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Incorporating Finite Element Analysis Into Component Life and Reliability

Richard August
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio

and

Erwin V. Zaretsky
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

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INCORPORATING FINITE ELEMENT ANALYSIS INTO

COMPONENT LIFE AND RELIABILITY

Richard August
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio 44142

Erwin V. Zaretsky*
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

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ABSTRACT

A method for calculating a component's design survivability by incorporating finite element analysis and probabilistic material properties was developed. The method evaluates design parameters through direct comparisons of component survivability expressed in terms of Weibull parameters. The analysis was applied to a rotating disk with mounting bolt holes. The highest probability of failure occurred at, or near, the maximum shear stress region of the bolt holes. Distribution of material failure as a function of Weibull slope affects the probability of survival. Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume Weibull parameters, as well as the stress-life exponent, in order to determine the effect of disk speed on the probability of survival.

NOMENCLATURE

c	material stress-life exponent
e	Weibull slope of material modulus
L	stress cycles to failure
L_i	stress cycles to failure of i^{th} component
L_o	stress cycles where 37.7 percent of specimens survive
L_s	stress cycles to failure of total system
L_u	stress cycles where all specimens survive
S	statistical fraction of specimens that survive a given number of stress cycles
S_s	total system survivability
V	stressed volume
τ	stress level applicable for failure theory

Subscripts:

a, b	states a and b, or bodies a and b

INTRODUCTION

A key component in any structural design process is establishing a minimum acceptable service life and then determining the potential of component or structural failure before that minimum life is achieved. Although components may be tested to failure to determine their functional life, this method is certainly impractical in terms of time requirements, the difficulty in simulating all possible loading conditions, and the expense involved in full-scale testing. Also, early-stage designs generally undergo quite a few iterations until a final set of specifications is formalized.

Many applications, especially in the aerospace industry, include size and weight constraints. This precludes utilizing life-enhancing design features, such as ultraconservative safety factors, or using stronger, but heavier, materials. In order to realize the lightest, most compact design, components are purposely designed for finite life (i.e., expected failure), with very high probability of survival throughout the entire duty cycle. The most efficient system operation then involves regularly scheduled component replacement, so that the predetermined time between replacements is only slightly shorter than the anticipated life of the component.

Often, empirical-based design techniques are employed for fatigue-tolerant designs. Although such procedures may result in successful designs with respect to fatigue considerations, the designs will probably not be efficient with respect to size or

*Fellow, ASME.

cost. Also, the use of new materials without an experience data base will pose a problem in making design decisions. For these reasons analytical-based life design methodologies are required to predict component life and survivability without the need for extensive testing or field experience.

First-order life prediction methods are generally based simply on yield and fatigue-limiting stresses. Often, they do not account for geometry and size effects, surface condition, duty cycles, and environmental factors. The material life data are also frequently misinterpreted. Stress-life curves are often assumed to be completely deterministic when in fact the stress-life data are presented as statistically averaged values. Finally, most life analyses are primarily concerned with a few critical areas in the component (i.e., the areas of highest stress). However, for designs with low stress gradients areas of probable failure are not intuitively obvious.

Finite-life component design requires a probabilistic approach that couples operating life with an expected rate of survival. Weibull (1939) demonstrated a statistical analysis that was particularly effective in describing experimental fatigue data. Lundberg and Palmgren (1947) applied Weibull analysis to contact-stress, high-cycle-fatigue bearing problems. Grisaffe (1965) demonstrated the applicability of a Weibull-based analysis to other types of durability problems. Ioannides and Harris (1985) and Zaretsky (1987) proposed a generalized Weibull-based methodology for structural fatigue life prediction based on a discrete-stressed-volume approach. Zaretsky et al. (1989) coupled this methodology to a stress field determined by finite element methods to predict the life and reliability of a generic, rotating disk. They also demonstrated the applicability of the methodology to the design process through parametric studies that showed component life sensitivities to such design variables as disk diameter and thickness and bolt hole size, number, and location. Nemeth et al. (1990) developed a computer program for quantifying reliability that is based on inherent flaws found in ceramics. The program calculates the fast fracture reliability of macroscopically isotropic ceramic components. This method is also based on the component's entire stress state, not just the maximum stress point.

Although these methods allow the determination of a total component life, it is often necessary to examine critical areas of the component in order to predict the probability of local failure. In this manner designs could potentially be optimized away from known critical areas through appropriate sizing. Therefore, the objectives of this investigation are (1) to extend the method of Zaretsky (1987) to allow for calculating the local probability of failure within any component's stressed volume as well as within the entire component and (2) to demonstrate the technique on a generic disk by examining the sensitivity of stressed-volume survivability to uncertainties in the material properties.

ANALYTICAL METHODOLOGY

Statistical Failure Theory

Experimental fatigue data can often be plotted as a straight line on Weibull probability paper. For a constant stress level the number of stress cycles to failure (i.e., life) is plotted on the abscissa. The percentile of specimens that survive at a given life is plotted on the ordinate. The transformation of stress cycles and survivability into Weibull coordinates is given by the equation

$$\ln \ln \frac{1}{S} = e \ln (L-L_u) - e \ln L_o \qquad (1)$$

where S is statistical fraction of specimens that survive a given number of stress cycles, e is the Weibull slope, L is stress cycles to failure, L_u is stress cycles where all specimens survive, and L_o is stress cycles where 37.7 percent of specimens survive. This is referred to as a three-parameter Weibull equation. A two-parameter Weibull equation is obtained by setting $L_u = 0$. The Weibull slope provides an indication of the scatter of the statistical properties, with $e = 1$, 2, or 3.57 being representative of exponential, Rayleigh, and Guassian distributions, respectively.

The number of stress cycles to failure at a given stress level can be related to stress cycles to failure at a new stress level by the equation

$$L_b = L_a \left(\frac{\tau_a}{\tau_b}\right)^c \qquad (2)$$

where L_a is the known life at stress level τ_a, and c is the stress-life exponent obtained from coupon testing. Weibull (1939, 1951) expressed the probability of survival as

$$\ln \frac{1}{S} \sim \tau^c L^e V \qquad (3)$$

where V is the stressed component volume. By using Eq. (3), the size effects on survivability for equally stressed volumes can be expressed as

$$S_b = S_a^{V_b/V_a} \qquad (4)$$

After component life and survivability have been adjusted on the basis of stress levels and stressed volume, all other combinations of life and survivability can be calculated. This allows valid comparison of relative component life at equal survivability or relative component survivability at equal life.

Using Eqs. (1) to (3), Zaretsky (1987) developed an expression for predicting life at a given survivability of a stressed component b relative to the known life of a stressed component a:

$$L_b = L_a \left(\frac{\tau_a}{\tau_b}\right)^c \left(\frac{V_a}{V_b}\right)^{1/e} \qquad (5)$$

This equation gives the component's life relative to another similar component when both have the same survivability. The system life can then be calculated from the individual component lives as follows:

$$\left(\frac{1}{L_s}\right)^e = \sum_{i=1} \left(\frac{1}{L_i}\right)^e \qquad (6)$$

where L_s is the total system life and L_i is the i^{th} component life (Zaretsky, 1987). Zaretsky's method can be extended further to examine relative component survivability at equal life. This is given by

$$S_b = S_a \qquad (7)$$

Equation (7) can be used to identify critical fatigue components within a system and to optimize noncritical ones. Because a minimum life is required of all components in the system, individual component survivability at that life can be assessed relative to a predetermined critical component. In this manner, components whose survivability is judged to be too low can be redesigned for greater survivability. Also, components whose survivability is much greater than that of the critical component can be redesigned for smaller size or lower weight while still maintaining adequate protection against failure.

Finally, the component's survivability is equal to the product of all the elements' survivabilities:

$$S_s = \prod_{i=1}^{n} S_i \qquad (8)$$

Incorporating Finite Element Analysis

The methodology just described can easily be incorporated into the finite element method to determine the life and survivability of a structural component. The entire component is analogous to a system. The finite element model of the component discretizes its geometry into elemental volumes. These elements are considered to be the base members of the "system." The time to crack initiation for each element is calculated from the elemental stress levels and volume as described previously to determine the component's incipient failure time. Elemental survivability is also calculated to find areas that have either too low survivability or are overdesigned.

A relative comparison approach is used in this methodology. A critical element in the model is selected on the basis of a maximum stress state. The selected stress failure criterion can be based on the material used in the component. If no fatigue data are available, the stressed element can be arbitrarily assigned a life and a survivability that is used to normalize the life and survivability of the other elements. This approach allows easy, qualitative comparisons between designs. Also, only one set of coupon fatigue tests at operating temperatures is necessary to fix the Weibull parameters and to establish quantitative component lives if the stress-life exponent of the material is already known. However, if the stress-life exponent is not known, at least two additional sets of coupon fatigue tests will be necessary.

A methodology for predicting elemental survivability at a fixed life is illustrated in Fig. 1. The critical element's life and survivability is set at unity and 90 percent, respectively (point 1). This is the analysis point to which all the other elements' lives and survivability will be referenced. Other designs can also be referenced to this point by using the same critical volume and stress. The expected survivability at any number of stress cycles can be determined by using Eq. (1). This is shown in the figure by drawing a line through the reference point with a slope equal to e, the material's Weibull slope.

Next, an element's survivability is adjusted based strictly on volume effects by using Eq. (4) (point 2). This adjustment is done independently of the stress levels. Because stress is not involved, elemental life is not affected, and the adjustment results in purely vertical displacement from the reference point. In the example shown, the second element's volume is greater than the critical volume and contains more potential crack initiation sites. Therefore, the adjusted survivability is lower.

The element's life is then adjusted strictly on the basis of elemental stress relative to the critical stress by using Eq. (2) (point 3). This adjustment is done independently of volume considerations. Consequently, elemental survivability is not affected, and the life adjustment results in purely horizontal displacement at constant survivability. In the example shown, the second element's stress is lower than the critical stress. Therefore, its life is greater and shifts to the right.

A new material Weibull line is now drawn through the adjusted life/survivability point. Because the compared element's material is the same as the critical element's material, the line is drawn parallel to the original line. However, if the elements had dissimilar materials, the new line would be drawn with the new Weibull slope. Thus, designs incorporating different materials can be compared with respect to survivability.

By using Eq. (7), the compared element's survivability is adjusted once more to the critical element's assigned life of unity (point 4). This time the displacement is along the Weibull line and gives the survivability of an element relative to the critical element, with both elements having the same life. Repeating this procedure for all the elements in the finite element model gives the survivability for any section of the design at any time.

It should be mentioned that volume-based elemental survivability will be somewhat influenced by the finite element mesh, especially with components that experience fairly uniform stress. For this case volumetric ratios will have more effect on elemental survivability. For components with nonuniform stress distributions, exponential stress ratios will have more influence on elemental survivability. For this case the methodology complements traditional finite element modeling philosophy, in that high-stress-gradient areas should be modeled with smaller elements. In this way sizing effects are minimized. In both cases an adaptive meshing technique could be used to explicitly optimize the finite element model sizing and to implicitly minimize the mesh size effects on elemental survivability. From Eqs. (4) and (8) it can be shown that component survivability remains unaffected by the individual element mesh.

RESULTS AND DISCUSSION

Parametric analytical studies were conducted to calculate speed effects on the survivability of a generic disk and to examine the probabilistic effects of material properties on disk survivability. The generic disk represents the first step in the investigation of aerospace propulsion turbine disks. It is 61 cm (24 in.) in diameter and has twelve 1.02-cm (0.40-in.) diameter bolt holes.

Figure 2(a) shows the finite element model of the disk used in these studies. From the axisymmetric conditions of the disk a 15 degree section of the disk is modeled with 62 grid points and 42 eight-noded solid elements. Boundary conditions, defined in terms of the model's cylindrical coordinates, constrain displacement in the circumferential direction and allow displacement in the radial and axial directions. The disk stresses are a result of the centrifugal loads due to disk rotation. No thermal loads were included.

MSC/NASTRAN was used for the finite element code. The linear elastic analysis option, SOL 24, was used to calculate the stresses. Although this analysis is limited to stress levels below the yield point, it can be used for high cycle fatigue where stresses are lower and the occurrence of failure is less deterministic and more probabilistic. However, because the statistical failure theory is based on stress levels and distribution, the life and reliability methodology can also be adapted for nonlinear analyses.

Table I lists the assumed isotropic material properties used in the analyses. The probabilistic aspect of the material strength is given in the L_{10} life of the material. This L_{10} life represents the life in the number of stress cycles in which 90 percent of the tested coupons survive at a constant maximum shear stress. By using the values of Table I, a finite element stress analysis was performed on the generic disk of Fig. 2(a) at a speed of 12 800 rpm. Figure 2(b) shows the results of the stress analysis. The maximum shear stress was approximately 275 800 kN/m^2 (40 ksi) at the bolt hole. The failure probability analysis for each of the model's elements was then calculated by using Eq. (7). The results shown in Fig. 2(c) indicate that the region with the lowest reliability (i.e., the highest probability of failure) occurred at, or near, the maximum shear stress region at the bolt hole. However, outside this region the disk had relatively high regions of survivability.

Figure 3 graphically shows the relation between life and survivability for the material in Table I at an assumed Weibull slope. For a Weibull slope of 3.57, 99 percent of the samples survived 6000 cycles, 90 percent survived 10 000 cycles, and 10 percent survived 20 000 cycles. A higher value of Weibull slope would demonstrate less scatter, with failures occurring over a smaller number of stress cycles. A lower value would show failures over a broader range of lives and therefore might be indicative of more uncertainty in the material lives.

Figure 4 shows the effect of a disk's rotational speed on its probability of achieving a life of 10 000 stress cycles. The speed effect was plotted for an assumed Weibull slope of 3.57 and a stress-life exponent of 9. The disk first experienced a maximum shear stress of 275 800 kN/m^2 (40 ksi) at a speed of 12 800 rpm. This disk had a probability of survival of about 96 percent at this speed, which was higher than the coupon survival rate of 90 percent at the same stress and number of cycles.

The higher probability of survival was due to the volumetric effects. The 275 800-kN/m^2 (40-ksi) stress region in the disk was much smaller than the gage volume of the coupon. The location of the maximum stress in the model occurred at the elements adjacent to the bolt hole, at the three o'clock position. Also, the stresses in the remainder of the disk elements were much lower than 275 800 kN/m^2 (40 ksi) and had individually high survivability. Therefore, they had hardly any effect on degrading the disk's survivability. As a result, the disk showed a probability of survival greater that the coupon L_{10} life.

The individual calculated elemental survivabilities can also be examined for design optimization. Regions with low-survivability elements can be redesigned to increase reliability. Similarly, regions containing elements with unnecessarily high survivability can be optimized through redesign to reduce weight.

Figure 5 shows the effect of speed on the cumulative probability of failure (i.e., 1 minus the survivability) with varying Weibull slopes. This figure demonstrates how the distribution of failure as a function of Weibull slope affects the probability of survival. All three curves intersect at 12 800 rpm, with a 4 percent failure (96 percent survival), because the L_{10} life reference stress of 275 800 kN/m^2 (40 ksi) first happens at 12 800 rpm. Above this speed the rate of failure was definitely higher with higher Weibull slope. The 50 percent failure probability occurred at 13 100 rpm for a Weibull slope of 2, at 13 300 rpm for a Weibull slope of 3.57, and at 14 000 rpm for a Weibull slope of 5. Below 12 800 rpm the value of the Weibull slope had little effect on survival probability.

Often, material data will be incomplete, if available at all, and designers must use engineering judgment in estimating material uncertainties. Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume the Weibull slope and the stress-life exponent, in order to determine the disk speed at which the probability of survival will be high. From Fig. 5, the estimate of the Weibull slope should be high for conservative design by anticipating a higher failure rate than would probably occur.

Figure 6 shows the effect of speed on the cumulative probability of failure with varying stress-life exponents. There is not the degree of divergence of the three curves above 12 800 rpm as there was with Weibull slope variations. The same general trends hold, higher failure rates with higher stress-life exponent, but not to the same extent as in Fig. 5. At high survivability values, the value of the stress-life exponent had little effect at any given speed.

CONCLUDING REMARKS

Probabilistic material properties expressed in terms of Weibull parameters were coupled with the stress field determined from MSC/NASTRAN finite element analysis to determine fatigue life based on crack initiation. This methodology can be applied to predict the probability of survival of the complete structural component as well as to identify critical failure regions of the component. A unique advantage of this approach is that only coupon fatigue testing is needed to establish the material fatigue parameters necessary for full-size component life and survivability analysis. Thus, this technique can be used in the early design stages to optimize life-based designs, thereby reducing the amount of full-size component testing required to validate these designs.

SUMMARY OF RESULTS

A method for calculating a component's design survivability that incorporates finite element analysis and probabilistic material properties was developed. The method evaluates design parameters through direct comparison of component survivability expressed in terms of Weibull parameters. The method considers the component's total stress state in the survivability calculations. Critical regions with respect to survivability can be identified for optimization purposes: Reliability can be improved at low-survivability regions; and weight can be reduced in high-survivability regions. The analysis was applied to a rotating disk with mounting bolt holes. The following results were obtained:

(1) The highest probability of failure for the disk occurred at, or near, the maximum shear stress region at the bolt hole.

(2) Distribution of failure as a function of Weibull slope affected the probability of survival. For speeds that induced stresses above the L_{10} life reference stress, higher Weibull slope predicted lower survival probability. However, the value of the Weibull slope had little effect at any given speed at high probabilities of survival.

(3) The stress-life exponent affected the reliability predictions. For speeds that induced stresses above the L_{10} life reference stress, higher stress-life exponents predicted lower survival probability. However, the value of the stress-life exponent had little effect at any given speed at high probabilities of survival.

(4) Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume Weibull parameters as well as the stress-life exponent in order to determine the disk speed at which the probability of survival will be highest.

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TABLE I. - ASSUMED ISOTROPIC MATERIAL PROPERTIES

Elastic modulus, kN/m^2 (psi)	1.10×10^{11} (16×10^6)
Poisson's ratio	0.33
Density, kg/m^3 (lb/in.^{3})	4.61×10^{-5} (0.16)
Gage volume, m^3 (in.^{3})	5.74×10^{-6} (0.35)
L_{10} life, number of cycles	10 000
Shear stress at L_{10}, kN/m^2 (ksi)	275 800 (40)

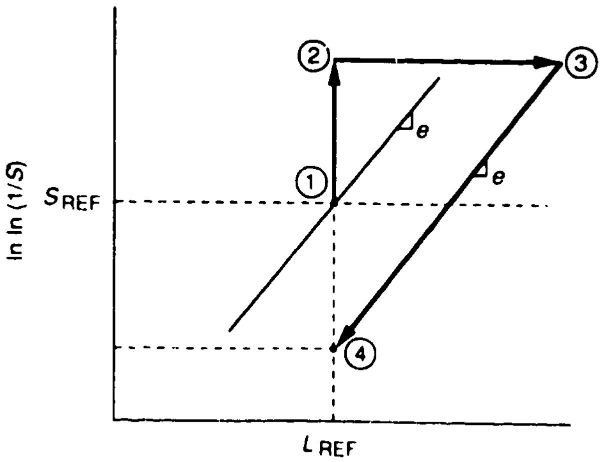
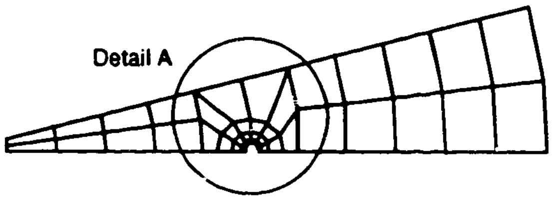
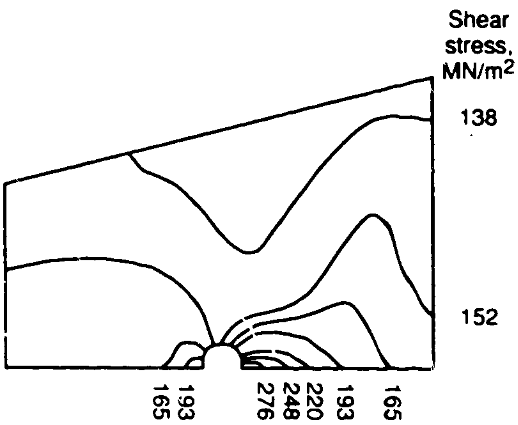


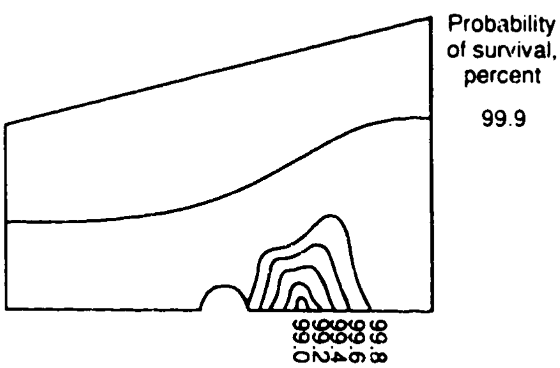
Figure 1.—Element survivability methodology.



(a) Disk finite element model.



(b) Shear stress contours at detail A.



(c) Probability of survival contours at detail A.

Figure 2.—Disk finite element model and analysis results. Nominal disk configuration: disk diameter, 0 061-m (24-in.); disk speed, 12 800 rpm; twelve 1.02-cm (0.40-in.) diameter bolt holes located on 0.030-m (12-in.) diameter.

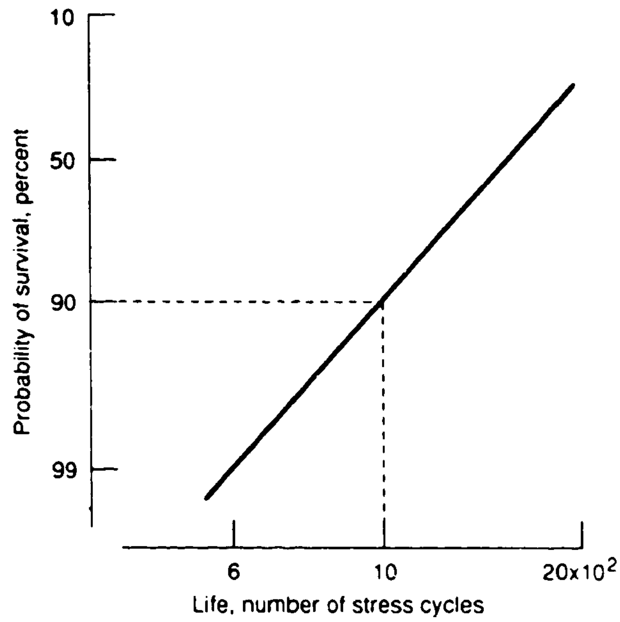


Figure 3.—Assumed material survivability at 275 800 kN/m² (40 ksi). Weibull slope, 3.57.

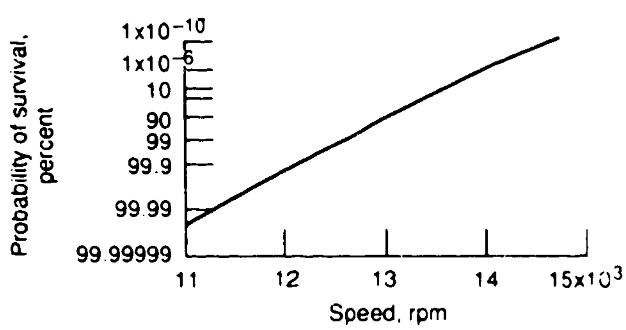


Figure 4.—Effect of speed on survivability at 10 000 cycles. Stress-life exponent, 9; Weibull slope, 3.57.

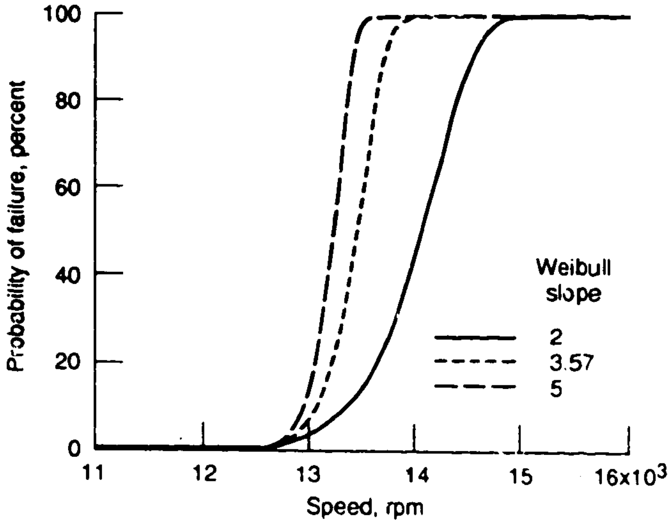


Figure 5.—Effect of Weibull slope on survivability at 10 000 cycles. Stress-life exponent, 9.

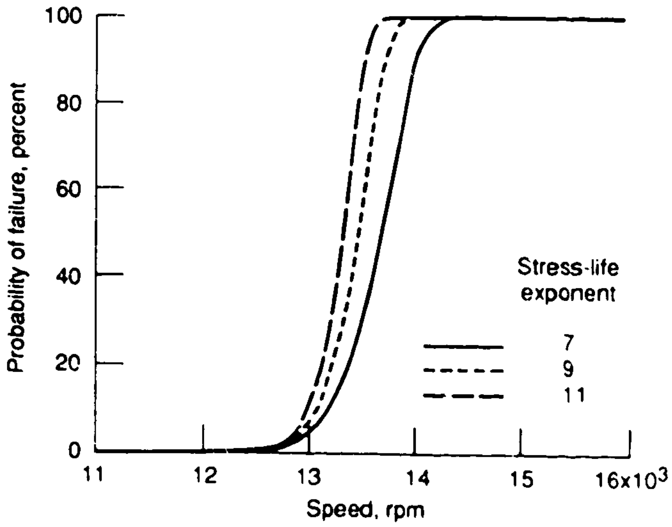


Figure 6.—Effect of stress-life exponent on survivability at 10 000 cycles. Weibull slope, 3.57.

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16. Abstract

A method for calculating a component's design survivability by incorporating finite element analysis and probabilistic material properties was developed. The method evaluates design parameters through direct comparisons of component survivability expressed in terms of Weibull parameters. The analysis was applied to a rotating disk with mounting bolt holes. The highest probability of failure occurred at, or near, the maximum shear stress region of the bolt holes. Distribution of failure as a function of Weibull slope affects the probability of survival. Where Weibull parameters are unknown for a rotating disk, it may be permissible to assume Weibull parameters, as well as the stress-life exponent, in order to determine the disk speed where the probability of survival is highest.

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