PROBABILISTIC BOUNDARY ELEMENT METHOD

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The purpose of the Probabilistic Structural Analysis Methods (PSAM) project is to develop structural analysis capabilities for the design analysis of advanced space propulsion system hardware. The PSAM effort consists of three major technical thrusts: probabilistic finite element methods (PFEM), probabilistic approximate analysis methods (PAAM), and probabilistic advanced analysis methods (PADAM). The boundary element method is used as the basis of the Probabilistic Advanced Analysis Methods (PADAM) this is discussed in this text. In particular, the BEST3D code developed under NASA/HOST program is modified for inclusion in the PSAM module as NESSUS/BEM.

The probabilistic boundary element method code (PBEM) is used to obtain the structural response and sensitivity results to a set of random variables. As such, PBEM performs analogous to other structural analysis codes such as finite elements in the PSAM system. The probabilistic analysis is performed by coupling PBEM and the Fast Probability Integrator (FPI) using a highly efficient mean based algorithm. Preliminary validation studies have shown PBEM to be an accurate tool for probabilistic analysis.

For linear problems, unlike the finite element method (FEM), the BEM governing equations are written at the boundary of the body only, thus, the method eliminates the need to model the volume of the body. However, for general body force problems, a direct condensation of the governing equations to the boundary of the body is not possible and therefore volume modelling is generally required. Since such volume modeling mostly eliminates the advantage of the BEM procedure, a surface transformation technique based on particular integrals is used to replace the volume integral by equivalent surface integrals in the current analysis.

To illustrate the particular integral procedure, consider the solution of thermoelastic problem. The surface transformation technique requires the evaluation of the particular integrals of the inhomogeneous Navier's equilibrium equations. However, in general, the temperature field is not known as a continuous functions, instead, the values of the temperatures are known at selected points at the surface and volume of the body. The procedure assumes that the temperature field then can be expressed in terms of interpolation functions and unknown *densities* associated with each collocation point. Since this procedure replaces the actual temperature field by an approximate field, the resulting integrals equations, while satisfying the equilibrium conditions, are only approximate. A plot of the interpolated temperature field on the surface of a cube subjected to uniform temperature shown in figure 4 indicates that the global interpolation results is in error except at collocation points. Nevertheless, the particular integral solution procedure uses nodal values interpolated by regular isoparametric shape functions in their computations which seems to minimize the interpolation error effect.

As a first example, consider the free vibration analysis of a cantilever beam. The first five normal modes shown in figure 5 using two BEM models agree well with FEM results, thus, validating the free-vibration analysis procedure. To validate the particular integral procedure for the thermoelastic problem, we considered the solution of hollow sphere with an external radius to internal radius ratio of 2 subjected to linear radial temperature variation. The normalized hoop stress compared to the theoretical results in figure 6 validates the particular integral procedure for thermal problems. To further investigate the accuracy of the procedure for higher temperature variations, we considered the same sphere subjected to cubic radial temperature variation. Again the agreement between the computational the theoretical results for hoop stresses is excellent as shown in figure 7. This shows that while the interpolated temperature field is in error, the solution of the corresponding boundary integral equations even for a higher order temperature field is still very accurate. As a final example, we considered a hollow cylinder subjected to linear temperature variation. We further assumed that the Young's modulus also varied linearly for this case. The resulting hoop stress compared to FEM in figure 8 validates the material inhomogeneity procedure.

Overview of NESSUS/BEM System
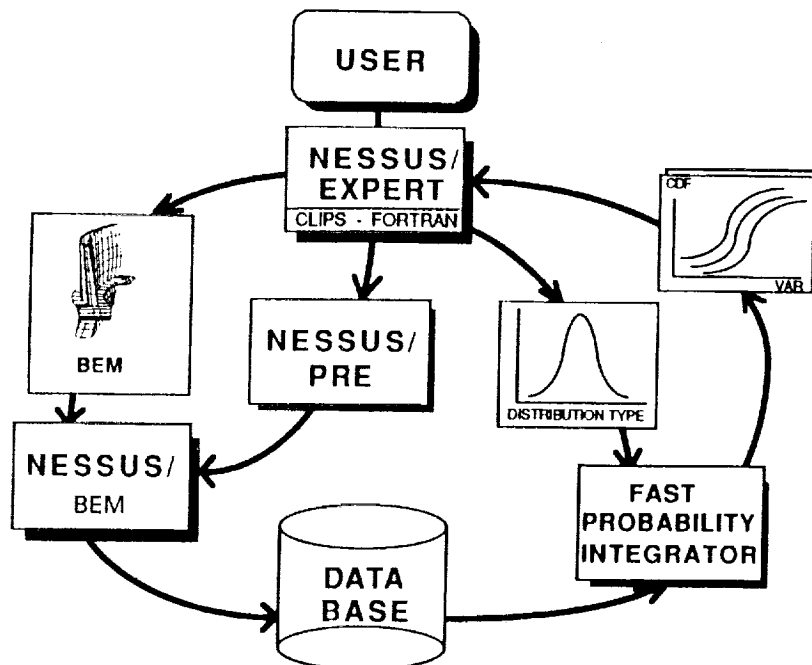
BEM Formulation for General Body Force Analysis Does Not Use Domain Modeling

- **Regular Domain BIE for Thermoelasticity**

$$\underline{c}\,\underline{u} + \int_s \underline{T}\,\underline{u}\,ds = \int_s \underline{U}\,\underline{t}\,ds + \int_V \underline{S}\,\theta\,dv$$

- **BIE for Particular Solution Replaces Domain Integral**

$$\underline{c}\,\underline{u}^p + \int_s \underline{T}\,\underline{u}^p\,ds - \int_s \underline{U}\,\underline{t}^p\,ds = \int_V \underline{S}\,\theta\,dv$$

- **Surface-Only BIE Replaces Thermoelastic BIE**

$$\underline{c}\,\hat{\underline{u}} + \int_s \underline{T}\,\hat{\underline{u}}\,ds = \int_s \underline{U}\,\hat{\underline{t}}\,ds$$

$$\hat{\underline{u}} = \underline{u} - \underline{u}^p(\theta)$$

$$\hat{\underline{t}} = \underline{t} - \underline{t}^p(\theta)$$

Particular Solutions Obtained for Approximating Body Forces

- **Navier's Equilibrium Equation (Thermoelastic)**

$$\underline{N} \bullet \underline{u}^p = m\nabla\underline{\theta}$$

- **Collocate $\underline{\theta}$ with Boundary Interpolation Functions**

$$\tilde{\theta}(q_m) = \sum_{n=1}^{N} K(q_m, Q_n)\,\varphi(Q_n) \qquad\qquad Q_n \varepsilon S$$

- **Particular Solution for Collocated Temperatures Replaces Domain Integral**

$$\underline{c}\,\tilde{\underline{u}}^p + \int_s \underline{T}\,\tilde{\underline{u}}^p\,ds - \int_s \underline{U}\,\tilde{\underline{t}}^p\,ds = \int_V \underline{S}\,\tilde{\theta}\,dV$$

- **Surface-Only BIE for Interpolated Temperatures**

$$\underline{c}\,\hat{\underline{u}} + \int_s \underline{T}\,\hat{\underline{u}}\,ds = \int_s \underline{U}\,\hat{\underline{t}}\,ds + \int_V \underline{S}\,(\theta - \tilde{\theta})\,dV$$
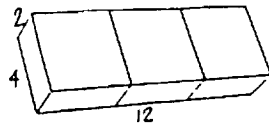
Interpolation Error Within the Domain

• Global Interpolation Functions

• Collocated at User-Defined Points (Surface and Interior)

• Particular Solution Method Minimizes Interpolation Error Effect

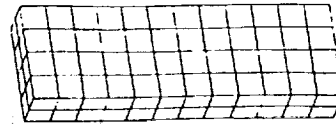• Particular Solution Approximation Uses BIE Shape Functions

Error in Global Interpolation Function for Constant Temperature

15.6
14.6
13.5
12.5
11.5
10.4
9.38
8.34
7.29
6.25
5.21
4.17
3.13
2.08
1.04
.00057

Collocation Points

Error
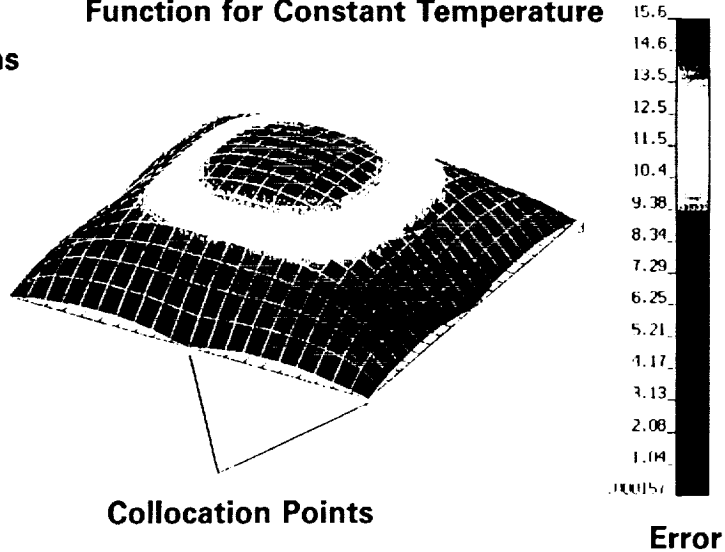
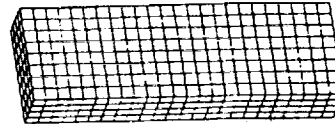# Normal Vibration Modes of BEM and FEM Models Are Compared

a) BEM Map 1

a) FEM Map 1

b) BEM Map 2

b) FEM Map 2

**First Five Modes of Vibration for the Cantilever Models**

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Nodes/ Elements	CPU (sec) VAX 8700
BEM-1	2686	5441	13485	16810	25382	44/14	352
BEM-2	2810	5333	12736	16255	24523	86/28	1676
FEM-1	3033	5449	13191	17227	25141	195/96	158
FEM-2	2885	5354	12778	16224	24522	1125/768	801

Results Are Accurate
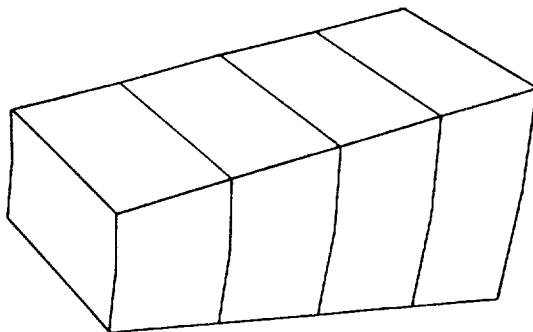# Even For Cubic Temperature Variation

Hollow Sphere
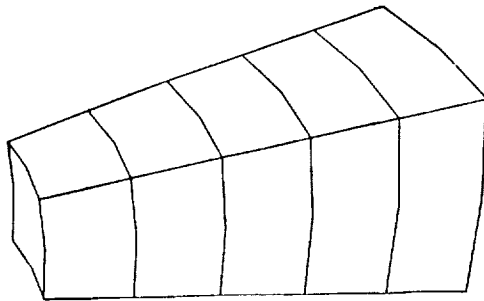
b/a = 2
a = 10
E = 2600

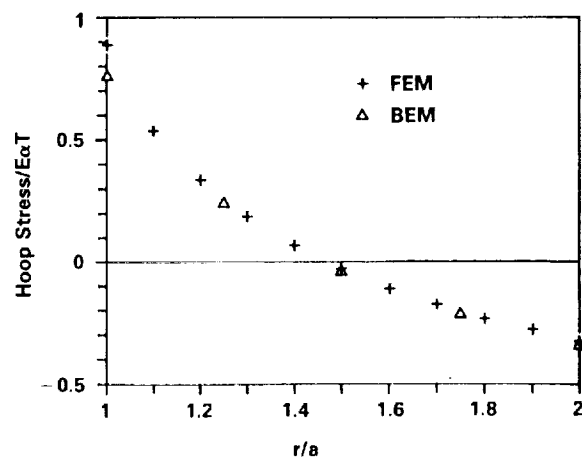$\nu = 0.3$
$\alpha = 10^{-4}$
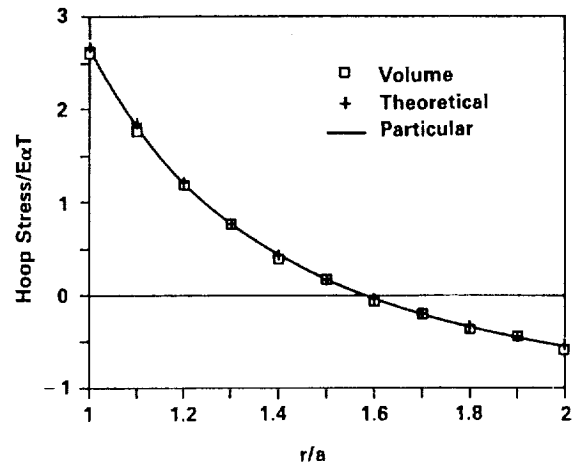
$T = 100 + 10r + r^2 + 0.2r^3$

Temperature-Dependent Thermal
Analysis Procedure Validated

Hollow Cylinder

b/a = 2
a = 10
$E_o = 30 \times 10^6$

$\nu = 0.3$
$\alpha = 10^{-4}$

$T = 10r$

$E = E_o - 10^4 \; T$

Current NESSUS/BEM Code Capabilities

- Linear Elastic Stress Analysis

- Centrifugal Loading Analysis

- Free-Vibration Analysis

- Thermal Analysis

- Thermal Analysis with Temperature-Dependent Material Properties

NESSUS/FEM Future Developments

- Additional Particular Integrals
  (e.g., Steep Thermal Gradient, Linear Temperature Field)

- Elastoplastic Analysis

- Large Deformation Analysis

- Transient Analysis