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# The Chaotic Dynamics of Comets and the Problems of the Oort Cloud

Roald Z. Sagdeev and G.M. Zaslavskiy
Institute of Space Research

**ABSTRACT**

This paper discusses the dynamic properties of comets entering the planetary zone from the Oort cloud. Even a very slight influence of the large planets (Jupiter and Saturn) can trigger stochastic cometary dynamics. Multiple interactions of comets with the large planets produce diffusion of the parameters of cometary orbits and a mean increase in the semi-major axis of comets. Comets are lifted towards the Oort cloud, where collisions with stars begin to play a substantial role. The transport of comets differs greatly from the customary law of diffusion and noticeably decelerates the average comet flow. The vertical tidal effect of the galaxy in this region of motion is adiabatic and cannot noticeably alter cometary distribution. A study of the sum of forces operating in the region to a $\sim 10^4$ AU does not permit us to explain at this time the existence of a sharp maximum, where a $\sim 10^4$ AU in the distribution of long-period comets. This is an argument in favor of the suggestion that it was caused by the close passage of a star several million years ago.

## INTRODUCTION

The solar system's new object, the Oort cloud, arose as a source of long-period comets (Oort 1950) in the planetary system's visible portion (r < 2 AU). Experimental material generated by processing the trajectory of a large number of long-period comets (Marsden *et al.* 1978; Marsden and Roemer 1982) determined for these comets the region in which they exist, which reaches a size of up to $\lesssim 2 \cdot 10^5$ AU. Oort proposed that collisions with stars passing fairly close to the Sun may be one of the primary causes for which comets attain the visible region. Research during the ensuing years greatly complicated the Oort cloud model, inputting Hills cloud (Hills 1981) into the analysis (with an upper boundary of $r \sim 2 \cdot 10^4$ AU) and the action of various forces such as the galactic tidal effect (Hills 1981; Heisler and Tremaine 1986; Morris and Muller 1986; Bailey 1986), collision with molecular clouds (Biermann and Lust 1978; Hut and Tremaine 1985), and interaction with the planets (Oort 1950; Khiper 1951).

Numerical analysis within the framework of the simplest, initial Oort model demonstrated the possibility of qualitatively explaining the reason for which comets enter the visible zone due to the effect of near stellar passages (Wiesmann 1982). Subsequent analysis showed that the action of the galaxy's vertical tidal effect may somewhat modify Oort cloud and Hills cloud parameters and the number of comets in them (Fernandez and Ip 1987; Duncan *et al.* 1988). Oort cloud mass and momentum may fluctuate more significantly if we assume certain, typical estimates for them, made after processing the results of the Halley's comet mission (Marochnik *et al.* 1988). The large mass of the Oort cloud ($M_o \sim 100\ M_\oplus$, $r > 2 \cdot 10^4$ AU) must affect in the most serious way models of the formation of the solar system.

It should be added here that the increase in the mass of the Hills cloud must also bring about an increase in its angular momentum (Marochnik *et al.* 1988). This must be reconciled with the approximate equality of the number of prograde and retrograde new comets. If it was not a question of new comets, this equality would be a sufficiently obvious consequence of the impact of random collisions of stars with comets with highly eccentric orbits. However, numerical simulation, where the initial angular momentum value of the cometary protocloud is taken into account, also reveals the considerable impact it exerts on the size of the final Oort cloud and on the number of comets in it (Lopatnikov *et al.* 1989). The obvious reason for this is tied to the different impacts of stellar collisions on circular and eccentric orbits.

In accounting for the final angular momentum of cometary distribution in the Hills and Oort clouds, an anisotropy is created in cometary dynamics at virtually all of its stages. Anisotropy in the distribution of cometary aphelia has been experimentally discovered (Delsemme 1987). It indicates the correlation between cometary distribution and the effect of galactic tidal forces. We can consider that these forces exert an influence on both cometary dynamics in the aphelion region (Heisler *et al.* 1987) and on their dynamics in the planetary zone. Hence, all characteristic regions of cometary orbits must participate in an interrelated manner in the formation of cometary zones. This makes it necessary to analyze more carefully all of the processes by which comets interact with the planets and the stars.

The interaction of comets with the stars is statistical. Therefore, the impact of individual collisions is averaged out, while the mechanism itself by which they have an impact on the zone of cometary aphelia is weakly dependent on individual details.

The passage of comets through the planetary zone is quite different. The influence of the large planets, Jupiter and Saturn, on cometary dynamics has proven more subtle. A significant portion of comets (about 50%) which pass close to Jupiter are thrown into hyperbolic orbits as early as the first passage. However, the phase space occupied by comets with a perhelion $\lesssim$ 2 AU is not very large. The phase space occupied by comets with a perihelion similar to the radius of Jupiter's orbit is substantially greater. The phenomena of chaos may arise for such comets (Petrosky 1986; Sagdeev and Zaslavskiy 1987; Petrosky and Broucke 1988; Sagdeev *et al.* 1988). They consist of the following: in the strictly dynamic, three-body problem (Sun-Jupiter-comet), the movement of the comets in varying conditions becomes unstable. This instability is seen, in particular, in the fact that Jupiter's phase at the moment when a comet passes through its perihelion is a sequence of random numbers. As a result, a mechanism accelerating comets begins to operate. This mechanism is analogous to Fermi's method of stochastic acceleration (Sagdeev *et al.* 1988). It produces diffusive alteration of all of the comet's parameters, a mean increase of the semi-major axis of cometary orbit, and the expulsion of the comet from the solar system. The process of stochastization of cometary movement is considered in detail in Natenson *et al.* (1989). Numerical analysis demonstrated that the region of cometary eccentricity values for which chaos arises is very broad. Orbits with $\epsilon \sim 0.5$ may already become stochastic. This circumstance should noticeably modify views of cometary interaction with the planetary zone.

We will discuss below the conditions in which the dynamics of comets with long-period orbits become stochastic, the role of such comets in the overall model of the Oort cloud, and the influence of the galactic tidal forces on cometary dynamics.

## THE DYNAMIC CHAOS OF COMETS

We can generate a straightforward idea of this chaos by looking at how a ball falls on a heavy plate in a gravity field (Figure 1, Zaslavskiy 1985) and if we consider their collision to be absolutely elastic. If the plate oscillates with an oscillation amplitude of $\bar{a}$ and a velocity amplitude of $\bar{v}$, then on the condition that:

$$2\bar{v}^2 \gtrsim \bar{a}g,$$

(g denotes acceleration in the gravity field), the oscillation phase of the

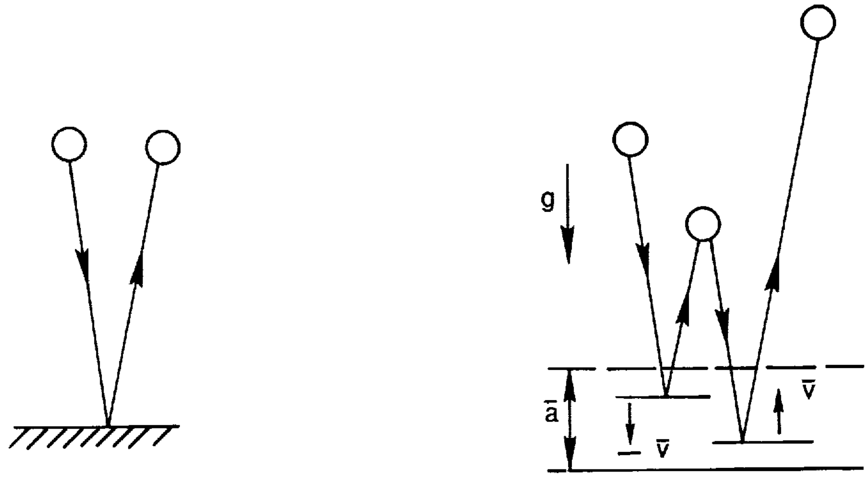


FIGURE 1 Illustration of the action of the "gravitational machine" (Zaslavskiy 1985), triggering the stochastic increase of the energy of a ball bouncing on a periodically oscillating plate.

plate at the moment of collision is random. The ball bounces irregularly over the plate and, on the average, rises increasingly higher. Its mean energy at the moment of impact behaves asymptotically, as in $<v^2> \sim t^{2/3}$, and the mean height of lift correspondingly increases, as in $<h^1> \sim t^{2/3}$. The time for the ball to return back to the plate, naturally, increases. However, the acceleration process does not stop.

This example somewhat clarifies what occurs with long-period comets whose perihelion is in the sphere of influence of, for example, Jupiter. Let M denote the orbital momentum of a comet, and $\psi$ be the phase of a comet's location relative to Jupiter in the comet's orbital plane. If the orbits of Jupiter and the comet lie in the same plane (the so-called flat, limited three-body problem), the Hamiltonian for the comet is equal to

$$H = \frac{1}{2}\left(p_r^2 + \frac{1}{r^2}p_\varphi^2\right) - \sigma p_\varphi - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} \tag{1}$$

$\sigma = \pm 1$ defines here either prograde or retrograde comets; $\mu$ denotes the ratio of Jupiter's mass to that of the Sun;

$$r_1 = [r^2 + 2\mu r\ cos(\varphi - \sigma w_J t) + \mu^2]^{1/2}$$

$$r_2 = [r^2 - 2(1-\mu) r\ cos(\varphi - \sigma w_J t) + (1-\mu)^2]^{1/2},$$

where the radius of Jupiter's orbit, $r_J$, is assumed to be equal to the unit, $w_J$ (Jupiter's rotational frequency). In this two-dimensional motion, $\psi = \psi - \sigma w_J t$, and a Jacoby motion integral exists:

$$G_o = H - \sigma w_J M. \tag{2}$$

Using ratio (2), we can make problem (1) a two-variable, nonstationary problem. We can select a canonically coupled pair (M, $\psi$) as the variables. Let, for example, $t_n$ denote the moment in time when the comet passes through its aphelion, $M_n$ be the orbital momentum during passage through aphelion, and $\psi_n$ denote the value of the Jovian phase during cometary passage through perihelion (preceding the time $t_n$). The relationship between the values ($M_{n+1}$, $\psi_{n+1}$) and $M_n$, $\psi_n$) then defines the expression (Petrosky 1986; Sagdeev and Zaslavskiy 1987)

$$\begin{aligned} M_{n+1} &= M_n + \Delta M \; sin\psi_n \\ \psi_{n+1} &= \psi_n + 2\pi\sigma \frac{w_j}{w(E_{n+1})}, \end{aligned} \tag{3}$$

where $\Delta$M denotes fluctuation in the orbital momentum over one cometary rotation, while the comet's energy, $E_n$ is determined using the motion integral (2):

$$E_n = G_o + \sigma w_J M_n. \tag{4}$$

The variables ($M_n$, $\psi$n) are canonically coupled. Expression (3) is only defined in the region of negative values of a comet's energy, H = E < 0, that is , according to (2)

$$G_o + \sigma w_J M < 0. \tag{5}$$

Inequality (5) is violated and expression (3) becomes meaningless when a comet is thrown into hyperbolic orbit. The value $\Delta$M in (3) is defined by the expression

$$\Delta M = max \int_{t_n}^{t_{n+1}} dt \frac{\partial H}{\partial \psi}, \tag{6}$$

where $t_n$, and $t_{n+1}$ denote the moments of time of two sequential passages of the apocenter by a comet. Estimates of the value of $\Delta$M in varying cases are provided in Petrosky 1986; Sagdeev and Zaslavskiy 1987; Petrosky and Broucke 1988; Natenson *et al.* 1989.

Expression (3) has a frequently encountered form, described in detail by Sagdeev *et al.* (1988) and Zaslavskiy (1985). If a comet does not pass too far from Jupiter, the duration of its interaction with Jupiter is on the order of a Jovian period of $2\pi/w_J$. This time scale determines the duration of a "collision." It is a great deal less than the time period between two "collisions" for long-period comets of $2\pi/w(E)$. In view of this circumstance, we can write a simple form of expression (3), in which the second formula simply describes alteration of the $\psi$ phase during the time between two sequential collisions. A similar expression also occurs for the model with the ball in Figure 1. The velocity, v, plays the role of a generalized pulse, M, while the collision frequency is proportional to 1/v. In this case, $w(E) = |2E|^{3/2}$.

We can produce a straightforward assessment of the stochastic dynamics of the comet for problem (3) from the condition that (Sagdeev *et al.* 1988; Zaslavskiy 1985):

$$K = \left| \frac{\partial \psi_{n+1}}{\partial \psi_n} - 1 \right| \gtrsim 1. \tag{7}$$

This gives us

$$K = 2\pi \frac{w_J}{w^2(E)} \left| \frac{\partial w(E)}{\partial E} \right| \cdot |\Delta M cos\psi| > 1. \tag{8}$$

Since the perihelion changes little as a result of the collision ($\Delta M \ll M$), while the comet's rotation frequency $w(E) = |2E|^{3/2} \rightarrow 0$ where $|E| \rightarrow 0$, condition (8) can be fulfilled for comets with sufficiently low binding energy ($|E| \rightarrow 0$) with a fixed value M. The phase portrait of cometary movement, corresponding to the Hamiltonian (1), with a fixed Jacoby integral value is in Figure 2 (Natenson *et al.* 1989). It was produced for true trajectories and demonstrates the complex structure of phase space with a large number of regions of stability. The region of global chaos is defined by the estimate in (7) and (8). A boundary with $\epsilon \approx 0.55$ and a semi-major axis of $a \approx 16.5$ AU corresponds to this. The comet's perihelion originally had a value of $q \approx 7.5$ AU.

Points in the region of global chaos in Figure 2, that is where $0 < E < -0.03$ AU, belong to one trajectory. If any initial condition in this region is selected, with the same Jacoby integral value, the corresponding movement of a comet will also be stochastic. This is where the significance of this region of stochasticity is manifested.

Two important consequences stem from the results of study (Natenson *et al.* 1989). The region of chaos is very significant, and even Halley's comet enters the zone where conditions of chaos are applicable. An analogous comment regarding Halley's comet, based on the use of representation (3), was made by Chirikov and Vecheslavov (1985). The region of chaos in Figure 2 also applies to medium-period comets. Therefore, the phase magnitude of comets with stochastic dynamics is an order more than the phase magnitude of comets appearing in the visible portion with $r < 2$ AU.

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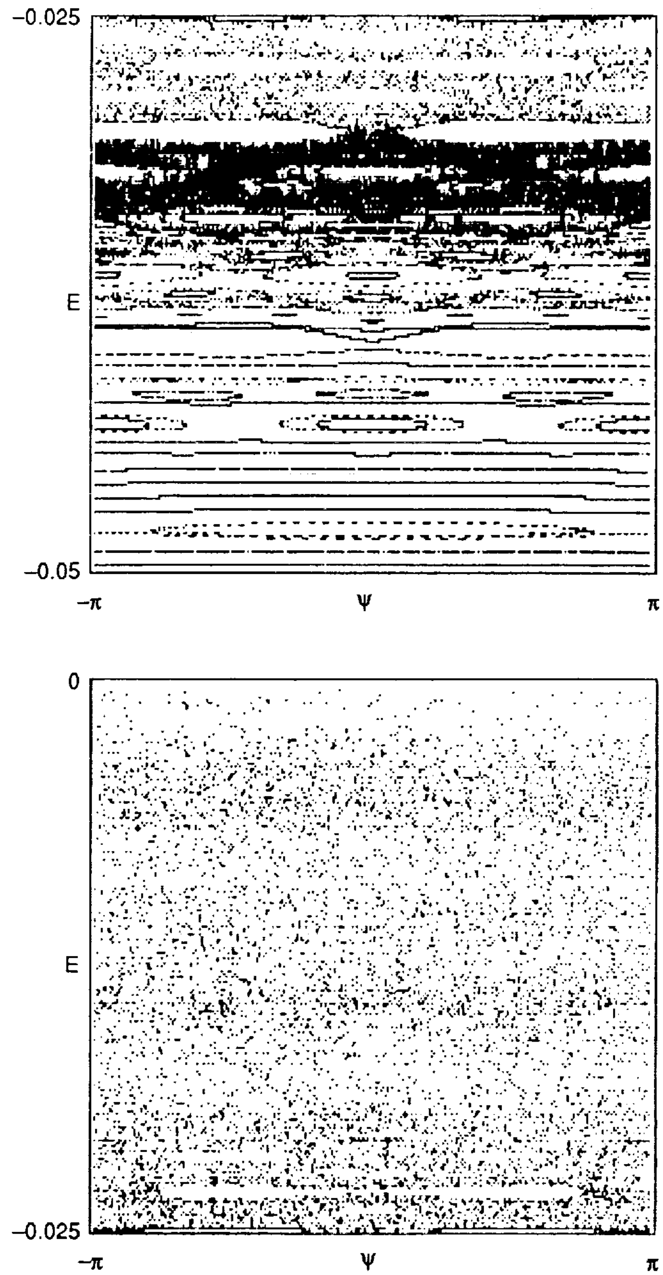


FIGURE 2 Phase portrait on the trajectory plane ($\psi$, E = $-1/2a$) of a comet. The orbit points are plotted at the moment in time when a comet passes through the aphelion point. For the sake of convenience, the portrait has been broken into two parts.

The second consequence is related to the nature of cometary diffusion. This process is extremely important, since it is the mechanism by which comets attain the region of semi-major axes. The usual diffusion formula (Yabushita 1980)

$$\frac{\partial f(M,t)}{\partial t} = \frac{1}{4\pi}\frac{\partial}{\partial M} < (\Delta M)^2 w(M) > \frac{\partial f(M,t)}{\partial M}, \tag{9}$$

is only justified in a region sufficiently remote from the chaos boundary. The impact of the chaos boundary and regions of stability (see Figure 2) significantly decelerates diffusion at the initial stage in comparison with the diffusion defined by formula (9) (Natenson *et al.* 1989).

## GENERAL COMETARY DYNAMICS AND THE INFLUENCE OF THE GALACTIC TIDE

The existence of a mechanism of dynamic chaos makes it necessary to reconsider the general dynamics of comets as they move from the Oort cloud to the visible zone. The customary route is that collisions with stars operate in the zone of aphelion of a comet's orbit. Those comets in the loss cone region enter the visible zone, originally having a $\gtrsim 10^4$ AU. Jupiter's influence throws about one half of these comets into hyperbolic orbit. Only a small portion of the comets may subsequently return again directly to the loss cone, in order to set out on the new path from the Oort cloud to the visible zone.

However, another avenue also exists.

A rather large portion of comets, those that first entered the invisible zone and have a perihelion comparable to the radius of Jupiter's orbit, enter the region of stochastic dynamics. The comet begins a long, diffusive path to the loss cone region. Therefore, an independent way of filling the loss cone is defined.

Other large planets of the solar system may also play the same role as Jupiter. Therefore, the portion of comets moving stochastically should be insignificant. The planetary "barrier," expelling part of the comets, concurrently makes the dynamics of others stochastic, thereby providing their route to the loss- cone. At the same time, the cometary perihelion changes very slightly, and the comets' orbits are as if "attached" in the zone of their perihelion.

Vertical tidal galactic forces must play an important role in the process of cometary stochastic transport described above (Bahcall 1984; Heisler and Tremaine 1986)

$$F_z = 1\pi m G \rho_s z, \tag{10}$$

where m denotes comet mass, G is the gravitational constant, $\rho s = 0.186$ $m_\odot/pc^3$ is stellar density, and z is the vertical coordinate in the galactic system of coordinates. The tidal force (10) considerably alters the perihelion. Therefore, it creates the drift of comets from the planetary zone during one phase of cometary orbit and, inversely, causes cometary perihelia to flow into the planetary barrier in another orbital phase. These two fluxes are approximately equal.

We will find the region, on the nonadiabatic influence of the tidal force $F_z$, from the condition that perhelion variation under its influence must be fairly strong. We will assume 10 AU as an example of the characteristic size of the planetary zone. Then the nonadiabatic condition means that

$$\Delta q > 10AU, \tag{11}$$

where $\Delta q$ denotes perihelion alteration under the influence of $F_z$ in a period of cometary orbit. For eccentric orbits

$$\Delta q \sim M^2/2m^2 m_\odot G.$$

Thus

$$\Delta q \sim M\Delta M/m^2 m_\odot G.$$

Assuming

$$\Delta M \sim F_z aT \sim 4\pi mG\rho_s a^2 T \sim nm_\odot Ga^2T/pc^3,$$

after substitution of all of these expressions in (11), we yield:

$$a \gtrsim 10^4 AU.$$

This estimate (Duncan *et al.* 1988) can also be clarified using a more careful input of numbers. However, it is clear that the effect of tidal forces in the region of a significant portion of cometary orbits is adiabatic. Therefore, the tidal forces cannot substantially alter cometary distribution in the region $\lesssim 10^4$ AU. However, they exert considerable influence on the near-boundary processes, where the planetary barrier operates, and along the border of the Oort cloud, where effective collisions with stars fill the loss cone (Fernandez and Ip 1987; Duncan *et al.* 1988).

## CONCLUSION

The dynamic chaos of comets with fairly eccentric orbits, moving in the Sun's field and perturbed by the fields of Jupiter and Saturn (or by the fields of other remote planets), forces us to reconsider individual elements of the Oort cloud theory. Cometary stochastization creates a mechanism by which comets move away from the planetary belt towards the Oort cloud, and may be an additional source by which the loss cone is filled. The process of diffusive cometary transport differs from the usual process of diffusion and retards the characteristic time scale for the flux of comets toward the semi-major axes a. The action of the tidal forces does not alter this time scale significantly and is adiabatic (with the exception of the regions near the belt of the large planets and near the inner boundary of the Oort cloud). Therefore, the internal mechanisms of cometary dynamics cannot explain the existence of a sharp maximum in the distribution of the observed comets from the Oort cloud (Marsden *et al.* 1978; Weismann 1982) with a period on the order of several million years. This gives us reason to suggest that the reason for the appearance of such comets may have been the last near-Earth passage of a star. This conclusion correlates with the conclusions of studies Biermann *et al.* 1983; Lust 1984) on the possible passage of a star or another large object in the region of cometary orbit with a $\sim 10^4$ AU, triggering the appearance of a coherently moving cometary cluster.

The global modeling of the dynamics of long-period comets must include the multiple interactions of comets with the large planets, if the perihelion of the comets does not greatly exceed the radii of planetary orbits. These issues and the existence of asymmetry in cometary cloud distribution are discussed in greater detail in Lopatnikov *et al.* (1989); Natenson *et al.* (1989).

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