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Abstract

This paper presents a sensitivity-based linearly varying scale factor used to reconcile results from simple and refined models for analysis of the same structure. The improved accuracy of the linear scale factor compared to a constant scale factor as well as the commonly used tangent approximation is demonstrated. A wing box structure is used as an example, with displacements, stresses and frequencies correlated. The linear scale factor could permit the use of a simplified model in an optimization procedure during preliminary design to approximate the response given by a refined model over a considerable range of design changes.

Introduction

The design optimization of an engineering system typically requires hundreds of analyses of that system. The use of approximations to the objective function and constraints during portions of the design process is quite common (e.g., [1]), because of the high computational cost of these analyses. Such design approximations can be divided into two classes. First there are local, derivative-based approximations such as the linear approximation based on a Taylor-series expansion about a design point. These approximations are typically based on an accurate model to obtain the system response and its derivatives. Second, there are global approximations that try to capture the behavior of the objective function or constraints over the entire design domain. Such approximations are often based on a simplified theory, a coarser model or both (e.g., [2]). Here such global approximations are referred to as simple-model approximations. Local approximations are typically very accurate near the design point where they are generated but, since they are based on an extrapolation procedure, their accuracy can deteriorate catastrophically at a distance. Simple-model approximations are intended to capture the physics of the problem at some lower, but acceptable, level of accuracy over the entire design domain. Hence, at a particular design point the simple-model approximations are generally less accurate than local approximations but on the other hand they typically do not experience the catastrophic deterioration in accuracy if significant design changes are made.

There has been much research into improving local approximations to extend their region of usefulness. In structural optimization the use of intervening variables has been found to be effective. For example, for many structural design problems (such as stress optimization) it was found that the linear approximation in the reciprocal of the design variables is more accurate than the ordinary linear approximation (e.g., [3]). Similarly, it was found that forces are approximated better than stresses (see [4]), so that a linear approximation of element forces followed by explicit calculation of stresses is more accurate than a linear approximation of the stresses. Similarly, in aerodynamics there have been efforts, such as coordinate stretch-

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ing [5], to improve local approximations for aerodynamic drag.

There has also been some research into improving global or simple-model approximations. One active area is the use of the so called reduced-basis approximations in structural dynamics. There, the order of the structural model is reduced by retaining only a small number of deformation shapes, e.g. vibration modes, and assuming that the structural response can be approximated as a linear combination of these shapes. Research has focused on the best selection of these shapes [6] as well as on methods that improve accuracy for a given set of shapes (e.g., [7]). The most commonly used global approximations, however, are based on coarser discretizations or less computationally expensive theoretical models for the same problem. For example, in the optimization of wing structures plate models have been used to replace the more expensive built-up finite-element models (e.g., [8]-[10]).

It may be expected that the advantages of local approximations and simple-model approximations can be combined. In fact, multi-grid analysis methods (e.g., [11]) have demonstrated the utility of working simultaneously with coarse and refined approximations. In [12] a method, called the global-local approximation, was proposed which uses sensitivity information from both a refined and a coarse model of the structure to construct an approximation that combines the advantages of local and global approximations. The method was used with coarse and refined finite-element models of a simple beam structure. The objective of the present paper is to show that the method can be used for better correlation of results obtained with models based on different analytical formulations. In particular, the method is applied to equivalent-plate and finite-element models of a candidate wing structure for a supersonic transport.

Sensitivity-Based Scaling Approximation

Scaling factors have been used in the past (e.g., [13]) for the correlation of simplified and refined models. Use of a scale factor involves comparing results from a coarse model or simplified approximation at a given design point to results for a more refined approximation, or to the exact result, if available. The ratio of results from the refined approximation to results from the coarse approximation is a scaling factor that is used to multiply the coarse approximation results at other points in the design domain. Recall that the coarse approximation is viewed as a global approximation. Then the scaling factor introduces some local flavor into the process in that it is most effective near the point in the design domain where the factor is calculated. It is possible, in cases where the value of the scaling factor changes considerably over the design domain, that the local improvement comes at the expense of reduced accuracy of the scaled coarse approximation far from the point where the scale factor is calculated.

The newly developed sensitivity-based correlation approach refines the traditional constant scaling factor by using a linearly varying scaling factor. The method can be viewed as a combination of the local aspect of derivative based approximations with the global aspect of coarse-model approximations. For simplicity, consider first a structural response from a refined analysis that is a function of a single design variable, $f_R(x)$ which is approximated by a simplified analysis as $f_S(x)$, and define a scaling factor, β calculated at a design point x_o as

$$\beta(x_o) = f_R(x_o) / f_S(x_o) \tag{1}$$

The scaling factor at any other point can be approximated as

$$\beta(x) = \beta(x_o) + (x - x_o)\, \beta'(x_o) \tag{2}$$

where the prime symbol denotes the derivative with respect to x. Taking the derivative of the expression for the scale factor, β in Eq. (1) gives

$$\beta'(x_o) = \beta(x_o) \left[f'_R(x_o)/f_R(x_o) - f'_S(x_o)/f_S(x_o) \right] \tag{3}$$

Substituting Eq. (3) into Eq. (2) gives the sensitivity based correlation factor, $\beta(x)$, and approximations to the global response using constant and linear scaling are given respectively by

$$f_{SRC}(x) = \beta(x_o)\, f_S(x) \quad \text{and} \quad f_{SRL}(x) = \beta(x)\, f_S(x) \tag{4}$$

This approach is applicable to a vector of design variables with the single scale factor, β, in Eq. (2) being replaced by the first-order Taylor series expansion of β.

Wing-Box Example

The proposed method is demonstrated for a wing-box model of a high speed civil transport shown in Fig. 1. The simplified-model of the wing box is analyzed using the equivalent-plate analysis method described in Refs. 9 and 10. The simplified model (Fig. 1(a)) is composed of 3 equivalent plates, referred to as panels I, II, and III, and 22 spar caps (not shown on figure). The cross-sectional geometry of the equivalent plate is given by the location of the midcamber surface and depth which are both defined as polynomial functions over the planform of the wing. The thicknesses of the upper and lower cover skins in each panel are given in polynomial form as

$$t = C_0 + C_1\xi + C_2\eta + C_3\xi\eta \qquad (5)$$

where ξ and η are the fraction of chord and fraction of semispan of each panel, and C_0, C_1, C_2, and C_3 are thickness coefficients which are used as design variables. The initial design point for the equivalent plate model has thickness coefficients for the three panels given in Table 1.

The refined analysis is performed with a finite-element program [14] using a built-up model, shown in Fig. 1(b), containing 60 membrane elements, 70 shear webs, and 180 bar elements to represent rib and spar caps and vertical posts between the covers. The thickness coefficients of the simplified model are used to generate the thickness of the membrane elements in the upper and lower skins in the finite-element model by averaging plate thickness values corresponding to the four nodes of each finite element. The wing is loaded by two 25,000 lb. loads in the z direction at the two extreme points of the tip of the model (nodes 1 and 7 in Fig. 1). The wing is assumed to be made of silicon carbide fiber in a titanium metal matrix (SCS6/Ti) stacked into a quasi-isotropic laminate. The material properties used here are a Young's modulus of 21.32×10^6 psi, Poisson's ratio of 0.325, shear modulus of 8.0×10^6 psi, and a density of 0.144 lb/in^2.

Values for the nominal displacements, stresses and lowest frequencies of the two models are given in Table 2 with the locations of the displacement and stress values shown in Fig. 1. These results indicate that the agreement between the calculated structural responses from the two models at the initial design point is quite good. The corresponding derivatives of tip displacement, stress value at the specified location, and lowest frequency with respect to the thickness coefficients in each panel are given in Table 3 for both models. These derivatives are needed to calculate the linearly varying scale factor, f_{SRL}. The values of the derivatives given by the simplified and refined models have a reasonable level of agreement, with the largest differences being in the stress derivatives.

Approximation Results and Discussion

The accuracy of the scaling methods are tested by making changes in the values of thickness coefficients and evaluating the structural responses given by the various approximations. The approximations are evaluated as the ratios of the thickness coefficients are increased up to factors of $C_i / C_i^0 = 5$, where the superscript ,0, denotes the original value. Results are presented for a subset of the design variables listed in Table 3. The coefficient C_0, the constant term in the thickness polynomial of Eq. 5, is selected for many of the results because the derivatives of the structural response quantities with respect to C_0 are large compared to derivatives with respect to other coefficients. The coefficent, C_2, is also used for some cases to illustrate the change of a linear term in the thickness polynomial.

A comparison of values of normalized tip displacement (again refer to Fig. 1) are shown in Figs. 2 and 3 for changes in C_0 in panel I and C_2 in panel II. In these figures f_R represents results from the refined (finite-element) analysis, f_S are results from the simplified (equivalent-plate) analysis, and f_{SRC} and f_{SRL} are the constant and linearly scaled approximations, respectively. The displacements are normalized by the value of the refined model at the initial design. The two figures show that the error associated with the simple model increases as the design variables move away from the initial design. The factor corresponding to the constant scale approximation f_{SRC} is small since the agreement between the two models is quite good at the initial design and this factor contributes very little to correcting the error at points away from the initial design. The linear scale factor approximation, f_{SRL}, on the other hand, almost completely eliminates the error due to change in design. The linear Taylor series (tangent) approximation, commonly used in optimization is also shown in Fig. 2 and 3. However, it is clear that such a tangent approximation is less accurate for the displacement here than either scale factor approximation. To keep an expanded scale on Fig. 2, the tangent approximation is not extended to the point at $C_i / C_i^0 = 5$.

Figures 4 and 5 show the same approximations applied to the stress at the point shown in Fig. 1 for changes in C_0 for panel I as before and C_0 instead of C_2 for panel II. For changes in C_0 for panel I (Fig. 4) the relative ranking of the approximations remains the same. The errors in f_S, though, are much larger, so that the improvement due to scaling is more evident. This time, however, it appears that while the linearly varying scaling is much better than a tangent approximation, the constant scale factor is about the same. For changes in C_0 for panel II (Fig. 5) the situation is different. In this case, the linearly varying scale factor is the most accurate approximation only near the initial design. Further away the constant scale factor is slightly better. This is an example where the local nature introduced by using derivatives asserts itself. Note, however, that even in this case both scaling methods are better than the tangent approximation at a distance from the initial design.

Figures 6 and 7 show the approximations applied to the first (lowest) natural frequency of the structure. As with the tip displacement, the frequencies of the refined and simple models are almost the same at the initial design. Therefore, very little improvement in accuracy is afforded by the constant scale factor approximation. Again the linear scale factor approximation is very accurate, and much better than the tangent approximation.

The previous figures show the effect of changing only one design variable at a time. These results do not check the effect of design variable interaction on the quality of the approximation. This effect was demonstrated by making a change to a selected combination of design variables. The combined change was made by simultaneously applying the changes given in the following four equations

$$
\begin{aligned}
C_0 &= C_0^0(1+\delta) \quad \text{in panel I} \\
C_0 &= C_0^0(1-\delta) \quad \text{in panel II} \\
C_2 &= C_2^0(1+\delta) \quad \text{in panel II} \\
C_0 &= C_0^0(1+\delta) \quad \text{in panel III}
\end{aligned}
\tag{6}
$$

Note that the fractional design variable change, δ, was applied in the negative direction for C_0 in panel II. Figures 8 to 10 show the effect of such a combined design variable change on displacement, stress and frequency. Again the same general trend of the linear scale factor giving better correlation than the constant scale factor is observed. However for large design changes, even the linear scale factor only accounts for approximately half the difference between the simplified and refined analysis results. Again, both scaling methods give better correlation than the tangent approximation at a sufficiently large distance from the initial design.

Implications for Design Optimization

Design optimization of complex structures is typically performed in procedures which solve a sequence of approximate optimization problems. With this approach each optimization is based on an approximation to the structural response. The traditional, most often used local approximation is the tangent approximation. Global approximations based on simpler theory or a coarser model are used with or without scale factors. The approximation is updated at the optimum of the approximate problem, and the process is repeated to convergence. The process of updating the approximation involves at least an analysis and possibly an analysis plus derivative calculation for the refined model. This updating often accounts for more than half of the total computational cost of the entire optimization procedure.

Periodic update of a constant scale factor requires only a single analysis of the refined model, however, updating the linear scale factor requires an analysis plus derivative calculation of the refined model. For effective structural design, the procedure should be selected that will result in lowest computational cost for the overall optimization procedure. This selection is application-, procedure-, and problem-dependent. In particular, it will depend strongly on the computational efficiency and level of accuracy of the simplified model. When the scale factor does not vary much over the design space the constant scale factor will be more effective because it does not require the calculation of expensive derivatives of the refined model. If the scale factor does vary substantially over the design space then the linear approximation is needed and is a valuable tool to provide the desired level of accuracy over a considerable region of the design domain. Actual experience in applying these scaling methods in a variety of problems is needed to establish guidelines for selecting the most effective approximation for a particular design problem. However, such applications are beyond the scope of this paper.

Concluding Remarks

A description is given of a new sensitivity-based scaling method for correlating structural response from different analytical models. Traditionally, a constant factor based on a single design point has been used to scale results between simple and refined models of the same structure. Here two different formulations are used to analyze the same structure; a simplified representation using an equivalent-plate analysis method, and a refined representation using a conventional finite-element analysis program. A wing box structure is used as an example, with displacements, stresses and frequencies correlated. The accuracy of the approximate responses given by the constant and linear varying scale factors as well as the commonly used tangent method are compared. In most cases presented, the linearly varying scale factor gave considerably better correlation than the constant scale factor. Both scaling methods gave results that were superior to the tangent approximation at a distance away from the initial design point. The linear scale factor was demonstrated to give good correlation with the refined analysis results over a considerable range of changes in design variables (up to a factor of 5 change).

This linear scaling method provides a new approach for approximating structural response which should prove to be effective when applied in overall design optimization procedures. Moreover, the method is not limited to structural applications and can be used by other disciplines to correlate responses from different analytical models.

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Table 1. Initial variables of the equivalent plates.

	Coefficient, inch			
Panel no.	C_0	C_1	C_2	C_3
I	0.008	-0.001	-0.001	-0.001
II	0.007	-0.001	-0.001	-0.001
III	0.006	-0.001	-0.001	-0.001

Table 2. Comparison of analysis results for simplified and refined analyses.

	Displacement inch	von Mises stress $\times 10^6$	First freq. Hz
Simplified model	13.30	7.48	1.80
Refined model	13.40	8.53	1.74

Table 3. Derivatives of displacement, stress and first frequency of the simplified and refined models.

		Simplified model			Refined model		
		Derivatives of			Derivatives of		
Panel no.	D.V.	Displ. $\times 10^2$	Stress $\times 10^8$	Freq. $\times 10^1$	Displ. $\times 10^2$	Stress $\times 10^8$	Freq. $\times 10^1$
	C_0	-1.4223	-1.3133	1.680	-1.1872	-0.8295	1.425
	C_1	-1.1413	-1.1691	1.255	-0.9186	-0.7299	1.093
I	C_2	-0.7116	-1.1691	0.913	-0.5993	-0.4185	0.718
	C_3	-0.5406	-0.7014	1.079	-0.4591	-0.3648	0.546
	C_0	-8.5849	-9.2335	7.533	-9.2122	-11.8865	7.931
	C_1	-4.5124	-3.9660	3.619	-4.7968	-10.5304	3.946
II	C_2	-4.9821	-3.8871	3.950	-5.2868	-4.8955	4.089
	C_3	-2.4329	-1.2359	2.083	-2.5387	-3.9610	1.808
	C_0	-8.0685	0.3918	-6.616	-7.8034	-0.0943	-6.997
	C_1	-3.9299	0.1940	-3.633	-3.8369	-0.0492	-4.137
III	C_2	-3.0114	-0.0733	-5.843	-2.9355	-0.0116	-5.557
	C_3	-1.4479	-0.0072	-2.981	-1.4237	-0.0063	-3.074

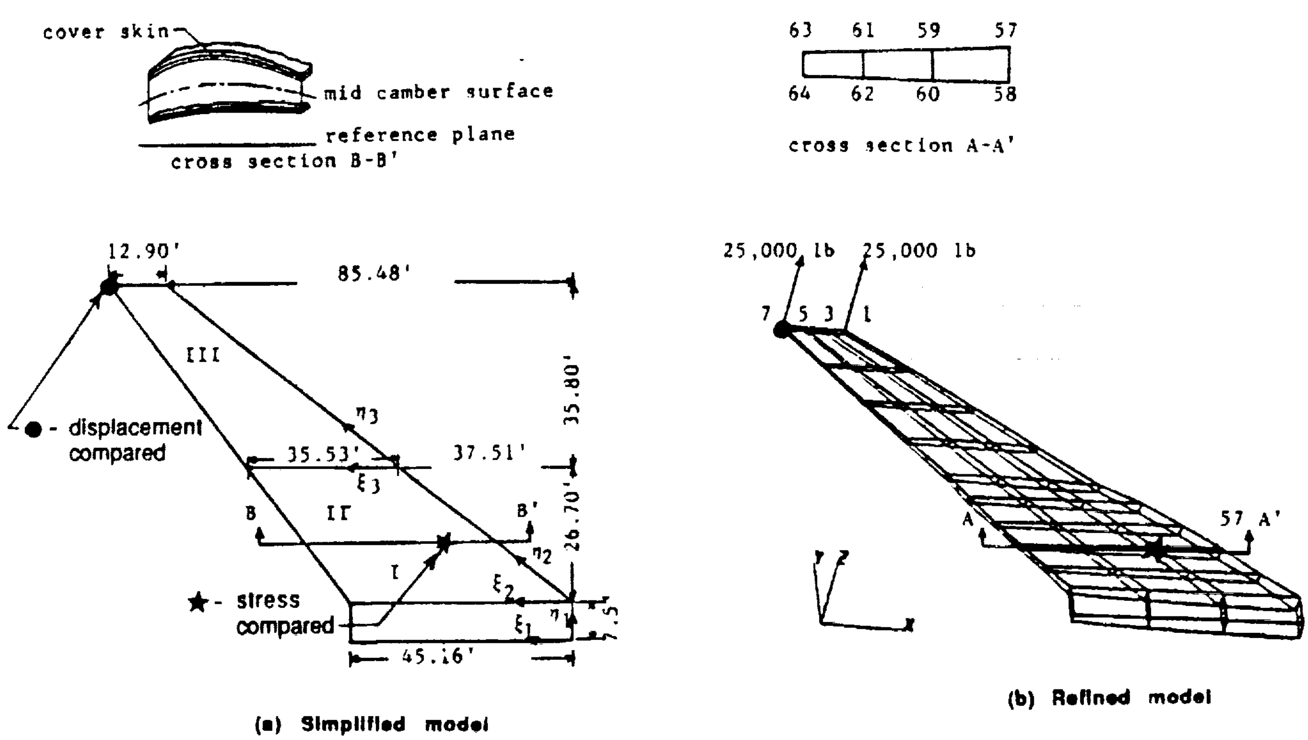


Figure 1. Simplified and refined models of wing-box structure.

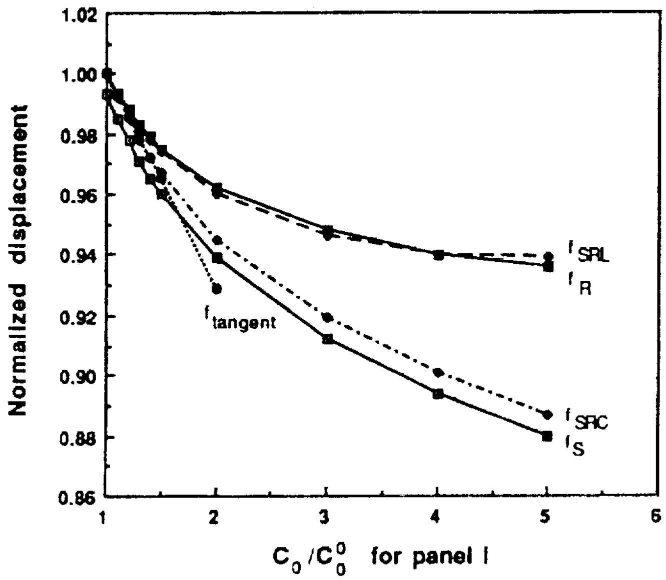


Figure 2. Approximations to wing tip displacement.

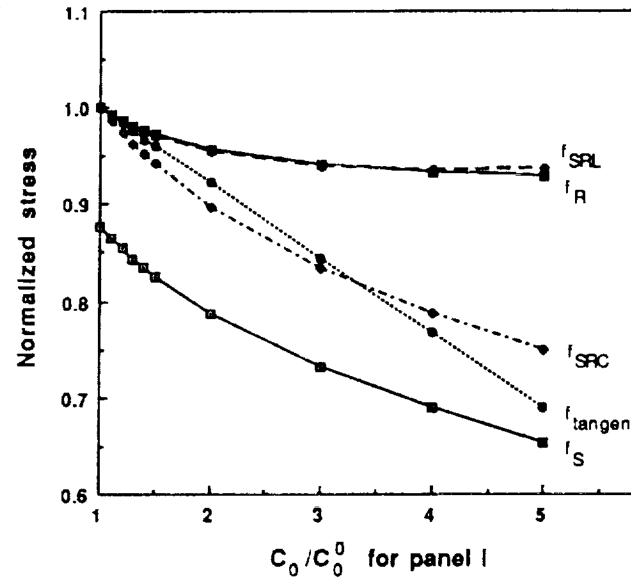
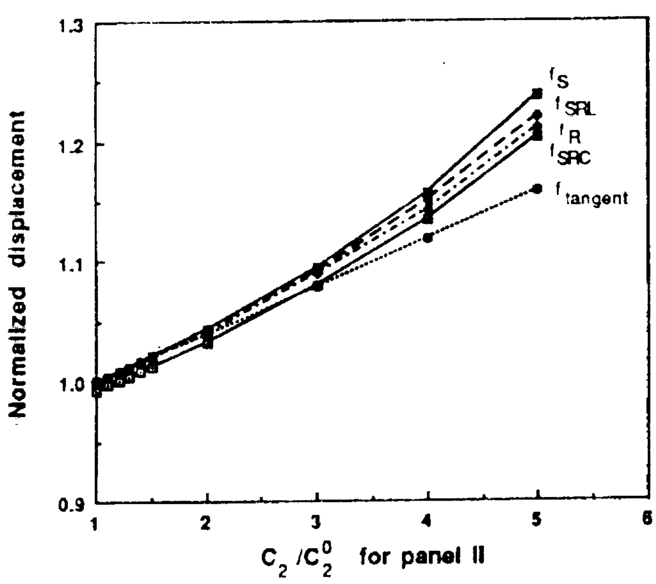


Figure 4. Approximations to von Mises stress at node 59.



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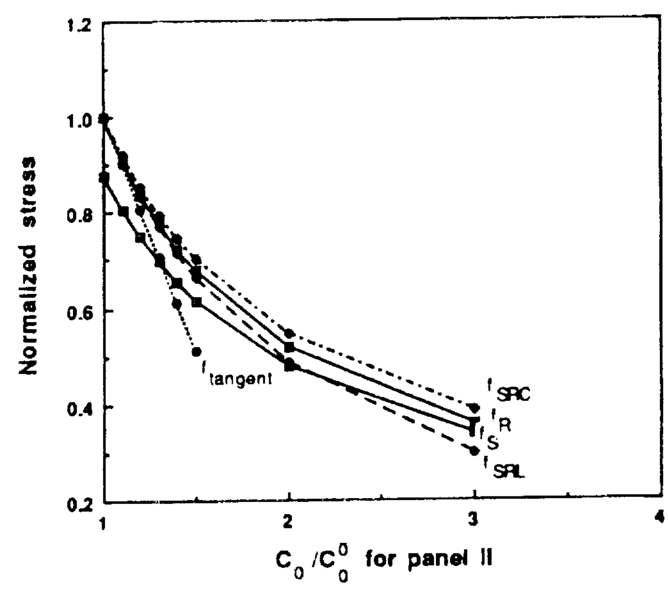

Figure 5. Approximations to von Mises stress at node 59.

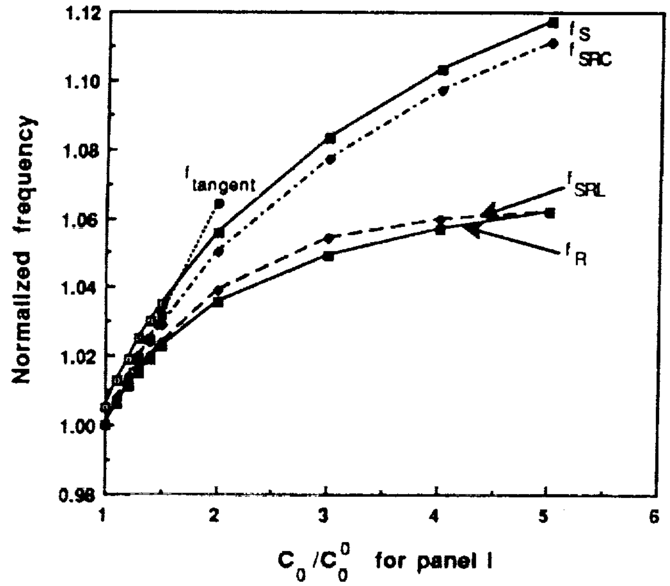

Figure 6. Approximations to first frequency.

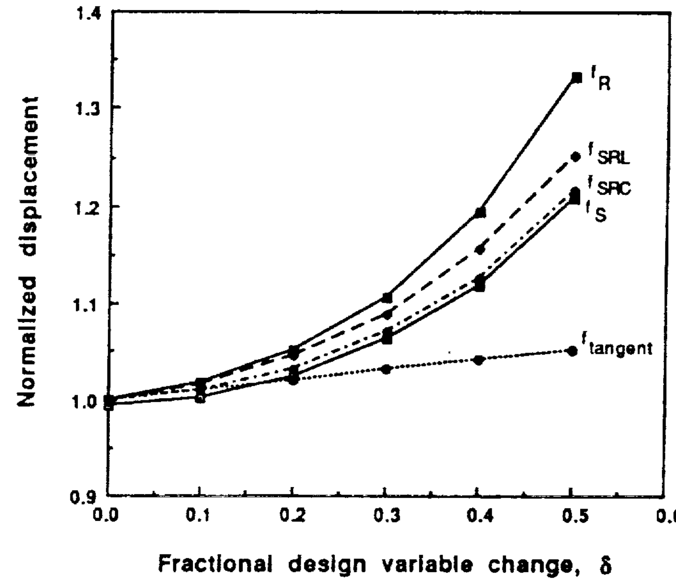

Figure 8. Approximations to wing tip displacement for combined changes of variables (see Eq. (6)).

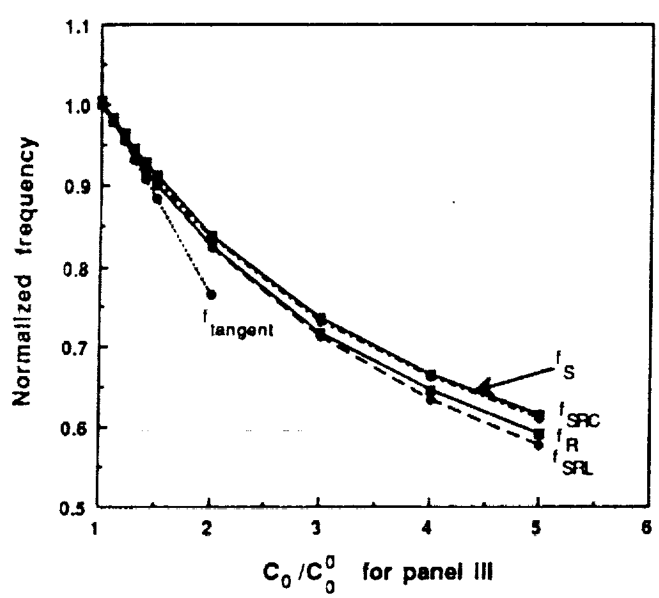

Figure 7. Approximations to first frequency.

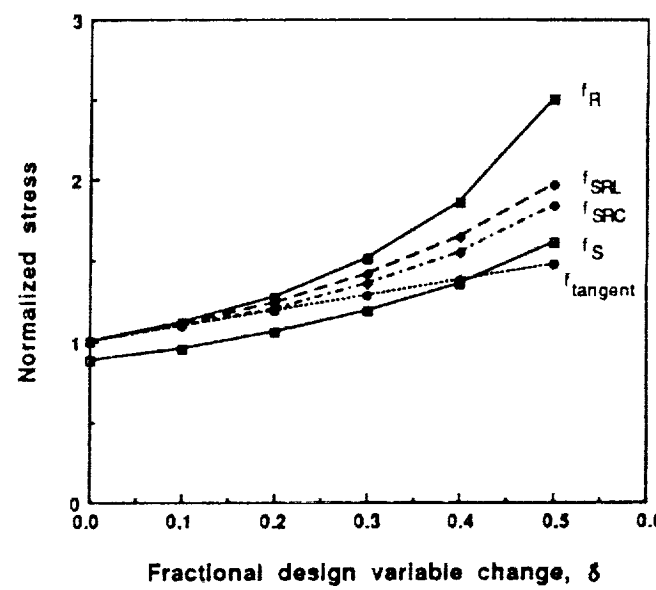

Figure 9. Approximations to von Mises stress for combined changes of variables (see Eq. (6)).

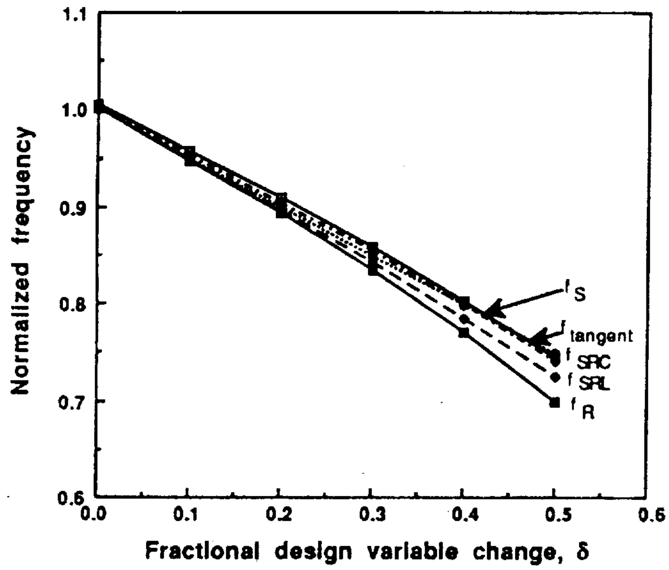

Figure 10. Approximations to the first frequency for combined changes of variables (see Eq. (6)).

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16. Abstract

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