measurements shows $\overline{w^2} < \overline{v^2}$ and this is not consistent with the measurements for other shear flows (say, the mixing layer, wake and axisymmetric jet). The calculated spreading rate (defined by $dY_{.5}/dx$, $Y_{.5}$ being the position where the velocity is the 50% of the centerline velocity of the jet) is 0.11 which is very close to the measurements.

Figures 10, 11, 12, 13 and 14 show the profiles for the mean velocity, shear stress and energy components in the axisymmetric jet. The calculations are compared with the measurements of Abbiss et al[24], Wygnanski & Fiedler[25] and Rodi[26]. The predictions for all the quantities are in good agreement with the measurements. The calculated spreading rate (defined as the same as in the planar jet) is 0.09 which is also very close to the measurements.

Finally, figures 15, 16 and 17 show the profiles for the mean velocity, shear stress and energy components in the two-dimensional wake. Usually the 2D wake is a strongly non-equilibrium flow. In our calculation, it takes more time for the solution to approach self preservation as compared with the solutions of the mixing layer and jets. The predictions of various quantities agree reasonably well with the measurements at the far field of the wake, even though the measured wakes are probably not becoming self-similar yet.

From the above calculations and comparisons, we conclude that the model based on the realizability concept performs quite well for typical boundary-free turbulent shear flows. The modeled equations are realizable and will not produce unphysical results, and therefore, we expect that the present model would be suitable for more complex flows.

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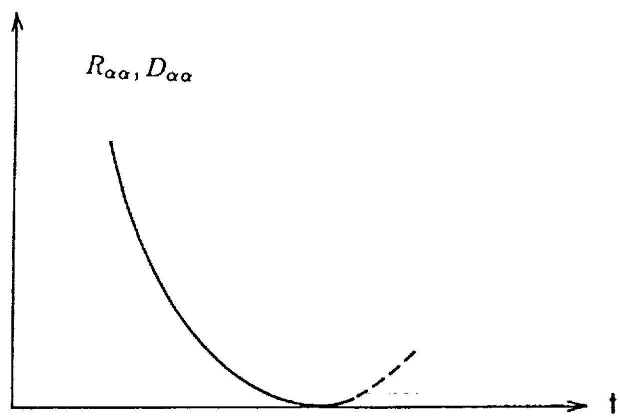

Fig. 1 Realizability condition when eigenvalues vanish.

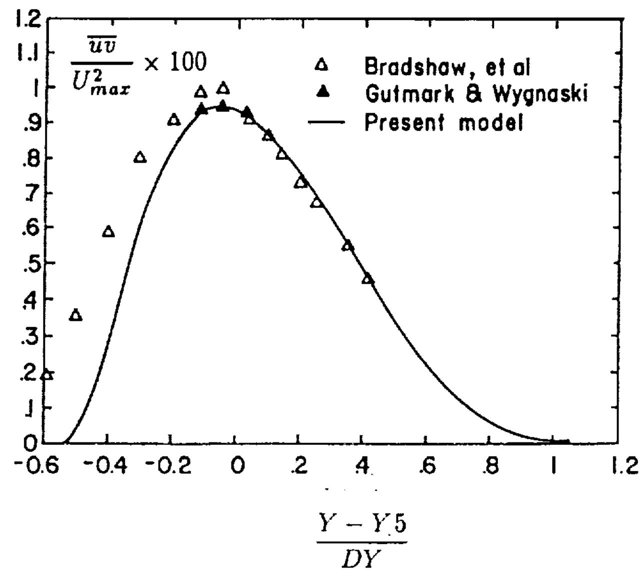

Fig. 3 Shear stress profiles in 2-D mixing layer. △: Bradshaw, et al[19], △: Gutmark & Wygnanski[21], —: Present model.

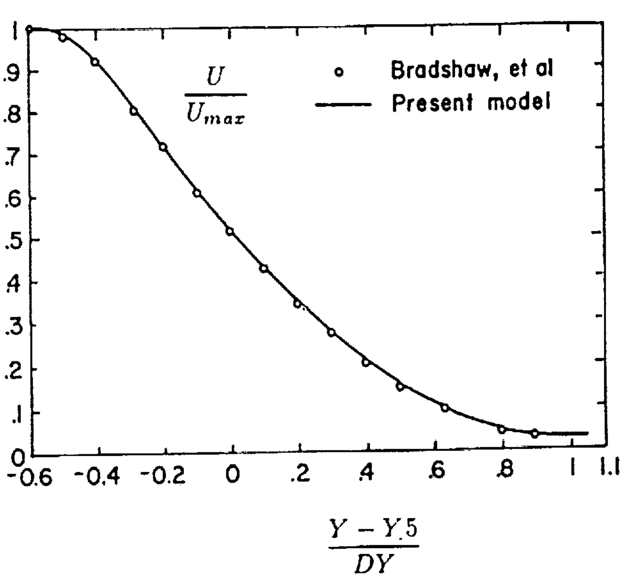

Fig. 2 Mean velocity profile in 2-D mixing layer. U_{max}: the free stream velocity, $Y_{.5}$: the position where $U = \frac{1}{2}U_{max}$. ○: Bradshaw, et al[19], —: Present model.

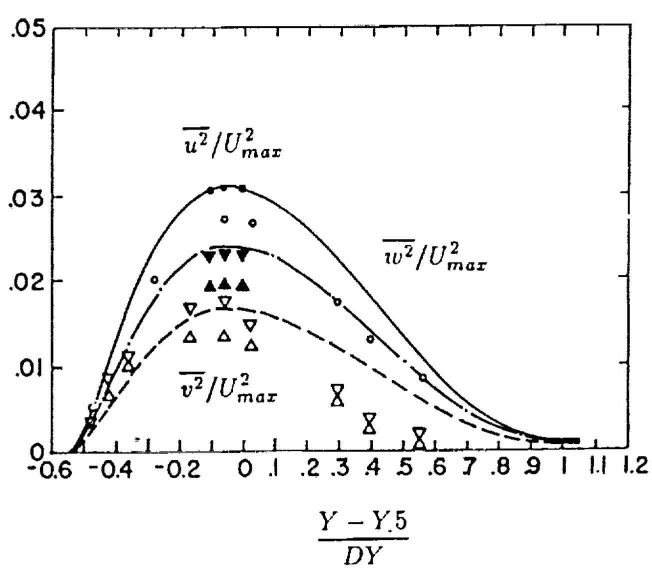

Fig. 4 Normal stress profiles in 2-D mixing layer. ○, △, ▽: Castro[20], ○, △, ▽: Gutmark & Wygnanski[21], The lines represent the present model.

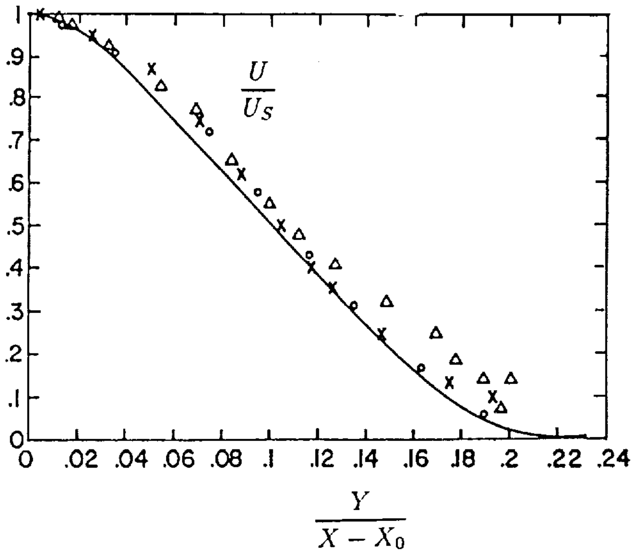

Fig. 5 Mean velocity profile in planar jet. U_S: the center line mean velocity. ○: Bradbury[22], △: Heskestad[23], X: Gutmark & Wygnanski[25], —: Present model.

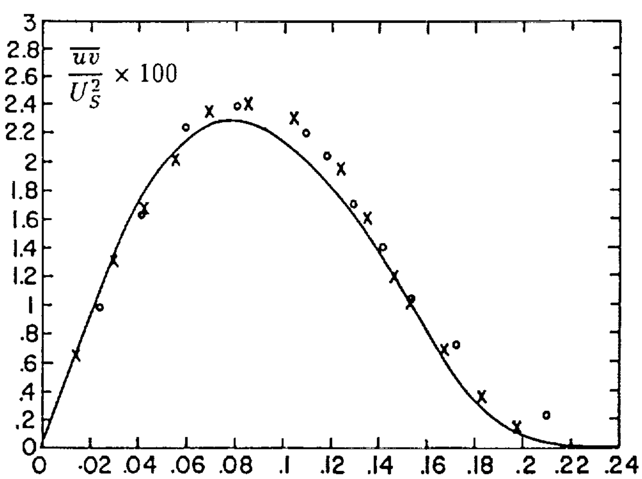

Fig. 6 Shear stress profile in planar jet. Legend as in Fig.5.

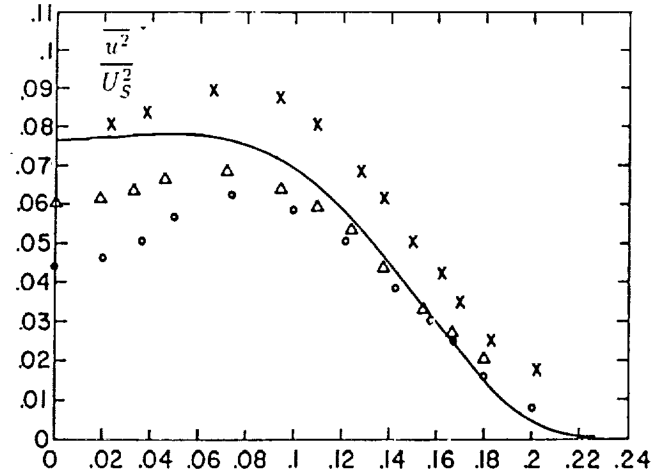

Fig. 7 Normal stress $\overline{u^2}/U_S^2$ profile in planar jet. Legend as in Fig.5.

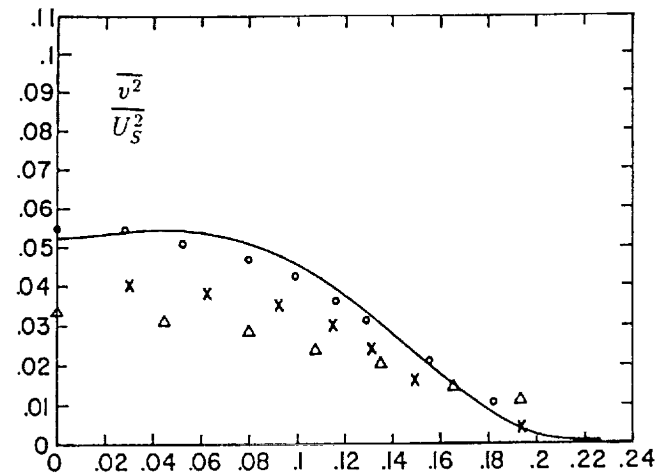

Fig. 8 Normal stress $\overline{v^2}/U_S^2$ profile in planar jet. Legend as in Fig.5.

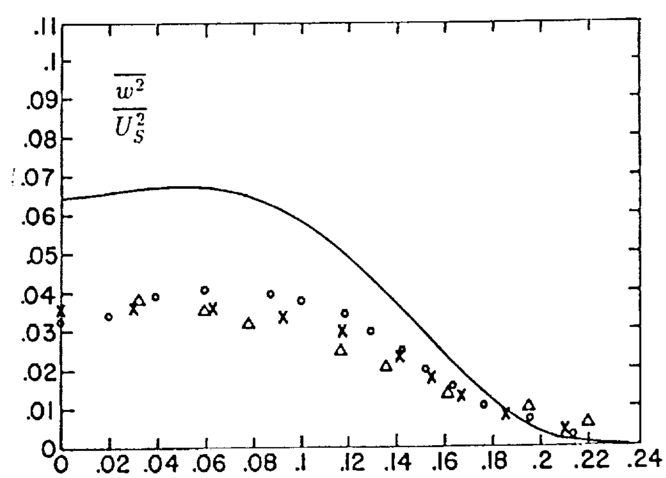

Fig. 9 Normal stress $\overline{w^2}/U_S^2$ profile in planar jet. Legend as in Fig.5.

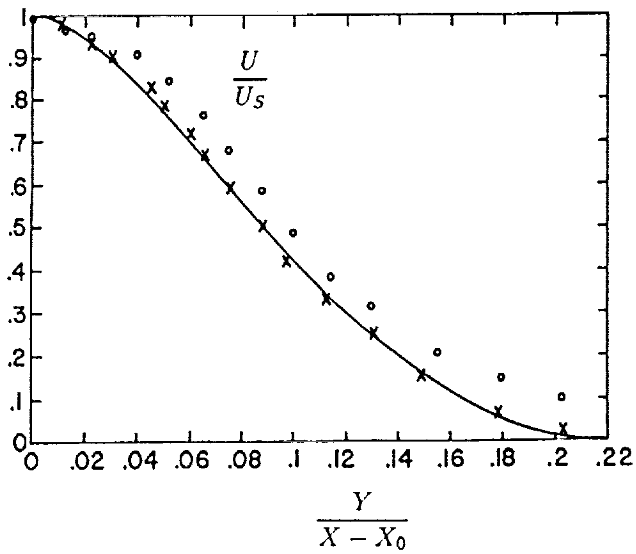

Fig. 10 Mean velocity U/U_S profile in axisymmetric jet. U_S: the centerline mean velocity. ○: Abbiss et al[24], X: Wygnanski & Fiedler[25], —: Present model.

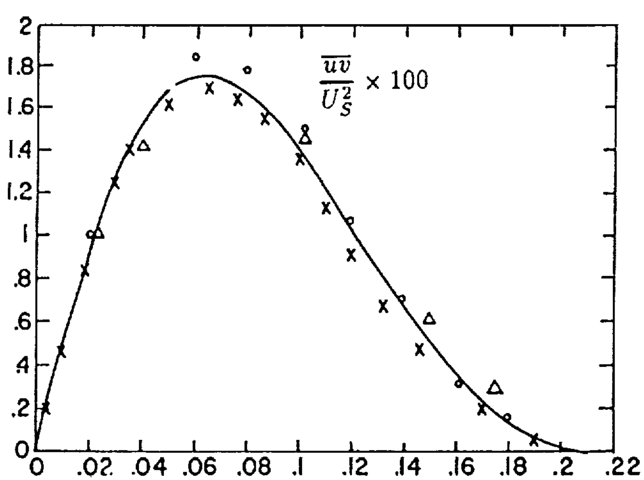

Fig. 11 Shear stress $\overline{uv}/U_S^2$ profile in axisymmetric jet. ○: Rodi[26], X: Wygnanski & Fiedler[25], △: Abbiss et al, —: Present model.

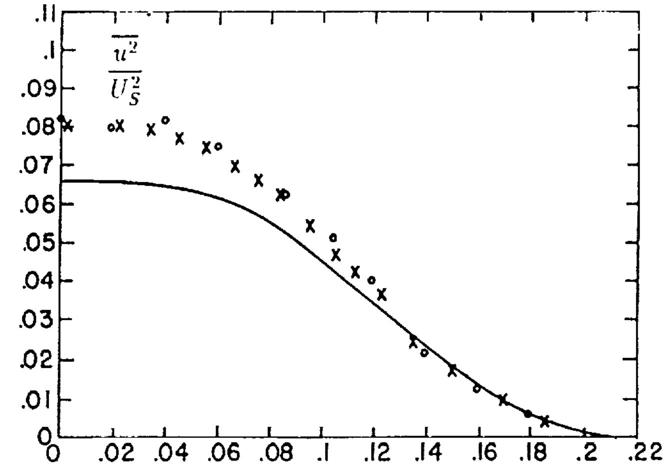

Fig. 12 Normal stress $\overline{u^2}/U_S^2$ profile in axisymmetric jet. Legend as in Fig.11.

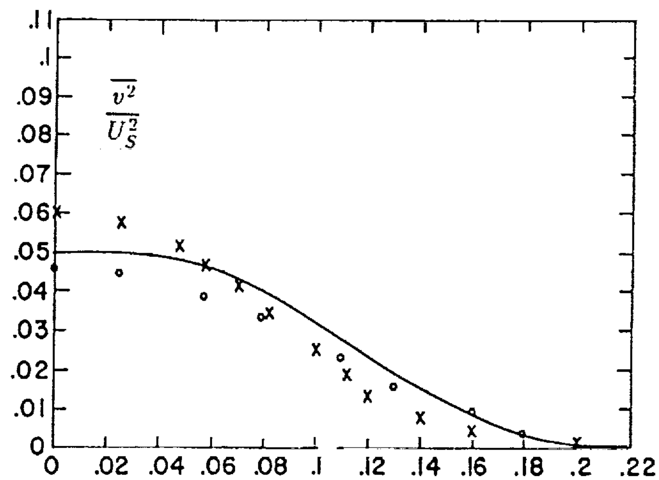

Fig. 13 Normal stress $\overline{v^2}/U_S^2$ profile in axisymmetric jet. Legend as in Fig.11.

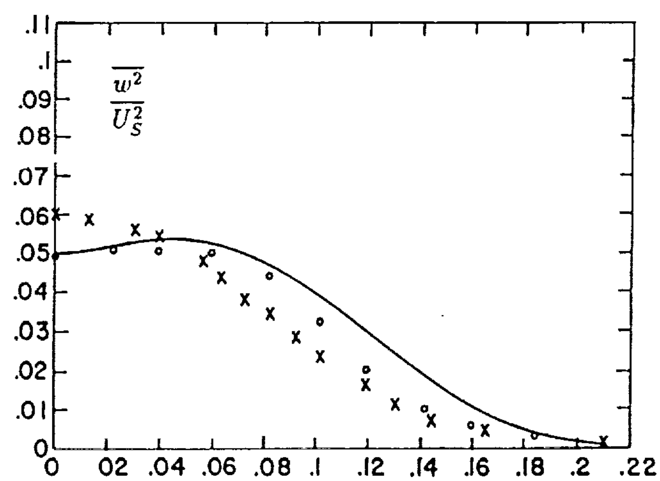

Fig. 14 Normal stress $\overline{w^2}/U_S^2$ profile in axisymmetric jet. Legend as in Fig.11.

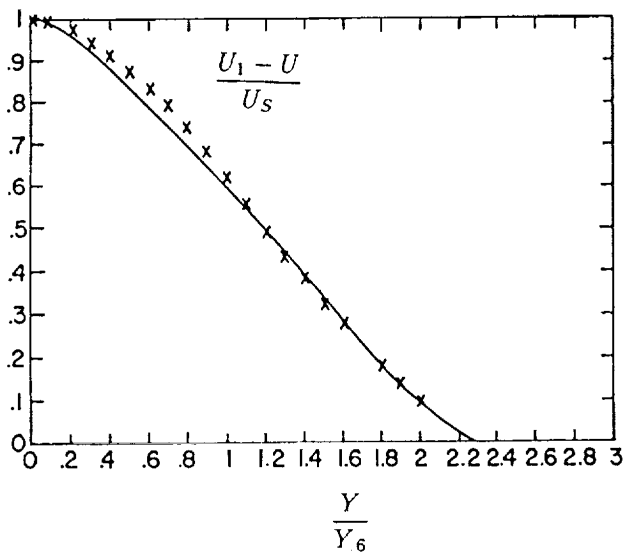

Fig. 15 Mean velocity profile $(U_1 - U)/U_S$ in 2-D wake. U_1: the free stream velocity, U_0: the centerline mean velocity, $U_S = U_1 - U_0$, $Y_{.6}$: the position where $U_1 - U = 0.6U_S$. X: Chevray & Kovasgnay[27], —: Present model.

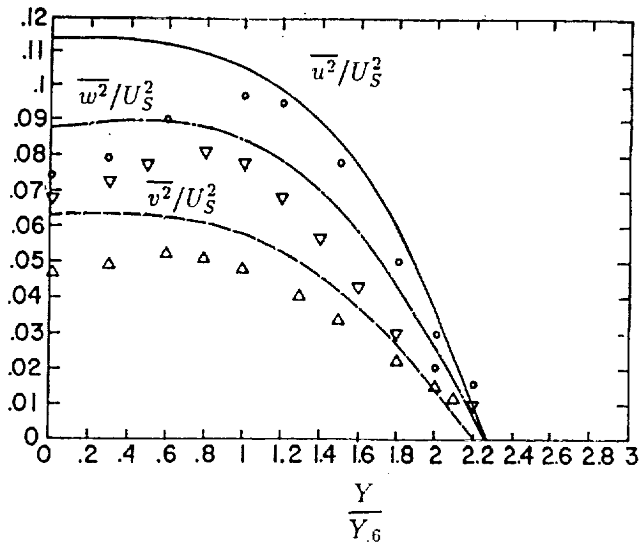

Fig. 17 Normal stress profile in 2-D wake. Chevray & Kovasynay[27]: ○: $\overline{u^2}/U_S^2$, △: $\overline{v^2}/U_S^2$, ▽: $\overline{w^2}/U_S^2$. The lines represent the present model.

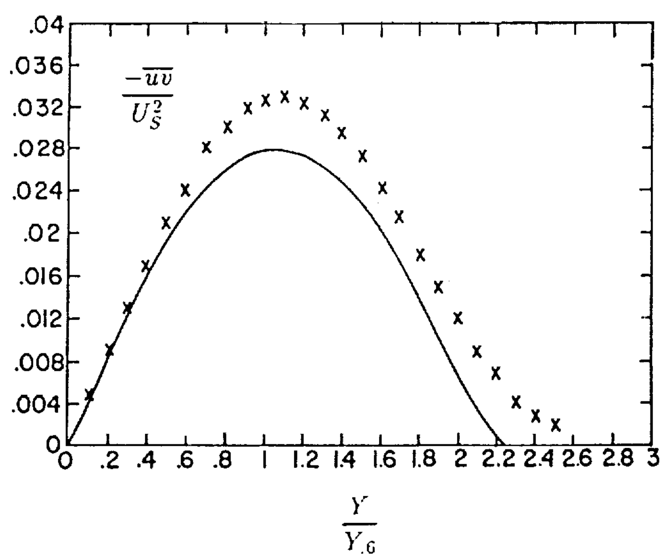

Fig. 16 Shear stress $\overline{uv}/U_S^2$ profile in 2-D wake. Legend as in Fig. 15.

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16. Abstract

This paper presents a set of realizable second order models for boundary free turbulent flows. The constraints on second order models based on the realizability principle are reexamined. The rapid terms in the pressure correlations for both the Reynolds stress and the passive scalar flux equations are constructed to exactly satisfy the joint realizability. All other model terms (return-to-isotropy, third moments and terms in the dissipation equations) already satisfy realizability (Lumley 1978, Shih and Lumley 1986). To correct the spreading rate of the axisymmetric jet, an extra term is added to the dissipation equation which accounts for the effect of mean vortex stretching on dissipation. The test flows used in this study are the mixing shear layer, plane jet, axisymmetric jet and plane wake. The numerical solutions show that the new unified model equations (with unchanged model constants) predict all these flows reasonably as the results compare well with the measurements. We expect that these model equations would be suitable for more complex and critical flows.

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