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FOR HOMOGENEOUS SHEAR FLOW

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ABSTRACT

In a recent paper [*Phys. Fluids* A2:1678-1684, 1990], the authors compared the performance of a variety of turbulence models including the $K - \varepsilon$ model and the second-order closure model derived by Yakhot and Orszag based on Renormalization Group (RNG) methods. The performance of these RNG models in homogeneous turbulent shear flow was found to be quite poor, apparently due to the value of the constant $C_{\varepsilon 1}$ in the modeled dissipation rate equation which was substantially lower than its traditional value. However, recently a correction has been made in the RNG based calculation of $C_{\varepsilon 1}$. It is shown herein that with the new value of $C_{\varepsilon 1}$, the performance of the RNG $K - \varepsilon$ model is substantially improved. On the other hand, while the predictions of the revised RNG second-order closure model are better, some lingering problems still remain which can be easily remedied by the addition of higher-order terms.

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A comparative study of the performance of nine independent turbulence models in rotating homogeneous shear flow was recently reported by Speziale *et al.*[1] Two of the models considered consisted of the $K - \varepsilon$ model and second-order closure model derived by Yakhot and Orszag[2] using Renormalization Group (RNG) methods. It was rather surprising how poorly the RNG models performed in homogeneous shear flow relative to the older, empirically based models of the same general type. The origin of the deficient predictions of the RNG models appeared to be largely due to the rather low value of the constant $C_{\varepsilon 1}$ in the modeled dissipation rate equation; the RNG value of $C_{\varepsilon 1}$ was 1.063 in contrast to the more traditional value of $C_{\varepsilon 1} = 1.44$. However, a recent re-examination of the RNG based calculation of $C_{\varepsilon 1}$ by Yakhot and Smith[3] has led to a correction – the new value of $C_{\varepsilon 1}$ is 1.42. Some minor changes in the values of other constants in the RNG $K - \varepsilon$ model were also made.[3] In light of these changes, it would be desirable to set the record straight in regard to what these Renormalization Group models now predict for homogeneous shear flow – a critical test case used to evaluate the performance of models. This establishes the motivation for the present paper.

In the RNG $K - \varepsilon$ model, the Reynolds stress tensor $\tau_{ij} \equiv \overline{u_i' u_j'}$ (given that u_i' is the fluctuating velocity and an overbar represents an ensemble mean) is modeled as follows:[2,3]

$$\tau_{ij} = \frac{2}{3}K\delta_{ij} - C_\mu \frac{K^2}{\varepsilon}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{1}$$

where $K \equiv \frac{1}{2}\overline{u_i' u_i'}$ is the turbulent kinetic energy, $\varepsilon \equiv \nu \overline{\partial u_i'/\partial x_j\, \partial u_i'/\partial x_j}$ is the turbulent dissipation rate, $\overline{u}_i$ is the mean velocity, and C_μ is a dimensionless constant which is calculated to be 0.085. In homogeneous turbulence, the turbulent kinetic energy is a solution of the transport equation

$$\dot{K} = -\tau_{ij}\frac{\partial \overline{u}_i}{\partial x_j} - \varepsilon \tag{2}$$

which is exact. The turbulent dissipation rate is obtained from the RNG derived transport equation

$$\dot{\varepsilon} = -C_{\varepsilon 1}\frac{\varepsilon}{K}\tau_{ij}\frac{\partial \overline{u}_i}{\partial x_j} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} \tag{3}$$

where $C_{\varepsilon 1} = 1.42$ and $C_{\varepsilon 2} = 1.68$ according to the recent calculations of Yakhot and Smith.[3] These new values constitute a correction to the earlier values of $C_{\varepsilon 1} = 1.063$ and $C_{\varepsilon 2} = 1.72$ reported by Yakhot and Orszag.[2] An additional production term was also uncovered by

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Yakhot and Smith[3] which they were unable to close. However, an order of magnitude analysis[3] indicated that this term is small unless there are large strain rates – a case which will not be considered herein. Hence, we will neglect this additional term in the present study. For the RNG second-order closure model, the eddy viscosity model (1) is replaced with a Reynolds stress transport model of the form[1]

$$\dot{\tau}_{ij} = -\tau_{ik}\frac{\partial \bar{u}_j}{\partial x_k} - \tau_{jk}\frac{\partial \bar{u}_i}{\partial x_k} - C_1\frac{\varepsilon}{K}\left(\tau_{ij} - \frac{2}{3}K\delta_{ij}\right) + C_2 K\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{2}{3}\varepsilon\delta_{ij} \qquad (4)$$

where C_1 and C_2 are constants that are calculated to be 1.59 and 2/15, respectively. Some clarifications are needed concerning the origin of this model which has not been published and was obtained from a private communication with V. Yakhot. We have come to learn that this was not intended to be a final model, but rather was the result of a low-order calculation of the pressure-strain correlation whose purpose was to merely demonstrate that the Rotta term – with a coefficient C_1 close to the well accepted value of 1.5 – could be formally obtained from RNG. Hence, the results predicted by this preliminary model should be judged accordingly.

In homogeneous shear flow, an initially isotropic turbulence where

$$\tau_{ij} = \frac{2}{3}K_0\delta_{ij}, \quad \varepsilon = \varepsilon_0 \qquad (5)$$

at time $t = 0$ is subjected to a constant shear rate S with the corresponding mean velocity gradient tensor

$$\frac{\partial \bar{u}_i}{\partial x_j} = \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \qquad (6)$$

In Figure 1, the time evolution of the turbulent kinetic energy (where $K^* = K/K_0$ and $t^* = St$) predicted by the new RNG $K-\varepsilon$ model is compared with the large-eddy simulation of Bardina et al.[4] for an initial condition of $\varepsilon_0/SK_0 = 0.296$. The predictions of the old version of the RNG $K-\varepsilon$ model (where $C_\mu = 0.0837$, $C_{\varepsilon 1} = 1.063$, and $C_{\varepsilon 2} = 1.72$) as well as the standard $K-\varepsilon$ model (where $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$ and $C_{\varepsilon 2} = 1.92$) are also shown in Figure 1. It is clear from these results that the revised RNG $K-\varepsilon$ model does the best overall job in reproducing the growth rate of the numerical experiment on homogeneous shear flow. Analytically, it can be shown why this is the case. From a straightforward calculation, it can be shown that the turbulent kinetic energy and dissipation rate grow exponentially in

homogeneous shear flow as follows:[1,5]

$$K^* \sim \exp(\lambda t^*), \quad \varepsilon^* \sim \exp(\lambda t^*)$$

where the dimensionless growth rate λ is given by

$$\lambda = \left[\frac{C_\mu (C_{\varepsilon 2} - C_{\varepsilon 1})^2}{(C_{\varepsilon 1} - 1)(C_{\varepsilon 2} - 1)} \right]^{1/2}. \tag{7}$$

Hence, the growth rate becomes singular when $C_{\varepsilon 1} = 1$ — a state of affairs that explains why the old version of the RNG $K - \varepsilon$ model, with $C_{\varepsilon 1} = 1.063$, overpredicted the growth rate of the turbulent kinetic energy by such a wide margin. The new version of the RNG $K - \varepsilon$ model predicts a growth rate of

$$\lambda = 0.142$$

which is extremely close to the range of values obtained from physical and numerical experiments.[6,7] On the other hand, the standard $K - \varepsilon$ model predicts the somewhat high value of $\lambda = 0.226$ which explains why this model overpredicts the LES data for K^* as shown in Figure 1. A more complete set of the equilibrium values predicted by these different versions of the $K - \varepsilon$ model will be provided later.

In Figure 2, the time evolution of the turbulent kinetic energy predicted by the revised RNG second-order closure model is compared with the large-eddy simulation of Bardina et $al.$[4] as well as with the predictions of the earlier version of the model and the Launder, Reece, and Rodi[8] (LRR) model. The new version of the RNG model does yield better predictions than the older version of the model since the previous value of $C_{\varepsilon 1} = 1.063$ was too close to $C_{\varepsilon 1} = 1$ which constitutes a bifurcation point of the dissipation rate transport equation as shown by Speziale.[9] However, there are still problems with the model which gives rise to points of inflection in the time evolution of K^* – a feature that makes it inferior to other second-order closure models such as the Launder, Reece, and Rodi model. The origin of this problem appears to be tied to the modeling of the pressure strain correlation. In the Launder, Reece, and Rodi model, the pressure strain correlation $\Pi_{ij} \equiv \overline{p'(\partial u_i'/\partial x_j + \partial u_j'/\partial x_i)}$ is modeled as follows

$$\begin{aligned}
\Pi_{ij} &= -2C_1 \varepsilon b_{ij} + 2C_2 K \overline{S}_{ij} + C_3 K \left(b_{ik}\overline{S}_{jk} + b_{jk}\overline{S}_{ik} - \frac{2}{3}b_{kl}\overline{S}_{kl}\delta_{ij} \right) \\
&\quad + C_4 K(b_{ik}\overline{W}_{jk} + b_{jk}\overline{W}_{ik})
\end{aligned} \tag{8}$$

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where

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right), \quad \overline{W}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} - \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{9}$$

$$b_{ij} = \left(\tau_{ij} - \frac{2}{3}K\delta_{ij}\right)/2K \tag{10}$$

(in the simplified form of the Launder, Reece, and Rodi model, $C_1 = 1.8$, $C_2 = 0.4$ and $C_3 = C_4 = 1.2$). This model satisfies two important consistency conditions: (a) the constant C_2 is equal to 2/5 – a result that follows from simple symmetry conditions for Π_{ij} as well as from Rapid Distortion Theory,[8] and (b) it represents a formal expansion of Π_{ij} to $O(\mathbf{b})$ in the anisotropy tensor. On the other hand, for this preliminary RNG second-order closure model we have

$$\Pi_{ij} = -2C_1\varepsilon b_{ij} + 2C_2 K\overline{S}_{ij} \tag{11}$$

where $C_1 = 1.59$ and $C_2 = \frac{2}{15}$. This model is not complete to $O(\mathbf{b})$ in the rapid pressure-strain term and violates the important symmetry constraint of $C_2 = 2/5$. The fixed points that the resulting nonlinear ODE's for these second-order closure models give rise to in homogeneous shear flow are of the focus type.[5] Significant deviations of C_2 from a value of 2/5 excites the imaginary parts associated with these fixed points, thus inducing inertial oscillations which are unphysical for the case of pure shear flow.

An overview of the performance of the models can be gleaned from Table 1 which compares the predicted equilibrium values with the most recent experimental data of Tavoularis and Karnik[7] for homogeneous shear flow (this data constitutes a mean over the stronger shear rate cases). Here $(\cdot)_\infty$ denotes the equilibrium value obtained in the limit as $t \to \infty$. Several observations concerning Table 1 are noteworthy:

(a) The revised RNG $K - \varepsilon$ model yields substantially better results than the old version of the model and is, on balance, better than the standard $K - \varepsilon$ model. This appears to explain why the models performed as they did in Figure 1 relative to the LES results.

(b) The only deficiency in the predictions of the new RNG $K - \varepsilon$ model for homogeneous shear flow are in the values of the normal components of the anisotropy tensor – a shortcoming of any model based on an isotropic eddy viscosity. However, the RNG based anisotropic eddy viscosity model of Rubinstein and Barton[10] – which predicts $(b_{11})_\infty = 0.260$ and $(b_{22})_\infty =$

−0.196 for the normal anisotropies in homogeneous shear flow – alleviates this deficiency to a large extent.

(c) The RNG second-order closure model does perform somewhat better with the new value of $C_{\varepsilon 1}$ (the model now predicts a weak exponential time growth of K^* whereas the old version of the model predicted a power law growth, with $(SK/\varepsilon)_\infty = \infty$, due to the close proximity of $C_{\varepsilon 1}$ to the bifurcation point $C_{\varepsilon 1} = 1$). However, this preliminary model still performs weakly in comparison to the more commonly used second-order closures such as the Launder, Reece, and Rodi model. The deficiency in this model is traced to the rapid part of the pressure-strain correlation which is $O(1)$ instead of $O(b)$ in the anisotropy tensor. In fact, the deviation of C_2 from 0.4 to 2/15 results from the model trying to compensate for the truncated $O(b)$ terms[11] (interestingly enough, if C_2 is set to 0.4 in Eq. (11), the predictions of the model deteriorate substantially). Hence, we have little doubt that if the RNG based calculation is extended to include the $O(b)$ terms, the resulting model would perform quite well in comparison to other second-order closures.

In conclusion, with the revised coefficients proposed by Yakhot and Smith[3], the RNG $K - \varepsilon$ model now performs well in homogeneous shear flow – particularly when the RNG based anisotropic eddy viscosity of Rubinstein and Barton[10] is used. The RNG second-order closure model needs further development, however. It would appear that an extension of the rapid pressure-strain correlation to include terms of $O(b)$ would resolve the remaining deficiency in this model. Consequently, our current assessment of RNG based turbulence models is now more optimistic than reported earlier.

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	$(b_{11})_\infty$	$(b_{12})_\infty$	$(b_{22})_\infty$	$\left(\dfrac{SK}{\varepsilon}\right)_\infty$
New RNG $K - \varepsilon$ Model	0	-0.185	0	4.38
Old RNG $K - \varepsilon$ Model	0	-0.489	0	11.70
Standard $K - \varepsilon$ Model	0	-0.217	0	4.82
New RNG Second-Order Closure	0.489	-0.091	-0.244	8.94
Old RNG Second-Order Closure	0.533	0	-0.267	∞
LRR Model	0.193	-0.185	-0.096	5.65
Experimental Data	0.21	-0.16	-0.13	4.8

Table 1. Comparison of the equilibrium values of the various models with the experimental data of Tavoularis and Karnik[7] on homogeneous shear flow.
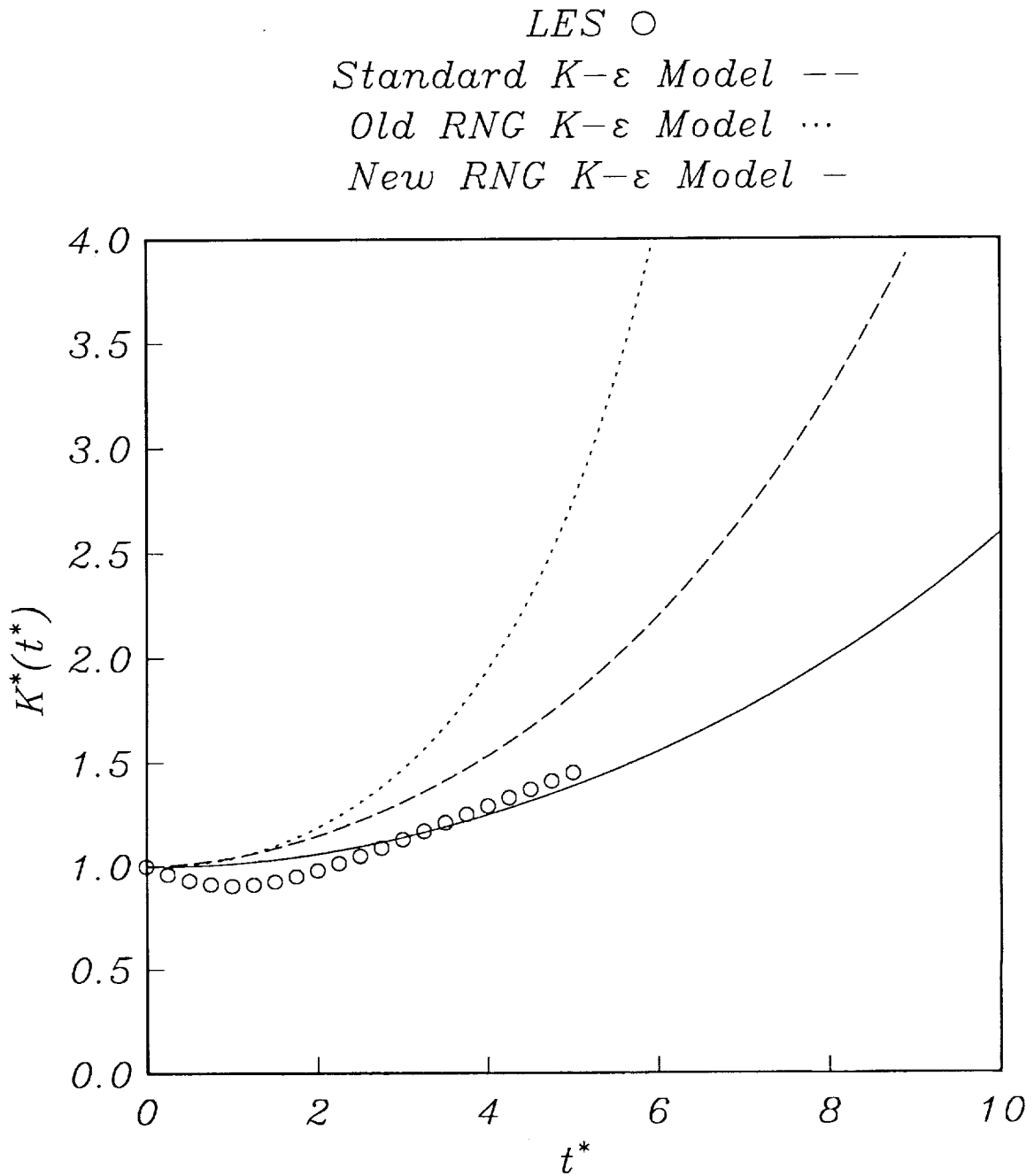
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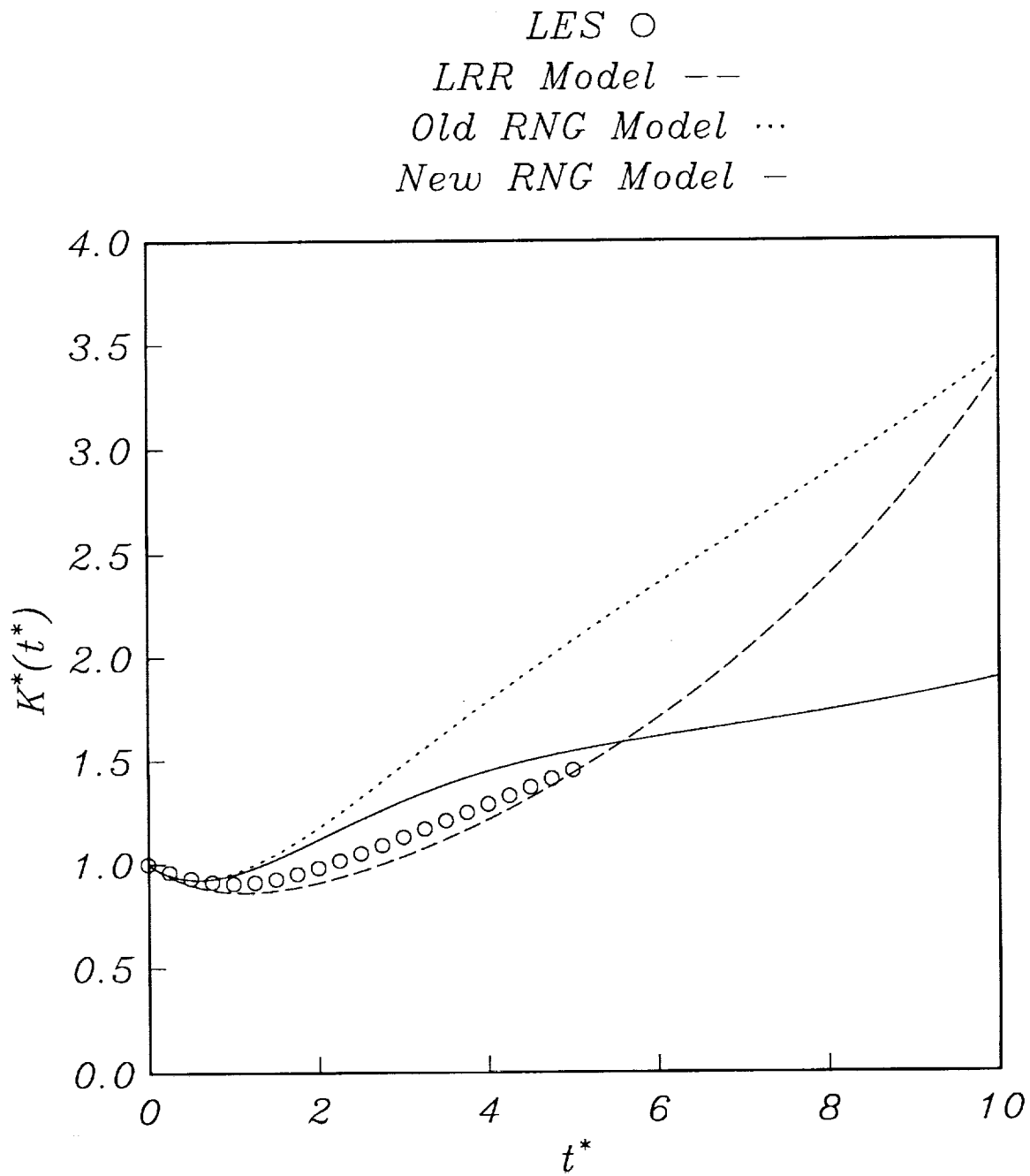
Figure 2. Time evolution of the turbulent kinetic energy in homogeneous shear flow: —— New RNG Second-Order Closure Model; ⋯ Old RNG Second-Order Closure Model; − − − Launder, Reece, and Rodi Model; o Large-Eddy Simulation.[4]

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16. Abstract

In a recent paper [Phys. Fluids A2:1678-1684, 1990], the authors compared the performance of a variety of turbulence models including the $K - \varepsilon$ model and the second-order closure model derived by Yakhot and Orszag based on Renormalization Group (RNG) methods. The performance of these RNG models in homogeneous turbulent shear flow was found to be quite poor, apparently due to the value of the constant $C_{\varepsilon 1}$ in the modeled dissipation rate equation which was substantially lower than its traditional value. However, recently a correction has been made in the RNG based calculation of $C_{\varepsilon 1}$. It is shown herein that with the new value of $C_{\varepsilon 1}$, the performance of the RNG $K - \varepsilon$ model is substantially improved. On the other hand, while the predictions of the revised RNG second-order closure model are better, some lingering problems still remain which can be easily remedied by the addition of higher-order terms.

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