N91-24328

AN APPROXIMATE METHODS APPROACH TO PROBABILISTIC STRUCTURAL ANALYSIS

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A major research and technology program in Probabilistic Structural Analysis Methods (PSAM) is currently being sponsored by the NASA Lewis Research Center with Southwest Research Institute as the prime contractor [1,2]. This program is motivated by the need to accurately predict structural response in an environment where the loadings, the material properties, and even the structure may be considered random. The heart of PSAM is a software package which combines advanced structural analysis codes with a fast probability integration (FPI) algorithm for the efficient calculation of stochastic structural response.

The most common structural analysis techniques in use today are finite element and boundary element methods. These techniques permit highly accurate three-dimensional modeling of structural geometry, thermal and mechanical load environment, and material properties. Unfortunately, these techniques can also be expensive and time-consuming. In the early stages of design, when geometries, loads, and materials are only tentative, it is not practical to assemble an exact, comprehensive three-dimensional model for every critical component. What is needed, typically, is some means of estimating the nature and rough magnitude of stresses, displacements, natural frequencies, etc. Of further value is the identification of those design parameters which exert the most influence on the total system performance, so that further design evolution is more efficient.

These needs are addressed in the PSAM software by the Probabilistic Approximate Analysis Methods (PAAM) module. The basic idea of PAAM is simple: make an approximate calculation of system response, including calculation of the associated probabilities, with minimal computation time and cost, based on a simplified representation of the geometry, loads, and material. The deterministic solution resulting should give a reasonable and realistic description of performance-limiting system responses, although some error will be inevitable. If the simple model has correctly captured the basic mechanics of the system, however, including the proper functional dependence of stress, frequency, etc. on design parameters, then the response sensitivities calculated may be of significantly higher accuracy. In other words, the calculated probabilistic distribution of the response variable may be in significant error only by some offset of the mean value.

Three factors make up the "approximate" analysis approach. The first is a simplified representation of the part geometry. Complex three-dimensional shapes are replaced by simple beams, shells, etc., with relatively few descriptive parameters. The second factor is a similar simplification of the applied mechanical and thermal loads. Point loads, uniform fields, and linear or parabolic distributions are used to describe these quantities. Often a complex load environment can be approximated through linear superposition of the simple descriptors. The third factor in the PAAM methodology is the solution technique itself. Complex numerical techniques with many degrees of freedom are replaced by simpler computational schemes. In some cases, it is possible to construct a simple mechanics-of-materials model, often in two dimensions. In other cases, more sophisticated closed-form solutions can be derived or (preferably) adapted from previous research results in the literature. These may be based on elasticity or may employ approximate energy methods.

The PAAM code employs the fast probability integration (FPI) algorithm of Wu and Wirsching [3], which has been demonstrated to consistently provide fast and accurate estimates of point probabilities for typical engineering response functions. This algorithm identifies a most-probable-point (or design point) on the response surface, establishes a quadratic polynomial approximation to the response function at that point, and then transforms the quadratic form into a linear one. Input variables are defined using probability distributions. An optimization routine is employed to approximate non-normal variates as equivalent three-parameter normals, thereby approximating the limit state as linear in normally distributed design factors. Solution of this simplified problem is straightforward. The FPI is repeatedly applied to a number of limit states to compute the cumulative distribution function (CDF).

The PSAM code is being demonstrated and validated by analyzing four representative critical components in the current Space Shuttle Main Engine (SSME). The same four components are considered in the PAAM code, which by its very nature is component-specific. It should be noted, however, that the architecture of the PAAM software permits new closed-form expressions for these or other components to be installed and evaluated quickly. In this presentation one of these demonstration components, the liquid oxygen (LOX) post, is used to illustrate typical PAAM solution strategies and results.

Two different simple models are used to analyze the LOX post. The steady temperature and pressure gradients between the internal and external radii are evaluated by modeling the post as an axisymmetric thick cylinder. This is a classic elasticity problem, and exact solutions are readily available in standard textbooks. Other loadings, including the transverse fluid flow, are considered by modeling the post as a beam with hollow circular cross-section and elastically restrained ends (i.e., a mechanics-of-materials approach). Note how this model is used to solve a vibration problem. The basic solutions for free vibration of fixed-fixed and fixed-pinned uniform beams are available in handbooks. Solutions for a limited number of intermediate end conditions are available in tabular form. These results can be fitted with a simple empirical relationship to estimate the frequency factor for any mode and any end condition. Subsequent calculations of mode shapes and stresses for forced vibration follow directly from simple equations.

Several sample PAAM/FPI calculations have been performed in order to validate and further demonstrate the approximate methods approach. Results are given in terms of cumulative distribution functions for critical response variables. FPI and Monte Carlo calculations are shown to agree closely. Also determined are the sensitivity factors of the random variables. These sensitivity factors are a function not only of the mathematical form of the equation, but also of the distribution types and coefficients of variation of each random variable. They are of particular value for guiding improvements in the analytical scheme or component design.

References

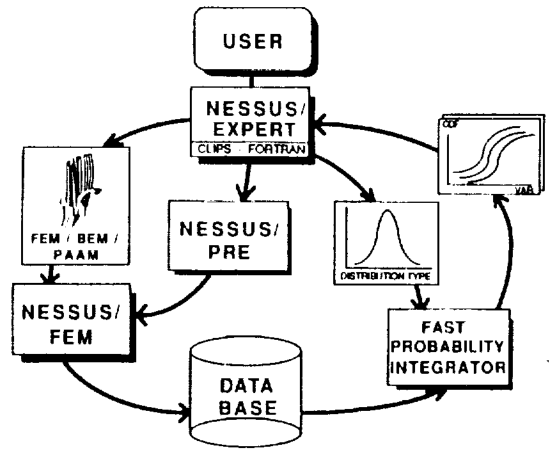
1. Burnside, O. H., "Probabilistic Structural Analysis for Space Propulsion System Components," *Advances in Aerospace Structural Analysis*, AD-09, ASME, 1985, pp. 87-102.

2. Cruse, T. A., Wu, Y.-T., Dias, B., and Rajagopal, K. R., "Probabilistic Structural Analysis Methods and Applications," *Computers and Structures*, Vol. 30, No. 1/2, 1988, pp. 163-170.

3. Wu, Y.-T., and Wirsching, P. H., "A New Algorithm for Structural Reliability Estimation," *Journal of Engineering Mechanics*, ASCE, Vol. 113, No. 9, September 1987, pp. 1319-1336.

AN OVERVIEW OF PROBABILISTIC STRUCTURAL ANALYSIS METHODS

- PSAM LINKS STRUCTURAL ANALYSIS WITH FAST PROBABILITY INTEGRATION FOR EFFICIENT CALCULATION OF STOCHASTIC STRUCTURAL RESPONSE WHEN LOADS, MATERIALS, AND GEOMETRY ARE RANDOM



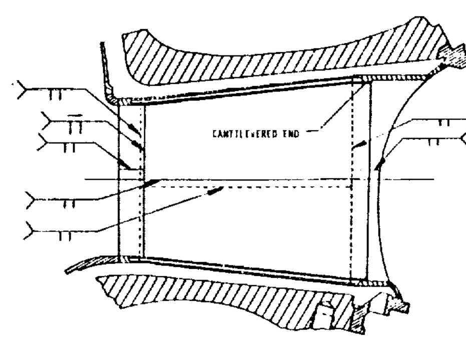
- FEM AND BEM ARE MOST COMMON STRUCTURAL ANALYSIS TECHNIQUES
  - DETAILED MODELS
  - ACCURATE 3-D SOLUTIONS
  - EXPENSIVE, TIME-CONSUMING
  - NOT APPROPRIATE FOR EARLY DESIGN/ANALYSIS

- NEED A RAPID, INEXPENSIVE ESTIMATE OF STRUCTURAL RESPONSE WHICH INCLUDES PROBABILISTIC INFORMATION

- "PROBABILISTIC APPROXIMATE ANALYSIS METHODS" (PAAM)

WHAT IS AN "APPROXIMATE METHOD" ?

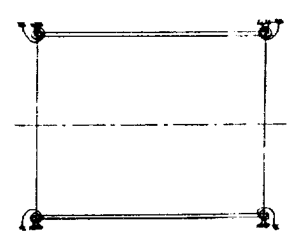
- SIMPLIFIED REPRESENTATIONS OF
  - COMPONENT GEOMETRY
  - APPLIED LOADS
  - MATERIAL PROPERTIES



- RAPID, EFFICIENT SOLUTION SCHEME
  - MECHANICS-OF-MATERIALS
  - SIMPLE ENERGY METHODS
  - CLOSED FORM OR TABULATED LITERATURE SOLUTIONS

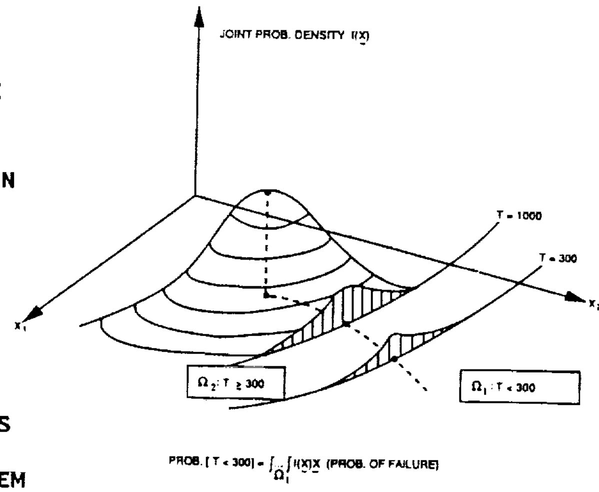
- CALCULATED MEAN VALUE OF RESPONSE FUNCTION CONTAINS SOME ERROR

- IF BASIC MECHANICS ARE CORRECT, RESPONSE DISTRIBUTIONS AND PARAMETER SENSITIVITIES WILL BE ACCEPTABLY ACCURATE



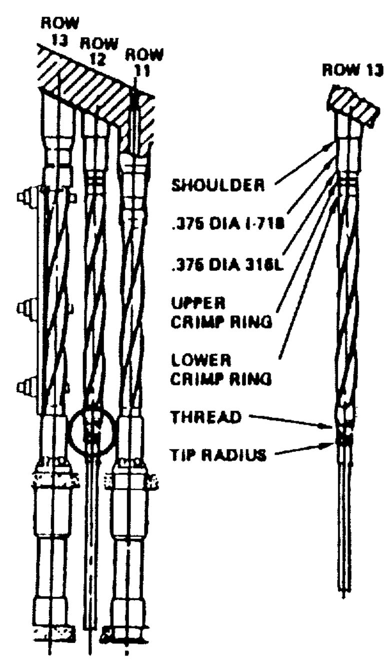
PAAM CODE USES FAST PROBABILITY INTEGRATION ALGORITHM

- o Wu-Wirsching FPI algorithm
- o Shared by other PSAM modules
- o Identify MPP on response surface for given probability level
- o Establish quadratic approximation to the response function at MPP and transform to a linear form
- o Input variables defined using probability distributions
- o Approximate non-normal variates as equivalent 3-parameter normals
- o Solve the resulting simple problem (linear with normal design factors)



PAAM CODE DEMONSTRATED WITH SSME LOX POST

- o One of four SSME demonstration components
- o A slender, hollow tube fixed at both ends
- o Important loadings include
  - ΔT and Δp across tube wall
  - steady and dynamic loads due to transverse fluid flow
- o Two different PAAM models shown
  - thick cylinder model
  - beam model
- o Although these equations are component-specific, the basic PAAM algorithm is component-independent



THICK CYLINDER MODEL USES KNOWN ELASTICITY SOLUTIONS

- THICK CYLINDER SUBJECTED TO DIFFERENT INTERNAL AND EXTERNAL PRESSURES AND TEMPERATURES
- EXACT CLOSED-FORM SOLUTIONS AVAILABLE IN STANDARD TEXTBOOKS
- CONSIDER THE HOOP STRESS:
  - PRESSURE SOLUTION

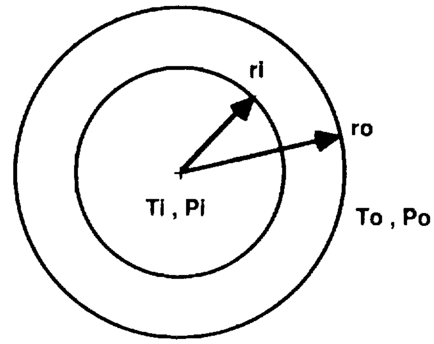
$$\sigma_0 = -\frac{(r_o/r)^2(p_o - p_i)}{k^2 - 1} + \frac{p_i - p_o k^2}{k^2 - 1}$$

  - THERMAL SOLUTION

$$\sigma_0 = -\beta\frac{(r_o/r)^2 + 1}{k^2 - 1} + \beta\frac{1 - \ln(r_o/r)}{\ln(k)}$$

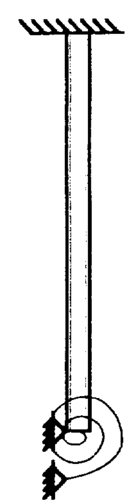
$$\beta = \frac{\alpha E(T_i - T_o)}{2(1 - \nu)}$$

$$k = r_o/r_i$$



BEAM MODEL USES A MECHANICS-OF-MATERIALS APPROACH

- SLENDER BEAM WITH HOLLOW CROSS-SECTION
  - FIXED AT ONE END
  - ELASTICALLY RESTRAINED AT THE OTHER
- CONSIDER THE VIBRATION PROBLEM
- BASIC EQUATIONS IN HANDBOOKS IN TERMS OF A FREQUENCY FACTOR λ
- LAMBDA DEPENDS ON END FIXITY AND MODE NUMBER
  - CLOSED FORM EXPRESSIONS FOR FIXED, PINNED
  - TABULATED SOLUTIONS FOR ELASTIC RESTRAINT
  - FIT THESE WITH AN EMPIRICAL EXPRESSION
- SIMPLE TO CALCULATE NATURAL FREQUENCIES, DISPLACEMENTS AND STRESSES DUE TO HARMONIC AND RANDOM FORCED VIBRATION



PAAM INPUT INCLUDES STATISTICAL DESCRIPTIONS OF RANDOM VARIABLES

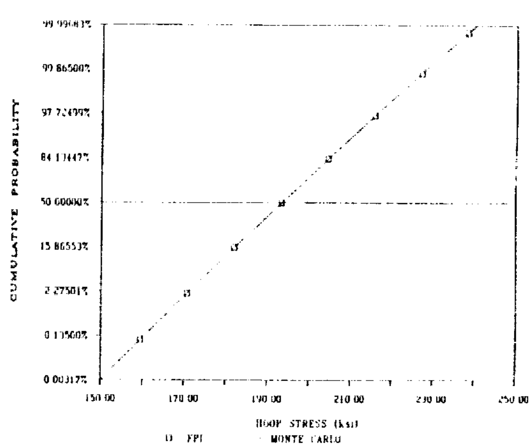
Input Variables for Lox Post Thick Cylinder Model

Variable	Distribution	Mean	COV
r_i	truncated Normal (±0.003)	0.094 in.	1.06%
r_o	truncated Normal (-0.002, +0.01)	0.110 in.	4.55%
E	Normal	3.40E+7 psi	2%
ν	Normal	0.3594	2%
α	Normal	5.65E-6 /R	5%
p_i	Lognormal	3077 psi	4%
p_o	Lognormal	3232 psi	4%
T_i	Lognormal	194 R	1.55%
T_o	Lognormal	1444 R	1.55%

- o USER SUPPLIES MEAN VALUE, DISTRIBUTION TYPE, AND COV FOR EACH RANDOM VARIABLE
- o MANY DISTRIBUTION TYPES POSSIBLE (INCLUDING TRUNCATED)
- o SET COV=0 FOR DETERMINISTIC VARIABLES
- o IF STATISTICAL INFORMATION UNKNOWN, MAKE ESTIMATES FROM DEFAULT SPECIFICATIONS

PAAM OUTPUT INCLUDES CDF OF RESPONSE FUNCTION

- o FPI CALCULATES CUMULATIVE DISTRIBUTION FUNCTION (CDF) OF RESPONSE VARIABLE
- o CDF BASED ON EQUIVALENT NORMAL DISTRIBUTION OF RESPONSE VARIABLE
- o EXCELLENT AGREEMENT BETWEEN FPI AND MONTE CARLO
- o FPI MUCH FASTER THAN MONTE CARLO



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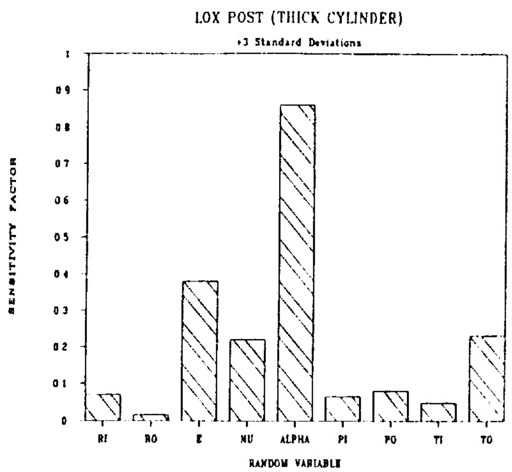
PAAM CALCULATES SENSITIVITY FACTORS OF RANDOM VARIABLES



- o SHOWS RELATIVE INFLUENCE OF RANDOM VARIABLES ON THE TOTAL UNCERTAINTY OF THE RESPONSE
- o FUNCTION OF BOTH
  - MATHEMATICS OF THE EQUATION
  - DISTRIBUTION TYPE AND COV OF THE RANDOM VARIABLES
- o HELPS THE DESIGNER/ANALYST TO DETERMINE REQUIREMENTS FOR FURTHER DESIGN/ANALYSIS
  - MORE ACCURATE INPUT STATISTICS
  - MORE EXACT ANALYSIS MODEL
  - TIGHTER TOLERANCES IN DESIGN
  - BEST OPPORTUNITIES FOR DESIGN IMPROVEMENTS

SUMMARY

- o PAAM MAKES A RAPID, INEXPENSIVE ESTIMATE OF THE PROBABILISTIC STRUCTURAL RESPONSE OF A SYSTEM
- o REQUIREMENTS:
  - SIMPLIFIED REPRESENTATION OF COMPONENT GEOMETRY AND LOADS
  - CLOSED-FORM SOLUTIONS DERIVED OR ADAPTED FROM THE LITERATURE
  - STATISTICAL DISTRIBUTIONS OF INPUT VARIABLES
- o PROBABILISTIC CALCULATION PERFORMED WITH FPI ALGORITHM
- o OUTPUT INCLUDES:
  - CDF OF RESPONSE VARIABLE
  - SENSITIVITY FACTORS FOR RANDOM VARIABLES
- o PAAM PARTICULARLY USEFUL IN EARLY STAGES OF DESIGN/ANALYSIS