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Numerical Two-Dimensional Calculations of the Formation of the Solar Nebula

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ABSTRACT

The protostellar phase of stellar evolution is of considerable importance with regard to the formation of planetary systems. The initial mass distribution and angular momentum distribution in the core of a molecular cloud determine whether a binary system or a single star is formed. A relatively slowly rotating and centrally condensed cloud is likely to collapse to a disk-like structure out of which planets can form. The above parameters then determine the temperature and density structure of the disk and the characteristics of the resulting planetary system.

There has been considerable recent interest in two-dimensional numerical hydrodynamical calculations with radiative transfer, applied to the inner regions of collapsing, rotating protostellar clouds of about 1 $M_{\odot}$. The calculations start at a density that is high enough so that the gas is decoupled from the magnetic field. During the collapse, mechanisms for angular momentum transport are too slow to be effective, so that an axisymmetric approximation is sufficiently accurate to give useful results. Until the disk has formed, the calculations can be performed under the assumption of conservation of angular momentum of each mass element. In a numerical calculation, a detailed study of the region of disk formation can be performed only if the central protostar is left unresolved.

With a suitable choice of initial angular momentum, the size of the disk is similar to that of our planetary system. The disk forms as a relatively thick, warm equilibrium structure, with a shock wave separating it from the surrounding infalling gas. The calculations give temperature and density

distributions throughout the infalling cloud as a function of time. From these, frequency-dependent radiative transfer calculations produce infrared spectra and isophote maps at selected viewing angles. The theoretical spectra may be compared with observations of suspected protostellar sources. These disks correspond to the initial conditions for the solar nebula, whose evolution is then driven by processes that transport angular momentum.

OBSERVATIONAL CONSTRAINTS ON THE PROPERTIES OF THE INITIAL SOLAR NEBULA

From observations of physical and cosmochemical properties of the solar system and from astronomical observations of star-forming regions and young stars, certain constraints can be placed on the processes of formation and evolution of the solar nebula.

a) Low-mass stars form by the collapse of initially cold (10 K), dense (10^5 particles cm^{-3}) cores of molecular clouds. The close physical proximity of such cores with T Tauri stars, with imbedded infrared sources which presumably are protostars, and with sources with bipolar outflows, presumably coming from stars in a very early stage of their evolution, lends support to this hypothesis (Myers 1987).

b) The specific angular momenta (j) of the cores, where observed, fall in the range 10^{20} – 10^{21} cm^2 s^{-1} (Goldsmith and Arquilla 1985; Heyer 1988). In the lower end of this range, the angular momenta are consistent with the properties of our solar system: for example, Jupiter's orbital motion has $j \approx 10^{20}$ cm^2 s^{-1}. In the upper end of the range, collapse with conservation of angular momentum would lead to a halt of the collapse as a consequence of rotational effects at a characteristic size of $\sim$ 2000 AU, far too large to account for the planetary orbits. In fact, hydrodynamical calculations suggest that collapse in this case would in fact lead to fragmentation into a binary or multiple system.

c) The infrared radiation detected in young stars indicates the presence of disks around these objects (Hartmann and Kenyon 1988). A particularly good example, where orbital motions have been observed, is HL Tau (Sargent and Beckwith 1987). The deduced masses of the disk and star are 0.1 $M_\odot$ and 1.0 $M_\odot$, respectively. The radius of the disk is about 2000 AU. Roughly half of all young pre-main-sequence stars are deduced to have disks, mostly unresolved, with masses in the range 0.01-0.1 $M_\odot$ and sizes from 10 to 100 AU (Strom *et al.* 1989).

d) The rotational velocities of T Tauri stars are small, typically 20 km s^{-1} or less (Hartmann *et al.* 1986). The distribution of angular momentum in the system consisting of such a star and disk is quite different from that in the core of a molecular cloud, which is generally assumed to be uniformly rotating with a power-law density distribution. Substantial angular momentum transport, from the central regions to the outer regions, must take place early in the evolution. The required transport is unlikely to occur during collapse; therefore it must occur during the disk evolution phase before the star emerges as a visible object.

e) The lifetime of the pre-main-sequence disks is difficult to determine from observation, but it probably does not exceed 10^7 years (Strom *et al.* 1989). The mechanisms for angular momentum transport, which deplete disk mass by allowing it to fall into the star, must have time scales consistent with these observations, as well as with time scales necessary to form gaseous giant planets.

f) The temperature conditions in the early solar nebula can be roughly estimated from the distribution of the planets' and satellites' mass and chemical composition (Lewis 1974). The general requirements are that the temperature be high enough in the inner regions to vaporize most solid material, and that it be low enough at the orbit of Jupiter and beyond to allow the condensation of ices. Theoretical models of viscous disks produce the correct temperature range, as do collapse models of disk formation with shock heating.

g) The evidence from meteorites is difficult to interpret in terms of standard nebular models. First, there is evidence for the presence of magnetic fields, and second, the condensates indicate the occurrence of rapid and substantial thermal fluctuations. Suggestions for explaining this latter effect include turbulent transport of material and non-axisymmetric structure (density waves) in the disk.

h) The classical argument, of course, is that the coplanarity and circularity of the planets' orbits imply that they were produced in a disk.

i) A large fraction of stars are observed to be in binary and multiple systems (Abt 1983); the orbital values of j in the closer systems are comparable to those in our planetary system. It has been suggested (Boss 1987; Safronov and Ruzmaikina 1985) that if the initial cloud is slowly rotating and centrally condensed, it is likely to form a single star rather than a binary. Pringle (1989) has pointed out that if a star begins collapse after having undergone slow diffusion across the magnetic field, it will be centrally condensed and will therefore form a single star. If, however, the collapse is induced by external pressure disturbances, the outcome is likely to be a binary. On the other hand, Miyama (1989) suggests that single star formation occurs in initial clouds with $j \approx 10^{21}$ cm^2 s^{-1}. After reaching a rotationally supported equilibrium that is stable to fragmentation, the cloud becomes unstable to nonaxisymmetric perturbations, resulting in angular momentum transport and collapse of the central regions.

THE PHYSICAL PROBLEM

The above considerations illustrate several of the important questions relating to the formation of the solar nebula: What are the initial conditions for collapse of a protostar? At what density does the magnetic field decouple from the gas? What conditions lead to the formation of a single star with a disk rather than a double star? Can the embedded IRAS sources be identified with the stage of evolution just after disk formation? What is the dominant mechanism for angular momentum transport that produces the present distribution of angular momentum in the solar system? The goal of numerical calculations is to investigate these questions by tracing the evolution of a protostar from its initial state as an ammonia core in a molecular cloud to the final quasi-equilibrium state of a central star, which is supported against gravity by the pressure gradient, and a circumstellar disk, which is supported in the radial direction primarily by centrifugal effects. A further goal is to predict the observational properties of the system at various times during the collapse. A full treatment would include a large number of physical effects: the hydrodynamics, in three space dimensions, of a collapsing rotating cloud with a magnetic field; the equation of state of a dissociating and ionizing gas of solar composition, cooling from molecules and grains in optically thin regions; frequency-dependent radiative transfer in optically thick regions; molecular chemistry; the generation of turbulent motions as the disk and star approach hydrostatic equilibrium; and the properties of the radiating accretion shock which forms at the edge of the central star and on the surfaces of the disk (Shu *et al.* 1987).

The complexity of this problem makes a general solution intractable even on the fastest available computing machinery. For example, the length scales range from 10^{17} centimeters, the typical dimension of the core of a molecular cloud, to 10^{11} centimeters, the size of the central star. The density of the material that reaches the star undergoes an increase of about 15 orders of magnitude from its original value of $\sim 10^{-19}$ g cm^{-3}. Also, the numerical treatment of the shock front must be done very carefully. The number of grid points required to resolve the entire structure is very large in two space dimensions; in three dimensions it is prohibitively large. Even if the detailed structure of the central object is neglected and the system is resolved down to a scale of 0.1 AU, the Courant-Friedrichs-Lewy condition in an explicit calculation requires that the time step be less than one-millionth of the collapse time of the cloud. Therefore, a number of physical approximations and restrictions have been made in all recent numerical calculations of nebular formation. For example, magnetic fields have not been included, on the grounds that the collapse starts only when the gas has become almost completely decoupled from the field because of the negligible degree of ionization at the relatively high densities and cold temperatures involved. Also, in most calculations, turbulence has been neglected during the collapse. Even if it is present, the time scale for transport of angular momentum by this process is expected to be much longer than the dynamical time. It turns out that angular momentum transport can be neglected during the collapse, and therefore an axisymmetric (two-dimensional) approximation is adequate during this phase. Three-dimensional effects, such as angular momentum transport by gravitational torques become important later, during the phase of nebular evolution. A further approximation involves isolating and resolving only specific regions of the protostar. In one-dimensional calculations (Stahler *et al.* 1980), it has been possible to resolve the high-density core as well as the low-density envelope of the protostar. However, in two space dimensions, proper resolution of the region where the nebular disk forms cannot be accomplished simultanously with the resolution of the central star. In several calculations the outer regions of the protostar are also not included, so the best possible resolution can be obtained on the length scale 1-50 AU. Thus the goal outlined above, the calculation of the evolution of a rotating protostar all the way to its final stellar state, has not yet been fully realized.

The stages of evolution of a slowly rotating protostar of about 1 $M_\odot$ can be outlined as follows:

a) The frozen-in magnetic field transfers much of the angular momentum out of the core of the molecular cloud, on a time scale of 10^7 years.

b) The gradual decoupling between the magnetic field and the matter allows the gas to begin to collapse, with conservation of angular momentum.

c) The initial configuration is centrally condensed. During collapse, the outer regions, with densities less than about 10^{-13} g cm^{-3} remain optically thin and collapse isothermally at 10 K. The gas that reaches higher densities becomes optically thick, most of the released energy is trapped, and heating occurs.

d) The dust grains, which provide most of the opacity in the protostellar envelope, evaporate when the temperature exceeds 1500 K. An optically thin region is created interior to about 1 AU. Further, at temperatures above 2000 K, the molecular hydrogen dissociates, causing renewed instability to collapse.

e) The stellar core and disk form from the inner part of the cloud. The remaining infalling material passes though accretion shocks at the boundaries of the core and disk; most of the infall kinetic energy is converted into radiation behind the shock. The surrounding infalling material is optically thick, and the object radiates in the infrared, with a peak at around 60-100 μm.

f) A stellar wind is generated in the stellar core, by a process that is not well understood. The wind breaks through the infalling gas at the rotational poles, where the density gradient is steepest and where most of the material has already fallen onto the core. This bipolar outflow phase lasts about 10^5 years.

g) Infall stops because of the effects of the wind, or simply because the material is exhausted. The stellar core emerges onto the Hertzsprung-Russell diagram as a T Tauri star, still with considerable infrared radiation coming from the disk.

h) The disk evolves, driven by processes that transfer angular momentum, on a timescale of 10^6 to 10^7 years. Angular momentum is transferred outwards through the disk while mass is transferred inwards. The rotation of the central object slows, possibly through magnetic braking in the stellar wind. Possible transport processes in the disk include turbulent (convectively driven) viscosity, magnetic fields, and gravitational torques driven by gravitational instability in the disk or by non-axisymmetric instabilities in the initially rapidly rotating central star.

The following sections describe numerical calculations of phases b through e, from the time when magnetic effects become unimportant to the time when at least part of the infalling material is approaching equilibrium in a disk.

REVIEW OF TWO-DIMENSIONAL CALCULATIONS OF THE FORMATION PHASE

Modern theoretical work on this problem goes back to the work of Cameron (1962, 1963), who discussed in an approximate way the collapse of a protostar to form a disk. In a later work, Cameron (1978) solved numerically the one-dimensional (radial) equations for the growth of a viscous accretion disk, taking into account the accretion of mass from an infalling protostellar cloud. The initial cloud was assumed to be uniformly rotating with uniform density. The hydrodynamics of the inflow was not calculated in detail; rather, infalling matter was assumed to join the disk at the location where its angular momentum matched that of the disk. A similar approach was taken by Cassen and Summers (1983) and Ruzmaikina and Maeva (1986), who, however, took into account the drag caused by the infalling material, which has angular momentum different from that of the disk at the arrival point. The latter authors discuss the turbulence that develops for the same reason (see also Safronov and Ruzmaikina 1985). This section concentrates on full two-dimensional calculations of the collapsing cloud during the initial formation of the disk.

One approach to this problem (Tscharnuter 1981; Regev and Shaviv 1981; Morfill *et al.* 1985; Tscharnuter 1987) is based on the assumption that turbulent viscosity operates during the collapse. The resulting transport of angular momentum out of the inner parts of the cloud might be expected to suppress the fragmentation into a binary system and to reduce the angular momentum of the central star to the point where it is consistent with observations of T Tauri stars. The procedure is to assume a simple kinematic viscosity $\nu = 0.33\ \alpha c_s L$, where c_s is the sound speed, L is the length scale of the largest turbulent eddies, and α is a free parameter. Subsonic turbulence is generally assumed, so that α is less than unity. Once the collapse is well underway, the sound crossing time is much longer than the free fall time, so that angular momentum transport is actually relatively ineffective. Binary formation is probably suppressed, but the material that falls into the central object still has high angular momentum compared with that of a T Tauri star.

Nevertheless, the model presented by Morfill *et al.* (1985) provides interesting information regarding the initial solar nebula. In contrast to the earlier calculation of Regev and Shaviv (1981), which used the isothermal approximation, this two-dimensional numerical calculation included the full hydrodynamical equations, applicable in both the optically thin and optically thick regions. Radiation transport was included in the Eddington approximation. The calculations started with 3 $M_{\odot}$ at a uniform density of 10^{-20} g cm^{-3} and with uniform angular velocity. Two different values for j were tested, 10^{21} and 10^{20} cm^2 s^{-1}, with qualitatively similar results. The case with lower angular momentum is the one of most interest. The collapse proceeds and results in the formation of a central condensation surrounded by a disk. However, the core does not reach hydrostatic equilibrium but exhibits a series of dynamical oscillations, driven, according to the authors, by heat generated through viscous dissipation in the region near the edge of the core. The dynamical expansion is accompanied by an outgoing thermal pulse.

So that the development of the disk could be studied, the computational procedure was modified to treat the region interior to 2×10^{12} cm as an unresolved core, and thereby to suppress the oscillations. Matter and angular momentum were allowed to flow into this central region but not out of it. The kinetic energy of infall was assumed to be converted into radiation at the same boundary. The calculation was continued until about 0.5 $M_{\odot}$ had accumulated in the core, and about 0.1 $M_{\odot}$ had collapsed into a nearly Keplerian disk, with radial extent of about 20 AU. The calculation was stopped at that point because of insufficient spatial resolution in the disk region, and because the ratio (β) of rotational kinetic energy to gravitational potential energy of the core exceeded 0.27, so dynamical instability to non-axisymmetric perturbations would be likely (Durisen and Tohline 1985). The development of a triaxial central object is likely to result in the transport of angular momentum by gravitational torques (Yuan and Cassen

1985; Durisen *et al.* 1986). Angular momentum would be transported from the central object to the disk, and the value of β for the core would be reduced below the critical value. However, its remaining total angular momentum would be still too large to allow it to become a normal star.

A further important feature of the calculation was its prediction of the temperatures that would be generated in the planet-forming region. Over a time scale of 3×10^4 years the temperature of material with the same specific angular momentum as that of the orbit of Mercury ranged from 400-600 K. The predicted temperature for Jupiter remained fairly constant at 100 K, while that for Pluto approximated 15 K. In the inner region of the nebula these temperatures are slightly cooler than those generally thought to exist during planetary formation or those calculated in evolving models of a viscous solar nebula (Ruden and Lin 1986).

A further calculation was made by Tscharnuter (1987) with similar physics but a different initial condition. A somewhat centrally condensed and non-spherical cloud of 1.2 $M_\odot$ starts collapse from a radius of 4×10^{15} cm, a mean density of 8×10^{-15} g cm^{-3}, and $j \approx 10^{20}$ cm^2 s^{-1}. A major improvement was a refined equation of state. The use of this equation of state to calculate the collapse of a spherically symmetric protostar starting from a density of 10^{-19} g cm^{-3} produces violent oscillations in central density and temperature after the stellar core has formed. The instability is triggered when the adiabatic exponent $\Gamma_1 = (\partial ln\ P/\partial ln\ \rho)_S$ falls below 4/3, and the source of the energy for the reexpansion is association of hydrogen atoms into molecules.

In the two-dimensional case, the much higher starting density and the correspondingly higher mass inflow rate onto the core, as well as the effects of rotation, are sufficient to suppress the instability. A few relatively minor oscillations, primarily in the direction of the rotational pole, dampen quickly. The numerical procedure uses a grid that moves in the (spherical) radial direction and thus is able to resolve the central regions well, down to a scale of 10^{10} cm. This calculation is carried to the point where a fairly well-defined core of 0.07 $M_\odot$ has formed, which is still stable to non-axisymmetric perturbations ($\beta = 0.08$). A surrounding disk structure is beginning to form, out to a radius of about 1 AU. The density and temperature in the equatorial plane at that distance are about 3×10^{-9} g cm^{-3} and 2500 K, respectively. Further accretion of material into the core region is likely to increase the value of β. The calculation was not continued because of the large amount of computer time required.

A different approach to the problem of the two-dimensional collapse of the protostar has been considered by Adams and Shu (1986) and Adams, *et al.* (1987). The aim is to obtain emergent spectra through frequency-dependent radiative transfer calculations. In order to bypass the difficulties of a full two-dimensional numerical calculation, they made several approximations. The initial condition is a "singular" isothermal sphere, in unstable equilibrium, with sound speed c_s, uniformly rotating with angular velocity Ω. In the initial state the density distribution is given by $\rho \propto R^{-2}$, where R is the distance to the origin, and the free-fall accretion rate onto a central object of mass M is given by $\dot{M} = 0.975\, c_s^3/G$. The hydrodynamical solution for the infalling envelope is taken to be that given by Terebey *et al.* (1984), a semianalytic solution under the approximation of slow rotation. The thermal structure and radiation transport through the envelope can be decoupled from the hydrodynamics (Stahler *et al.* 1980). The model at a given time consists of an (unresolved) core, a circumstellar disk, and a surrounding infalling, dusty, and optically thick envelope. The radiation produced at the accretion shocks at the core and disk is reprocessed in the envelope, and emerges at the dust photosphere, primarily in the mid-infrared. The thermal emission of the dust in the envelope is obtained by approximating the rotating structure as an equivalent spherical structure; however, the absorption in the equation of transfer is calculated taking the full two-dimensional structure into account. The model is used to fit the observed infrared radiation from a number of suspected protostars, by variation of the parameters M, c_s, Ω, η_D, and η_*, where the last two quantities are the efficiencies with which the disk transfers matter onto the central star and with which it converts rotational energy into heat and radiation, respectively. These models provide good fits to the spectra of the observed sources for typical parameters $M = 0.2$ -$1.0\ M_\odot$, $c_s = 0.2$ - 0.35 km s^{-1}, $\Omega = 2 \times 10^{-14}$ - 5×10^{-13} rad s^{-1}, $\eta_D = 1$ and $\eta_* = 0.5$. Of particular interest is the fact that in many cases the deduced values of Ω fall in the range $j \approx 10^{20}$ cm^2 s^{-1}, which is appropriate for "solar nebula" disks. The contribution from the disk broadens the spectral energy distribution and brings it into better agreement with the observations than does the non-rotating model. More recent observational studies of protostellar sources (Myers *et al.* 1987; Cohen *et al.* 1989) also are consistent with the hypothesis that disks have formed within them.

RECENT MODELS WITH HYDRODYNAMICS AND RADIATIVE TRANSPORT

Full hydrodynamic calculations of the collapse, including frequency-dependent radiative transport, have recently been reported by Bodenheimer *et al.* (1988). The purpose of the calculations was to obtain the detailed structure of the solar nebula at a time just after its formation and to obtain spectra and isophotal contours of the system as a function of viewing angle and time. Because of the numerical difficulties discussed above, the protostar was resolved only on scales of 10^{13} - 10^{15} cm. These calculations have now been redone with the extension of the outer boundary of the grid to 5×10^{15} cm, with improvements in the radiative transport, and with a somewhat better spatial resolution, about 1 AU in the disk region (Bodenheimer *et al.* 1990).

The initial state, a cloud of 1 $M_{\odot}$ with a mean density of 4×10^{-15} g cm^{-3}, can be justified on the grounds that only above this value does the magnetic field decouple from the gas and allow a free-fall collapse, with conservation of angular momentum of each mass element, to start (Nakano 1984; Tscharnuter 1987). The initial density distribution is assumed to be a power law, the temperature is assumed to be isothermal at 20 K, and the angular velocity is taken to be uniform with a total angular momentum of 10^{53} g cm^2 s^{-1}. Because the cloud is already optically thick at the initial state, the temperature increases rapidly once the collapse starts. The inner region with $R \leq 1$ AU is unresolved; the mass and angular momentum that flow into this core are calculated. At any given time, a crude model of this material is constructed under the assumption that it forms a Maclaurin spheroid. From a calculation of its equatorial radius R_e, the accretion luminosity $L = GM\dot{M}/R_e$ is obtained. For each timestep Δt the accretion energy $L\Delta t$ is deposited in the inner zone as internal energy and is used as an inner boundary condition for the radiative transfer. Most of the energy radiated by the protostar is provided by this central source. During the hydrodynamic calculations, radiative transfer is calculated according to the diffusion approximation, which is a satisfactory approximation for an optically thick system. Rosseland mean opacities were taken from the work of Pollack *et al.* (1985). After the hydrodynamic calculations were completed, frequency-dependent radiative transfer was calculated for particular models according to the approach of Bertout and Yorke (1978), with their grain opacities which include graphite, ice, and silicates.

The results of the calculations show the formation of a rather thick disk, with increasing thickness as a function of distance from the central object. As a function of time the outer edge of the disk spreads from 1 AU to 60 AU, because of the accretion of material of higher angular momentum. The shock wave on the surface is evident, and the internal motions in the disk are relatively small compared with the collapse velocities. At the end of the calculation the mass of the disk is comparable to that of the central object, and it is not gravitationally unstable according to the axisymmetric local criterion of Toomre (1964). The central core of the protostar, inside 10^{13} cm, contains about 0.6 $M_{\odot}$ and sufficient angular momentum so that $\beta \approx 0.4$. This region is almost certainly unstable to bar-like perturbations. Theoretical spectra show a peak in the infrared at about 40μ; when viewed from the equator the wavelength of peak intensity shifts redward from that at the pole. A notable difference between equator and pole is evident in the isophotal contours. At 40 μm, for example, the peak intensity shifts spatially to points above and below the equatorial plane because of heavy obscuration there. This effect becomes more pronounced at shorter wavelengths. Maximum temperatures in the midplane of the disk reached 1500 K in the distance range 1-10 AU. At the end of the calculation, after an elapsed time of 2500 years, these temperatures ranged from 700 K at 2 AU to 500 K at 10 AU and were decreasing with time.

FURTHER EVOLUTION OF THE SYSTEM

In the preceding example most of the infalling material joined the disk or central object on a short time scale, because of the high initial density. For a lower initial density, processes of angular momentum transport in the disk would begin before accretion was completed. The problem of the rapidly spinning central regions is apparently not solved by including angular momentum transport by turbulent viscosity during the collapse phase. Furthermore, no plausible physical mechanism for generating turbulence on the appropriate scale has been demonstrated. The angular momentum transport resulting from gravitational torques arising from the non-axisymmetric structure of the central regions is likely to leave them with values of β near 0.2 (Durisen *et al.* 1986). Therefore, even further transport is required. A related mechanism has been explored by Boss (1985, 1989). He has made calculations of protostar collapse, starting from uniform density and uniform angular velocity, with a three-dimensional hydrodynamic code, including radiation transport in optically thick regions. Small, initial non-axisymmetric perturbations grow during the collapse, so that the central regions, on a scale of 10 AU, become significantly non-axisymmetric even before a quasi-equilibrium configuration is reached. The deduced time scales for angular momentum transport depend on the initial conditions but range from 10^3 to 10^6 years for systems with a total mass of 1 $M_\odot$. However, since the evolution has not actually been calculated over this time scale, it is not clear how long the non-axisymmetry will last or how it will affect the angular momentum of the central core.

It is likely that some additional process is required to reduce j of the central object down to the value of 10^{17} characteristic of T Tauri stars. The approach of Safronov and Ruzmaikina (1985) is to assume that the initial cloud had an even smaller angular momentum ($j \approx 10^{19}$ cm^2 s^{-1}) than that assumed in most other calculations discussed here. The cloud would then collapse and form a disk with an equilibrium radius much less than that of Jupiter's orbit. Outward transport of angular momentum into a relatively small amount of mass is then required to produce the solar nebula. Magnetic transport could be important in the inner regions, which are warm and at least partially ionized (Ruzmaikina 1981). However, outside about 1 AU (Hayashi 1981) the magnetic field decays faster than it amplifies, and magnetic transport is ineffective. A supplementary mechanism must be available to continue the process. One possibility is the turbulence generated in the surface layers of the disk, caused by the shear between disk matter and infalling matter. Another possibility is that the initial cloud had higher j, and the rapidly spinning central object is braked through a centrifugally driven magnetic wind which can remove the angular momentum relatively quickly (Shu *et al.* 1988).

As far as the evolution of the disk itself is concerned, other important mechanisms that have been suggested include (a) gravitational instability; (b) turbulent viscosity induced by convection, and (c) sound waves and shock dissipation. The former can occur if the disk is relatively massive compared with the central star or if the disk is relatively cold. Although it is still an open question whether this instability can result in the formation of a binary or preplanetary condensations, the most likely outcome is the spreading out of such condensations, because of the shear, into spiral density waves (Larson 1983). Lin and Pringle (1987) have estimated the transport time to be about 10 times the dynamical time. Processes (b) and (c) have time scales more in line with the probable lifetimes of nebular disks. Convective instability in the vertical direction (Lin and Papaloizou 1980), induced by the temperature dependence of the grain opacities, gives disk evolutionary times of about 10^6 years (Ruden and Lin 1986). An alternate treatment of the convection (Cabot *et al.* 1987a,b) gives a time scale longer roughly by a factor of 10. Sound waves induced by various external perturbations give transport times in the range 10^6 to 10^7 years (Larson 1989). A complete theory of how the disk evolves after the immediate formation phase may involve several of the mechanisms just mentioned, and its development will require a considerable investment of thought and numerical calculation.

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