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RATE DEPENDENT STRESS-STRAIN BEHAVIOR OF ADVANCED POLYMER MATRIX COMPOSITES

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Summary

This paper describes the formulation of an elastic/viscoplastic constitutive model which was used to predict the measured behavior of graphite/thermoplastic and graphite/bismaleimide composite materials at elevated temperature. The model incorporates the concepts of overstress and effective stress/strain to provide a simple formulation which was able to account for material behavior under monotonic tension or compression loads over a temperature range of 23°C to 200°C. Observed behavior such as stress relaxation and steady state creep, in off-axis tension and compression tests, were predicted by the model. Material constants required by the model were extracted from simple off-axis test data.

NOMENCLATURE

A - quasistatic elastic/plastic material constant
a_{66} - potential function material constant
E - elastic Young's modulus
F - time dependent inelastic strain rate function
f - potential function
G - elastic shear modulus
H - overstress
h(θ) - function defined in equation 20
K - elastic/viscoplastic material constant
m - elastic/viscoplastic material constant
n - quasistatic elastic/plastic material constant
Q - stiffness matrix
S - compliance matrix
ϵ - strain
$\dot{\epsilon}$ - strain rate
$\bar{\epsilon}$ - effective strain
$\dot{\bar{\epsilon}}$ - effective strain rate
σ - stress
$\dot{\sigma}$ - stress rate
$\bar{\sigma}$ - effective stress
$\dot{\bar{\sigma}}$ - effective stress rate
$\sigma^\star$ - quasistatic stress
$\bar{\sigma}^\star$ - effective quasistatic stress
θ - fiber angle relative to loading axis
λ - proportionality constant
Φ - overstress function

Subscripts

i,j,k,l = 1 - 6, lamina principal directions
x - laminate axial loading direction

Superscripts

e - elastic
qp - quasistatic plastic
p - plastic
in - inelastic
vp - viscoplastic

INTRODUCTION

An elastic/viscoplastic constitutive model for polymer matrix composites was developed in order to meet two research objectives. The first objective was to provide analytical and experimental procedures for comparing the relative rate dependent response of several composite material systems under conditions of elevated temperature and high loads. The second objective was to help develop the analytical foundation for predicting various rate dependent phenomenon such as creep, stress relaxation and strain rate sensitivity in a laminated structure.

In order to address these objectives, an elastic/viscoplastic constitutive model which incorporates the concepts of overstress and effective stress/strain was formulated. The model assumed that the lower bound of the rate dependent stress/strain behavior is described by rate independent (quasistatic) expressions. The rate dependent strain was decomposed into two terms to account for both loading and unloading. Material constants required by the model were extracted form simple off-axis tension or compression test data over a range of test temperatures.

CONSTITUTIVE MODEL DESCRIPTION

A variety of constitutive models have been suggested in recent years to model nonlinear, rate dependent stress/strain behavior in advanced composite material systems. A recent survey of several of these models and their applications to polymer matrix composites can be found in Gates [1]. These models have been examined to determine their suitability for predicting the nonlinear, rate dependent stress/strain behavior of several advanced polymer matrix composites (PMC's) at elevated temperature. In order to provide a means of analytically modeling the observed material behavior, the author has developed a modified form of the time dependent elastic/viscoplastic constitutive model given by Gates and Sun [2]. The quasistatic elastic/plastic constitutive model given by Chen and Sun [3] has also been used in the model development. This paper presents the mathematical formulation, assumptions and possible shortcomings of the model. In addition, model predictions are compared to experimental results for two advanced PMC materials.

Rate Dependent Elastic/Viscoplastic Behavior

For the elastic/viscoplastic behavior, the time dependent strain is assumed to be composed of elastic and viscoplastic components. For the general multiaxial case, this is written as:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}^{e}_{ij} + \dot{\epsilon}^{vp}_{ij} \tag{1}$$

where we have the individual constitutive relations:

$$\{\dot{\epsilon}\} = [S]^{e}\{\dot{\sigma}\} \qquad \text{or} \qquad \dot{\epsilon}^{e}_{ij} = S^{e}_{ijkl}\dot{\sigma}_{kl} \qquad \text{elastic} \tag{2}$$

and

$$\{\dot{\epsilon}\} = [S]^{vp}\{\dot{\sigma}\} \qquad \text{or} \qquad \dot{\epsilon}^{vp}_{ij} = S^{vp}_{ijkl}\dot{\sigma}_{kl} \qquad \text{viscoplastic} \tag{3}$$

If we further decompose the viscoplastic component into time dependent plastic and inelastic components, we can write:

$$\dot{\epsilon}^{vp}_{ij} = \dot{\epsilon}^{p}_{ij} + \dot{\epsilon}^{in}_{ij} \tag{4}$$

and this implies the constitutive relations:

$$\Rightarrow \qquad \dot{\epsilon}^{p}_{ij} = S^{p}_{ijkl}\dot{\sigma}_{kl} \qquad \text{and} \qquad \dot{\epsilon}^{in}_{ij} = S^{in}_{ijkl}\dot{\sigma}_{kl} \tag{5}$$

It is noted that for this formulation, the superscript (p) refers to a rate dependent term while the rate independent, or quasistatic plastic, term is designated by the superscript (qp).

Using a procedure similar to that used in rate independent plasticity, a form of the associated flow rule for the rate dependent plastic strain can be written as:

$$\dot{\epsilon}^{p}_{ij} = \frac{\partial f}{\partial \sigma_{ij}}\dot{\lambda} \tag{6}$$

where $\dot{\lambda}$ is a proportionality factor. Using the formulation given in Chen and Sun [3] and Kenaga [4], the potential function (f), which accounts for material anisotropy, is formed by assuming elastic behavior along the fiber direction and plane stress conditions.

$$2f(\sigma_{ij}) = \sigma_{22}^{2} + 2a_{66}\sigma_{12}^{2} \tag{7}$$

Where σ_{22} and σ_{12} are the inplane transverse and shear stress components respectively. The single material constant is given by the a_{66} term and can be found from test results on off-axis specimens. It is noted that as a_{66} decreases, a higher shear stress to transverse stress ratio is required to satisfy the yield condition.

Using the overstress concept, the effective plastic strain rate is written as a function of overstress given by:

$$\dot{\bar{\epsilon}}^{p} = < \Phi(H) > \tag{8}$$

where H is the "overstress" and is defined as:

$$H \equiv (\bar{\sigma} - \sigma^{\star}) \tag{9}$$

where (σ) is the rate dependent stress and $(\sigma^{\star})$ is the rate independent or quasistatic stress. Using the potential function above, the effective stress is defined by:

$$\bar{\sigma} \equiv \sqrt{3f(\sigma_{ij})} \tag{10}$$

and $\bar{\sigma}^{\star}$ is the effective quasistatic stress. A power law is assumed for the effective stress, effective quasistatic plastic strain relation. Functionally, this can be written as:

$$\bar{\epsilon}^{qp} = A(\bar{\sigma}^{\star})^{n} \tag{11}$$

where A and n are material constants found from fitting the power law to the effective stress, effective plastic strain data. The Macaulay (<>) brackets imply a conditional statement which can be written in a general sense as

$$< \Phi(H) > = \begin{cases} \Phi(H) & \text{if } H > 0 \\ 0 & \text{if } H \leq 0 \end{cases} \tag{12}$$

The concept of overstress and its relationship to viscoplastic strain in isotropic metallics has been attributed to Malvern[5] and his work on high strain rate conditions during wave propagation. Additional references to overstress and its use in constructing viscoplastic models can be found in the work of Eisenberg and Yen[6,7] and Krempl and Hong[8].

The effective stress quantity allows for the dependence of the state of stress on the angle between the load and fiber directions and can be represented in a general sense by equation (10). Use of effective stress and strain along with the potential function allows for the generation of master curves for elastic/plastic and elastic/viscoplastic material constants.

If we let

$$\gamma = \frac{3}{2}\bar{\sigma} \tag{13}$$

then use of the overstress function (Φ) allows equation (6) to be written as:

$$\dot{\epsilon}^p_{ij} = \gamma \frac{\partial f}{\partial \sigma_{ij}} < \Phi(H) > \tag{14}$$

In terms of a functional relationship, we can use a power law expression to model test data and write the effective plastic strain rate as:

$$\dot{\bar{\epsilon}}^p = < \Phi(H) > = \langle \left[\frac{H}{K} \right] \rangle^{(1/m)} \tag{15}$$

where K and m are material constants found from test data on off-axis uniaxial specimens.

Further examination of equation (14) leads to the following:

$$\gamma \frac{\partial f}{\partial \sigma_{ij}} < \Phi(H) > = S^p_{ijkl} \dot{\sigma}_{kl} \tag{16}$$

In terms of the in-plane components, this can be expanded to give:

$$\frac{3}{2}\bar{\sigma} \left\{ \begin{array}{c} 0 \\ \sigma_{22} \\ 2a_{66}\sigma_{12} \end{array} \right\} < \Phi(H) > = \frac{9}{4\bar{\sigma}^2} \frac{d\bar{\epsilon}^p}{d\bar{\sigma}} \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \sigma_{22}^2 & 0 \\ 0 & 0 & 4a_{66}^2\sigma_{12}^2 \end{array} \right] \left\{ \begin{array}{c} \dot{\sigma}_{11} \\ \dot{\sigma}_{22} \\ \dot{\sigma}_{12} \end{array} \right\} \tag{17}$$

This allows the compliance matrix for the multiaxial plastic strain rate constitutive relation to be written as:

$$[S]^p = \frac{3}{2\bar{\sigma}} \langle \left[\frac{H}{K} \right] \rangle^{(1/m)} \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \frac{\sigma_{22}}{\dot{\sigma}_{22}} & 0 \\ 0 & 0 & \frac{2a_{66}\sigma_{12}}{\dot{\sigma}_{12}} \end{array} \right] \tag{18}$$

Working at the lamina level and assuming plane stress conditions for an off-axis type of test under a state of uniaxial tension or compression (σ_x), the values of the principal stress components are:

$$\sigma_{11} = \sigma_x \cos^2\theta, \qquad \sigma_{22} = \sigma_x \sin^2\theta, \qquad \sigma_{12} = -\sigma_x \sin\theta\cos\theta \tag{19}$$

where θ is the angle of the fiber direction relative to the load direction. Sun and Chen [3] showed that a function which depends on θ and a_{66} can be formed and used to help define effective stress and strain quantities. This function was defined as:

$$h(\theta) = \left[\frac{3}{2}\left(\sin^4\theta + 2a_{66}\sin^2\theta\cos^2\theta\right)\right]^{\frac{1}{2}} \tag{20}$$

Using $h(\theta)$, the effective plastic strain rate can be given as:

$$\bar{\dot{\epsilon}}^p = \frac{\dot{\epsilon}_x^p}{h(\theta)} \tag{21}$$

The overstress can also be written in terms of the axial stress components.

$$H = h(\theta)(\sigma_x - \sigma_x^\star) \tag{22}$$

where $\sigma_x^\star$ is the quasistatic stress and σ_x is the instantaneous applied stress. The quasistatic stress $\sigma_x^\star$ can be found by solving Sun and Chen's [3] rate independent quasistatic constitutive relation:

$$\epsilon_x = \frac{\sigma_x^\star}{E_x} + h(\theta)^{(n+1)}A(\sigma_x^\star)^n \tag{23}$$

where A and n are material constants found from test data.

We note that the conditional statement on the overstress function (Φ) can now be written as:

$$\bar{\dot{\epsilon}}^p = <\Phi(H)> = \begin{cases} \Phi & \text{if } \sigma_x > \sigma_x^\star \\ 0 & \text{if } \sigma_x \leq \sigma_x^\star \end{cases} \tag{24}$$

Expanding Φ we have:

$$\Phi(H) = \left[\frac{h(\theta)(\sigma_x - \sigma_x^\star)}{K}\right]^{(1/m)} \tag{25}$$

This implies the axial plastic strain rate can now be completely written.

$$\Rightarrow \quad \dot{\epsilon}_x^p = [h(\theta)]^{(1+\frac{1}{m})}\left(\frac{1}{K}\right)^{1/m}(\sigma_x - \sigma_x^\star)^{1/m} \tag{26}$$

If we let the quantity β be given by;

$$\beta = [h(\theta)]^{(1+\frac{1}{m})}\left(\frac{1}{K}\right)^{1/m} \tag{27}$$

we have a compact form for the axial plastic strain rate.

$$\dot{\epsilon}_x^p = \beta(\sigma_x - \sigma_x^*)^{1/m} \tag{28}$$

For the time dependent inelastic behavior, we begin by utilizing the general multiaxial form;

$$\dot{\epsilon}^{in} = F(\sigma, \epsilon, sign\dot{\sigma})\dot{\sigma} \tag{29}$$

noting that this implies the following relationships.

$$F(\sigma, \epsilon, sign\dot{\sigma}) = \begin{cases} F(\sigma, \epsilon) & \text{if } \dot{\sigma} > 0 \\ 0 & \text{if } \dot{\sigma} \leq 0 \text{ or } \sigma \leq \sigma^* \end{cases} \tag{30}$$

Assume that $F(\sigma, \epsilon)$ comes from the quasistatic elastic/plastic expressions. We can make use of the quasistatic plastic compliance matrix [3]:

$$[S]^{qp} = \Psi \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22}^2 & 0 \\ 0 & 0 & 4a_{66}^2\sigma_{12}^2 \end{bmatrix} \tag{31}$$

where the term Ψ can be written:

$$\Psi = \frac{9}{4\bar{\sigma}^2}\frac{d\bar{\epsilon}^{qp}}{d\bar{\sigma}} = \frac{9}{4}nA\bar{\sigma}^{(n-3)} \tag{32}$$

Performing the necessary differentiation gives;

$$\frac{d\epsilon_{11}^{qp}}{dt} = 0 \tag{33}$$

and

$$\frac{d\epsilon_{22}^{qp}}{dt} = \frac{\partial\epsilon_{22}^{qp}}{\partial\bar{\sigma}}\frac{d\bar{\sigma}}{dt} + \frac{\partial\epsilon_{22}^{qp}}{\partial\sigma_{22}}\frac{d\sigma_{22}}{dt} \tag{34}$$

$$\frac{d\epsilon_{22}^{qp}}{dt} = \dot{\bar{\sigma}}(n-3)\frac{\sigma_{22}^3}{3\bar{\sigma}} + \Psi\sigma_{22}^2\dot{\sigma}_{22} \tag{35}$$

and

$$\frac{d\epsilon_{12}^{qp}}{dt} = \frac{\partial\epsilon_{12}^{qp}}{\partial\bar{\sigma}}\frac{d\bar{\sigma}}{dt} + \frac{\partial\epsilon_{12}^{qp}}{\partial\sigma_{12}}\frac{d\sigma_{12}}{dt} \tag{36}$$

$$\frac{d\epsilon_{12}^{qp}}{dt} = \dot{\bar{\sigma}}\frac{2}{3}(n-3)a_{66}^{2}\Psi\frac{\sigma_{12}^{3}}{\bar{\sigma}} + 2\Psi a_{66}^{2}\sigma_{12}^{2}\dot{\sigma}_{12} \tag{37}$$

Collecting terms gives the function F as;

$$F = \frac{\dot{\bar{\sigma}}(n-3)}{3\bar{\sigma}}\Psi\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\sigma_{22}^{3}}{\dot{\sigma}_{22}} & 0 \\ 0 & 0 & \frac{4a_{66}^{2}\sigma_{12}^{3}}{\dot{\sigma}_{12}} \end{bmatrix} + \Psi\begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{22}^{2} & 0 \\ 0 & 0 & 4a_{66}^{2}\sigma_{12}^{2} \end{bmatrix} \tag{38}$$

where the effective stress rate is:

$$\dot{\bar{\sigma}} = \frac{3}{2\bar{\sigma}}(\sigma_{22}\dot{\sigma}_{22} + 2a_{66}\sigma_{12}\dot{\sigma}_{12}) \tag{39}$$

It should be noted that the quasistatic plastic compliance matrix $[S]^{qp}$ appears as the second term in equation (38).

Using equation (38), the multiaxial constitutive relation can be written in a more compact form as;

$$\left\{\dot{\epsilon}^{in}\right\} = \frac{\dot{\bar{\sigma}}(n-3)}{3\bar{\sigma}}[S]^{in}\{\dot{\sigma}\} + [S]^{qp}\{\dot{\sigma}\} \tag{40}$$

As before, assuming the case of uniaxial loading of an off-axis laminate, the inelastic strain rate can be written as:

$$\dot{\epsilon}_{x}^{in} = F(\sigma_x, \epsilon_x, sign\dot{\sigma}_x)\dot{\sigma}_x \tag{41}$$

Using the form of the uniaxial quasistatic elastic/plastic expression, we have;

$$\dot{\epsilon}_{x}^{in} = .2[h(\theta))]^{(n+1)}An(\sigma_x)^{(n-1)}\dot{\sigma}_x \tag{42}$$

This allows the F to be given in a specific sense as;

$$F(\sigma_x, \epsilon_x, sign\dot{\sigma}_x) = \begin{cases} .2[h(\theta)]^{(n+1)}An(\sigma_x)^{(n-1)} & \text{if } \dot{\sigma}_x > 0 \text{ (loading)} \\ 0 & \text{if } \dot{\sigma}_x \leq 0 \text{ (unloading)} \\ & \text{or } \sigma_x < \sigma_x^{*} \end{cases} \tag{43}$$

where the multiplying factor of .2 gave good correspondence to test data for the material systems investigated.

CONSTITUTIVE MODEL, UNIAXIAL FORM

Considering uniaxial loading of an off-axis laminate, it was assumed that the rate dependent strain could be decomposed into elastic and viscoplastic components.

$$\dot{\epsilon}_x = \dot{\epsilon}_x^e + \dot{\epsilon}_x^{vp} \tag{44}$$

Where the elastic term is;

$$\dot{\epsilon}_x^e = \frac{\dot{\sigma}_x}{E_x} \tag{45}$$

and the viscoplastic term can be written in a general sense as;

$$\dot{\epsilon}_x^{vp} = \dot{\epsilon}_x^{in} + \dot{\epsilon}_x^p \tag{46}$$

Using the above expressions, four cases can be given to account for different aspects of a test regime. Specifically, for tension loading, these cases are;

1. *Quasistatic;*

$$\text{given;} \quad \sigma_x = \sigma_x^\star \quad \Rightarrow \quad h(\theta) < \Phi >= 0 \quad \text{and} \quad \dot{\sigma}_x = 0$$

$$\text{we have;} \quad \dot{\epsilon}_x{}^p = 0, \quad \dot{\epsilon}_x^{in} = 0, \quad \epsilon_x = \frac{\sigma_x}{E_x} + h(\theta)^{n+1} A(\sigma_x)^n$$

2. *Loading;*

$$\text{given;} \quad \dot{\epsilon}_x > 0 \quad \Rightarrow \quad \dot{\epsilon}_x = h(\theta) < \Phi > + \dot{\sigma}_x \left(\frac{1}{E_x} + F \right)$$

$$\text{we have;} \quad \dot{\sigma}_x = \left(\frac{\dot{\epsilon}_x - < \beta(\sigma_x - \sigma_x^\star)^{1/m} >}{\left(\frac{1}{E_x} + .2[h(\theta)]^{n+1} An(\sigma_x)^{n-1} \right)} \right)$$

3. *Stress Relaxation;*

$$\text{given;} \quad \dot{\epsilon}_x = 0 \quad \Rightarrow \quad \dot{\sigma}_x < 0 \quad \Rightarrow \quad F = 0$$

$$\text{we have;} \quad \dot{\epsilon}_x^{vp} = h(\theta) < \Phi >= -\dot{\epsilon}_x^e \quad \text{or;} \quad h(\theta) < \Phi >= \frac{-\dot{\sigma}_x}{E_x}$$

4. *Creep;*

$$\text{given;} \quad \dot{\sigma}_x = 0, \quad \dot{\epsilon}_x > 0 \quad \Rightarrow \quad F = 0, \quad \dot{\epsilon}^e_x = 0$$

$$\text{we have;} \quad \dot{\epsilon}_x = < \beta(\sigma_x - \sigma^*_x)^{1/m} >$$

It is noted that the expressions for creep will not allow for prediction of creep recovery after unloading below the quasistatic stress level.

LAMINATION THEORY

The model presented above can be used to describe the nonlinear, rate dependent behavior in a laminated composite. For such a laminated composite, we can use typical notation from lamination theory so that Q_{ij} is the stiffness matrix and S_{ij} is the corresponding compliance matrix. We also note that the transformed stiffness and compliance matrices are given by $\bar{Q}_{ij}$ and $\bar{S}_{ij}$ respectively. This gives the constitutive equations for the quasistatic elastic/plastic case to be;

$$\{d\sigma\}_k = [\bar{Q}]^e_k \{d\epsilon^e\}_k \qquad \text{elastic} \tag{47}$$

$$\{d\sigma\}_k = [\bar{Q}]^{qp}_k \{d\epsilon^{qp}\}_k \qquad \text{plastic} \tag{48}$$

The subscript k refers to the individual or k'th layer in the laminate. If we let

$$[\bar{Q}]^{ep} = [\bar{Q}]^e + [\bar{Q}]^{qp} \tag{49}$$

then, the combined expressions may be written for the quasistatic elastic/plastic case as;

$$\left\{ \begin{array}{c} dN \\ \\ dM \end{array} \right\} = \left[\begin{array}{c|c} A & B \\ & \\ B & D \end{array} \right]^{eqp} \left\{ \begin{array}{c} d\epsilon^\circ \\ \\ d\kappa \end{array} \right\} \tag{50}$$

Similarly, for the elastic/viscoplastic case, we can write;

$$[\bar{Q}]^{evp} = [\bar{Q}]^e + [\bar{Q}]^{vp} \tag{51}$$

then, the combined expressions may be written for the elastic/viscoplastic case as;

$$\left\{ \begin{array}{c} \frac{dN}{dt} \\ - \\ \frac{dM}{dt} \end{array} \right\} = \left[\begin{array}{ccc} A & | & B \\ - & & - \\ B & | & D \end{array} \right]^{evp} \left\{ \begin{array}{c} \frac{d\epsilon^{\circ}}{dt} \\ - \\ \frac{d\kappa}{dt} \end{array} \right\} \tag{52}$$

MATERIALS TESTING

Material constants and related functions needed by this model were found by performing uniaxial tension or compression tests on off-axis laminates under isothermal conditions. By using effective values and collapsing this data into master curves, five experimentally derived constants were incorporated into the constitutive model. Additional information on the test procedures and equipment can be found in reference [9].

Test Specimens and Material

Two polymer matrix composite material systems were investigated in this study. The first, a graphite/thermoplastic was composed of Hercules[1] IM7 fiber and Amoco[1] 8320 matrix. The second material under study was a graphite/bismaleimide composed of Hercules IM7 fibers and Narmco[1] 5260 matrix. Both material systems had glass transition temperatures (T_g) listed by the manufacture to be approximately 220°C.

Rectangular test specimens similar to those described in ASTM specification D3039-76 were cut from the finished panels. For the elastic material constants, tests were run on $[0^{\circ}]_{12}$, $[90^{\circ}]_{12}$ and $[\pm 45^{\circ}]_{2s}$ specimens in order to determine E_1, ν_{12}, E_2 and G_{12}. Procedures similar to those outlined in ASTM specifications D3039-76 and D3518-76 were used to generate the constants.

For the elastic/viscoplastic material constants, off-axis tests on 15°, 30° and 40° coupons were performed using the rectangular specimen geometry described above. The specimens had an aspect ratio of 9.5:1 measured between the specimen ends. Figure 1 shows a photograph of the test apparatus.

All tests were conducted under isothermal conditions using either monotonic tension or compression loads. The four temperatures selected for study

[1]The use of trade names in this paper does not constitute endorsement, either expressed or implied, by the National Aeronautics and Space Administration.

were $23°, 70°, 125°$ and $200°C$. The tests were run using strain or load control as appropriate.

Material Constants

Five material constants are required by the analytical model for any given temperature and load direction. These constants are: a_{66} for the potential function, A and n for the quasistatic elastic/plastic relations, and K and m for the rate dependent elastic/viscoplastic relations. These constants were all found using data from simple off-axis tension or compression tests.

As shown in reference [9], using a strain controlled test and an approach similar to that outlined by Yen [10], all of the constants were extracted from uniaxial tests with repeated holds built into the test to allow for stress relaxation to occur. The idea behind this approach is that during the period of stress relaxation, the stress will decrease rapidly towards some limiting value. This limiting value is assumed to be the quasistatic stress and represents the stress needed to solve the elastic/plastic constitutive equation for a given strain. By repeating these periods of stress relaxation during the course of the test, enough quasistatic points are found to allow an effective stress, effective plastic strain curve to be constructed. This curve represents the quasistatic behavior. The effective curves from different off-axis tests can then be plotted together and collapsed into a single curve by selecting the appropriate value of a_{66}. A power law fit to this curve determines A and n.

During stress relaxation, the total strain rate is zero, therefore, as shown by the expressions in case 3, the viscoplastic term can be equated to the stress rate divided by the elastic modulus. During relaxation, the overstress and the stress rate can be defined for any point in time. This allows the construction of a overstress (H) versus effective viscoplastic strain rate (Φ) graph. These master curves are fit with a power law expression which gives the constants K and m.

RESULTS

Not all specimens were tested to failure, so failure levels are not reported. Optical observations and edge replication revealed no matrix cracking prior to specimen failure.

A review of material properties is provided in table 1, which also gives a complete list of all the constants. The material constant a_{66} which appears in the potential function was found to be independent of temperature. However, it was found to be dependent upon material system. In addition, a_{66} for IM7/8320 was found to be dependent on whether the material was in tension or compression.

From the quasistatic master curves such as those shown in figures 2-3, it is apparent that the IM7/5260 material shows less tendency towards nonlinear elastic/plastic behavior than the IM7/8320 system. Both systems show a definite trend towards increased ductility as temperature increases.

From the rate dependent master curves such as shown in figures 4-5, one can see from a comparison of the two systems that the IM7/5260 material shows a greater tendency towards higher overstress than the IM7/8320 does for an equivalent increase in plastic strain rate. This trend implies that the IM7/5260 is exhibiting less viscoplastic behavior than the IM7/8320. Both material systems indicate a slight increase in the viscoplastic behavior with an increase in temperature.

For both material systems, the master curves for tension differ than those for compression at the same temperature. This may imply that the deformation mechanisms in tension and compression are not the same.

On the lamina level, the uniaxial elastic/viscoplastic constitutive model was solved numerically using the fourth order Runge-Kutta technique with a modified Newton technique to find the roots of the quasistatic equation. Comparing the uniaxial test results against the analytical model shows that the model does well in predicting several phenomenon including: stress relaxation, short term creep, linear elastic unloading and variable strain rate loading. Typical comparisons between test and predictions are given in figures 6-9.

Short term creep behavior of a 15° off-axis specimen at 70°C is shown in figure 6. The applied stress history, resultant strain history and predicted strain history are given. The correlation between test and predicted strain is typical for short term creep tests performed on both material systems.

The prediction of creep behavior is a good verification of the model since the material constants used for the creep prediction were found from the stress relaxation procedures described previously.

Figure 7 shows the relationships between stress, strain and time for a 25° off-axis tension specimen at 70°C. In this case, the test was run under strain control and the resultant stress was measured. Periods of constant strain rate, stress relaxation and unloading comprise the input strain history. The predicted stress/time and stress/strain behavior is plotted against the measured values in figures 7a and 7b. A good correlation between test and predicted values is evident. Figure 8 is similar to figure 7 except for the higher test temperature. Once again, good correlation is shown. It should be noted that the 25° data was not used to construct the master curves and therefore correlation between test and prediction can be used to help verify the model.

Figure 9 gives test and predicted values for a strain controlled, uniaxial compression test of a 30° off-axis specimen at 125°C. The results indicate a reasonable correlation, but as was typical with most of the compression results, the comparison between test and predicted values was not as good as the tension cases.

In general, comparison of test versus predicted values gave good results for both materials, in tension and compression, and throughout the range of test temperatures.

SUMMARY

An elastic/viscoplastic constitutive model was developed to describe the observed nonlinear, rate-dependent behavior in advanced polymer matrix composites at elevated temperatures. Formulations for the general multiaxial case and the specific uniaxial case were given. Both uniaxial tension and compression loading of off-axis specimens were investigated experimentally.

Based upon the need for material constants in the elastic/viscoplastic constitutive model outlined above, test procedures were developed for the determination of the required parameters. The test methods used were found to be reliable and repeatable. Simple off-axis tests with repeated periods of stress relaxation were found to be useful for generating the necessary data. The specimen geometry and end tabs minimized the extension-shear

coupling, however, test temperatures close to T_g will enhance this behavior.

Comparisons between test data and predicted values indicated that the model provides a reasonable prediction of the behavior of load or strain controlled tests. Periods of loading, unloading, stress relaxation and creep were accounted for. Observed material behavior that the model does not account for include: creep recovery after unloading and nonlinear time dependent behavior during unloading. A complete description of cyclic behavior would require that these phenomenon be correctly modeled.

It is expected that further work with this constitutive model will support the development of a laminate level computer code which can be used to predict the elastic/viscoplastic behavior of any arbitrary laminate under conditions of combined mechanical and thermal loads. Further testing on IM7/5260 and IM7/8320 laminates should provide data for additional verification of the analysis procedures.

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Material Type	°C	Elastic $E_1(GPa)$	$E_2(GPa)$	$G_{12}(GPa)$	ν_{12}	Elastic/Plastic a_{66}	$A(MPa)^{-n}$	n	Elastic/Viscoplastic $K(MPa)$	m
IM7/5260 Tension	23	152.8	8.7	5.2	.30	.60	5.07E-10	3.34	1.80E+05	.92
	70	161.7	9.2	5.8	.31	.60	1.13E-09	3.35	2.09E+05	.98
	125	156.5	8.8	5.2	.36	.60	1.18E-09	3.63	3.06E+05	1.03
	200	154.3	7.5	5.1	.35	.60	2.17E-17	9.64	9.83E+04	.94
IM7/5260 Compress.	23	152.8	8.7	5.2 *	.30	.60	1.59E-09	3.02	1.24E+06	1.12
	70	161.7	9.2	5.8 *	.31	.60	4.20E-09	2.85	3.42E+04	.83
	125	156.5	8.8	5.2 *	.36	.60	4.33E-13	5.25	2.59E+05	1.01
	200	154.3	7.5	5.1 *	.35	.60	4.30E-14	6.91	9.73E+04	.92
IM7/8320 Tension	23	157.9	7.1	4.3	.32	.30	8.86E-12	5.48	8.40E+03	.79
	70	153.8	7.9	4.3	.34	.30	2.19E-08	3.36	3.33E+04	.86
	125	142.0	7.5	3.2	.35	.30	3.61E-11	5.50	3.77E+03	.72
	200	147.3	5.5	2.6	.35	.30	6.16E-05	2.66	1.40E+03	.64
IM7/8320 Compress.	23	157.9	7.1	4.3 *	.32	.15	1.08E-07	3.03	3.37E+03	.71
	70	153.8	7.9	4.3 *	.34	.15	8.25E-08	3.09	1.29E+04	.83
	125	142.0	7.5	3.2 *	.35	.15	1.03E-11	5.89	1.42E+04	.83
	200	147.3	5.5	2.6 *	.35	.15	7.93E-05	2.52	2.40E+04	.90

Table 1: Material Properties and Constants. * Assumed from tension data

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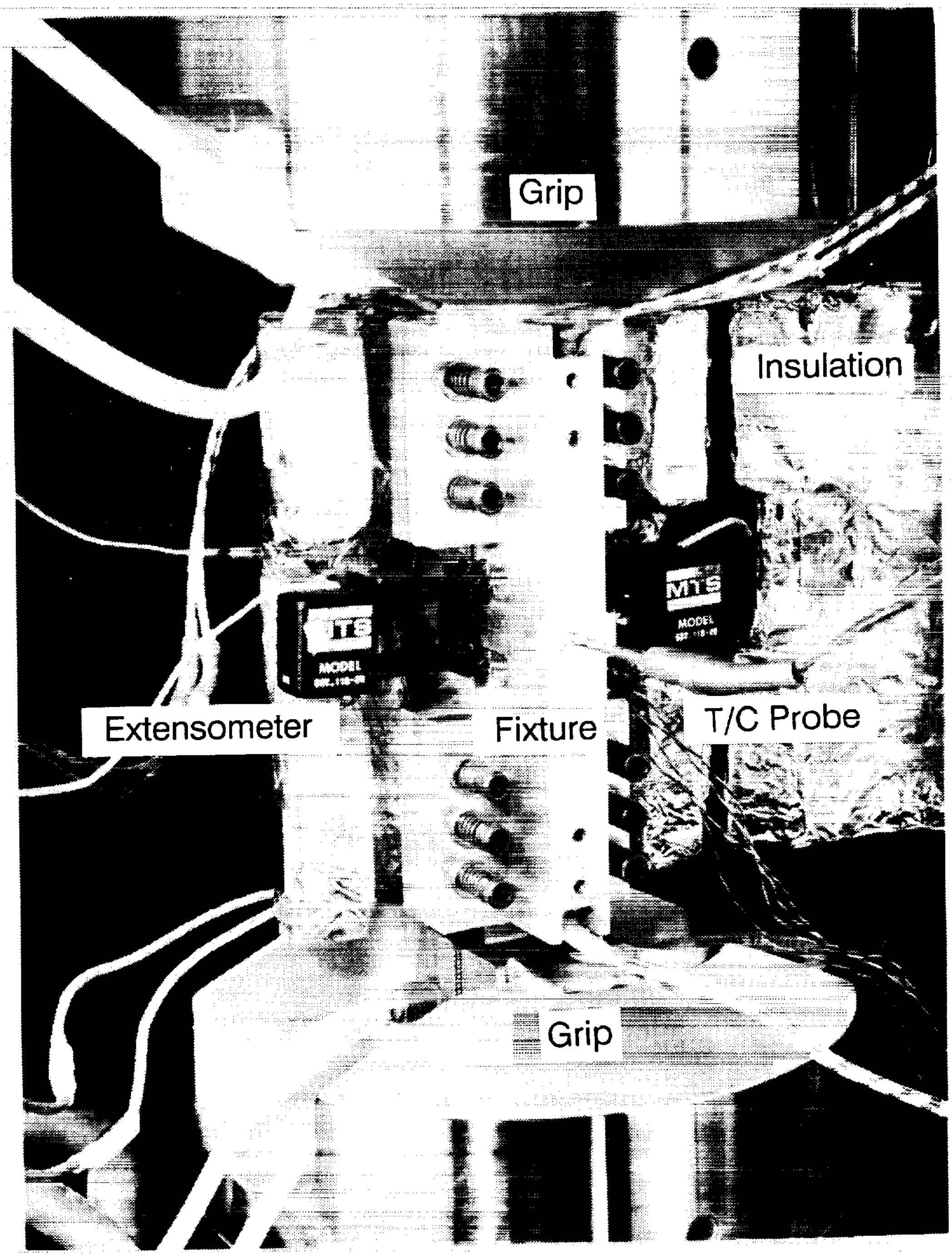


Figure 1: Test specimen and fixture mounted in the test machine.

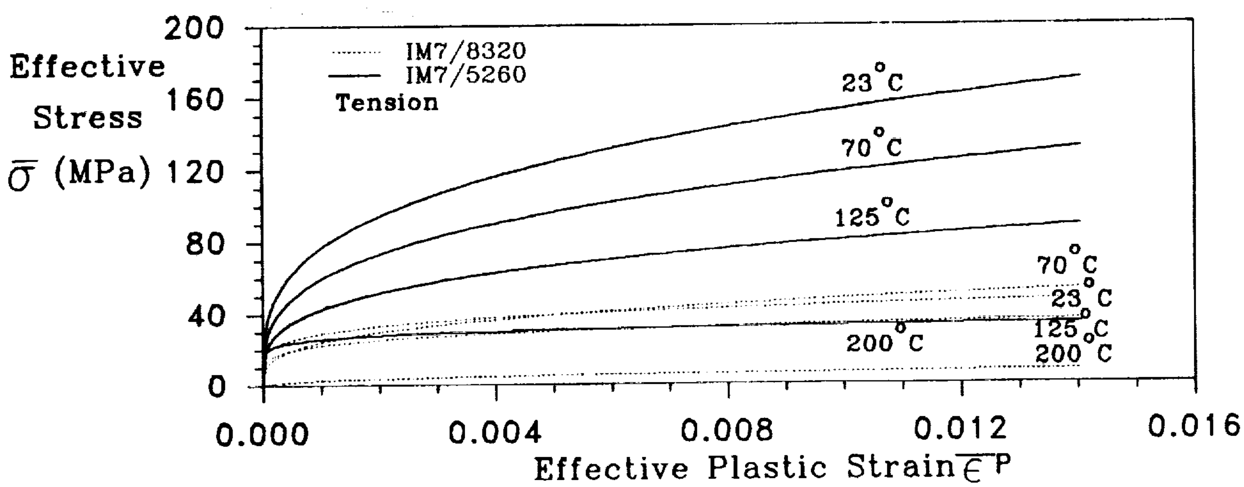


Figure 2: Elastic/plastic master curves for tension loading.

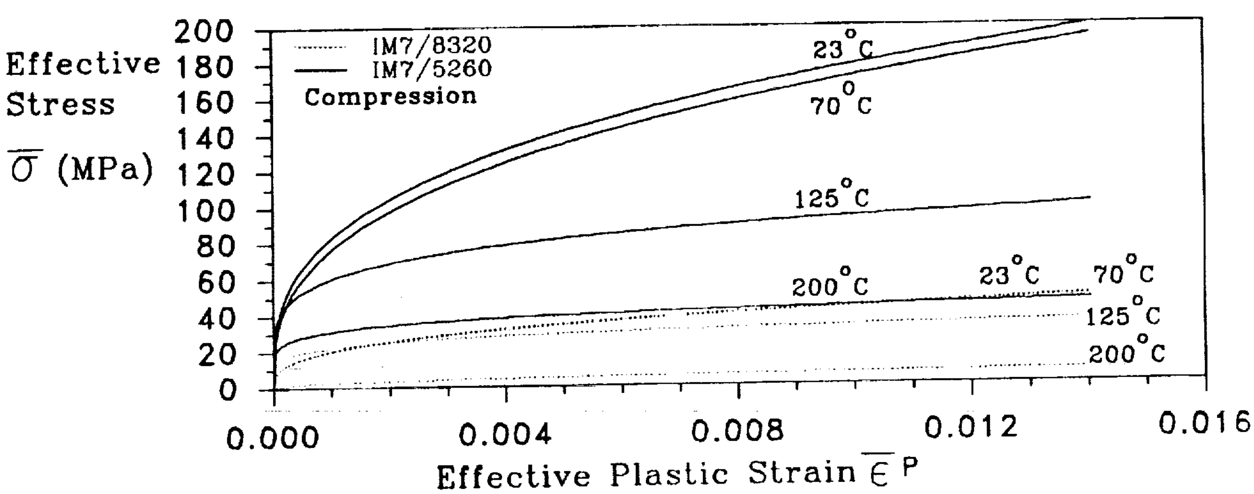


Figure 3: Elastic/plastic master curves for compression loading.

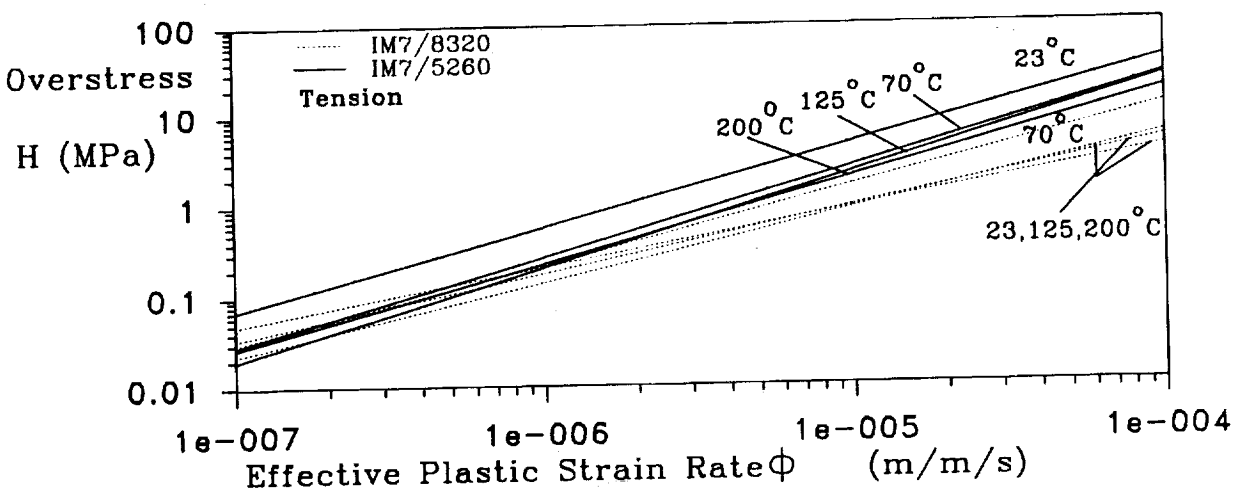


Figure 4: Elastic/viscoplastic master curves for tension loading.

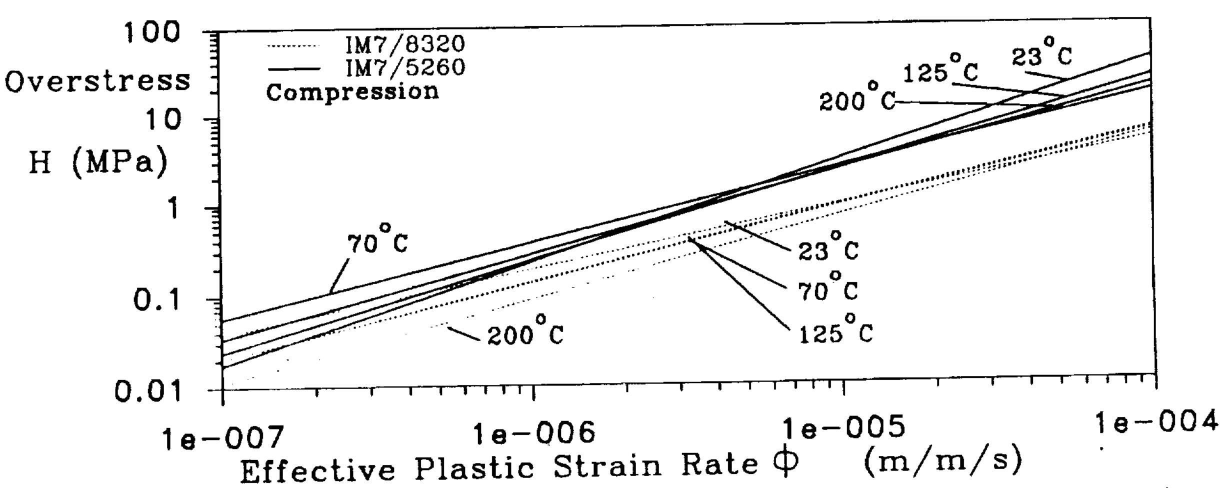


Figure 5: Elastic/viscoplastic master curves for compression loading.

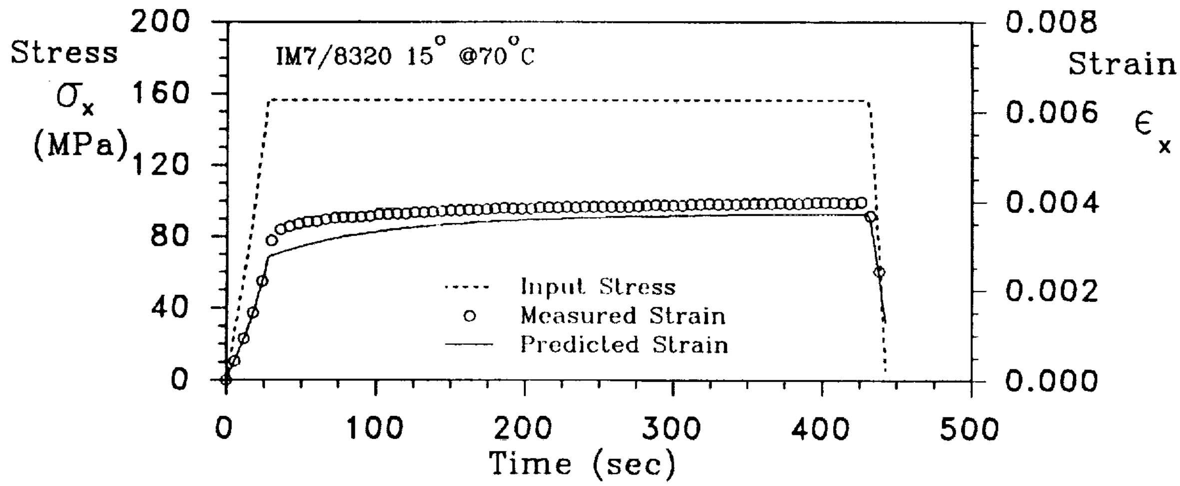
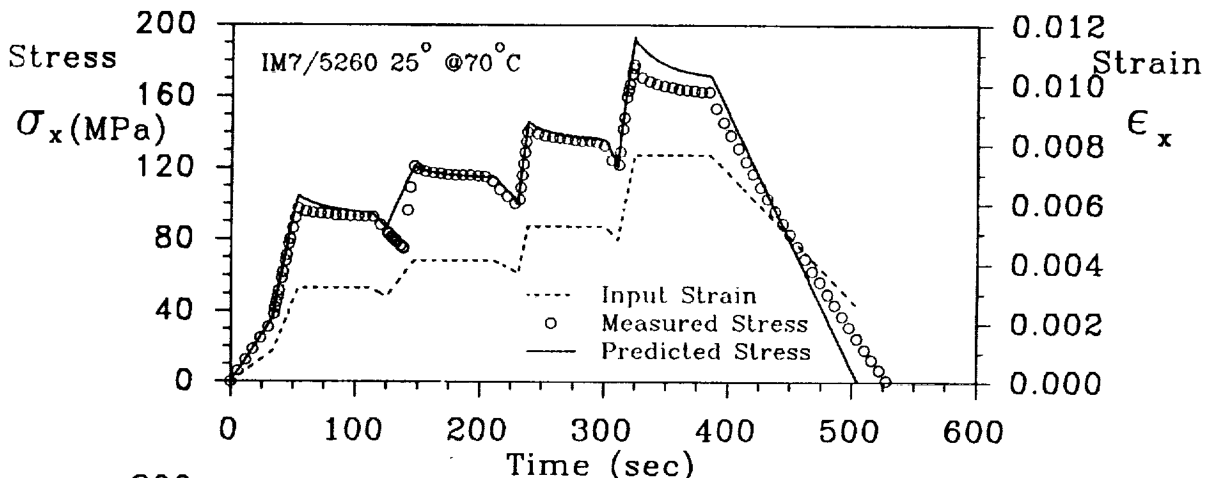


Figure 6: Comparison of test results and model predictions for creep in tension, of a typical IM7/8320 15° off-axis specimen at 70°C under load control.



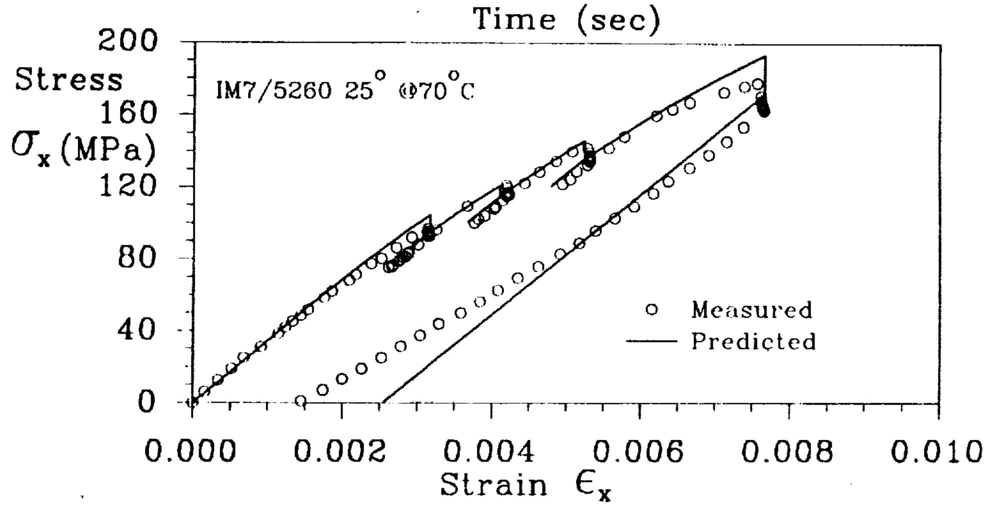


Figure 7: Comparison of test results and model predictions of a typical IM7/5260 25° off-axis tension specimen at 70°C under strain control.

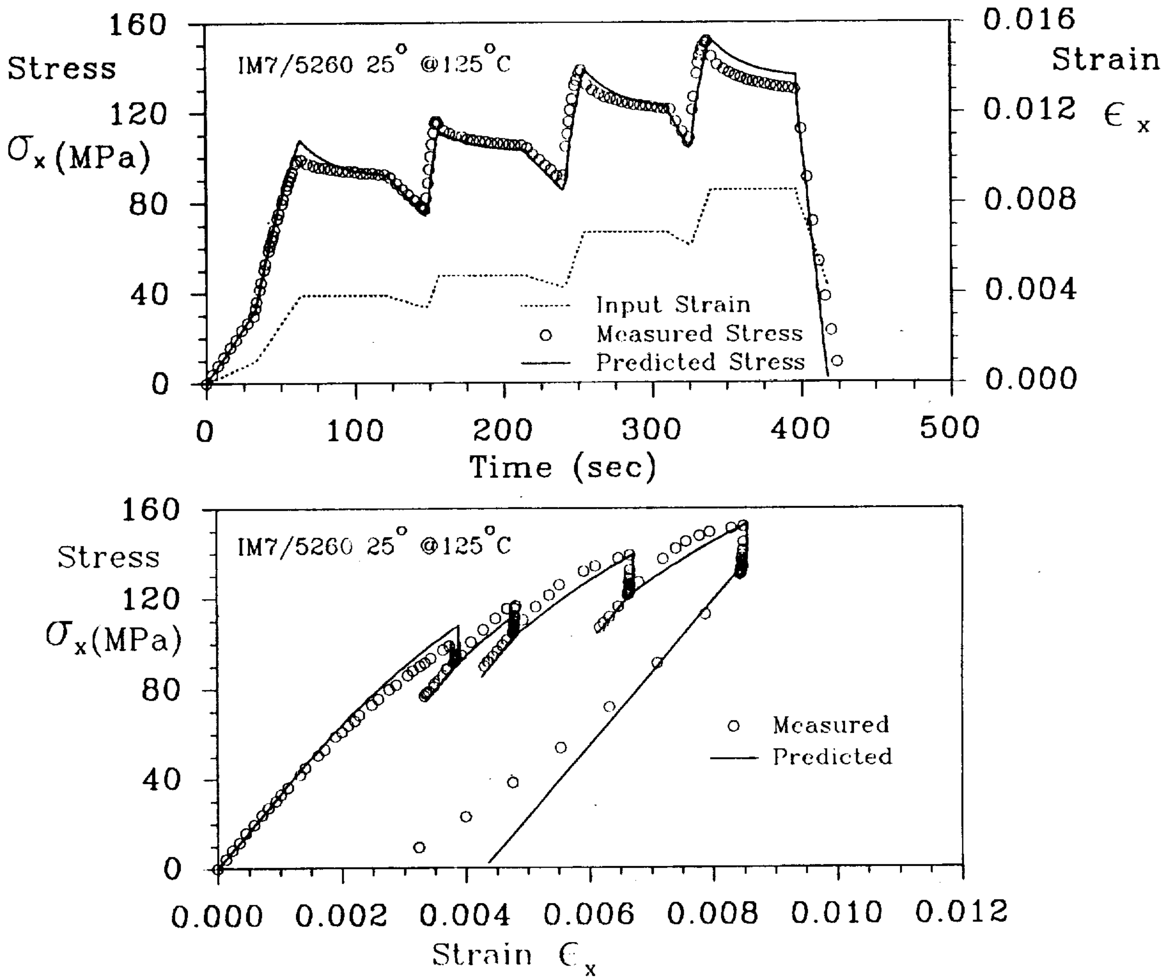


Figure 8: Comparison of test results and model predictions of a typical IM7/5260 25° off-axis tension specimen at 125°C under strain control.

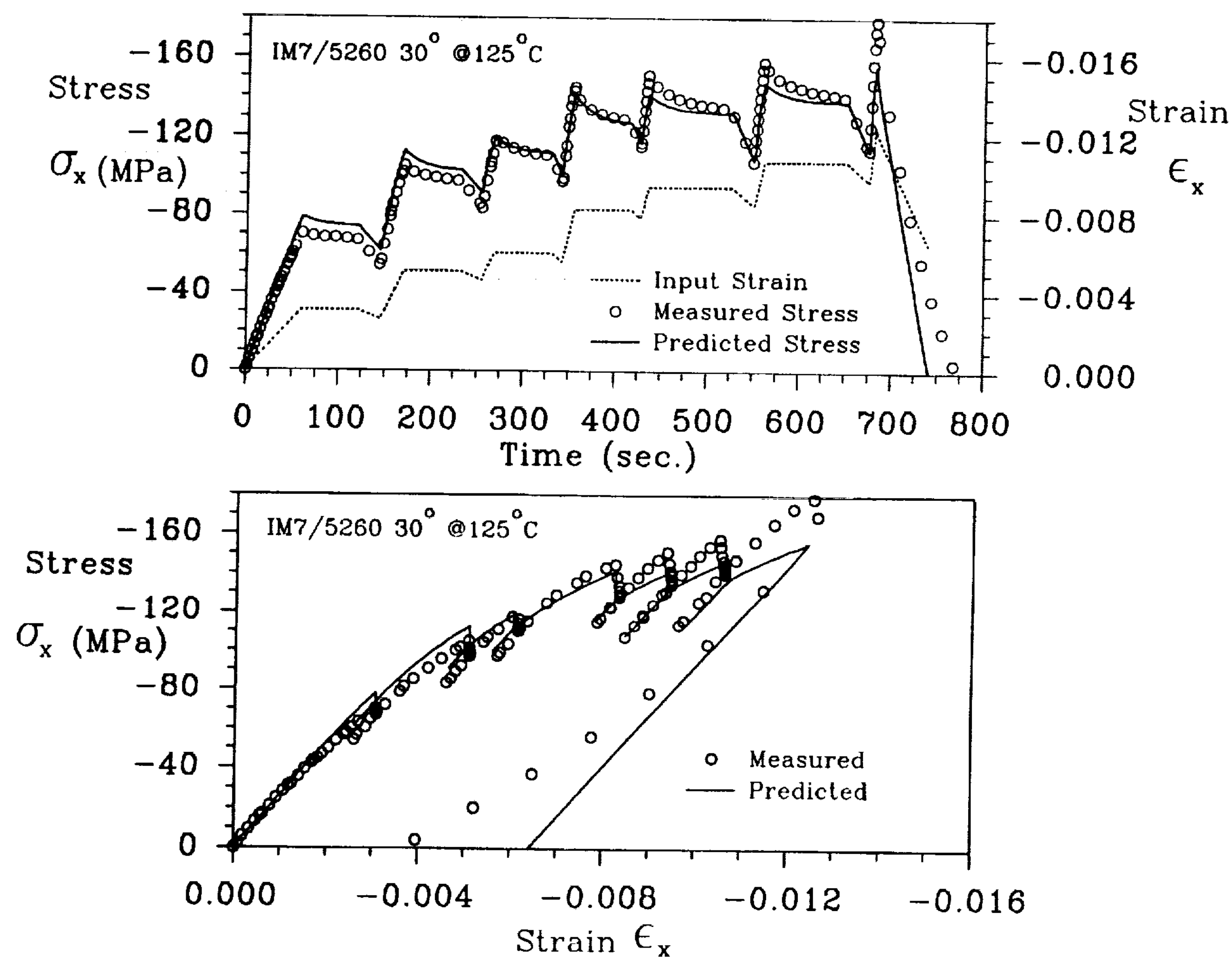


Figure 9: Comparison of test results and model predictions of a typical IM7/5260 30° off-axis compression specimen at 125°C under strain control.

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15. Supplementary Notes

16. Abstract

This paper describes the formulation of an elastic/viscoplastic constitutitve model which was used to predict the measured behavior of graphite/thermoplastic and graphite/bismaleimide composite materials at elevated temperature. The model incorporates the concepts of overstress and effective strain/strain to provide a simple formulation which was able to account for material behavior under monotonic tension or compression loads over a temperature range of 23°C to 200°C. Observed behavior such as stress relaxation and steady state creep, in off-axis tension and compression tests, were predicted by the model. Material constants required by the model were extracted from simple off-axis test data.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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