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Modal Analysis of Multistage Gear Systems Coupled With Gearbox Vibrations

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MODAL ANALYSIS OF MULTISTAGE GEAR SYSTEMS

COUPLED WITH GEARBOX VIBRATIONS

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ABSTRACT

This paper presents an analytical procedure to simulate vibrations in gear transmission systems. This procedure couples the dynamics of the rotor-bearing gear system with the vibration in the gearbox structure. The modal synthesis method is used in solving the overall dynamics of the system, and a variable time-stepping integration scheme is used in evaluating the global transient vibration of the system. Locally each gear stage is modelled as a multimass rotor-bearing system using a discrete model. The modal characteristics are calculated using the matrix-transfer technique. The gearbox structure is represented by a finite element model, and modal parameters are solved by suing NASTRAN. The rotor-gear stages are coupled through nonlinear compliance in the gear mesh while the gearbox structure is coupled through the bearing supports of the rotor system. Transient and steady state vibrations of the coupled system are examined in both time and frequency domains. A typical three-geared system is used as an example for demonstration of the developed procedure.

NOMENCLATURE

$A_i(t)$	Modal function of the ith mode in x-direction
$A_{ti}(t)$	Modal function of the ith mode in θ-direction
$B_i(t)$	Modal function of the ith mode in y-direction
$[C_{bx}]$ $[C_{by}]$ $[C_{bz}]$	Gearbox damping matrices

$[C_{xx}]$, $[C_{xy}][C_{yx}]$, $[C_{yy}]$	Bearing direct and cross-coupling damping matrices
$[C_T]$	Torsional damping matrix
F_{Bx}, F_{By}	Bearing excitation forces
$F_x(t)$, $F_y(t)$	External excitation forces
$F_T(t)$	External excitation moment
$F_{Gx}(t)$, $F_{Gy}(t)$	Gear mesh force in x- and y-directions
$F_{Gt}(t)$	Gear mesh torque
$[G_V]$	Gyroscopic-angular rotation matrix
$[G_A]$	Gyroscopic-angular acceleration matrix
$[I]$	Identity matrix
$[J]$	Rotational mass moment of inertia matrix
$[K]$	Average stiffness matrix
$[K_{bx}]$ $[K_{by}]$ $[K_{bz}]$	Gearbox stiffness matrix
K_{dx}, K_{dy}	Compensation matrices in x- and y-direction
K_{tik}	Gear mesh stiffness between i^{th} and k^{th} rotor
K_s	Shaft stiffness matrix
$[K_{xx}]$, $[K_{xy}][K_{yx}]$, $[K_{yy}]$	Bearing direct and cross-coupling stiffness matrix
$[K_T]$	Torsional stiffness matrix
$[M]$	Mass-inertia matrix of rotor
$[M_b]$	Mass-inertia matrix of gearbox
R_{ci}	Radius of gear in the ith rotor
T_F	Gear generated torque
X, Y	Generalized motion in x- and y-directions
X_b, Y_b, Z_b	Gearbox motion in x-, y- and z-directions
X_{bs}, Y_{bs},	Gearbox motion at bearing supports
X_{ci}, Y_{ci}	Gear displacements in x- and y-directions of the ith rotor
X_F, Y_F	Gear forces in x- and y-directions

X_s, Y_s,	Motion of rotor at bearing support
α_{ki}	Angle of tooth mesh between k^{th} and i^{th} rotor
θ_{ci}	Rotational displacement of the ith gear
$[\Lambda^2]$, $[\Lambda_t^2]$	Lateral and torsional eigenvalue diagonal matrices
$[\Phi]_k$, $[\Phi_t]_k$	Lateral and torsional orthonormal eigenvector matrices of the kth rotor
ϕ_{kjl}	j^{th} orthonormal mode of kth stage at l^{th} node

INTRODUCTION

Recently there has been an increase in the use of gear transmissions in both defense and commercial applications. The ever increasing speed and torque requirements of newer transmission systems, often result in excessive noise and vibration at both the gear stages and the gearbox structure. Today there is a wealth of literature concerning noise and vibration reduction through gear tooth modification and design. However, the study of the dynamics and acoustics of the overall gear transmission system is somewhat limited.

August (1986) studied gear system vibrations for a planetary gear system. Boyd and Pike (1987) and Choy (1988b) used the work done by Cornell (1981) to study the dynamics of various gear transmission systems. Mitchell (1985) and David (1987, 1988) simulated the dynamics of multistage gear systems using the matrix transfer method. Choy et al. (1989, 1990) calculated the dynamics of the multistage gear systems with effects of base motion using the modal method. Ozguven and Houser (1988) and Kahraman et al. (1990) used a finite element model to predict the dynamics of multistage gear systems. In term of gearbox vibration analysis, some work has been reported by Lim (1990) using finite element analysis. Very little work, however, has been cited in the literature concerning gearbox coupled vibration in gear transmission systems.

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The work presented in this paper is the development and application of a combined approach of using the modal synthesis and finite element method in analyzing the dynamics of multistage gear systems coupled with the gearbox structure. Modal equations of motion are developed for each rotor-bearing-gear stage, using the matrix transfer method, to evaluate the modal parameters. The modal characteristics of the gearbox structure are evaluated using a finite element model on NASTRAN. The modal equations for each rotor stage and the gearbox structure are coupled through bearing supports and gear meshings. The modal equations are solved simultaneously with the appropriate initial conditions. The modal accelerations are integrated using a variable time-stepping integration scheme to obtain the transient vibration of the system. A typical three stage gear system is used as an example for this analysis. Results are presented in both time and frequency domain and in both modal and generalized coordinates to facilitate a complete representation of the dynamic characteristics of the system.

Development of Equations of Motion

The equations of motion for a single stage multi-mass rotor-bearing-gear system with the coupling effects of gear-box vibrations and the rotor inertia-gyroscopic effects can be written in matrix form for the ith stage (Choy, 1987; 1989) for the X-Z plane as:

$$[M]_i\{\ddot{X}\}_i + [G_v]_i\{\dot{Y}\}_i + [C_{xx}]_i\{\dot{X} - \dot{X}_b\}_i + [C_{xy}]_i\{\dot{Y} - \dot{Y}_b\}_i$$

$$+ [G_A]_i\{Y\}_i + [K_{xx} + K_s]_i\{X\}_i - [K_{xx}]_i\{X_{bs}\}_i$$

$$+ [K_{xy}]_i\{Y - Y_{bs}\}_i = \{F_x(t)\}_i + \{F_{Gx}(t)_i\} \qquad (1)$$

and in the Y-Z plane as:

$$[M]_i\{\ddot{Y}\}_i - [G_V]_i\{\dot{X}\}_i + [C_{yx}]_i\{\dot{X} - \dot{X}_b\}_i + [C_{yy}]_i\{\dot{Y} - \dot{Y}_b\}_i$$

$$- [G_A]_i\{X\}_i + [K_{yy} + K_s]_i\{Y\}_i - [K_{yy}]_i\{Y_{bs}\}_i$$

$$+ [K_{yx}]_i\{Y - Y_{bs}\}_i = \{F_y(t)\}_i + \{F_{Gy}(t)\}_i \qquad (2)$$

Here F_x and F_y are force excitations from the effects of mass imbalance

and shaft residual bow in both X- and Y-directions. F_{Gx} and F_{Gy} are the X

and Y gear mesh forces induced from the gear teeth interaction with other

coupled gear stages. The bearing forces are evaluated through the relative

motion between the rotor $\{X\}$, $\{Y\}$ and the gearbox $\{X_b\}$, $\{Y_b\}$ at the bearing

locations (Choy, 1987). The mass-inertia and gyroscopic effects are

incorporated in the mass matrix [M] and the gyroscopic matrices $[G_V]$ and $[G_A]$.

The coupled torsional equations of motion for the single rotor-bearing-gear

system can be written as:

$$[J]_i\{\ddot{\theta}\}_i + [C_T]_i\{\dot{\theta}\}_i + [K_T]_i\{\theta\}_i = \{F_T(t)\}_i + \{F_{Gt}(t)\}_i \qquad (3)$$

In Eq. 3, $\{F_T(t)\}$ represents the externally applied torque and $\{F_{Gt}(t)\}$

represents the gear mesh induced moment. Note that Eqs. (1) to (3) repeat for

each single gear/rotor stage. The gear mesh forces couple the force equations

of each stage to each other as well as the torsional equations to the lateral

equations (Choy, 1989; Cornell, 1981; and David, 1987; 1988). The coupling

relationships between the torsional and the lateral vibrations and the

dynamics of each individual gear/rotor stage are derived in the next section.

In addition, there are equations of motion for the gearbox which couple the

various rotor stages through the bearing supports. The gearbox equations can be written as:

X-equation

$$[M_b]\{\ddot{X}_b\} + [C_{bx}]\{\dot{X}_b\} + [C_{xx}]\{\dot{X}_b - \dot{X}_s\} + [C_{xy}]\{\dot{Y}_b - \dot{Y}_s\} + [K_{bx}]\{X_b\}$$
$$- [K_{xx}]\{X_b - X_s\} + [K_{xy}]\{Y_b - Y_s\} = 0 \qquad (4)$$

Y-equation

$$[M_b]\{\ddot{Y}_b\} + [C_{by}]\{\dot{Y}_b\} + [C_{yx}]\{\dot{X}_b - \dot{X}_s\} + [C_{yy}]\{\dot{Y}_b - \dot{Y}_s\} + [K_{by}]\{Y_b\}$$
$$- [K_{yy}]\{Y_b - Y_s\} + [K_{yx}]\{X_b - X_s\} = 0 \qquad (5)$$

and Z-equation

$$[M_b]\{\ddot{Z}_b\} + [C_{bz}]\{\dot{Z}_b\} + [K_{bz}]\{Z_b\} = F_{bz}(t) \qquad (6)$$

where $F_{bz}(t)$ is an excitation function due to external forces in the axial direction. Since the bearing is assumed to be uncoupled in the Z-direction, Eq. (6) can be solved independently without considering shaft motion.

Coupling of Gear Meshes

The torsional and lateral vibrations of a single individual rotor and the dynamic relationships between each gear stage are coupled through the nonlinear interactions in the gear mesh. Gear mesh forces and moments are evaluated as functions of relative motion and rotational between two meshing gears and the corresponding gear mesh stiffnesses. These gear mesh stiffnesses vary in a repeating nonlinear pattern with each tooth pass engagement period (August, 1986 and Cornell, 1981) and can be represented by a high order polynomial (Cornell, 1981 and Boyd, 1987). A sixth order polynomial curve is used in this study to simulate the stiffness changes for contacting gear pairs (zero stiffness are input for noncontacting pairs). The repeatability of such nonlinear mesh stiffnesses can also act as a source of steady state type of excitation to the gear system. With the coordinate

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system as shown in Fig. 1, the following gear mesh coupling equations can be established by equating forces and moments (Choy, 1989). For the kth stage gear of the system, summing forces in the X-direction results in:

$$F_{Gxk} = \sum_{i=1,i \neq k}^{n} K_{tki}[-R_{ci}\theta_{ci} -R_{ck}\theta_{ck} + (X_{ci} - X_{ck})\cos \alpha_{ki}$$
$$+ (Y_{ci} - Y_{ck}) \sin \alpha_{ki}] \cos \alpha_{ki} \qquad (7)$$

Summing forces in the Y-direction results in:

$$F_{Gyk} = \sum_{i=1,i \neq k}^{n} K_{tki}[-R_{ci}\theta_{ci} -R_{ck}\theta_{ck} + (X_{ci} - X_{ck})\cos \alpha_{ki}$$
$$+ (Y_{ci} - Y_{ck}) \sin \alpha_{ki}] \sin \alpha_{ki} \qquad (8)$$

Summing moments in the Z-direction results in:

$$F_{Gyk} = \sum_{i=1,i \neq k}^{n} R_{ck}\{K_{tki}[(-R_{ci}\theta_{ci} -R_{ck}\theta_{ck}) + (X_{ci} - X_{ck})\cos \alpha_{ki}$$
$$+ (Y_{ci} - Y_{ck}) \sin \alpha_{ki}]\} \qquad (9)$$

where n is the number of stages in the system.

Modal Analysis

To reduce the computational effort, the number of degrees-of-freedom of the system are reduced through modal transformation. Orthonormal modes for each individual rotor-bearing stage are obtained by solving the uncoupled system homogeneous characteristic equations. Using the modal expansion approach (Choy, 1987; 1988a; and 1989), the motion of the system can be expressed as:

$$\{X\} = \sum_{i=1}^{m} A_i\{\varnothing_i\}, \quad \{X_b\} = \sum_{i=1}^{m} A_{bi}\{\varnothing_{bxi}\}$$

$$\{Y\} = \sum_{i=1}^{m} B_i\{\varnothing_i\}, \quad \{Y_b\} = \sum_{i=1}^{m} B_{bi}\{\varnothing_{byi}\}$$

$$\{\theta\} = \sum_{i=1}^{m} A_{ti}\{\varnothing_{ti}\}, \quad \{Z_b\} = \sum_{i=1}^{m} D_{bi}\{\varnothing_{zi}\} \qquad (10)$$

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where m is the number of modes used to define each motion. The orthogonality conditions of the modes can be expressed as:

$$[\Phi]^T [K] [\Phi] = [\Lambda^2] \qquad (11)$$

where

$$K = (K_{xx} + K_{yy})/2 + K_s$$

$$[\Phi_b]^T [K_b] [\Phi_b] = [\Lambda_b^2] \qquad (12)$$

$$[\Phi_t]^T [K_T] [\Phi_t] = [\Lambda_t^2] \qquad (13)$$

$$[\Phi]^T [M] [\Phi] = [\Phi_b]^T [M_b][\Phi_b] = [I] \qquad (14)$$

Using the modal expansion and the orthogonality conditions, with the bearing forces due to the base motion expressed in the right hand side of the equation, the modal equations of motion for the rotor bearing system (Choy, 1989) can be written as:

X-Z equation

$$\{\ddot{A}\} + [\Phi]^T [G_V][\Phi]\{\dot{B}\} + [\Phi]^T [C_{xx}][\Phi]\{\dot{A}\} + [\Phi]^T [C_{xy}][\Phi]\{\dot{B}\}$$

$$+ [\Phi]^T [G_A][\Phi]\{B\} + [\Lambda^2]\{A\} + [\Phi]^T [K_{dx}][\Phi]\{A\}$$

$$+ [\Phi]^T [K_{xy}][\Phi]\{B\} = [\Phi]^T \{F_x(t) + F_{Gx}(t) + F_{Bx}(T)\} \qquad (15)$$

where

$$[K_{dx}] = [K_{xx}] + [K_s] - [K] \qquad (16)$$

and,

$$F_{Bx}(t) = [C_{xx}]\{\dot{X}_b\} + [C_{xy}]\{\dot{Y}_b\} + [K_{xx}]\{X_b\} + [K_{xy}]\{Y_b\} \qquad (17)$$

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which can also be expanded into modal parameters as

$$F_{Bx}(t) = [C_{xx}][\Phi_{bx}]\{\dot{A}_b\} + [C_{xy}][\Phi_{by}]\{\dot{B}_b\}$$

$$+ [K_{xx}][\Phi_{bx}]\{A_b\} + [K_{xy}][\Phi_{by}]\{B_b\} \qquad (18)$$

Y-Z equation

$$\{\ddot{B}\} - [\Phi]^T[G_V][\Phi]\{\dot{A}\} + [\Phi]^t[C_{yx}][\Phi]\{\dot{A}\} + [\Phi]^T[C_{yy}][\Phi]\{\dot{B}\}$$

$$-[\Phi]^T[G_A][\Phi]\{A\} + [\Lambda^w]\{B\} + [\Phi]^T[K_{dy}][\Phi]\{B\}$$

$$+ [\Phi]^T[K_{yx}][\Phi]\{A\} = [\Phi]^T\{F_y(t) + F_{Gy}(t) + F_{By}(t)\} \qquad (19)$$

where

$$[K_{dy}] = [K_{yy}] + [K_s] - [K] \qquad (20)$$

$$F_{By}(t) = [C_{yx}]\{\dot{X}_b\} + [C_{yy}]\{\dot{Y}_b\} + [K_{yx}]\{X_b\} + [K_{yy}]\{Y_b\} \qquad (21)$$

which can also be expanded into modal parameters as

$$F_{By}(t) = [C_{yx}][\Phi_{bx}]\{\dot{A}\} + [C_{yy}][\Phi_{by}]\{\dot{B}\} + [K_{yx}][\phi_{bx}]\{A\} + [K_{yy}][\Phi_{by}]\{B\} \qquad (22)$$

and the θ-equation can be expressed as

$$\{\ddot{A}_t\} + [\Phi_t]^T[C_t][\Phi_t]\{\dot{A}_t\} + [\Lambda_t^2]\{A_t\} = [\Phi_t]^T\{F_t(t) + F_{Gt}(t)\} \qquad (23)$$

The gear mesh forces and moments can also be expressed in the modal form, for the kth stage with the gear location at the lth node, as:

$$[\Phi]_k^T\{F_{Gx}\} = \sum_{j=1}^m \phi kjl \left\{ \sum_{i=1, i \neq k}^n K_{tki}[-R_{ci}\theta_{ci} - R_{ck}\theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} \right.$$

$$\left. + (Y_{ci} - Y_{ck}) \sin \alpha_{ki}] \cos \alpha_{ki} \right\} \qquad (24)$$

$$[\Phi]_k^T \{F_{Gy}\} = \sum_{j=1}^{m} \varnothing kjl \left\{ \sum_{i=1, i \neq k}^{n} K_{tki}[-R_{ci}\theta_{ci} -R_{ck}\theta_{ck} + (X_{ci} - X_{ck}) \cos \alpha_{ki} \right.$$

$$\left. + (Y_{ci} - Y_{ck}) \sin \alpha_{ki}] \sin \alpha_{ki} \right\} \tag{25}$$

$$[\Phi]_k^T \{F_{Gt}\} = \sum_{j=1}^{m} \varnothing kjl \left\{ \sum_{i=1, i \neq k}^{n} R_{ck}\left[K_{tki}[(-R_{ci}\theta_{ci} -R_{ck}\theta_{ck}) + (X_{ci} - X_{ck}) \right. \right.$$

$$\left. \left. + \cos \alpha_{ki} + (Y_{ci} - Y_{ck}) \sin \alpha_{ki}] \right\} \right\} \tag{26}$$

where k is the stage number, j is the mode number, and l is the station location of the gear mesh.

A set of modal equations of motion can also be written for the gearbox as:

X-equation

$$[\Phi_{bx}]^T [M_b][\Phi_{bx}]\{\ddot{A}_b\} + [\Phi_{bx}]^T [C_{bx}][\Phi_{bx}]\{\dot{A}_b\} + [\Phi_{bx}]^T [K_{bx}][\Phi_{bx}]\{A_b\}$$

$$+ [\Phi_{bx}]^T \{[K_{xx}]\{X_b - X_s\} + [C_{xx}]\{\dot{X}_b - \dot{X}_s\}$$

$$+ [K_{xy}]\{Y_b - Y_s\} + [C_{xy}]\{\dot{Y}_b - \dot{Y}_s\}\} = 0 \tag{27}$$

For proportional damping in the gearbox model, the equation can further be reduced to:

$$[I]\{\ddot{A}_b\} + [C_x]\{\dot{A}_b\} + [\Lambda_b^2]\{A_b\} + [\Phi_{bx}]^T \{F_{Bx}(t)\} - [\Phi_{bx}]^T \{[C_x][\Phi]\{\dot{A}\}$$

$$+ [C_{xy}][\Phi]\{\dot{B}\} + [K_{xx}][\Phi]\{A\} + [K_{xy}][\Phi]\{B\}\} = 0 \tag{28}$$

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Similarly, the Y-equation can be written as

$$[I]\{\ddot{B}_b\} + [C_{by}]\{\dot{B}_b\} + [\Lambda_b^2]\{B_b\} + [\Phi_{by}]^T\{F_{By}(t)\} - [\Phi_{by}]^T\{[C_{yx}][\Phi]\{\dot{A}\}$$

$$+ [C_{yy}][\Phi]\{\dot{B}\} + [K_{yx}][\Phi]\{A\} + [K_{yy}][\Phi]\{B\}\} = 0 \qquad (29)$$

and the Z-equation as

$$[I]\{\ddot{D}_b\} + [C_{bz}]\{\dot{D}_b\} + [\Lambda]\{D_b\} = [\Phi_{bz}]^T\{F_{bz}(t)\} \qquad (30)$$

Solution Procedure

In order to obtain a solution for the overall dynamics of the gear transmission system, a three phase solution procedure is used, namely, (1) (the evaluation of modal characteristics of the rotor-bearing systems and the gearbox structure, (2) transient vibration solution of the overall dynamics in modal coordinates, and (3) the transformation of force and vibration data from modal into generalized coordinates. Discussions of the three solution phases are presented in following paragraphs.

The natural frequencies and mode shapes required in the transformation of the equations of motion for the rotor-bearing system (Eqs. (1) to (3)) into the modal equations (Eqs. (15), (19), and (23)) are evaluated using the matrix transfer method (Choy, 1989 and 1990) on a discretized lumped mass model. Using an average stiffness value for the bearing supports, the undamped modes are calculated using a marching search technique for the assigned frequency range. The natural frequencies and mode shapes for the gearbox structure are obtained by the finite element analysis using NASTRAN. The modal data from the gearbox analysis are used to transform the gearbox equations of motion (Eqs. (4) to (6)) into their modal coordinates (Eqs. (28) to (30)).

The second phase of the solution procedure involves the solution of the coupled modal equations of motion between the rotor stages and the gearbox structure. The coupling effects of the gear mesh and the bearing supports are

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also expressed in modal coordinates such that the global equations are solved simultaneously in modal form. Using a set of initial conditions for both displacement and velocity of the global system, calculated from the steady state conditions at the rotor-bearing systems and zero vibration at the gearbox, the modal accelerations A, B, A_t, A_b, B_b, and D_b of the system can be evaluated (Eqs. (15), (19), (23), and (28) to (30)). A variable time stepping integration scheme (the Newmark-Beta Method) is used to integrate the modal acceleration to evaluate the modal velocities and displacements at the next time step. A regular time interval of 200 points per shaft revolution is used in this study with a refined region of smaller steps at the gear mesh transition period for single and multiple teeth contact.

The modal acceleration, velocity, and displacement calculated from the transient integration scheme are transformed back into the generalized coordinates (Eq. (10)). The nonlinear bearing forces and gear mesh forces can be evaluated from the velocity and displacement differentials between the rotor stages and the gearbox structure. Results from this solution procedure are demonstrated in the following section using a prototypical gear transmission system.

Discussion of Results

To demonstrate the application of the discussed analytical approach, a three-gear transmission system given in Fig. 2 is used as an example. Note that the gearbox is assumed to have uniform thickness throughout the enclosed walls and is fixed to the ground at the four lower corners. Figure 3 shows the arrangement and orientation of the gear stages inside the gearbox structure. While all three gears have an identical 36-tooth gear and a mesh contact ratio of 1.6, the driver (stage 1) is longer and larger in diameter than the two other stages, which are identical in geometry. Stage 1 is supported by two bearings located at the end plates while gear stages 2 and 3

are supported by bearings located at both end plates and at the middle of the gearbox, as shown in Fig. 3. An operating speed of 3000 rpm is used for all three gears.

In order to calculate the global dynamics of the gearbox system, the modal characteristics for both the gearbox structure and the rotor-bearing-gear stages are evaluated. The finite element approach (NASTRAN) is used to model the gearbox structure using a combination of plate elements. Table I presents the results of the first nine natural frequencies of the gearbox structure with their corresponding three-dimensional mode shapes given in Figs. 4 to 6. The dynamics of each individual rotor-bearing-gear stage are modelled using the matrix-transfer method. Using an averaged bearing stiffness, as discussed in the previous sections, the lateral natural frequencies and mode shapes of each rotor stages are evaluated. A similar procedure is repeated to evaluate the torsional vibrations of the rotor stages. Table II presents the results of both torsional and lateral natural frequencies for all three rotor stages. Some of the lateral and torsional mode shapes for the rotor stages are given in Figs. 7(b), 8(b) and 9(b).

Using mass imbalance in all three rotor stages as excitation input and a nonlinear gear mesh compliance between the gear stages, the global transient dynamics of the system are evaluated using zero initial conditions. Figures 10 to 13 present the gearbox vibrations in terms of modal excitations of the first 8 natural frequencies. These modal excitations (Choy, 1987; 1988a; and 1989) represent the excitability of the particular mode and are expressed in the frequency domain using an FFT procedure to transform the modal time variables into the frequency domain. Note that the major component excited in each mode is at its own natural frequency. The highest component of excitation is in the x-direction. A moderate excitation is seen in the y-direction, while a very small magnitude of excitation exists in the

z-direction. A closer examination shows that the two highest x components

occur at the second and fifth mode (462 and 575 Hz) while the highest in the

y-direction occurs at the third mode (509 Hz). Figure 14 presents the total

vibration at the upper corner of the gearbox (node 82) in both time and

frequency domains. Note that three dominant components excited in the

x-direction are located at frequencies of 462, 509, and 575 Hz with the

highest two located at 462 and 575 Hz. The Y-direction has only one major

component a 509 Hz while no significant vibration is detected in the

z-direction. This further confirms the use of modal excitation parameters

in representing the dynamic behaviors of the global system.

Figure 15 shows the orbiting motion of the gear stages during the initial

transient period. Note that the vibration of the system eventually settles

into a steady state motion. The smaller first stage orbit is due to the large

stiffness in the rotor-bearing system. The gear mesh forces between gears in

stages 1-2 and 1-3 in both time and frequency domains are given in Fig. 16.

Three major frequency components exist in the gear mesh forces at 0, 50, and

1800 Hz. They represent the static gear load at 0 Hz, the vibratory frequency

at rotational speed of 50 Hz, and the tooth pass frequency at 1800 Hz. The

existence of sidebands at 1800 Hz is mainly because of the shifts in the

toothpassing frequency due to the vibration of the rotor-gear system. A

further amplification of such frequency components can be seen in Figs. 7(a),

8(a), and 9(a). The modal excitation of the first lateral mode in Fig. 7(a)

has its largest component at 50 Hz due to mass imbalance at rotational speed.

The modal components of the first torsional mode in Fig. 8(a) show that the

gear load at zero frequency and the tooth pass frequency vibration are excited

in stages 2 and 3. Figure 9(a) shows that the zero gear load frequency is

excited in stage 1 at the second mode. This again further confirms the use

of the modal synthesis approach in analyzing dynamic behaviors in gear transmission systems.

SUMMARY

This paper presents a vibration analysis of a gear transmission system in which the dynamics of the gearbox structure is coupled with the vibration of the rotor-bearing-gear stages. The analysis combines the modal characteristics of the gear stages developed through the matrix transfer method with the modal parameters of the gearbox evaluated by a finite element model. The major content of this work can be summarized as follows:

1. A comprehensive procedure is developed to combine the dynamics of the rotor-gear system with vibrations of the gearbox structure to determine the global system response.

2. The modal method is used to transform the equations of motion into modal coordinates before the synthesis of the global systems of equations of motion to reduce the degrees of freedom of the system.

3. The use of modal excitation functions in the frequency domain provide good insights of the dynamic behavior of the gear transmission system. Such knowledge is crucial for designing transmissions with improved performance and durability.

4. The sensitivity of the gearbox vibration, due to rotor mass imbalance and gear mesh nonlinearity, can be evaluated using the developed methodology.

5. The initial transient dynamic analysis approach developed in this study can also be applied to simulate conditions in which a sudden excitation is applied to the gear transmission system.

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TABLE I. - GEARBOX
NATURAL
FREQUENCIES

Mode	Hz
1	412
2	462
3	509
4	540
5	575
6	697
7	852
8	893
9	911

TABLE II. - ROTOR-GEAR STAGE NATURAL FREQUENCIES

	Lateral mode, Hz	Torsional mode, Hz	Axial mode, Hz
Stage 1	131	661	3 931
	297	2086	5 208
	1047	4197	11 724
Stage 2	123	186	1 918
	187	866	4 860
	390	1686	8 186
Stage 3	123	186	1 918
	187	866	4 860
	390	1686	8 186
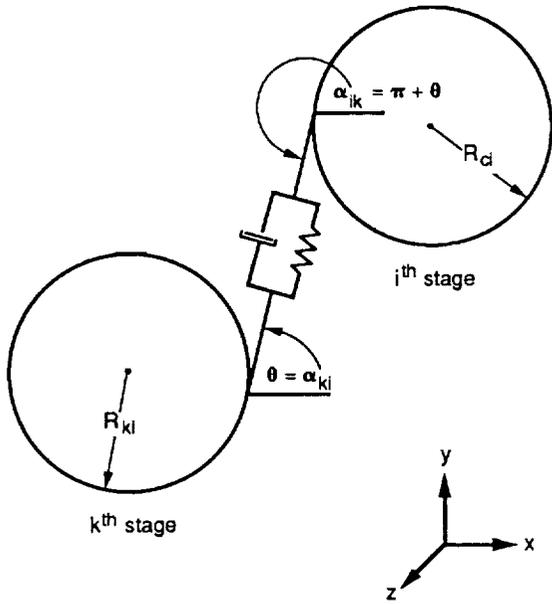
Figure 1.—Coordinate system for gear mesh force.
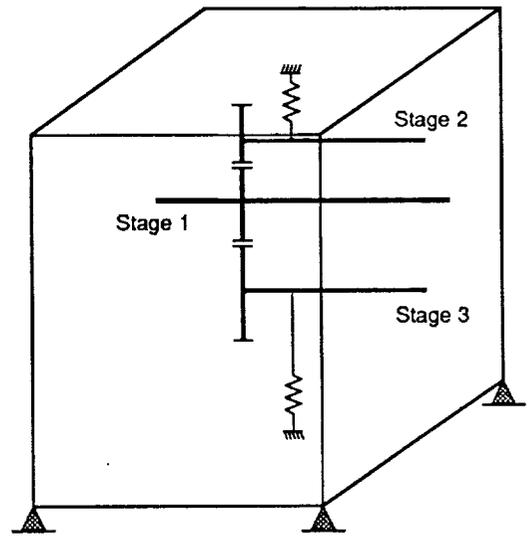
Figure 2.—Gear transmission assembly.

Gear box assembly
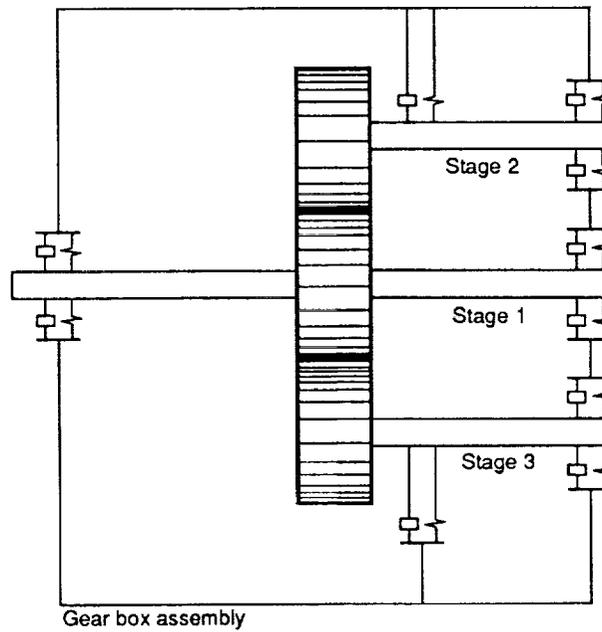
Figure 3.—Three-stage rotor-bearing-gear system.
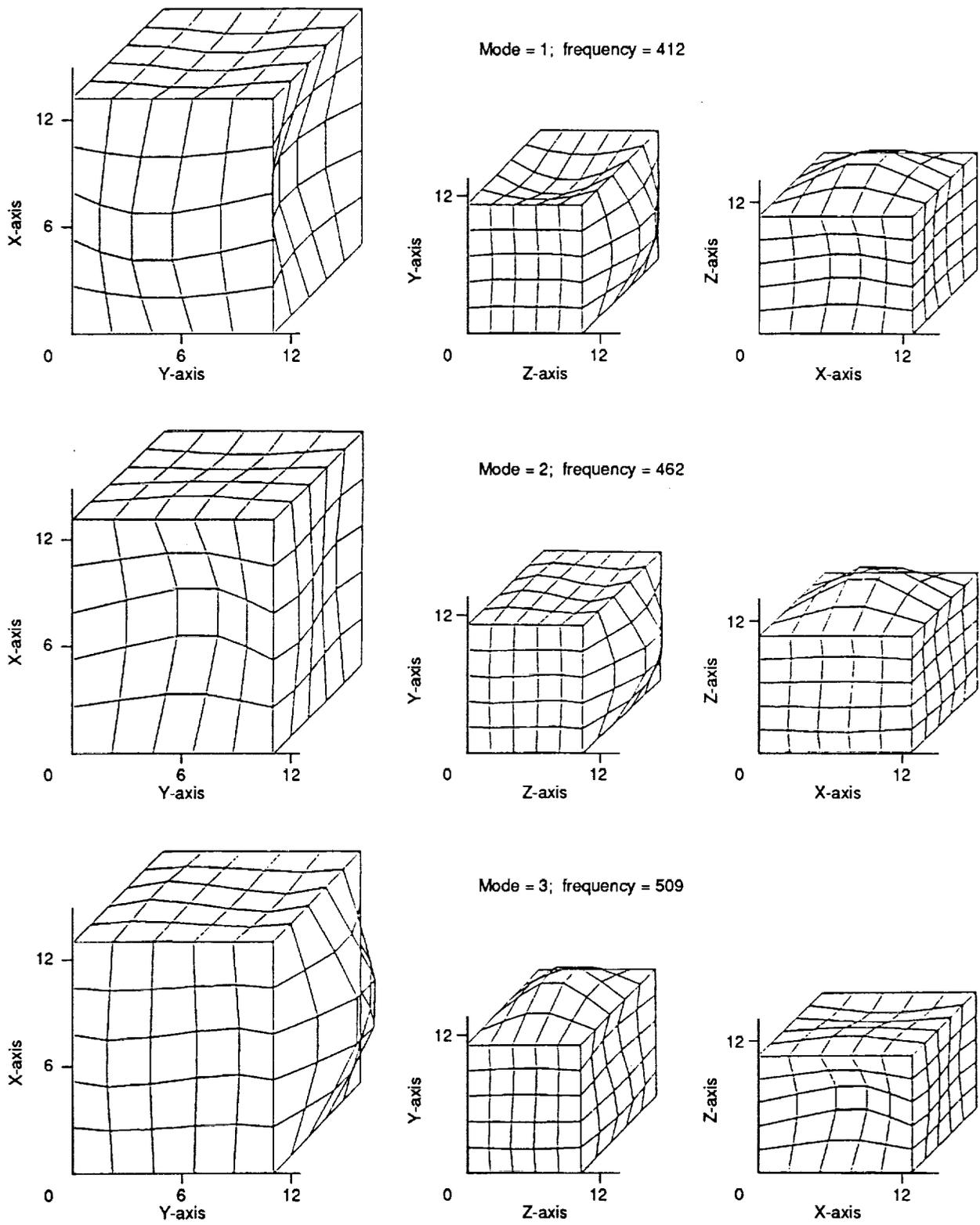
Figure 4.—Gearbox 3-D mode shapes for modes 1, 2, and 3.
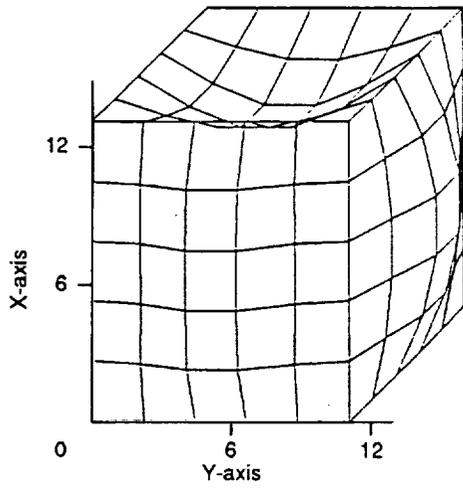
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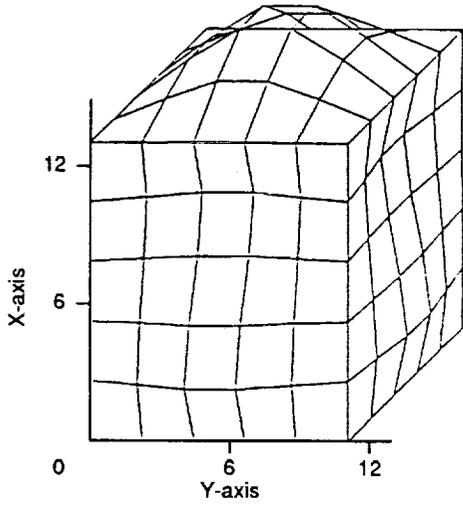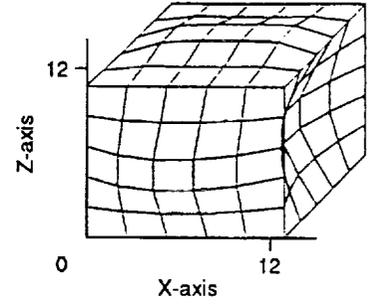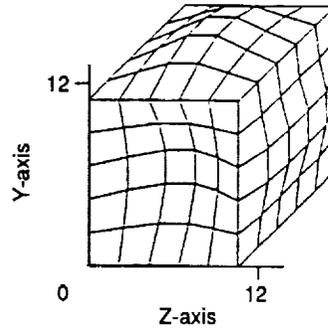
Mode = 5; frequency = 575
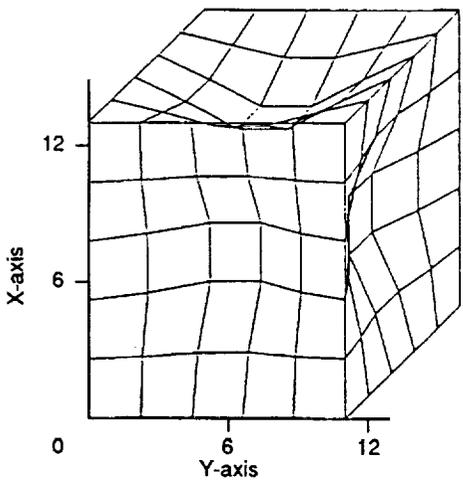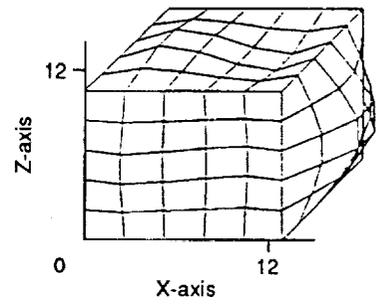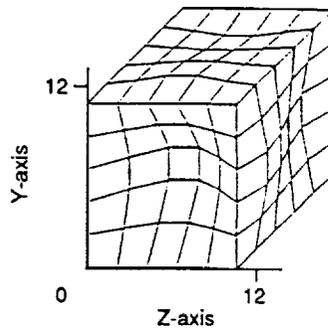
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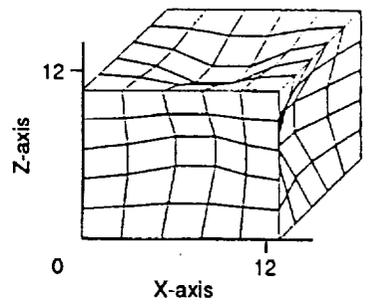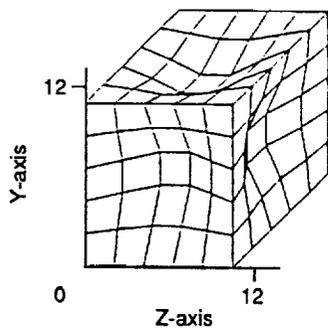
Figure 5.—Gearbox 3-D mode shapes for modes 4, 5, and 6.
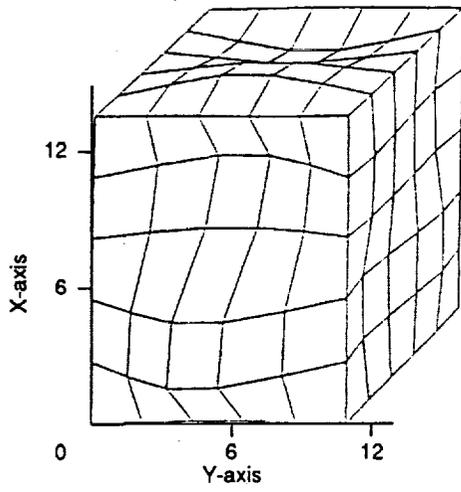
Mode = 7; frequency = 852
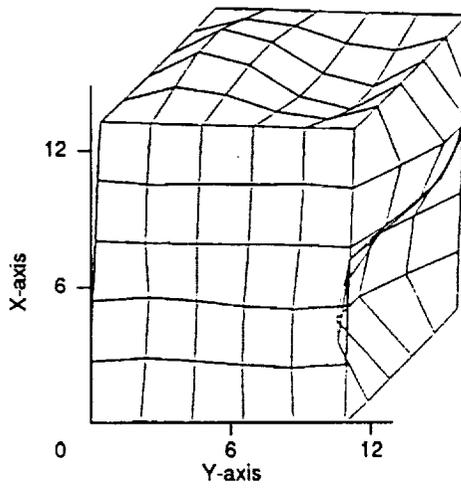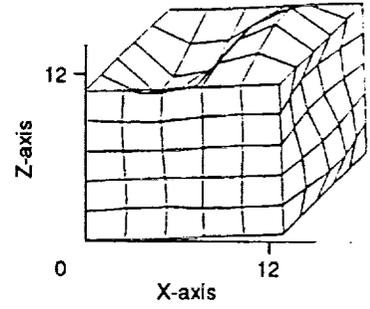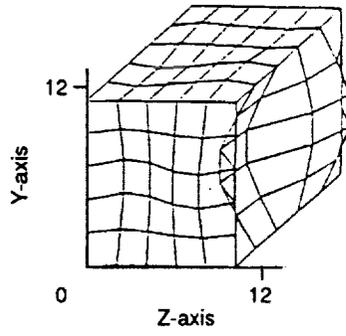
Mode = 8; frequency = 893
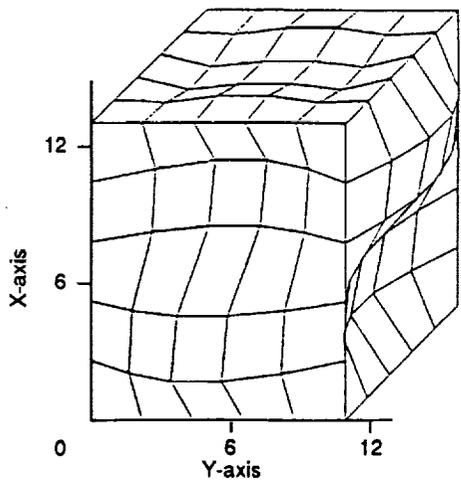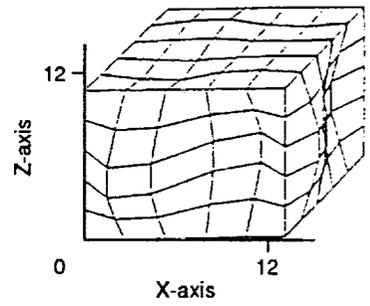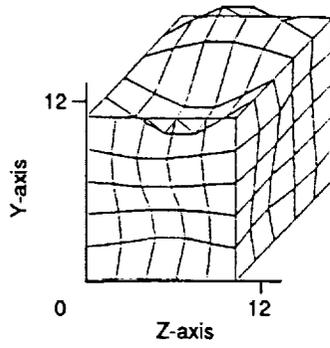
Mode = 9; frequency = 911
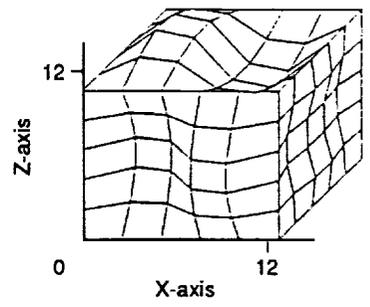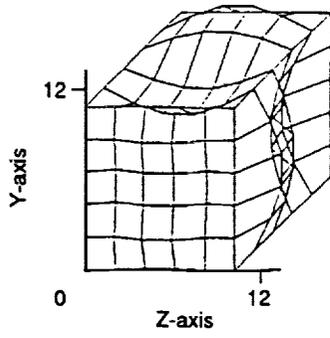
Figure 6.—Gearbox 3-D mode shapes for modes 7, 8, and 9.
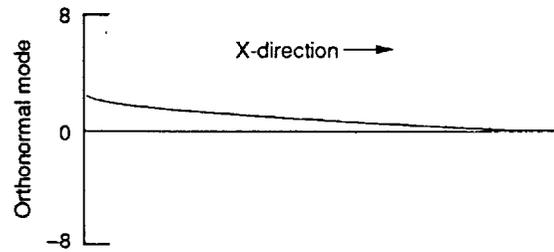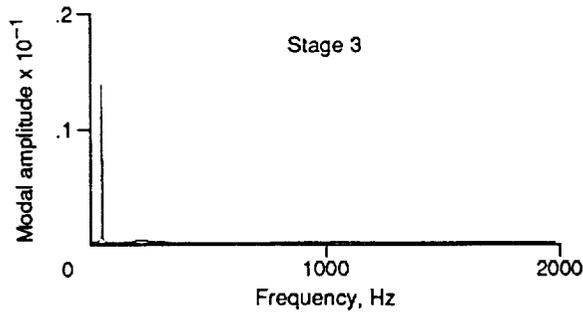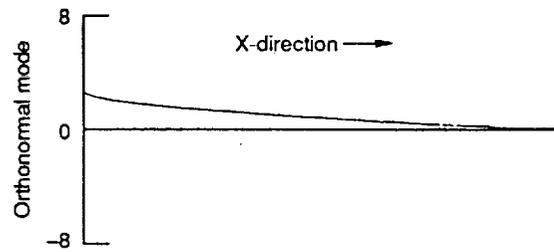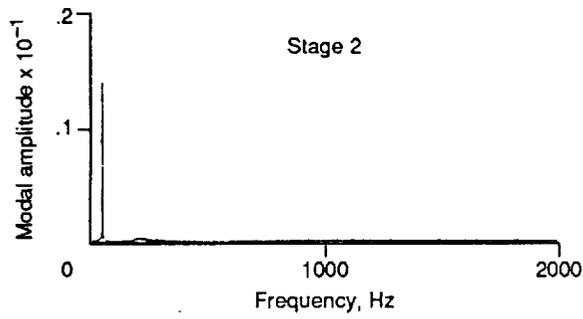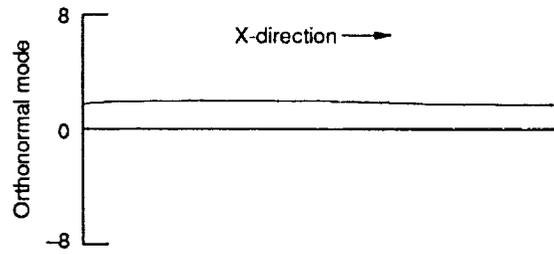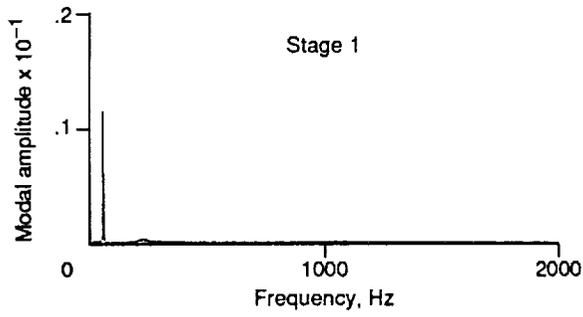
(a) X-mode = 1.

(b) Modal shape of rotor.

Figure 7.—Rotor first lateral modal excitation and mode shapes for stages 1, 2, and 3.
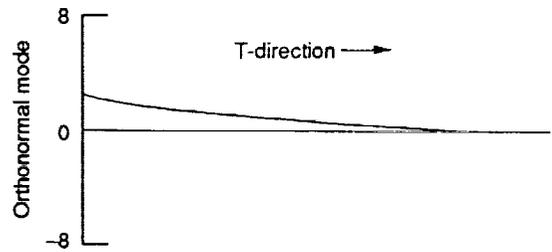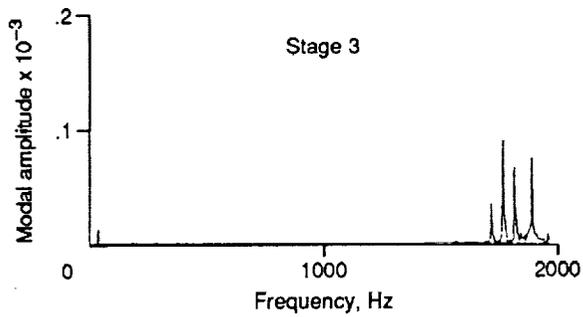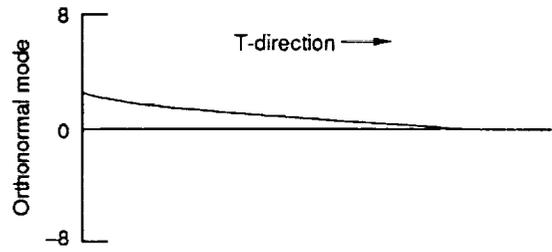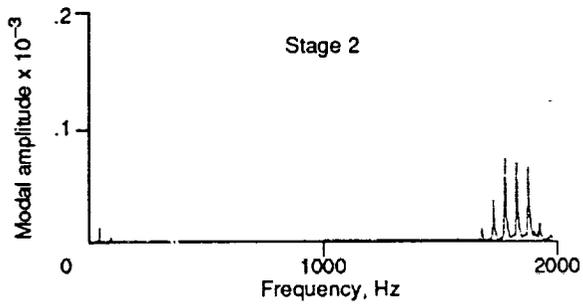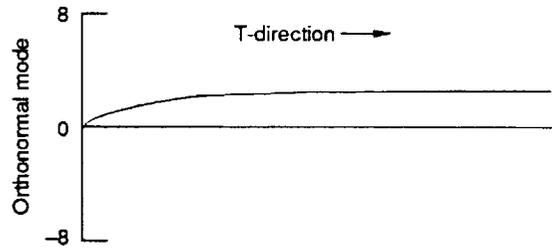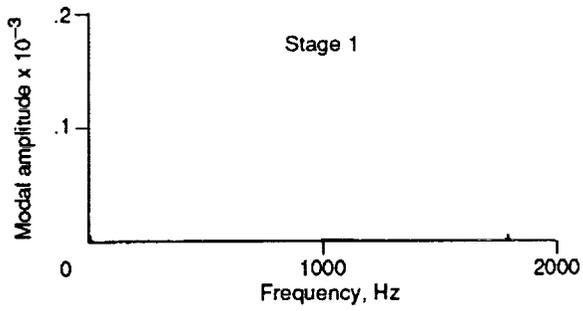
(a) Torsional mode = 1.   (b) Modal shape of rotor.

Figure 8.—Rotor first torsional modal excitation and mode shapes for stages 1, 2, and 3.
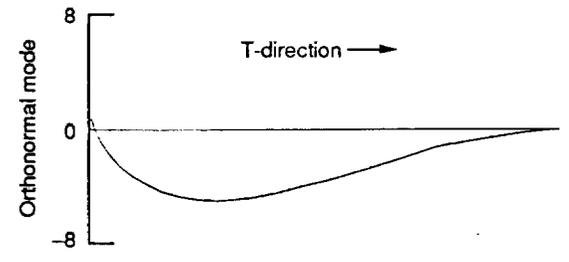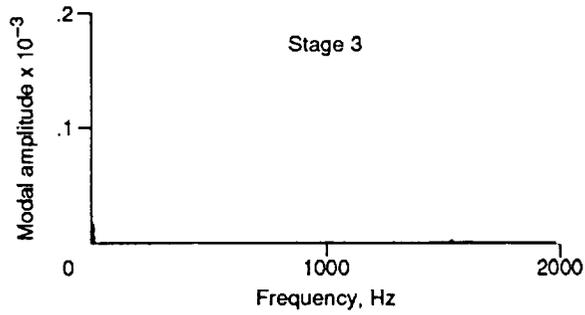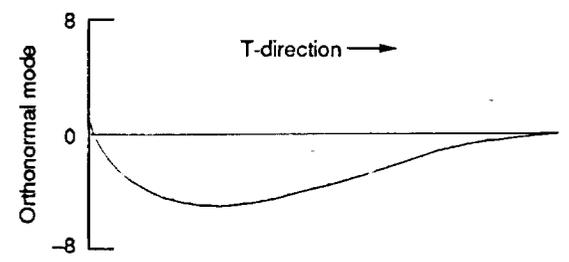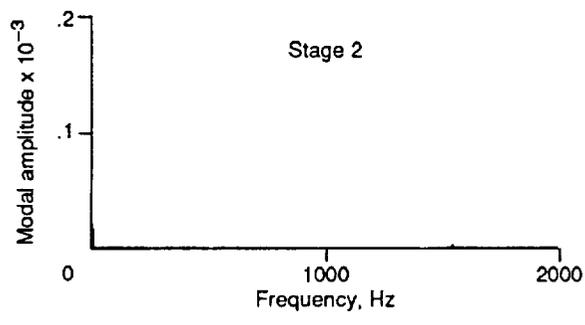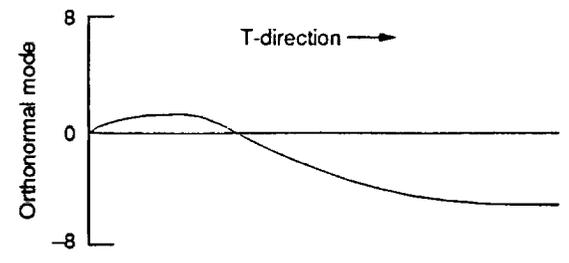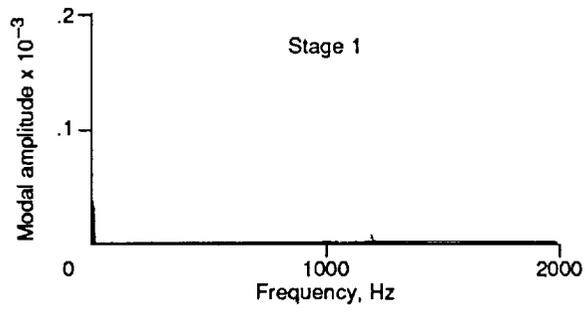
(a) Torsional mode = 2.

(b) Modal shape of rotor.

Figure 9.—Rotor second torsional modal excitation and mode shapes for stages 1, 2, and 3.
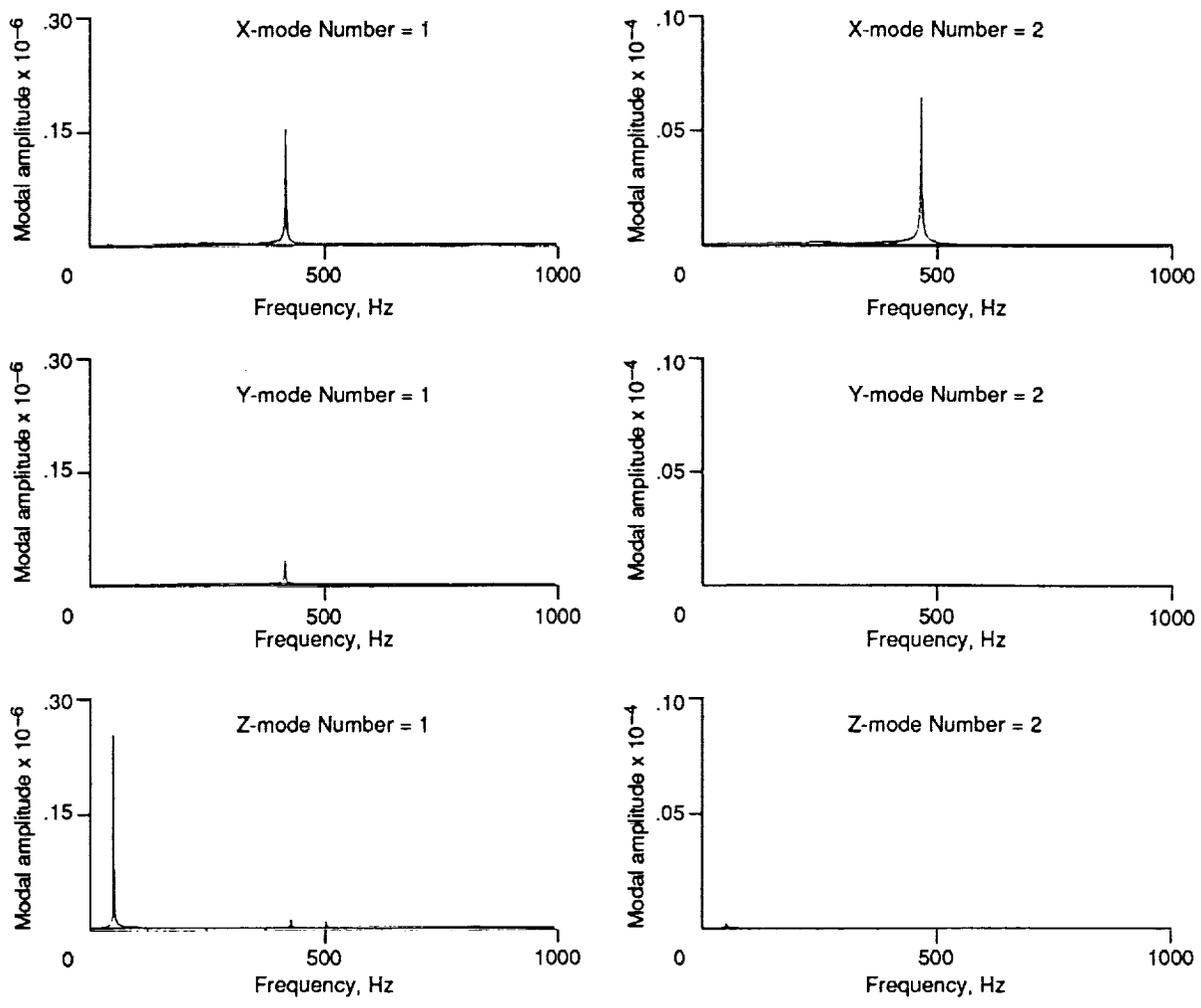
Figure 10.—Gearbox modal excitation for modes 1 and 2.

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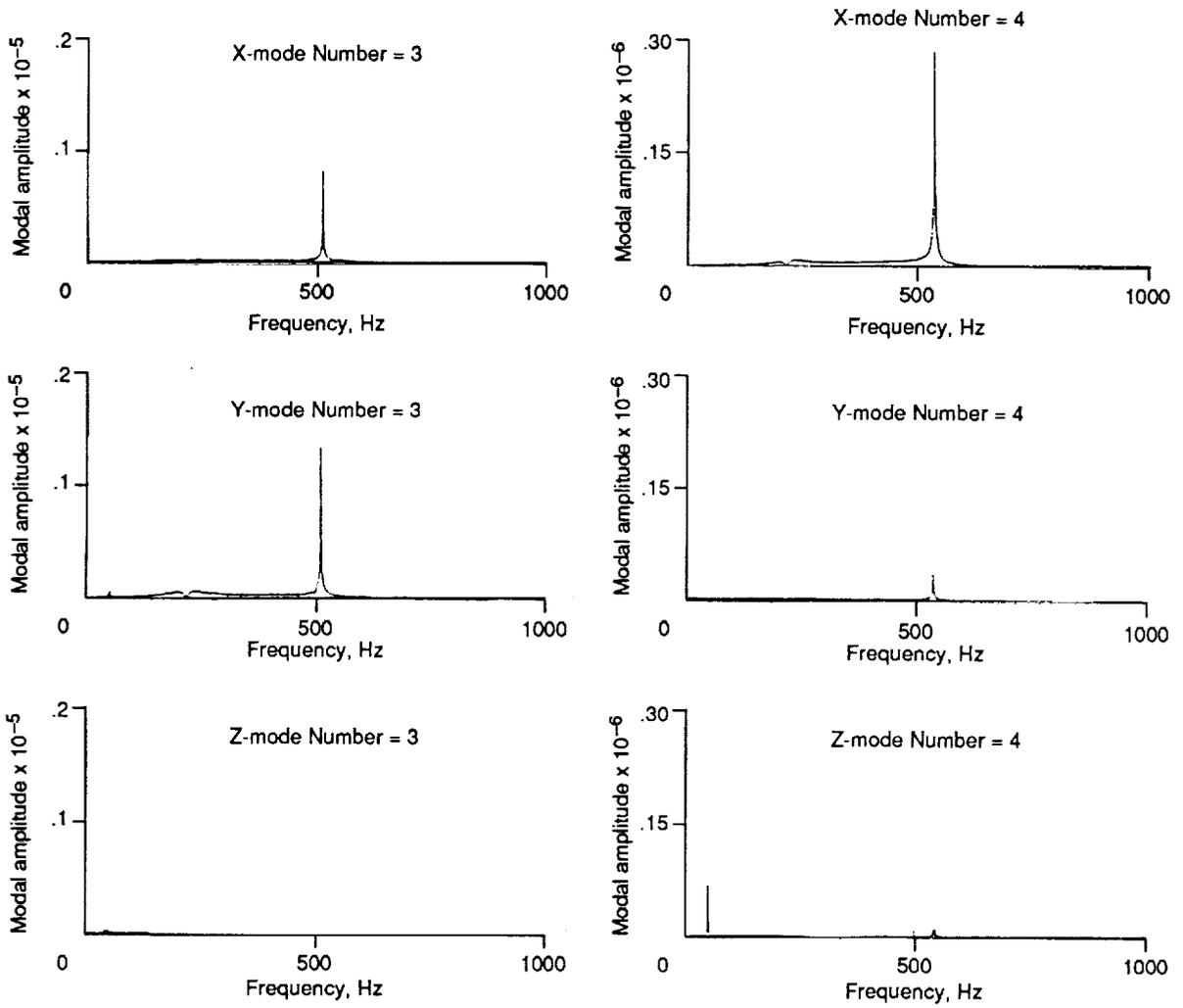
Figure 11.—Gearbox modal excitation for modes 3 and 4.
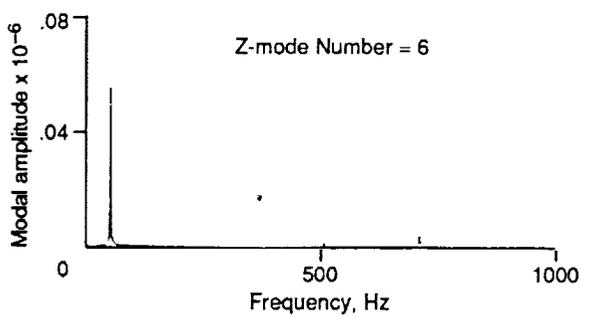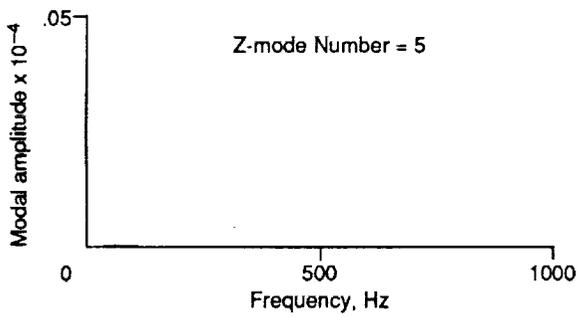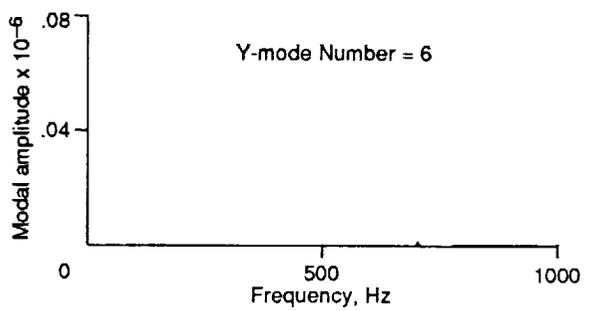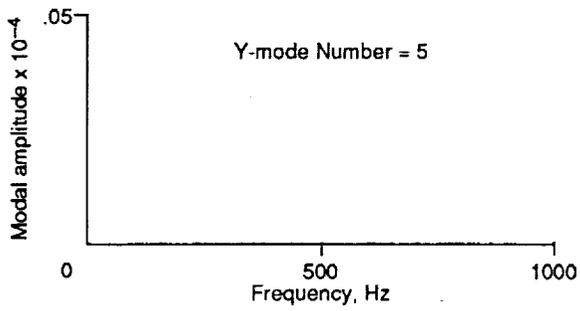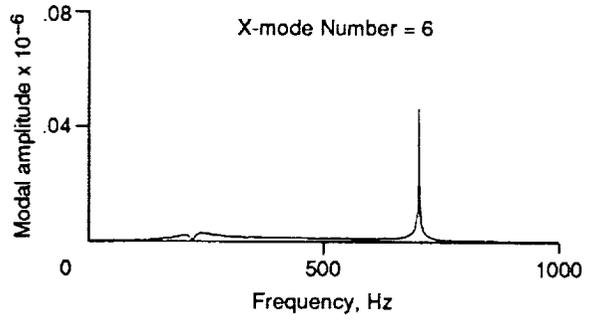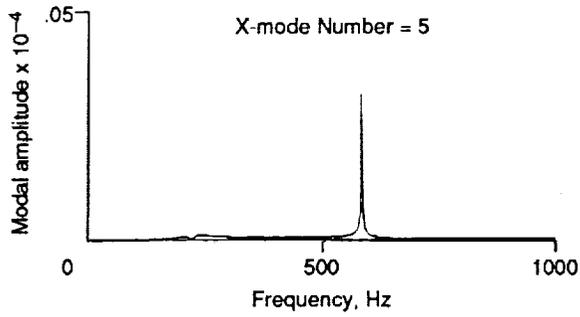
Figure 12.—Gearbox modal excitation for modes 5 and 6.
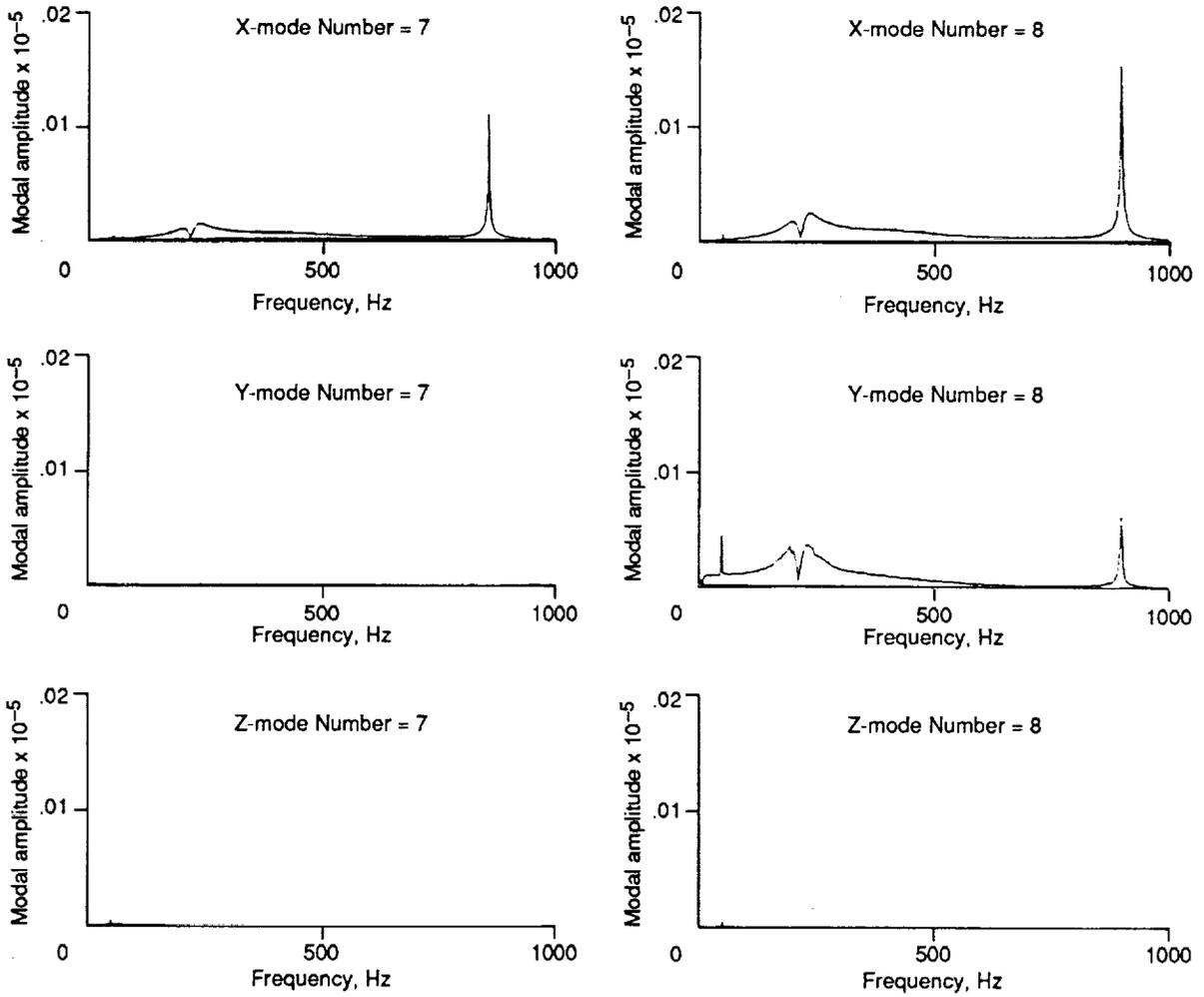
Figure 13.—Gearbox modal excitation for modes 7 and 8.
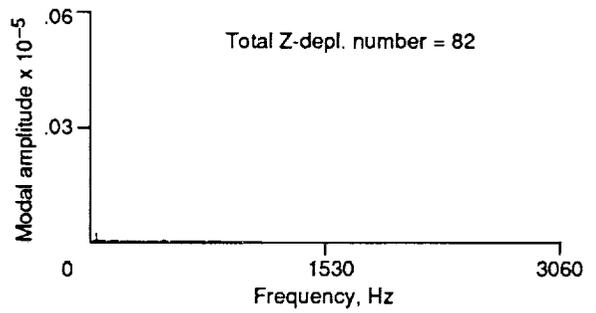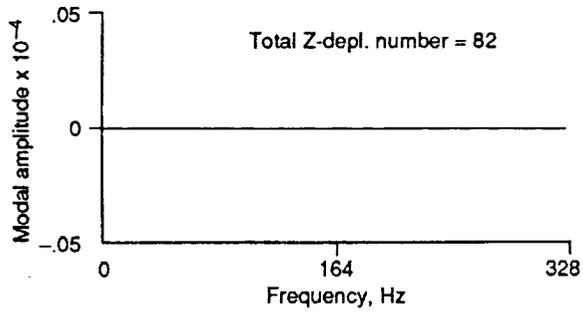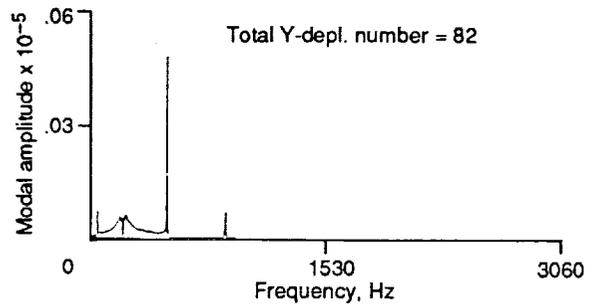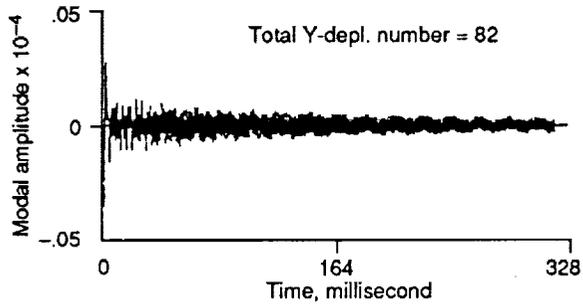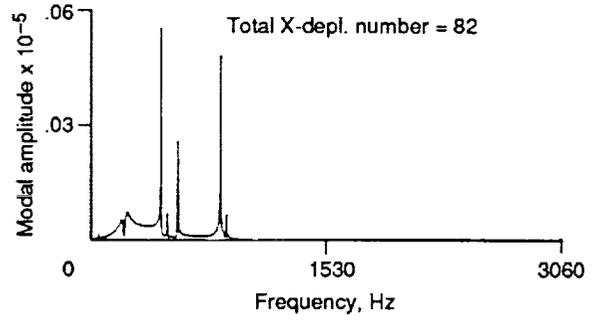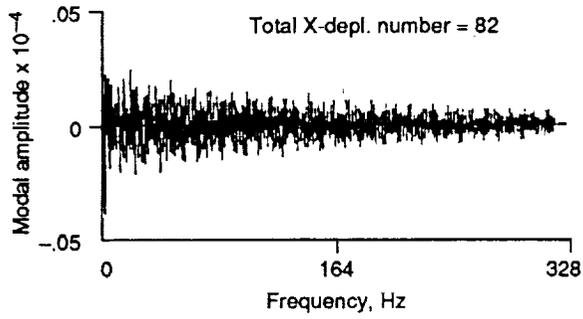
Figure 14.—Total vibration at upper corner of gearbox (node 81).
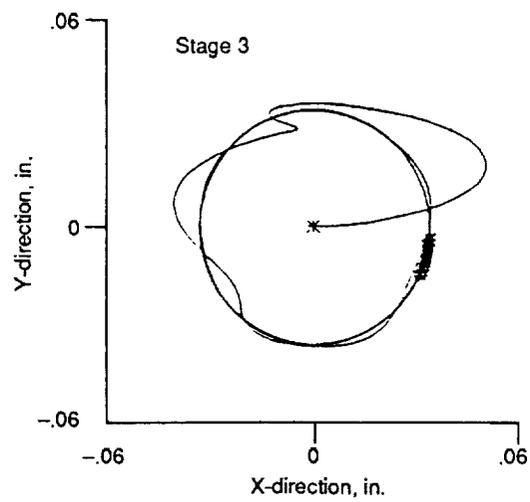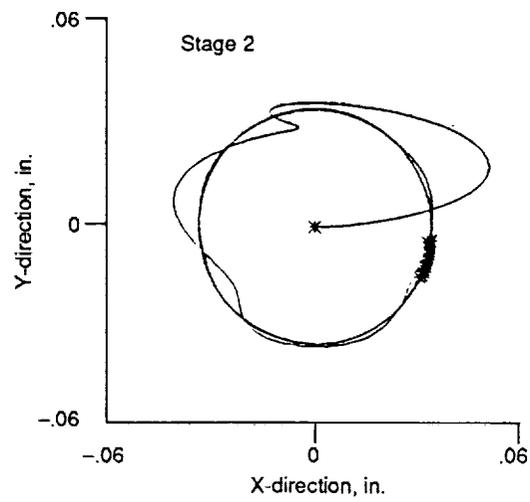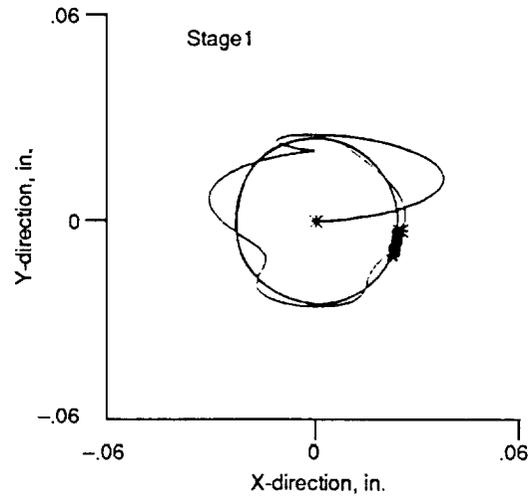
Figure 15.—Vibrational orbit at gear mesh for
rotors 1, 2, and 3.
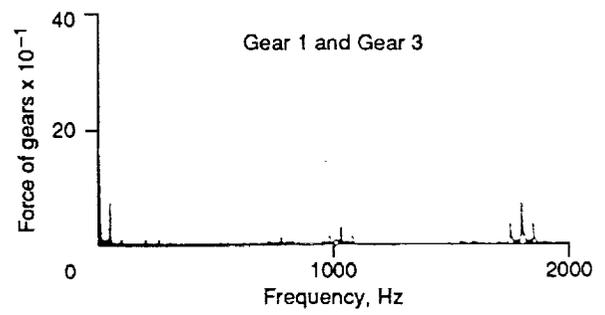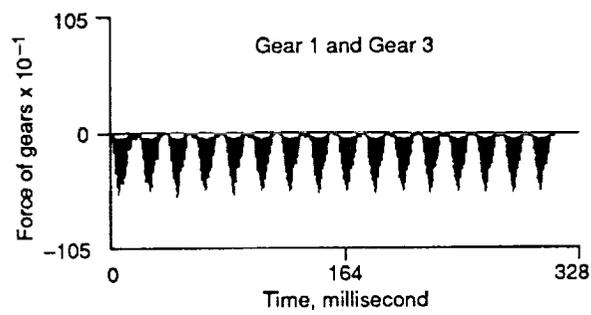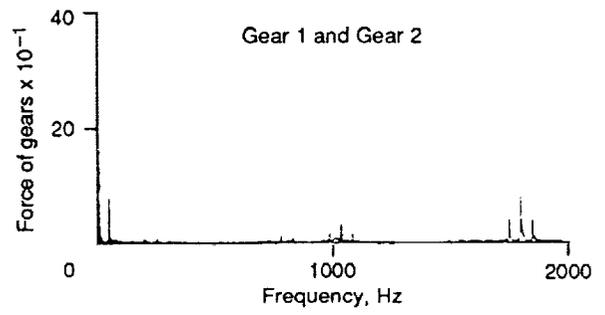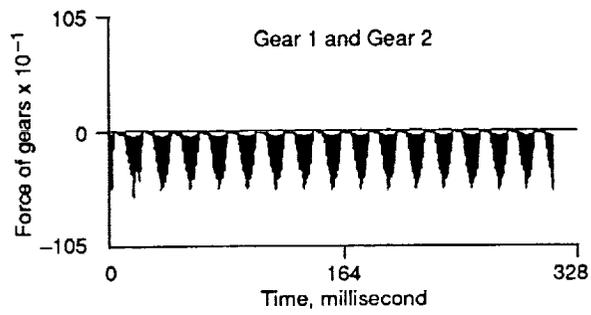
Figure 16.—Time and frequency domain gear forces at gear meshes 1-2 and 1-3.

1. Report No. NASA TM-103797 AVSCOM TR 90 - C -033	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Modal Analysis of Multistage Gear Systems Coupled With Gearbox Vibrations		5. Report Date
		6. Performing Organization Code
7. Author(s) F.K. Choy, Y.F. Ruan, Y.K. Tu, J.J. Zakrajsek, and D.P. Townsend		8. Performing Organization Report No. E - 6085
		10. Work Unit No. 505 - 63 - 36 1L162211A47A
9. Performing Organization Name and Address NASA Lewis Research Center Cleveland, Ohio 44135 - 3191 and Propulsion Directorate U.S. Army Aviation Systems Command Cleveland, Ohio 44135 - 3191		
		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546 - 0001 and U.S. Army Aviation Systems Command St. Louis, Mo. 63120 - 1798		
		14. Sponsoring Agency Code

16. Abstract

This paper presents an analytical procedure to simulate vibrations in gear transmission systems. This procedure couples the dynamics of the rotor-bearing gear system with the vibration in the gear box structure. the modal synthesis method is used in solving the overall dynamics of the system, and a variable time-stepping integration scheme is used in evaluating the global transient vibration of the system. Locally each gear stage is modelled as a multimass rotor-bearing system using a discrete model. The modal characteristics are calculated using the matrix-transfer technique. The gearbox structure is represented by a finite element model, and modal parameters are solved by suing NASTRAN. The rotor-gear stages are coupled through nonlinear compliance in the gear mesh while the gearbox structure is coupled through the bearing supports of the rotor system. Transient and steady state vibrations of the coupled system are examined in both time and frequency domains. A typical three-geared system is used as an example for demonstration of the developed procedure.

17. Key Words (Suggested by Author(s)) Modal resopnse Vibration Gears	18. Distribution Statement Unclassified - Unlimited Subject Category 37		
19. Security Classif. (of the report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 34	22. Price* A03