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AN UNCONDITIONALLY STABLE RUNGE-KUTTA METHOD FOR UNSTEADY ROTOR-STATOR INTERACTION

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A quasi-three-dimensional analysis has been developed for unsteady rotor-stator interaction in turbomachinery. The analysis solves the unsteady Euler or thin-layer Navier-Stokes equations in a body-fitted coordinate system. It accounts for the effects of rotation, radius change, and stream-surface thickness. The Baldwin-Lomax eddy-viscosity model is used for turbulent flows. The equations are integrated in time using an explicit four-stage Runge-Kutta scheme with a constant time step. Implicit residual smoothing is used to increase the stability limit of the time-accurate computations. The scheme is described, and stability and accuracy analyses are given.

Results are shown for the stage of the space shuttle main engine high pressure fuel turbopump. Two stators and three rotors were used to model the 41:63 blade ratio of the actual machine. Implicit residual smoothing was used to increase the time step limit of the explicit scheme by a factor of six with negligible differences in the unsteady results. We feel that the implicitly smoothed Runge-Kutta scheme is easily competitive with implicit schemes for unsteady flows while retaining the simplicity of an explicit scheme.

REFERENCE

Jorgenson, P.C.E.; and Chima, R.V.: An Unconditionally Stable Runge-Kutta Method for Unsteady Flows. AIAA paper 89-0205, Jan. 1989.

UNSTEADY ROTOR-STATOR INTERACTION CODE

DEVELOPED BY P. C. E. JORGENSON & R. V. CHIMA

DESCRIPTION

- UNSTEADY THIN-LAYER NAVIER-STOKES SOLVER FOR ROTOR-STATOR INTERACTION

FEATURES

- QUASI-3-D FORMULATION
  INCLUDES ROTATION, RADIUS CHANGE, & STREAM SURFACE THICKNESS
- C-TYPE GRIDS OVERLAP AT INTERFACE
- MULTI PASSAGE CAPABILITY
- EXPLICIT RUNGE-KUTTA SCHEME + IMPLICIT RESIDUAL SMOOTHING
  UNCONDITIONALLY STABLE (PRACTICAL LIMIT $CFL = O(20)$)
  2nd ORDER TIME ACCURATE
  FIRST USE OF IMPLICIT SMOOTHING FOR UNSTEADY PROBLEMS

RESULTS

- SPACE SHUTTLE MAIN ENGINE TURBINE

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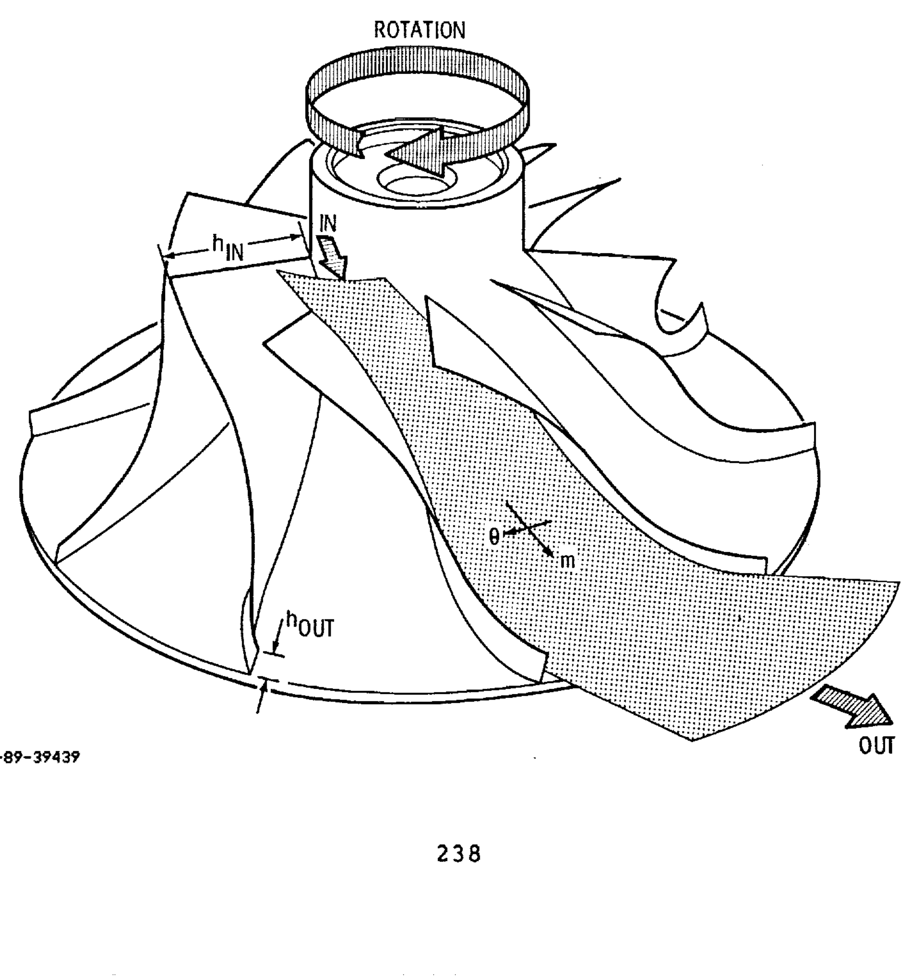
QUASI-THREE-DIMENSIONAL STREAM SURFACE



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QUASI-3-D THIN LAYER NAVIER-STOKES EQUATIONS

$$\partial_t \hat{q} + \partial_\varepsilon \hat{E} + \partial_\eta (\hat{F} - Re^{-1} \hat{S}) = \hat{K}$$

$$\hat{q} = 1/\bar{J} \begin{bmatrix} \rho \\ \rho v_m \\ \rho v_\theta r \\ e \end{bmatrix}, \quad \hat{E} = 1/\bar{J} \begin{bmatrix} \rho W^\varepsilon \\ \rho v_m W^\varepsilon + \varepsilon_m p \\ (\rho v_\theta W^\varepsilon + \bar{\varepsilon}_\theta p) r \\ (e + p) W^\varepsilon + r\Omega \bar{\varepsilon}_\theta p \end{bmatrix} \quad \hat{F} = 1/\bar{J} \begin{bmatrix} \rho W^\eta \\ \rho v_m W^\eta + \eta_m p \\ (\rho v_\theta W^\eta + \bar{\eta}_\theta p) r \\ (e + p) W^\eta + r\Omega \bar{\eta}_\theta p \end{bmatrix} \quad \hat{K} = 1/\bar{J} \begin{bmatrix} 0 \\ K_2 \\ 0 \\ 0 \end{bmatrix} \quad \hat{S} = 1/\bar{J} \begin{bmatrix} 0 \\ S_2 \\ S_3 r \\ S_4 \end{bmatrix}$$

WHERE

$$\left.\begin{aligned} W^\varepsilon &= \varepsilon_m v_m + \bar{\varepsilon}_\theta w_\theta \\ W^\eta &= \eta_m v_m + \bar{\eta}_\theta w_\theta \end{aligned}\right\} = \text{CONTRAVARIANT VELOCITY COMPONENTS IN RELATIVE SYSTEM}$$

$w_\theta = v_\theta - r\Omega$ = RELATIVE TANGENTIAL VELOCITY

Ω = BLADE ROTATION SPEED

$K_2 = (\rho v_\theta^2 + p - Re^{-1} \sigma_{22}) \partial_m r/r + (p - Re^{-1} \sigma_{33}) \partial_m h/h$

= CENTRIFUGAL FORCE TERM + $p\partial_m$ (AREA)

$$\left.\begin{aligned} \bar{\varepsilon}_\theta &= \varepsilon_\theta / r \\ \bar{\eta}_\theta &= \eta_\theta / r \end{aligned}\right. \quad : \theta\text{-METRICS SCALED BY } 1/r$$

$1/\bar{J} = rh/J$: JACOBIAN SCALED BY AREA

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ROTOR-STATOR INTERACTION CODE DETAILS

INITIAL CONDITIONS

ANALYTIC 1-D SOLUTION OF FLOW EQUATIONS WITH AREA CHANGE

BOUNDARY CONDITIONS

INLET

P_0, T_0, AND v_θ SPECIFIED

$R^- = v_m - \dfrac{2a}{\gamma - 1}$ EXTRAPOLATED

EXIT

P SPECIFIED

3 CONSERVATION VARIABLES EXTRAPOLATED

WALLS

INVISCID FLOW = TANGENCY, ρ EXTRAPOLATED

VISCOUS FLOW = NO-SLIP, T_W SPECIFIED

NORMAL MOMENTUM EQUATION FOR PRESSURE

PERIODIC BOUNDARIES

SOLVED LIKE INTERIOR POINTS

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MULTISTAGE RUNGE-KUTTA ALGORITHM

GOVERNING EQUATIONS

$$\partial_t q = -J\,[R_I - (R_V + D)]$$

$R_I =$ INVISCID RESIDUAL

$R_V =$ VISCOUS RESIDUAL

D = ARTIFICIAL DISSIPATION TERM

MULTISTAGE SCHEME

$$q_0 = q_n$$

$$q_1 = q_0 - \alpha_1 J \Delta t\,[R_I\, q_0 - (R_V + D)\, q_0]$$

$$\vdots$$

$$q_k = q_0 - \alpha_k J \Delta t\,[R_I\, q_{k-1} - (R_V + D)\, q_0]$$

$$q_{n+1} = q_k$$

R_V & D EVALUATED AT FIRST STAGE ONLY

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ARTIFICIAL DISSIPATION

NONCONSERVATIVE VERSION OF JAMESON FORMULATION

$$Dq = (D_\xi + D_\eta)\, q$$

ξ-DIRECTION OPERATOR

$$D_\xi q = C\,(V_2 q_{\xi\xi} - V_4 q_{\xi\xi\xi\xi})$$

WHERE:

$$C = \frac{1}{J \Delta t_{i,j}}$$

(LOCAL $\Delta t_{i,j}$ MINIMIZES DISSIPATION)

$$V_2 = \mu_2 \max(\nu_{i+1}, \nu_i, \nu_{i-1})$$

$$V_4 = \max(0, \mu_4 - V_2)$$

AND

$$\nu_{i,j} = \frac{|P_{i+1,j} - 2P_{i,j} + P_{i-1,j}|}{|P_{i+1,j} + 2P_{i,j} + P_{i-1,j}|}$$

$$\mu_2 = O(1)$$

$$\mu_4 = O(\tfrac{1}{16})$$

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IMPLICIT RESIDUAL SMOOTHING

USE A TIME STEP GREATER THAN THE STABILITY LIMIT
MAINTAIN STABILITY BY SMOOTHING THE RESIDUAL IMPLICITLY

REWRITE STAGE (K) OF MULTISTAGE SCHEME AS

$$\Delta q^{(k)} \equiv q^{(k)} - q^{(0)} = -\alpha_k \overline{J} \Delta t \left[Rq^{(k-1)} - Dq^{(0)} \right]$$

IMPLICIT SMOOTHING STEP

$$(1 - \epsilon \delta_{\xi\xi})(1 - \epsilon \delta_{\eta\eta}) \overline{\Delta q^{(k)}} = \Delta q^{(k)}$$

$$q^{(k)} = q^{(0)} + \overline{\Delta q^{(k)}}$$

UNCONDITIONALLY STABLE IF

$$\epsilon \geq \frac{1}{4} \left[\left(\frac{CFL}{CFL^*} \right)^2 - 1 \right]$$

WHERE
CFL^* IS COURANT LIMIT OF THE UNSMOOTHED SCHEME
CFL IS THE LARGER OPERATING COURANT NUMBER

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IMPLICIT RESIDUAL SMOOTHING

TWO-DIMENSIONAL LOCAL SMOOTHING PARAMETER

$$\epsilon_{ij} = max \left\{ 0, \frac{1}{4} \left[\left(\frac{CFL_{ij}}{CFL^*} \right)^2 - 1 \right] \right\}$$

WHERE
CFL^* IS COURANT LIMIT OF THE UNSMOOTHED SCHEME
CFL_{ij} IS THE LOCAL COURANT NUMBER

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IMPLICIT RESIDUAL SMOOTHING

FEATURES

- IRS DOES NOT EFFECT FORMAL TIME ACCURACY OF R-K SCHEME
- SMOOTHING ONLY OCCURS WHERE LOCAL COURANT NUMBER CFL_{ij} IS GREATER THAN THE COURANT LIMIT OF THE R-K SCHEME CFL^* (EG - VISCOUS REGION)
- CAN USE $CFL = O(10-20)$ IN VISCOUS REGIONS WHILE $CFL = O(.1)$ IN INVISCID CORE
- ϵ_{ij} EASY TO COMPUTE, OR CAN BE STORED
- IRS REQUIRES SCALAR TRIDIAGONAL INVERSION ALONG EACH GRID LINE
  ADDS APPROXIMATELY 26% TO CPU TIME BUT BOOSTS TIME STEP BY 652% FOR A NET DECREASE OF 460% IN CPU TIME
- EXPLICIT R-K SCHEME WITH IRS COMPETETIVE WITH IMPLICIT SCHEMES FOR UNSTEADY PROBLEMS

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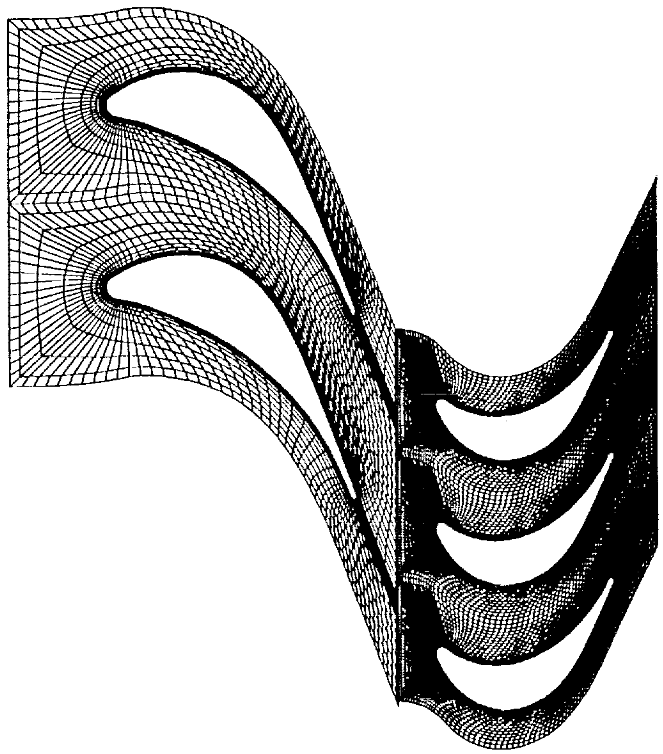
SSME STATOR AND ROTOR GRIDS



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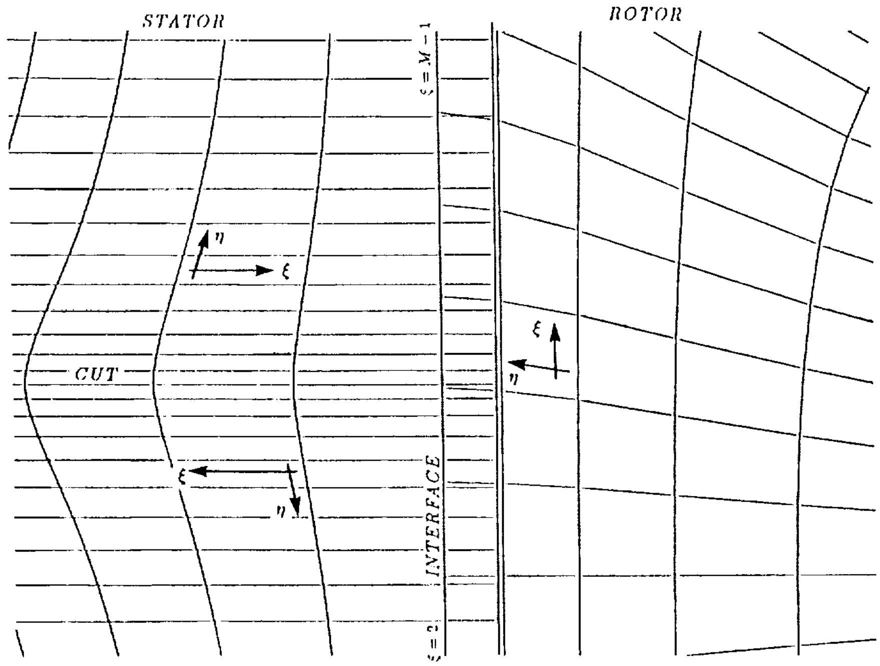
DETAIL OF STATOR/ROTOR GRID OVERLAP



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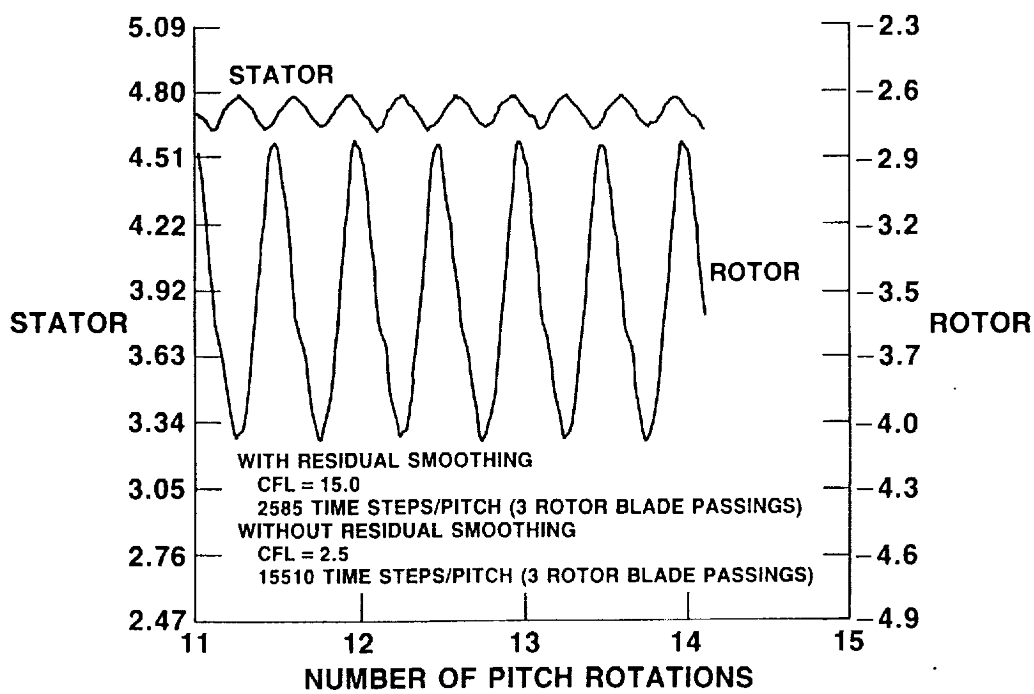
UNSTEADY LIFT COEFFICIENTS ON STATOR AND ROTOR



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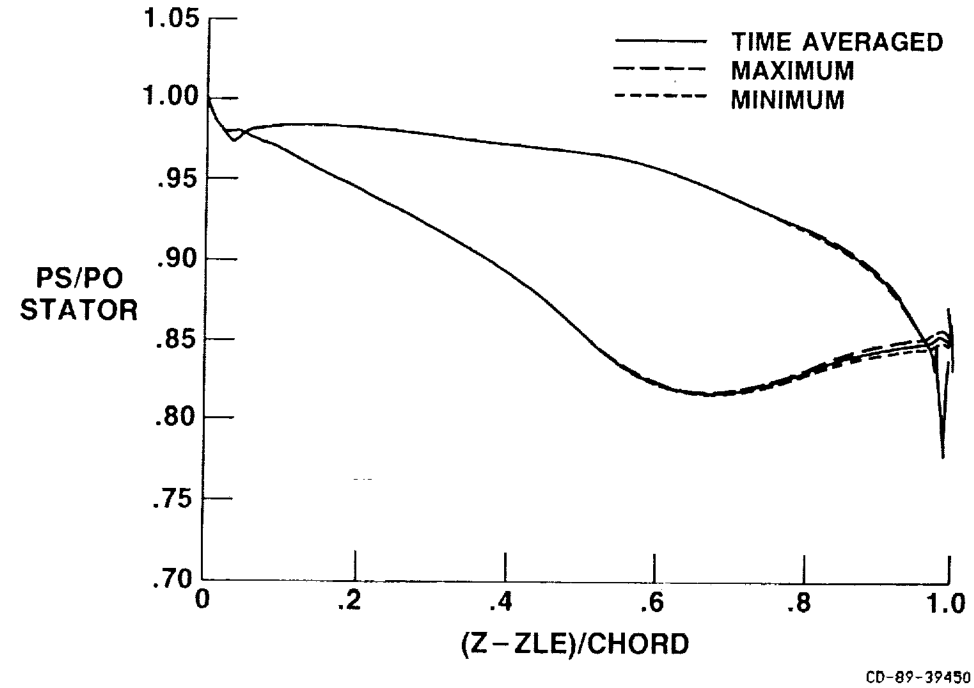
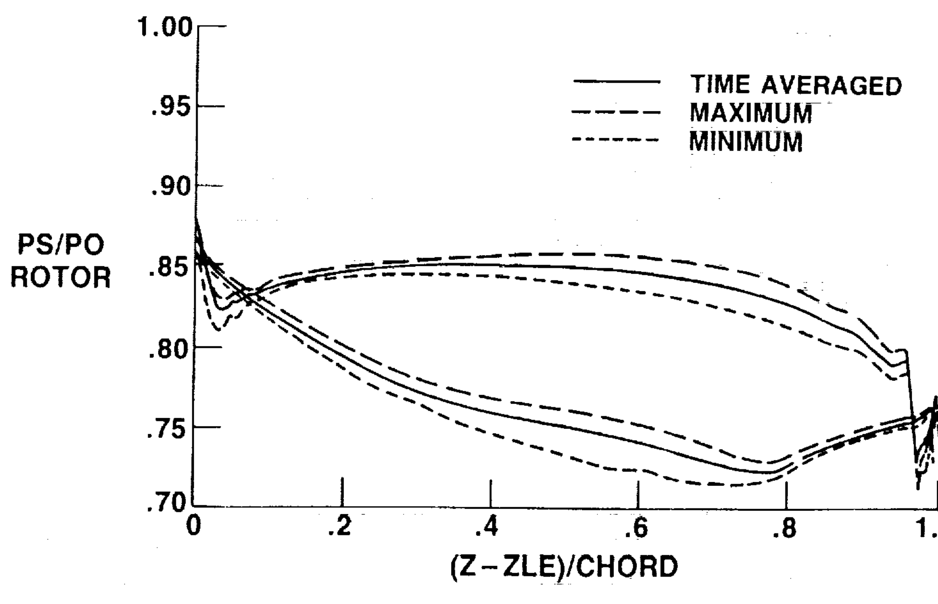
PRESSURE DISTRIBUTION ENVELOPE ON STATOR



PRESSURE DISTRIBUTION ENVELOPE ON ROTOR



ABSOLUTE MACH NUMBER CONTOURS

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ENTROPY CONTOURS

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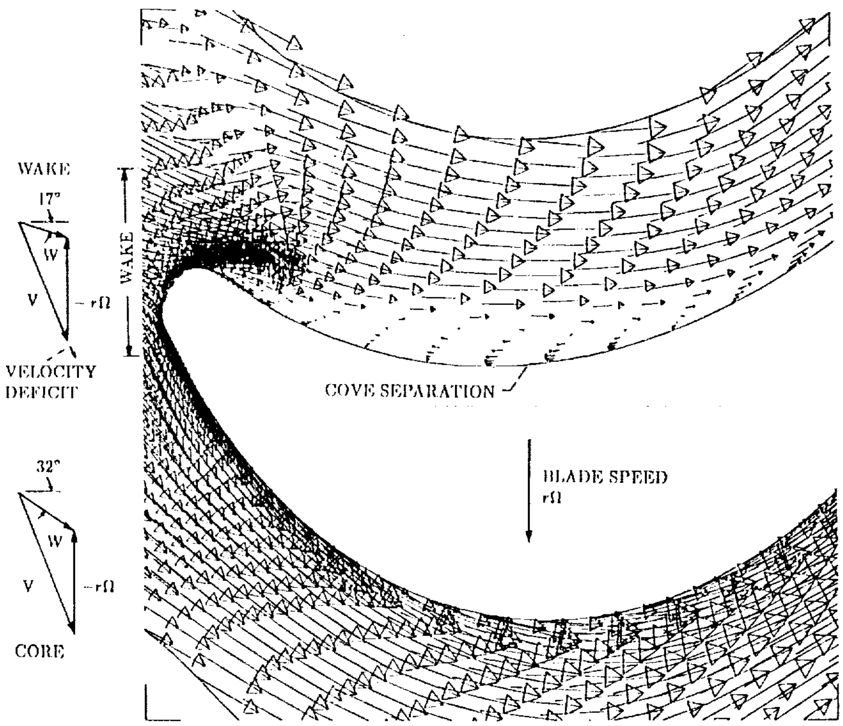
VELOCITY VECTORS AROUND ROTOR



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UNSTEADY ROTOR/STATOR INTERACTION CODE

CONCLUDING REMARKS

- TIME-ACCURATE ROTOR/STATOR INTERACTION CODE HAS BEEN DEVE OPED

- CODE IS APPLICABLE TO AXIAL OR RADIAL MACHINES

- VISCOUS RESULTS FOR SSME TURBINE SHOW
  UNSTEADY LOADING & SEPARATION, & MIGRATION OF STATOR WAKES

- IMPLICIT RESIDUAL SMOOTHING
  DOES NOT EFFECT FORMAL TIME ACCURACY OF R-K SCHEME

  ADDS 26% TO CPU TIME
  INCREASED TIME STEP BY A FACTOR OF 6.0
  RESULTS IN A NET DECREASE OF ABOUT 450% IN CPU TIME

  MAKES EXPLICIT R-K SCHEME COMPETETIVE WITH IMPLICIT SCHEMES
  FOR UNSTEADY PROBLEMS

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