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Abstract

Optimization procedures allow one to design a spur gear reduction for maximum life and other end use criteria. A modified feasible directions search algorithm permits a wide variety of inequality constraints and exact design requirements to be met with low sensitivity to initial guess values. The optimization algorithm is described and the models for gear life and performance are presented. The algorithm is compact and has been programmed for execution on a desk top computer. Two examples are presented to illustrate the method and its application.

Nomenclature

A	velocity factor
B	material surface constant, psi
b	design parameter scaling constant
c	design problem constant
C	dynamic capacity, lb
d	design parameter scaling coefficient
E	elastic modulus, psi
F	force, lb
f	face width, in.
∇f	feasible direction gradient vector
∇h	total violated constraint gradient vector
J	AGMA bending stress geometry factor
ℓ	life (10^6 cycles or hours)
M	merit function
∇m	merit function gradient vector
N	number of teeth
R	gear radius, in.
R	reliability
RMS	surface finish
S_{ac}	allowable surface strength, psi
ΔS	optimizing step size
T	temperature, °F
V	constraint
V	velocity, in./sec
$\underline{V}$	stress volume, $in.^3$
∇v	constraint gradient vector
W	weight, lb
X	unscaled design vector
X	AGMA factor
Y	scaled design vector
z_o	depth to critical stress, in.
α	angle of approach, deg
θ	characteristic life (10^6 cycles or hours)
μ	friction coefficient
ν	Poisson's ratio
ρ	surface radius of curvature, in.
σ	contact stress, psi
τ_o	shear stress, psi
ϕ	pressure angle, deg
ω	angular velocity, rad/sec

Exponents

b	Weibull slope
c	proportionality factor
h	proportionality factor
p	load-life exponent

Subscripts

10	90 percent probability of survival (reliability)
ag	gear addendum
B	base
b	bending
c	center distance
d	total dynamic
f	flash
g	gear
H	Hertzian
i	independent parameter index
j	search step index
k	constraint index
Li	parameter lower estimate
pl	pinion radius to gear addendum on line of action
s	sliding
t	tooth
Ui	parameter upper estimate
w	weight

Introduction

The optimal design of a spur gear mesh is a problem of considerable interest in mechanical design.[1-4] When fracture of the gear teeth due to bending is the primary mode of failure, the minimum number of teeth which avoids interference offers the strongest gear set for a given size.[5] However, as speeds increase, so do the prospects for pitting and scoring modes of failure. Pitting at and below the pitch point on the pinion tooth is a failure mode which limits the life of the gear teeth.[6] Procedures have been presented to design a gear set for minimum size and minimum weight, considering pitting fatigue as well as bending and scoring.[2,3] These procedures determine an optimal gear set from a weight or size standpoint alone.

One promise of optimization is adaptability to the application at hand. For example, the shaft center distance may be fixed by other considerations in the design of a machine. Given this situation, one would like to apply optimization theory to determine a design which maximizes the gear set life for a given center distance.

The computer is useful here, and the accessibility of the personal computer makes this use attractive. A modified feasible directions search algorithm is applied in a continuous design space in a form which is memory and time efficient. Using a fixed search step, which is halved only when the search passes a solution, simplifies the computations. The method is an improvement of the basic gradient search method.[7,8] It checks the gradients in the inequality constraints as well as the merit function to calculate a feasible search direction which improves the merit function while staying within the acceptable design region. After finding the continuous optimum design, the program allows the designer to check alternate designs which may be more practical. These designs can have parameter values which obey the discrete parameter limitations and are close to the continuous optimal design's parameter values.

This method is applied to the problem of designing for maximum life at a given center distance as an example. Also, it is applied to a similar problem of minimum center distance with a given life requirement for comparison purposes.

Optimization Method

As with most optimization techniques, the procedure begins with several vectors. These vectors are the independent design variables, X; the inequality constraints, V; the parameters of the merit function, P; and the constants which define the specific problem, C. An optimization solution is the design variable values, X, which minimize or maximize the merit function value while maintaining all constraint values, V, inside their specified limits. A procedure starts with a guess for the design variable, X, and iterates with some logical procedure to find the optimal design variable.

Scaling

To maintain balance among the independent design parameters, the design space is scaled into a continuous, dimensionless design space.[9] The scaled design parameters, Y, vary from -1.0 to 1.0 as specified by upper and lower bounds on the independent design parameters, X, such that:

$$-1.0 < Y_i < 1.0 \tag{1}$$

as

$$X_{Li} < X_i < X_{Ui} \tag{2}$$

Thus:

$$Y_i = d_i X_i + b_i \tag{3}$$

where

$$d_i = \frac{2}{X_{Ui} - X_{Li}} \tag{4}$$

and

$$b_i = -\frac{X_{Ui} + X_{Li}}{X_{Ui} - X_{Li}} \tag{5}$$

The actual design variable, X_i, can always be retrieved from the scaled variable, Y_i, by:

$$X_i = \frac{Y_i - b_i}{d_i} \tag{6}$$

Gradient

Central to the method is the gradient calculation. This is performed with small perturbations in the design variables from the nominal position. The gradient in the merit function, ∇M, is calculated as:

$$\nabla M = \begin{bmatrix} \frac{\partial M}{\partial Y_1} \\ \frac{\partial M}{\partial Y_2} \\ \frac{\partial M}{\partial Y_3} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{7}$$

where,

$$\frac{\partial M}{\partial Y_i} = \frac{M(Y_1,\ldots,Y_i + \Delta Y,\ldots,Y_n) - M(Y_1,\ldots,Y_i,\ldots,Y_n)}{\Delta Y} \tag{8}$$

In Eq. (8), M is the merit function, Y_i is a scaled design variable, and ΔY is the small change which is made in each Y_i. In the program, ΔY is set at 0.001 which is 0.05 percent of the full range of a scaled design parameter.

The magnitude of the gradient vector is given by:

$$|\nabla M| = \left[\sum_{i=1}^{n} \left(\frac{\partial M}{\partial Y_i} \right)^2 \right]^{1/2} \quad (9)$$

For minimization, the direction of change in Y which reduces the merit function, M, at the greatest rate is determined by the unit vector, ∇m:

$$\nabla M = -\frac{\nabla M}{|\nabla M|} \quad (10)$$

For maximization, the sign in Eq. (10) reverses.

In the simple gradient method, Eq. (10) defines the direction for the step change in the scaled design vector.

$$Y_{j+1} = Y_j + \Delta s\ \nabla m \quad (11)$$

where Δs is the scalar magnitude of the step. If no constraints are violated, this will be the next value for Y in the search.

Step Size

Step size, Δs, is a significant element of any optimization procedure.[8] For stability and directness, a fixed step size is used. Initially, the step size is 0.1, which is 5 percent of the range of a single design parameter. But the procedure halves the step whenever a local minimum is reached or the search is trapped in a constraint corner. To complete the simple procedure, the search declares a solution when the percent change in the merit function, M, is less than a pre-set limit of 0.0001.

$$\left| \frac{M_{j+1} - M_j}{M_j} \right| < 0.0001 \quad (12)$$

Initial Value

The optimization procedure described above is scaled, fixed step, and steepest decent. When the initial guess is in the acceptable design space, and the optimum is a relative minimum, this method works quite well. However, placing the initial guess in the acceptable design region is often a problem, and in many cases, the best design is determined by a "trade-off" among conflicting design constraints at the edge of the design space.

These problems are addressed with a second gradient. Just as one can calculate the gradient in the merit function, one can calculate the gradient in a constraint variable:

$$\nabla v_k = -\frac{\nabla V_k}{|\nabla V_k|} \quad (13)$$

where ∇v_k is a unit vector in the direction of decreasing value in the constraint, V_k. For upper bound constraints, moving through the design space in the direction of ∇v_k will reduce the constraint value V_k. For lower bound constraints, a sign reversal in Eq. (13) will produce an increase in the constraint value, V_k, for motion in the gradient direction. The vector sum of the gradients in the violated constraints, ∇h, is the second gradient of the algorithm:

$$\nabla h = \frac{\sum_k \nabla v_k}{\left| \sum_k \nabla v_k \right|} \quad (14)$$

The gradient in the violated constraints, ∇h, points towards the acceptable design space from the unacceptable design space. By itself, it enables the algorithm to turn an unacceptable initial guess into an acceptable initial guess by a succession of steps:

$$Y_{j+1} = Y_j + \Delta s\ \nabla h \quad (15)$$

Feasible Direction

Once inside the acceptable design region, the algorithm proceeds along the steepest descent direction until the calculated step places the next trial outside the acceptable design space. To avoid this condition, the algorithm selects a feasible direction for the next step. Figure 1 shows a sloped constraint intersecting contour lines of improving merit function values. The figure shows gradients in the merit function, ∇m, and the impending constraint, ∇h. The two gradient vectors are placed at the last viable design step, although the constraint gradient is calculated at the trial location - one step in the merit gradient direction from the indicated point. The feasible direction selected, ∇f, is the unit vector sum of these two gradients in the merit function, ∇m, and the violated constraints, ∇h:

$$\nabla f = \frac{\nabla m + \nabla h}{|\nabla m + \nabla h|} \quad (16)$$

And the next step becomes:

$$Y_{j+1} = Y_j + \Delta s\ \nabla f \quad (17)$$

Algorithm Use

By using subroutines to calculate the merit function and constraint values for each design trial, the procedure separates the logic of the algorithm from the analysis necessary to define the problem. This allows the design problem to be changed easily without concern for the optimization procedure. The directness of the procedure adds additional steps, but enables the program to run on a personal computer.

An additional benefit of separating the analysis routines from the optimization logic is the ability to modify the design at execution and verify the characteristics of similar, more practical designs with the same program. The optimization procedure works with a continuous design space, which includes gears with fractions of teeth and nonstandard sizes. By allowing the user to see the ideal continuous variable solution and to modify this to designs with whole numbers of teeth and standard sizes, the procedure enables a designer to determine a practical optimum design easily.

Input Data

The algorithm requires the user to provide an ASCII data file with four groups of data: the constants, design variables, constraints, and merit function weighting coefficients. All constants, variables, limits and coefficients have names and units in the data file to make the program output more understandable. In addition, the data file includes a title for the optimization task.

The constants define the specific problem outside of the analysis subroutines. With their labeling names and units, these constants define the specific problem being optimized in the output data file. The constants also enable similar problems to be solved by merely changing the constant values without recompiling and linking the analysis subroutines to the optimizing routines.

The design variables are the independent parameters for which the algorithm seeks values. Three values are included for each variable: a lower estimate, an upper estimate, and an initial guess. The lower and upper estimates determine the scaling range as the span between them is set to two dimensionless units by the optimizing procedure. However, they do not set hard limits for the search. If hard limits are needed to avoid indeterminate calculations, these may be set in the constraint limits.

The constraint limits include a bound value and a direction: lower or upper. Both a lower and an upper bound may be set on the same property by including two separate limits in the constraint list. Since the program reports the constraint values for the optimal and check designs, one can add inactive constraints which are always satisfid to obtain a printout of additional properties in the constraint list. This also gives one the opportunity to switch the controlling constraints to obtain different designs for different physical conditions without changing the program.

The last group of data are the parameters which may be included in the merit function and their weighting coefficients. These weighting coefficients may convert different properties to a common measure such as cost. They may be order of magnitude corrections to keep one parameter from dominating the design. Or they may all be zero except one, to select that parameter as the quantity to be optimized, so the same program can be used to obtain different designs for different requirements by switching the merit function parameter with the non-zero coefficient. This switch changes the objective of optimization, while the switch of active constraints changes the environment of optimization.

Runtime Options

In running the program, the user is requested to enter the data file name prefix. The program will look for this name on an input data file with the suffix ".IN" and will write the output to a file with this prefix and the suffix ".OUT" as well as to the screen as it runs. Since the program calls the analysis subroutines many times, it writes information to the screen and the output file at each step to document the search path and assure the user that it is running properly.

Once an optimum is found, the program writes the design variable values that produced the optimum, the merit function value and its component parameter array, and the full constraint variable array to the screen and the output file. It pauses for user inspection, and on resumption gives the user a chance to enter a different set of design variable values. This allows the user to include practical, near optimal design trials in the computer analysis record. Since the best design should be of this type, this provides full documentation to the additional trials. When the user declines to enter more trials, the program closes the output file and ends execution.

Analysis Subroutines

Two analysis subroutines apply the optimization procedure to the spur gear design problem. The routines are subroutine BOUNDS and subroutine VALUES. Subroutine BOUNDS analyzes a gear design for its constraint variable values. Subroutine VALUES analyzes a gear design for its merit function parameter values. These subroutines and an input data file treat the two problems of designing for maximum life at a given center distance and of designing for minimum center distance at a given life.

Bounds

Included in the 16 constraint variables evaluated by subroutine BOUNDS are: some distances, the pinion weight and loading, and some stress, life and scoring limits. The involute interference variable is the distance along the line of action from the base circle of the pinion to the addendum circle of the gear. This distance must be positive for the gear tooth tip to contact the pinion tooth on its involute surface and avoid interference.

The pinion weight is the product of the density of steel times the volume of a disk with a diameter equal to the pinion pitch diameter and a thickness equal to the gear and pinion face width. The transmitted load is the pinion torque divided by the base radius of the pinion, which is the nominal force acting between the gears along the line of action. The pitch line velocity is the rotational speed of the pinion times its pitch radius.

In this analysis, the dynamic load estimate is the AGMA velocity factor model. In terms of a gear quality number, Q_v, the AGMA estimate of the sum of the transmitted load and the dynamic load is:

$$F_d = F_t\left(\frac{A + \sqrt{V}}{A}\right) \qquad (18)$$

where

$$A = 50 + 56\left(1 - \frac{[12 - Q_v]^{2/3}}{4}\right) \qquad (19)$$

In Eq. (18), F_d is the total dynamic load, F_t is the nominal transmitted load and V is the pitch line velocity of the gears. In Eq. (19), the gear quality number, Q_v, may have a value between 6 and 11 with 11 corresponding to the higher quality gear. All gear stresses and lives are calculated using this total dynamic load, with a quality number, $Q_v = 9$.

Gear tooth bending fatigue and gear tip scoring are modes of failure considered in the program. The bending fatigue model uses the AGMA J factor to estimate the bending stress with the load at the highest point of single tooth loading on the pinion. The formula for the bending stress is:

$$\sigma_b = \frac{F_d \cdot P_d}{f \cdot J} \tag{20}$$

where J is the AGMA J factor[14] and f is the effective gear tooth face width.

The maximum contact stress and gear tip Hertzian pressure are calculated as:

$$\sigma_H = \left[\frac{F_d}{\pi f \cos\phi}\left(\frac{1/\rho_p + 1/\rho_g}{\dfrac{1-\nu_p^2}{E_p} + \dfrac{1/\nu_g^2}{E_g}}\right)\right]^{1/2} \tag{21}$$

where the ρ's are the radii of curvature of the two gear tooth surfaces at the point of contact, the ν's are the Poisson ratios for the two gear's materials and the E's are the elastic moduli for the two gear's materials. The maximum contact stress occurs at the lowest point of full load contact on the pinion tooth. The gear tip Hertzian pressure uses one half of the total dynamic load since the load is shared between two tooth pairs at this point.

The gear tip scoring model includes the pressure times velocity factor and the critical oil scoring temperature model from lubrication theory. The normal pressure times sliding velocity is proportional to the frictional power loss of the gear set. This factor is the highest for contact at the gear tip, so the normal pressure is the gear tip Hertzian pressure. the sliding velocity at the gear tip is given by:

$$V_s = \omega_g R_{ag} \sin(\phi + \alpha_{ag}) - \omega_p R_{p1} \sin(\phi + \alpha_{p1}) \tag{22}$$

where the ω's are the angular velocities of the gears, the R's are the radii to the contact point on the two gears, ϕ is the nominal pressure angle of the gear set and the α's are the angles of approach on the two gears.

The lubricating oil flash temperature is another factor used to monitor gear tooth scoring. One estimate of this temperature[14] is given by:

$$T_f = T_B + \left(\frac{X_\Gamma F_d}{f}\right)^{3/4}\left(\frac{0.45\,\mu_m X_M X_G \sqrt{V}}{(R_p + R_g)^{1/4}}\right) \tag{23}$$

where T_B is the base temperature of the oil, μ_m is the average surface friction coefficient, X_Γ is the load sharing factor, X_M is a thermal - elastic factor and X_G is a geometry factor.

The pinion life and mesh life calculations are described in the gear life model section which follows.

Values

The second analysis subroutine, VALUES, calculates the three components of the merit function: the mesh life in thousand hours, the center distance in inches, and the pinion weight in pounds. The total merit function is:

$$M = C_\ell \ell + C_c(R_p + R_g) + C_w W \tag{24}$$

where ℓ is the mesh life, the radii sum is the center distance and W is the pinion weight. The three weighting function coefficients: C_ℓ, C_c and C_w are constants defined in the input data file. By assigning different values to these coefficients, the user can change the problem being solved. With C_ℓ not equal to zero and the other two coefficients equal to zero, the optimization will seek out a solution which maximizes the gear mesh life. With C_ℓ and C_w equal to zero and C_c negative, the optimization will seek out a solution which minimizes the center distance.

Gear Life Model

The gear life model, based on surface pitting, comes from rolling element bearings.[6] Surface pitting of gear teeth follow a similar pattern to bearing race pitting, with the possible difference of surface initiation. Lundberg and Palmgren proposed the model in the late 1930's.[10] They assumed that the log of the reciprocal of the reliability, R, of a bearing is proportional to its life, ℓ, and some stress parameters. These parameters are: the stress level, τ_o; the depth to the maximum shear stress, z_o; and the stress volume, $\underline{V}$. The relationship is:

$$\mathrm{Ln}\left(\frac{1}{R}\right) \sim \tau_o^c z_o^{-h}\, \underline{V} \ell^b \tag{25}$$

In relation (25) b is the Weibull slope and c and h are exponents of proportionality to be found experimentally. This is the equation for the two-parameter Weibull distribution with the addition of stress and size factors. In general terms, the two-parameter Weibull distribution is:

$$\mathrm{LN}\left(\frac{1}{R}\right) = \left(\frac{\ell}{\theta}\right)^b \tag{26}$$

where b is the Weibull slope or shape factor and θ is the characteristic life of the distribution. To replace the characteristic life with a 90 percent probability of survival life, ℓ_{10}, solve Eq. (26) for the characteristic life, θ.

$$\theta = \ell\left[\frac{1}{\mathrm{Ln}\left(\frac{1}{R}\right)}\right]^{1/b} \tag{27}$$

At $R = 0.9$, the life is ℓ_{10}. Substituting this into Eq. (26) gives the two parameter Weibull distribution in terms of the ℓ_{10} life as:

$$\mathrm{Ln}\left(\frac{1}{R}\right) = \mathrm{Ln}\left(\frac{1}{0.9}\right) \cdot \left(\frac{\ell}{\ell_{10}}\right)^{b} \qquad (28)$$

The life to reliability relationship of Eq. (28) is for a specific load which determines the ℓ_{10} life. This load, F, is related to the component dynamic capacity, C, as:

$$\ell_{10} = \left(\frac{C}{F}\right)^{p} \qquad (29)$$

Here, the dynamic capacity of the component, C, is the load which has a 90 percent reliability life of one million cycles, the load on the component is F, and the power, p, is the load-life exponent. Since the life at the dynamic capacity is one million load cycles or unity, it does not appear as a variable in the equation.

Because gear tooth life behaves similar to bearing life, engineers at the NASA Lewis Research Center formulated a model for gear tooth life similar to the bearing life model.[6,11] Starting with Eq. (25) Coy, Townsend and Zaretsky developed a model for the reliability and life of a spur gear. The model uses both the two-parameter Weibull distribution of Eq. (28) and the Palmgren load-life relation of Eq. (29). With statistically replicated data, they showed that these models predict gear tooth pitting.

From Eq. (25), they determined a relationship for the dynamic capacity, C_t, of a spur gear tooth. Rounding the exponents to whole numbers gives the dynamic capacity as a function of Buckingham's load-stress factor, B.[12,13]

$$C_t = B\left(\frac{f}{\Sigma 1/\rho}\right) \qquad (30)$$

In Eq. (30), B is a material strength which has the dimensional units of stress, the effective face width of the tooth is f, and the curvature sum at the failure point for the contacting teeth is $\Sigma 1/\rho$. The curvature sum is:

$$\Sigma 1/\rho = \frac{1}{\rho_g} + \frac{1}{\rho_p} \qquad (31)$$

Here, ρ_g is the radius of curvature of the gear tooth surface at the failure point, and ρ_p is the radius of curvature of the pinion tooth surface at the failure point. With the dynamic capacity expressed in this form, the material strength factor serves the role of the surface fatigue strength, S_{ac}, of the AGMA design code.[14] A relation for the material strength factor in terms of the surface fatigue strength is:

$$B = \pi S_{ac}^{2}\left(\frac{1 - \nu_p^2}{E_p} + \frac{1 - \nu_g^2}{E_g}\right) \qquad (32)$$

The dynamic capacity of the whole gear is lower than that of a single tooth. In a single pass, fixed axis gear set, each rotation of the gear subjects every tooth on the gear to a single load cycle. The gear fails when any single tooth on the gear fails, and the fatigue damage in each tooth accumulates independently of the damage in the other teeth. In successive coin tosses, the probability of a specific combined event is the product of the probabilities of each coin toss. So too, the reliability of the gear, R_g, is the product of the reliabilities of each tooth in the gear.

$$R_g = R_t^{N_g} \qquad (33)$$

In Eq. (33), N_g is the number of teeth on the gear, and R_t is the reliability of a single tooth on the gear. The reliability of any tooth on a gear is equal to the reliability of any other tooth on the gear. To transform Eq. (33) into a life relationship, substitute Eq. (28) for the two reliabilities into the log of Eq. (33).

$$\ell_{10,g} = \left(\frac{1}{N_g^{1/b}}\right)\ell_{10,t} \qquad (34)$$

The gear life, $\ell_{10,g}$, has units of million gear rotations.

The gear dynamic capacity, C_g, is found by substituting Eq. (34) into Eq. (29) for the tooth to obtain the analog of Eq. (29) for the gear. This produces:

$$C_g = \frac{C_t}{N_g^{1/(b \cdot p)}} \qquad (35)$$

Gear Design

Consider the design of a gear set to transmit 80 hp from a shaft turning at 5000 rpm to an output shaft turning at 2500 rpm. The life of the gears as a pair in hours should be maximized at a center distance of 5.0 in.

Table 1 is the program output listing of the problem defining constants and the design variable range and initial value. The gears are to have a 20° nominal pressure angle and be made of high strength, heat treated steel with a tooth surface finish of 32 rms. The material surface constant of 9800 psi corresponds to a surface compression endurance strength of 200 000 psi at 10^7 fatigue cycles and a reliability of 90 percent. The load-life factor of 8.93 is from the ANSI/AGMA 2001 B88 Standard,[14] and the Weibull slope of 2.5 is from the NASA Lewis gear test data.[6,11] The three design variables to be found are the number of teeth on the pinion, the diametral pitch and the face width of the gears. For the gear, the number of teeth will be twice that of the pinion.

The gears should have a face width which gives the pinion a length to pitch diameter ratio between 0.2 and 0.5. Additional strength limits placed on the design are: a tooth bending stress limit of 40 000 psi, a surface contact stress limit of

150 000 psi, a pressure times velocity factor of 100 million psi-ft/min, and an oil flash temperature limit of 275 °F. Involute interference should be avoided as well. These limits are shown in Table 2 which lists the design constraints and merit function weighting factors in the output data file.

The controlling constraints are the request for a center distance no larger than 5.0 in. and the maximum pinion length to diameter ratio. Included in the list of constraints are properties which have limits that any design will satisfy, such as: pinion torque, transmitted load, dynamic load, pinion life, and the oil flash temperature. These constraints enable the program to analyze each design for their values and display them as part of the design summary for each optimum and check design. A velocity factor equation calculates the dynamic load as prescribed by ANSI/AGMA 2001 B88 with a quality number of 9. All calculated stresses and contact pressures use the dynamic load.

Table 3 summarizes the program's optimal solution for the maximum life design including the design variable values, the merit function parameter values and all constraint values and limits. For this ideal design, the mesh life is 28 000 hr. Table 4 gives a similar summary for a selected maximum life design. Figure 2 shows this design which has 46 and 92 teeth on the two gears, a diametral pitch of 14 and a face width of 1.625 in. It has a life of 18 000 hr with all other constraints satisfied. The design has a pinion weight of 3.9 lb, a transmitted load of 650 lb and a total dynamic load of 1840 lb.

To illustrate the flexibility of the design program, the design is changed to request a minimum life of 2000 hr to see how compact the design can become.

Six changes are made to the input file. The center distance limit changes from an upper bound of 5.0 in. to a lower bound of 0.0 in. The mesh life limit changes from a lower bound of 0.0 hr to a lower bound of 2000 hr. The merit function direction changes from MAX to MIN. And, the nonzero weighting factor switches from mesh life to center distance. Labeling changes for the file itself and the problem title make up the last two changes which enable the program to preserve and distinguish the input and output records for the two design requests.

Table 5 summarizes the ideal optimum design for the minimum size design. In this design, the gear center distance is reduced to below 5.5 in. by the request to reduce the mesh life by an order of magnitude. Table 6 summarizes the nearest practical design, which is shown in Fig. 3. In this design, the pinion and the gear have 36 and 72 teeth, and both have a diametral pitch of 12 and a face width of 1.5 in. The center distance is 4.5 in. and the mesh life is 2200 hr.

Although the bending and contact stresses have increased over those for the maximum life design, they are still within the acceptable criteria. The scoring limit values also have increased to acceptably larger values. Had any of these constraints been reached, the bounding constraints would have influenced the ideal and final optimal designs.

It should be noted that all the designs have the maximum possible face width. Designs based on pinion weight rather than center distance also required the maximum possible face width. Minimizing pinion weight for a life of 2000 hr produced the same optimal design as that found by minimizing the gear center distance.

Conclusions

By combining the power of optimization with the access of the desk top computer, a practical spur gear life design program has been written. The program uses a fixed step, modified feasible directions algorithm to search for the optimum design. Basic relations for the optimization algorithm are presented.

Extensive labels and keyboard interactions give the designer a record of the ideal optimal design and any user specified designs entered after the optimum has been found and reported. Small changes in the input data file enable one to redirect the design objective from maximum life to minimum size and to change the controlling design constraints.

Two gear analysis subroutines and the compatible input data file apply the optimization procedure to the gear life design problem. The first subroutine analyzes a gear design for the constraint variable values. And the second subroutine analyzes a gear design for the merit function parameter values. The input data file defines these values, sets the limits for the design constraints, and selects the active merit function parameters.

Tooth surface pitting fatigue life produces the finite life of the gear set. A fatigue life model for this mode of failure is presented. Statistical variations in gear life as predicted by the two parameter Weibull distribution are given also.

Two designs illustrate the design calculations of the program. One gear set is designed for maximum life with a required center distance, and a second design is obtained for a minimum center distance at a minimum acceptable life. The design procedure considers other constraints as well to obtain practical gear designs. Minimum weight and minimum center distance designs were found to be identical.

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TABLE 1. - GEAR DESIGN CONSTANTS AND VARIABLES

FOR SPUR GEAR LIFE MAXIMIZATION

[Design with modified gradient optimization using a maximum step limit and scaled variables.]

(a) Fixed design requirements

Poisson's ratio	0.25
Elastic modulus, psi	$3x10^7$
Pressure angle, deg	20
Gear ratio	2
Transmitted power, hp	80
Pinion speed, rpm	5000
Material weight density, lb/in.^{3}	0.283
Material surface constant, psi	9800
Weibull slope	2.5
Load-life factor	8.93
Reliability	0.9
Base temperature, °F	120
Tooth surface fiish, rms	32

(b) Estimated values of the three independent design variables

	Low	High	Initial
Pinion teeth	10	100	40
Diametral pitch, in.$^{-1}$	4	28	14
Face width, in.	0.5	5	2.5

TABLE 2. - 16 MAXIMUM LIFE CONSTRAINTS AND MERIT FUNCTION

(a) The 16 constraint limits

Constraint	Limit	Type of bound
Involute interference, in.	0.001	Lower
Lower face width-to-diameter ratio	0.2	Lower
Upper face width-to-diameter ratio	0.5	Upper
Pinion weight, lb	0	Lower
Center distance, in.	5.0	Upper
Pinion torque, lb-in.	0	Lower
Transmitted load, lb	0	Lower
Total dynamic load, lb	0	Lower
AGMA bending stress, psi	0.4×10^5	Upper
Full load contact stress, psi	1.5×10^5	Upper
Gear tip hertz pressure, psi	1.5×10^5	Upper
Pinion life, cycles	0	Lower
Mesh life, hr	0	Lower
Pitch line velocity, ft/min	0	Lower
PV factor, psi-ft/min	100	Upper
Flash temperature, °F	275	Upper

(b) Maximize the objective function (OBJ), which has three terms. OBJ = the linear sum of

Component	Unit	Multiplied by
Mesh life	in thousands of hours	1
Center distance	in inches	0
Pinion weight	in pounds	0

TABLE 3. - MAXIMUM LIFE OPTIMAL DESIGN

[Optimization successful in 50 steps.]

(a) The final design vectors

Design parameter	Scaled vector, Y_i	Actual vector, X_i
Pinion teeth	-0.24857	43.81443
Diametral pitch, in.$^{-1}$	-.23797	13.14433
Face width, in.	-.48148	1.66667

(b) Components of the maximum objective function, 28.1361

Component	Value	Multiplied by
Mesh life, hrx10^3	28.136	1
Center distance, in.	5.0000	0
Pinion weight, lb	4.1161	0

(c) The 16 constraint values

Constant	Value	Limit	Type of bound
Involute interference, in.	0.23224	1×10^{-3}	Lower
Lower face width-to-diameter ratio	0.5	0.2	Lower
Upper face width-to-diameter ratio	0.5	0.5	Upper
Pinion weight, lb	4.1161	0	Lower
Center distance, in.	5.0	5.0	Upper
Pinion torque, lb-in.	1008.4	0	Lower
Transmitted load, lb	643.87	0	Lower
Total dynamic load, lb	1825.9	0	Lower
AGMA bending stress, psi	0.34422×10^5	0.4×10^5	Upper
Full load contact stress, psi	1.2290×10^5	1.5×10^5	Upper
Gear tip hertz pressure, psi	0.80590×10^5	1.5×10^5	Upper
Pinion life, cycles	9.5274×10^9	0	Lower
Mesh life, hr	28.136×10^3	0	Lower
Pitch line velocity, ft/min	4100.2	0	Lower
PV factor, psi-ft/min	67.268×10^6	100×10^6	Upper
Flash temperature, °F	203.96	275	Upper

TABLE 4. - MAXIMUM LIFE DESIGN CHECK

(a) Design check

Design parameter	Scaled vector, X_i
Pinion teeth	46.000
Diametral pitch, in.$^{-1}$	14.000
Face width, in.	1.625

(b) The components of the maximum objective function, 18.1249

Component	Value	Multiplied by
Mesh life, hrx10^3	18.125	1
Center distance, in.	4.9286	0
Pinion weight, lb	3.8993	0

(c) The 16 constraint values

Constraint	Value	Limit	Type of bound
Involute interference, in.	0.23810	1×10^{-3}	Lower
Lower face width-to-diameter ratio	0.49457	0.2	Lower
Upper face width-to-diameter ratio	0.49457	0.5	Upper
Pinion weight, lb	3.8993	0	Lower
Center distance, in.	4.9286	5.0	Upper
Pinion torque, lb-in.	1008.4	0	Lower
Transmitted load, lb	653.20	0	Lower
Total dynamic load, lb	1843.8	0	Lower
AGMA bending stress, psi	0.37648×10^5	0.4×10^5	Upper
Full load contact stress, psi	1.2582×10^5	1.5×10^5	Upper
Gear tip hertz pressure, psi	0.81864×10^5	1.5×10^5	Upper
Pinion life, cycles	6.1374×10^9	0	Lower
Mesh life, hr	18.125×10^3	0	Lower
Pitch line velocity, ft/min	4041.6	0	Lower
PV, factor, psi-ft/min	64.271×10^6	100×10^6	Upper
Flash temperature, °F	202.37	275	Upper

TABLE 5. - MINIMUM SIZE OPTIMAL DESIGN

[Optimization successful in 27 steps.]

(a) The final design vectors

Design parameter	Scaled vector, Y_i	Actual vector, X_i
Pinion teeth	-0.45045	34.72992
Diametral pitch, in.$^{-1}$	-.36590	11.60917
Face width, in.	-.55778	1.49501

(b) The components of the minimum objective function, 4.48739

Component	Value	Multiplied by
Mesh life, hrx10^3	2.0236	0
Center distance, in.	4.4874	1
Pinion weight, lb	2.9739	0

(c) The 16 constraint values

Constraint	Value	Limit	Type of bound
Involute interference, in.	0.20055	1×10^{-3}	Lower
Lower face width-to-diameter ratio	0.49919	0.2	Lower
Upper face width-to-diameter ratio	0.49919	0.5	Upper
Pinion weight, lb	2.9699	0	Lower
Center distance, in.	4.4870	0	Lower
Pinion torque, lb-in.	1008.4	0	Lower
Transmitted load, lb	717.48	0	Lower
Total dynamic load, lb	1965.3	0	Lower
AGMA bending stress, psi	0.38256×10^5	0.4×10^5	Upper
Full load contact stress, psi	1.4325×10^5	1.5×10^5	Upper
Gear tip hertz pressure, psi	0.98539×10^5	1.5×10^5	Upper
Pinion life, cycles	0.67728×10^9	0	Lower
Mesh life, hr	2.0001×10^3	2.0×10^3	Lower
Pitch line velocity, ft/min	3679.5	0	Lower
PV factor, psi-ft/min	92.242×10^6	100×10^6	Upper
Flash temperature, °F	234.28	275	Upper

TABLE 6. - MINIMUM SIZE DESIGN CHECK

(a) The final design vectors

Design parameter	Actual vector, X_1
Pinion teeth	36.000
Diametral pitch, $in.^{-1}$	12.000
Face width, in.	1.500

(b) The components of minimum objective function, 4.500

Component	Value	Multiplied by
Mesh life, hrx10^3	2.1964	0
Center distance, in.	4.5000	1
Pinion weight, lb	3.0006	0

(c) The 16 constraint values

Constraint	Value	Limit	Type of bound
Involute interference, in.	0.20588	$1x10^{-3}$	Lower
Lower face width-to-diameter ratio	0.5	0.2	Lower
Upper face width-to-diameter ratio	0.5	0.5	Upper
Pinion weight, lb	3.0006	0	Lower
Center distance, in.	4.5	0	Lower
Pinion torque, lb-in.	1008.4	0	Lower
Transmitted load, lb	715.41	0	Lower
Total dynamic load, lb	1961.4	0	Lower
AGMA bending stress, psi	$0.38980x10^5$	$0.4x10^5$	Upper
Full load contact stress, psi	$1.4239x10^5$	$1.5x10^5$	Upper
Gear tip hertz pressure, psi	$0.97093x10^5$	$1.5x10^5$	Upper
Pinion life, cycles	$0.74373x10^9$	0	Lower
Mesh life, hr	$2.1964x10^3$	$2.0x10^3$	Lower
Pitch line velocity, ft/min	3690.2	0	Lower
PV factor, psi-ft/min	$88.071x10^6$	$100x10^6$	Upper
Flash temperature, °F	230.41	275	Upper

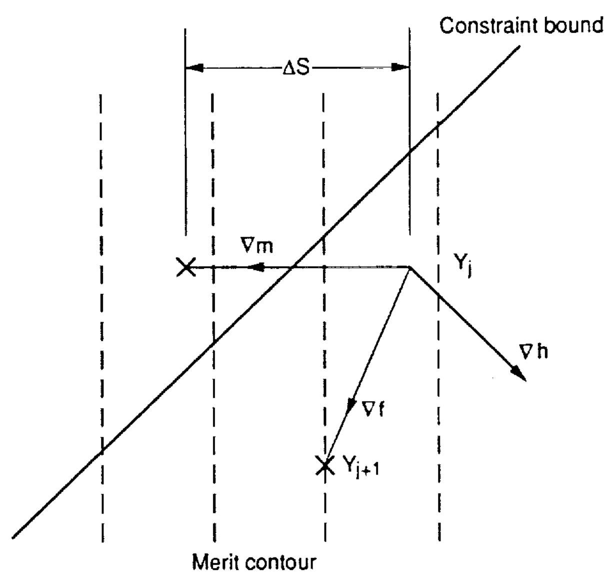

Figure 1.—Gradient sum to find feasible search direction.

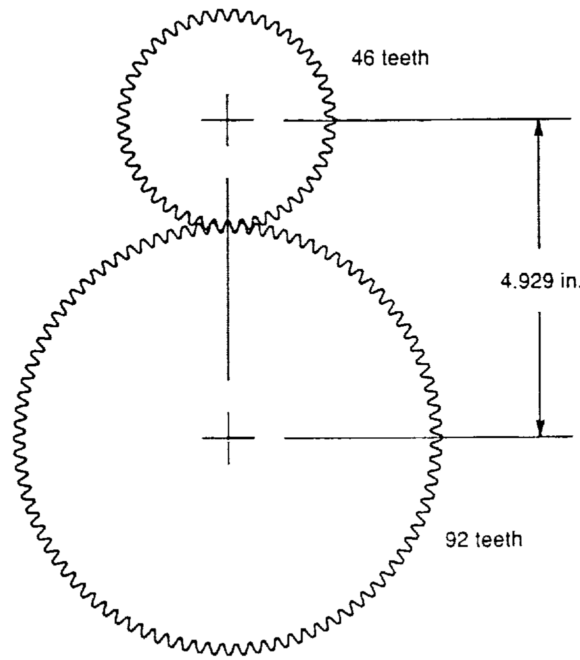

Figure 2.—Maximum life spur gear design with a center distance of 5 in.

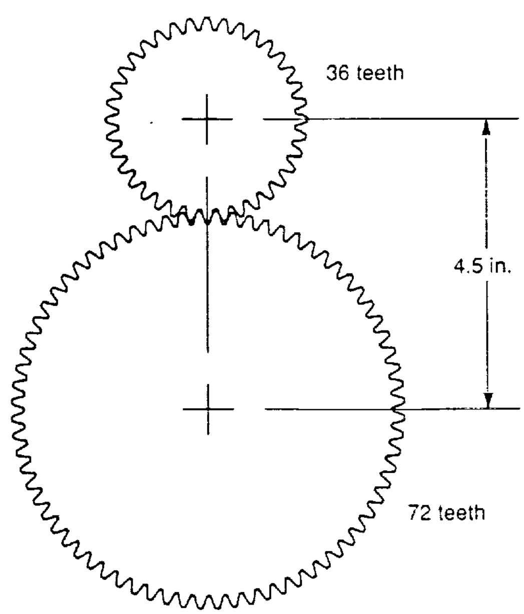

Figure 3.—Minimum size spur gear design for a life of 2,000 hrs.

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16. Abstract
Optimization procedures allow one to design a spur gear reduction for maximum life and other end use criteria. A modified feasible directions search algorithm permits a wide variety of inequality constraints and exact design requirements to be met with low sensitivity to initial guess values. The optimization algorithm is described and the models for gear life and performance are presented. The algorithm is compact and has been programmed for execution on a desk top computer. Two examples are presented to illustrate the method and its application.

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