Fermi National Accelerator Laboratory

FERMILAB-Pub-91/136-A
May 1991

NAGW-1340
IN-72-CR
19356
P-15

DIRAC NEUTRINOS AND SN 1987A

Michael S. Turner

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Batavia, IL 60510-0500
and
Departments of Physics and Astronomy & Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637-1433

Abstract. Previous work has shown that the cooling of SN 1987A excludes a Dirac-neutrino mass greater than $\mathcal{O}(20\,\mathrm{keV})$ for ν_e, ν_μ, or ν_τ. We re-examine the emission of wrong-helicity, Dirac neutrinos from SN 1987A, and conclude that due to neutrino degeneracy and additional emission processes ($N + N \to N + N + \nu\bar{\nu}$, $\pi^- + p \to n + \nu\bar{\nu}$) the effect of a Dirac neutrino on the cooling of SN 1987A has been *underestimated.* We believe that the limit that follows from the cooling of SN 1987A is better—probably much better—than 10 keV. This result is significant in light of the recent evidence for a 17 keV mass eigenstate that mixes with the electron neutrino.

(NASA-CR-188448) DIRAC NEUTRINOS AND SN 1987A (Fermi National Accelerator Lab.) 15 p CSCL 20H

N91-24865

Unclas
G3/72 0019356

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. Introduction

Very-weakly interacting particles can be produced in the core of a newly born, hot neutron star, carry away energy, and accelerate the initial cooling process. Their observable effect is the shortening of the duration of the neutrino burst associated with the early cooling phase—and of course, the neutrino burst from SN 1987A was detected by the Kamiokande II (KII) and Irivine-Michigan-Brookhaven (IMB) water Cherenkov detectors. The potential shortening of the neutrino burst associated with SN 1987A has been used to severely constrain the properties of axions,[1] "righthanded" neutrinos, and other weakly interacting particles. Our interest here is in Dirac neutrinos; several authors have argued that a Dirac mass for any of the three neutrinos in the range of $\mathcal{O}(20\,\mathrm{keV})$ to $\mathcal{O}(300\,\mathrm{keV})$ is excluded by SN 1987A.[2]

The key to the argument involves the additional helicity states a Dirac neutrino has. While a majorana neutrino has but two helicity states: negative helicity (ν_-), which corresponds to the neutrino state, and positive helicity (ν_+), which corresponds to the antineutrino state, a Dirac neutrino has four: two helicity states associated with the neutrino (ν_-, ν_+) and two associated with the antineutrino ($\bar{\nu}_-$, $\bar{\nu}_+$). For a massless neutrino the helicity states, the eigenstates of a freely propagating neutrino, coincide with the chirality states (left and right), the weak-interaction eigenstates of the neutrino: $\nu_L = \nu_-$ and $\nu_R = \nu_+$ (or $\bar{\nu}_+$). In this case the additional, "wrong-helicity" states of a Dirac neutrino have no interactions—and are irrelevant.

The situation changes if the neutrino has mass: the chirality and helicity states no longer coincide. In the ultrarelativistic limit the projection of ν_- ($\bar{\nu}_+$) onto ν_L ($\bar{\nu}_R$) is order unity, and these helicity states have ordinary weak interactions. On the other hand, the wrong-helicity states, ν_+ ($\bar{\nu}_-$), have but a small projection, order $m_\nu/2E_\nu$, on to the chirality states ν_L ($\bar{\nu}_R$), and to a first approximation are sterile. Owing to their small projection onto the weak-interaction chirality states they can be produced through ordinary weak interactions ("spin-flip" production): $\nu_- \to \nu_+$ or $\bar{\nu}_+ \to \bar{\nu}_-$. Of course it is also possible that the wrong-helicity states have other, new interactions (e.g., righthanded interactions). We will not address that possibility here.

(Once produced, wrong-helicity neutrinos do interact through their projections onto the proper-helicity states; however for $m_\nu \lesssim 300\,\mathrm{keV}$, the mean free path for such interactions is large compared to the size of a neutron star. For $m_\nu \gtrsim 300\,\mathrm{keV}$ wrong-helicity neutrinos should become trapped like their proper-helicity counterparts, and for a sufficiently large mass, their effect on the cooling will be comparable to that of the proper-helicity state neutrinos. The mass at which trapping is sufficient to make a Dirac species "supernova safe" must be greater than 300 keV, and an accurate determination of this mass requires a careful treatment of wrong-helicity neutrino transport. This is a formidable task. For our purposes it suffices to say that the value of the "supernova safe" mass must certainly be greater than 300 keV, the mass where trapping sets in, and that for $m \lesssim 300\,\mathrm{keV}$

wrong-helicity neutrinos once produced stream out.)

The most detailed study of the effect of Dirac neutrinos on the cooling of SN 1987A is that of Gandhi and Burrows.[3] In numerical models of the early cooling of SN 1987A they included the cooling effect of wrong-helicity neutrinos produced by the spin-flip-scattering processes, $\nu_- + N \to \nu_+ + N$ and $N + \bar{\nu}_+ \to N + \bar{\nu}_-$. For neutron-star models cooled by both proper- and wrong-helicity neutrinos they computed the flux of proper-helicity neutrinos and the response of the KII and IMB detectors to this flux. They concluded that the duration of the detected neutrino bursts exclude a Dirac mass greater than about 14 keV. In fact, their mass limit was extremely conservative; the effect of 14 keV Dirac neutrino was to reduce the burst duration expected to less than about 1 sec in either detector. Had one instead insisted that the neutrino burst duration expected be no shorter than half the duration of the actual burst, their limit would have been about 9 keV.

On the face of it their work seems to preclude a Dirac neutrino of mass 17 keV for example. Of course 17 keV is a very interesting mass since several β-decay experiments have found evidence for a 17 keV neutrino-mass eigenstates that mixes with the electron neutrino at the 1% level ($\sin^2\theta \simeq 0.01$).[4] Moreover, the absence of neutrinoless double-β decay in several isotopes strongly suggests that the 17 keV mass eigenstate is of the Dirac type. Unfortunately, Gandhi and Burrows[3] recently discovered a simple factor of four error in the rate they used for the emission of wrong-helicity neutrinos, which has the effect of *doubling* their mass limit—raising their original limit to 28 keV (and the less conservative limit that one could derive from their results to about 18 keV). The motivation for re-examining the emission of wrong-helicity neutrinos from SN 1987A hardly needs to be mentioned.

To summarize our results briefly, we find that due to a number of effects the volume emissivity ($\mathrm{erg\,cm^{-3}\,sec^{-1}}$) of wrong-helicity neutrinos is at least as large as—and probably much larger than—that used originally by Gandhi and Burrows, implying that their original "conservative limit" of 14 keV stands. In particular the production of wrong-helicity neutrinos due to nucleon-nucleon, neutrino-pair bremsstrahlung is at least as important as spin-flip-scattering production. Since the cores of neutron stars are on the verge of pion condensation negative pions are likely to be present in great numbers. If this is the case, the process $\pi^- + p \to n + \nu\bar{\nu}$ is likely to be even more important than the bremsstrahlung process (although less certain since the pion density depends critically upon the equation of state). Finally and probably most importantly, if there is significant mixing between the massive Dirac-neutrino (greater than few 0.1%), then deep in core of the neutron star the mass Dirac-neutrino species is degenerate (with chemical potential $\mu_\nu \gtrsim 200\,\mathrm{MeV}$) rather than nondegenerate as previously assumed; when this fact is taken into account the rate for wrong-helicity neutrino emission increases by a factor of order $(\mu_\nu/T)^4 \sim 10^4$.

While it is premature to quote a definitive limit to the mass of a Dirac neutrino based upon the cooling of SN 1987A, it seems clear that when all of the additional effects discussed here are incorporated into detailed numerical models of the early phase of neutron-star

cooling the mass limit will be more stringent than 10 keV, probably more like 1 keV.

II. Spin-flip-scattering Production of Wrong-helicity Neutrinos

Nondegenerate neutrinos

Positive-helicity neutrinos (and negative-helicity antineutrinos) are produced by the helicity-flip scattering processes $\nu_- + N \to \nu_+ + N$ and $\bar{\nu}_+ + N \to \bar{\nu}_- + N$, where N is a nucleon. The matrix-element squared for this process has been computed by Gaemers et al.,[5]

$$|\mathcal{M}_{\rm SF}|^2 = 8G_F^2 m_\nu^2 m^2 \left[(c_V^2 + 3c_A^2) - (c_V^2 - c_A^2)\cos\theta\right], \tag{1}$$

where $|\mathcal{M}_{\rm SF}|^2$ has been summed over initial and final nucleon spins, θ is the angle between the incoming and outgoing neutrinos, m is the nucleon mass, m_ν is the Dirac-neutrino mass, $G_F \simeq 1.17 \times 10^{-5}\,{\rm GeV}^2$ is the Fermi constant, and $c_V(p) = (1 - 4\sin^2\theta_W)/2 \simeq 0$, $c_A(p) \simeq g_A/2$, $c_V(n) = -1/2$, $c_A(n) \simeq -g_A/2$, and $g_A = 1.26$. Unless stated otherwise we work in units where $\hbar = k_B = c = 1$.

The volume emissivity ($\rm erg\,cm^{-3}\,sec^{-1}$) of wrong-helicity neutrinos is given by

$$\dot{\varepsilon}_{\rm SF} = \int |\mathcal{M}_{\rm SF}|^2 (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3}$$

$$\times \frac{d^3k_1}{2k_1(2\pi)^3} \frac{d^3k_2}{2k_2(2\pi)^3} f_1(1 - f_2) f_\nu k_2, \tag{2}$$

where p_i is the four momentum of the incoming ($i = 1$)/outgoing ($i = 2$) nucleon, k_i is the four momentum of the incoming ($i = 1$)/outgoing ($i = 2$) neutrino, $f_\nu = [\exp(k_1/T) + 1]^{-1}$ is the phase-space distribution function of the incoming neutrino, and $f_i = [\exp(E_i/T - \mu_i/T) + 1]^{-1}$ are the phase-space distribution functions of the nucleons. Note that we have allowed for nucleon degeneracy, but we have assumed that neutrinos are nondegenerate (in the standard scenario, a good assumption for ν_μ and ν_τ, but not ν_e). Shortly we will return to the important issue of neutrino degeneracy.

Making three reasonable assumptions this 12-dimensional integral can be reduced to a single integral. They are: (1) nonrelativistic nucleons; (2) negligible three momentum carried by the neutrinos (compared to the nucleons); and (3) incoming and outgoing neutrinos have the same energy (elastic scattering). The volume emissivity for the process $\nu_- + N \to \nu_+ + N$ can then be written as

$$\dot{\varepsilon}_{\rm SF} = \frac{G_F^2 m_\nu^2 (c_V^2 + 3c_A^2)\rho_\nu}{4\pi} \left\{ \frac{\sqrt{2}(mT)^{3/2}}{\pi^2} \frac{\partial}{\partial y} \int_0^\infty \frac{\sqrt{u}\,du}{e^{u-y} + 1} \right\}, \tag{3}$$

where $y = (\mu - m)/T$ and $\rho_\nu = 7\pi^2 T^4/240$ is the energy density in thermal neutrinos. In the nondegenerate limit, $y \ll -1$, the final term reduces to n_N. Taking the nondegenerate

limit and taking into account the antineutrino process as well, the total volume emissivity becomes

$$\dot{\varepsilon}_{\rm SF} = 1.2 \times 10^{32} m_{100}^2 \rho_{14} T_{10}^4 \, [0.9 + 0.2 X_n] \, {\rm erg\,cm^{-3}\,sec^{-1}}, \tag{4}$$

where m_{100} is the neutrino mass in units of 100 keV, ρ_{14} is the total mass density in units of $10^{14}\,{\rm g\,cm^{-3}}$, T_{10} is the temperature in units of 10 MeV, and X_n is the neutron fraction (the proton fraction $X_p = 1 - X_n$). For arbitrary nucleon degeneracy,

$$\dot{\varepsilon}_{\rm SF} = \frac{7 G_F^2 m_\nu^2 m^{3/2} T^{11/2}}{2^{9/2} \cdot 15 \cdot \pi} \, [1.2 I(y_p) + 1.4 I(y_n)] \,, \tag{5a}$$

$$\simeq 2.6 \times 10^{31} \, m_{100}^2 \, (m/0.94\,{\rm GeV})^{3/2} \, T_{10}^{11/2} \, [1.2 I(y_p) + 1.4 I(y_n)] \; {\rm erg\,cm^{-3}\,sec^{-1}}, \tag{5b}$$

where $I(y) \equiv (\partial/\partial y) \int_0^\infty \sqrt{u} du / [\exp(u - y) + 1]$ and we have displayed explicitly the dependence upon the nucleon mass m because the effective nucleon mass in nuclear matter is expected to be reduced by a factor of order 1/2. In the nondegenerate limit $\dot{\varepsilon}_{\rm SF}$ does not depend upon the value of the nucleon mass, cf. Eq. (3); in the degenerate limit $I(y) \propto y^{1/2}$ and $y \propto m^{-1}$, so that $\dot{\varepsilon}_{\rm SF} \propto m^{1/2}$. The conditions at the core of the neutron star are expected to be closer to nondegenerate than degenerate and $\dot{\varepsilon}_{\rm SF}$ should be insensitive to the effective value of the nucleon mass.

The following is a simple fit to $I(y)$ that is accurate to better than 12% (typically accurate to a few %):

$$I_{\rm fit}(y)^{-1} = \frac{2 e^{-y}}{\sqrt{\pi}} + \frac{1}{\sqrt{1 + |y|}} - \frac{1}{8(1 + |y|)^{3/2}}.$$

Before going on to consider neutrino degeneracy, we should compare our expression for $\dot{\varepsilon}_{\rm SF}$ with that used by Burrows and Gandhi.[3] After correcting the spin-flip cross section in their paper for the errant factor of four, we find that our volume emissivity (for $X_n = 1/2$) is a factor of about 1.9 larger than theirs. Most of the difference traces to one fact: We use $|c_A(n,p)| = g_A/2 = 0.63$ and they use 0.5. We believe that $g_A/2$ is the appropriate value to use.[6] The remaining discrepancy appears to involve round off (the spin-flip cross section used by Gandhi and Burrows is only given to one significant figure).

Neutrino degeneracy

At the core of a newly born hot neutron star, where the density is several times nuclear density and the temperature is of order 30 MeV, electrons are highly degenerate with $\mu_e \simeq p_F(e) \simeq 240\,(X_p \rho_{14})^{1/3}$ MeV. On the other hand, neutrons and protons are only semi-degenerate, with $\mu_n, \mu_p \sim \mathcal{O}(30\,{\rm MeV})$. From this it follows that electron neutrinos are highly degenerate, as β-equilibrium ($n + \nu_e \leftrightarrow p + e^-$) enforces: $\mu_{\nu_e} = \mu_e + \mu_p - \mu_n \simeq \mu_e$. In the absence of interactions that interconvert neutrinos of different flavor, μ and τ neutrinos should be nondegenerate. Since we know that electron-neutrino mass is less than about 10 eV (more precisely, the mass of the dominant mass eigenstate associated with ν_e), only the

production of wrong-helicity μ and τ neutrinos is of interest in setting a Dirac mass limit. In the absence of flavor-changing interactions they should be nondegenerate, justifying the previous assumption of nondegenerate neutrinos.

However, mixing changes the story. If there is mixing between ν_e and ν_μ, ν_τ, then μ and τ neutrinos can become degenerate through $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\tau$ oscillations. Since the matter-oscillation length of a neutrino is much less than its mean free path between weak interactions, the probability that an electron neutrino "next interacts as a μ (or τ) neutrino" is simply $\sin^2 2\theta_m/2$, where θ_m is the mixing angle in matter between ν_e and ν_μ (or ν_τ). The matter mixing angle is related to the vacuum mixing angle by:[7]

$$\sin 2\theta_m \simeq \sin 2\theta_0 \min\left[1,\ \Delta_0/V\right], \qquad \Delta_0/V \simeq 0.05 \left(\frac{\delta m^2}{10^8\,\mathrm{eV}^2}\right)\left(\frac{100\,\mathrm{MeV}}{E_\nu}\right)\left(\frac{3}{\rho_{14}}\right);$$

$$\Delta_0 = \frac{\delta m^2}{2E_\nu} \qquad V \simeq 3\,\rho_{14}\,\mathrm{eV},$$

where E_ν is the neutrino energy, δm^2 is the difference of the mass squared between the two mass eigenstates, and V is the (weak-interaction) energy difference associated with the interaction of electron neutrinos, and μ and τ neutrinos with the background (neutrons, protons, electrons, positrons, and neutrinos in the core of the neutron star). (Note, by using the $\min[1, \Delta_0/V]$ we have not allowed for the possibility of resonant conversion; it would only further enhance neutrino mixing as $\sin^2 2\theta_m \to 1$ in this case.)

If neutrino mixing is effective, then chemical equilibrium will be established between the two (or three) neutrino species: μ_{ν_μ} (and μ_{ν_τ}) $= \mu_{\nu_e}$. One can estimate the rate at which neutrino mixing populates a degenerate sea of μ and τ neutrinos: $\Gamma \sim \Gamma_\nu \sin^2 2\theta_m/2$, where $\Gamma_\nu \sim n_N G_F^2 E_\nu^2/\pi$ characterizes the rate at which neutrinos scatter off nucleons (the dominant neutrino interaction). Assuming $\Delta_0/V \lesssim 1$, we estimate that the time required to populate a degenerate sea of μ (or τ) neutrinos is

$$\tau \sim \Gamma^{-1} \sim 10^{-3}\,\mathrm{sec}\left(\frac{\rho_{14}}{3}\right)\left(\frac{0.01}{\sin^2 2\theta_0}\right)\left(\frac{10^8\,\mathrm{eV}^2}{\delta m^2}\right).$$

That is, the mixing of a μ or τ neutrino of mass of order 10 keV with the electron neutrino at the 1% level is sufficient to very rapidly populate a degenerate sea of μ or τ neutrinos in the core of a hot neutron star. (Rapid here means much less than the cooling time of the neutron star: $\tau \lesssim 1$ sec). Note that neutrino oscillations mix flavors, but not helicity states, and so the degenerate sea of massive Dirac neutrinos filled by neutrino oscillations are proper-helicity neutrinos. The wrong-helicity neutrinos must still be produced by spin-flip processes.

Degeneracy of μ (and/or τ) neutrinos will of course modify the "chemical composition" of the neutron star. While chemical equilibrium enforces $\mu_{\nu_e} = \mu_{\nu_\mu} = \mu_{\nu_\tau}$ and $\mu_{\nu_e} = \mu_p - \mu_n + \nu_e$, charge conservation, $n_e = n_p$, and lepton-number conservation, $n_L - n_{\bar{e}} + n_{\nu_e} - n_{\bar{\nu}_e} + n_{\nu_\mu} - n_{\bar{\nu}_\mu} + n_{\nu_\tau} - n_{\bar{\nu}_\tau} = n_e - n_{\bar{e}} + 3(n_{\nu_e} - n_{\bar{\nu}_e}) = \mathrm{const}$, must also be observed.

Together these conditions determine all the chemical potentials. Qualitatively we can see that additional protons will have to decay to supply the additional neutrinos in the degenerate μ and τ seas: this will increase μ_n and decrease μ_p, μ_e, and μ_ν. Since $n \propto \mu^3$ for a highly degenerate species, the neutrino and electron chemical potentials should not change by a large amount. However, a careful treatment of the effect of a massive Dirac neutrino that mixes with the electron neutrino on the initial cooling of a hot neutron star must take this into account. For our purposes we will assume that $\mu_e \sim \mu_\nu \sim 200\,\mathrm{MeV}$.

If the massive Dirac neutrinos are degenerate then our calculation of $\dot{\epsilon}_{\rm SF}$ must be revised: The neutrino distribution function must be changed to $f_\nu = [\exp(E_\nu/T - \mu_\nu/T) + 1]^{-1}$. (No blocking factor need be added for the final-state neutrino since it is a wrong-helicity neutrino.) Making the same approximations as before, Eq. (3) is unchanged; however, the energy density of the neutrino ρ_ν is now

$$\rho_\nu = \frac{T^4}{2\pi^2}\int_0^\infty \frac{u^3 du}{e^{u-y}+1} \longrightarrow \frac{\mu^4}{8\pi^2} \qquad (\text{for } \mu \gg T),$$

where $y = \mu_\nu/T$. In the highly degenerate limit, the energy density of neutrinos is much larger, order μ^4 rather than order T^4—there are more neutrinos and they have higher energies—and the volume emissivity is increased relative to the previous result, cf. Eq. (3), by a factor of

$$\frac{30}{7\pi^4}\frac{\mu^4}{T^4} \simeq 7\times 10^3 (\mu/200\,\mathrm{MeV})^4/T_{10}^4.$$

In the highly degenerate limit the process involving antineutrinos is severely suppressed as $\mu_{\bar\nu} = -\mu_\nu$, implying that

$$\rho_{\bar\nu} = \frac{T^4}{2\pi^2}\int_0^\infty \frac{u^3 du}{e^{u+y}+1} \ll \rho_\nu.$$

Bringing everything together, if we allow the neutrino sea associated with the massive Dirac neutrino to be degenerate,

$$\dot{\epsilon}_{\rm SF} = \frac{G_F^2 m_\nu^2 m^{3/2} T^{11/2}}{2^{5/2}\cdot\pi^5}\,[1.2I(y_p) + 1.4I(y_n)]\left\{\int u^3 du\left(\frac{1}{e^{u-y}+1} + \frac{1}{e^{u+y}+1}\right)\right\}, \quad (6a)$$

$$\longrightarrow 9.2\times 10^{34}\, m_{100}^2\,(m/0.94\,\mathrm{GeV})^{3/2}\,(\mu_\nu/200\,\mathrm{MeV})^4\, T_{10}^{3/2}$$
$$\times[1.2I(y_p) + 1.4I(y_n)]\ \mathrm{erg\,cm^{-3}\,sec^{-1}} \quad (\text{for } \mu_\nu \gg \mathrm{T}), \quad (6b)$$

Note that the effect of neutrino degeneracy is always to increase $\dot{\epsilon}_{\rm SF}$, because $(\rho_\nu + \rho_{\bar\nu})$ achieves its minimum for $\mu_\nu = 0$.

Provided that the massive Dirac neutrino is degenerate, the volume emissivity of wrong-helicity neutrinos is increased by a factor of $\mathcal{O}(10^4)$, which naively should improve the mass limit by a factor of 100. Holding the mixing angle fixed at 1%, we see that the timescale for populating the degenerate sea becomes of order 10^{-1} sec for a mass of 1 keV, in which

case there is barely enough time to populate the degenerate sea of massive Dirac neutrinos. The SN 1987A mass limit clearly depends upon the mixing angle, and for 1% mixing it should be about 1 keV.

III. Bremsstrahlung-pair Production of Wrong-helicity Neutrinos

Wrong-helicity neutrinos can be produced through another spin-flip process: nucleon-nucleon, neutrino-pair bremsstrahlung, $N + N \to N + N + \nu\bar{\nu}$, where $\nu\bar{\nu} = \nu_-\bar{\nu}_-$ or $\nu_+\bar{\nu}_+$ and N is a nucleon. The rate for this process can be found by using the matrix-element squared calculated by Friman and Maxwell[8] and the phase-space volume calculated by Brinkmann and Turner.[9]

The volume emissivity for $N + N \to N + N + \nu_+ + \bar{\nu}_+$ is given by

$$\dot{\varepsilon}_{\rm BREM} = \int S|\mathcal{M}|^2(2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4 - q_1 - q_2)$$

$$\times \frac{d^3p_1}{2E_1(2\pi)^3}\cdots\frac{d^3p_4}{2E_4(2\pi)^3}\frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3} f_1 f_2(1-f_3)(1-f_4)\omega_1, \tag{7}$$

where p_i are the four momenta of the nucleons, $f_i = [\exp(E_i/T - \mu_i/T) + 1]^{-1}$ are the nucleon phase-space distribution functions, $q_{1,2}$ are the four momenta of the neutrino/antineutrino, $\omega_{1,2}$ are the energies of the neutrino/antineutrino, ω_1 is the energy of the wrong-helicity neutrino, S is the symmetry factor (a factor of 1/2! for any pair of identical particles in the initial or final state), and the matrix-element squared is to be summed over initial and final nucleon spins. To begin we will assume that neutrinos are nondegenerate.

Friman and Maxwell[8] have calculated the matrix element for the ordinary (no spin flip) pair-production bremsstrahlung process in the one-pion exchange approximation; their matrix element can be used to obtain the matrix element for the spin-flip process by multiplying by $m_\nu/2\omega_1$. Doing so and pulling out the only factor in the matrix element that depends upon the neutrino energies the desired matrix element can be written as

$$|\mathcal{M}|^2 = |\mathcal{M}'_{\rm FM}|^2 \left(\frac{m_\nu}{2\omega_1}\right)^2 \left(\frac{\omega_1\omega_2}{\omega^2}\right), \tag{8}$$

where $\omega = \omega_1 + \omega_2$ is the total energy carried off by the neutrino and antineutrino.

There are actually three different bremsstrahlung processes: neutron-neutron, proton-proton, and neutron-proton. The matrix elements for the first two are the same. Calculating the matrix-element squared is a tedious process involving the square of the sum of eight different diagrams (four direct and four exchange). Friman and Maxwell[8] have computed the square of the sum of the direct diagrams and the square of the sum of the exchange diagrams, but not the interference term. From previous experience with nucleon-nucleon, axion bremsstrahlung,[9] for which the matrix element has a similar structure, we know that in the nondegenerate limit the interference term is very small, while in the degenerate limit

the contribution of the interference term *increases* $|\mathcal{M}|^2$ by about 50% (over the incoherent sum of the direct and exchange terms). Based on this we will ignore the interference terms (thereby likely *underestimating* the matrix element in the degenerate limit). The two matrix-elements squared (and summed over nucleon spins) are

$$|\mathcal{M}'_{\rm FM}({\rm nn,pp})|^2 = 2\cdot 2^{10} G_F^2 g_A^2 f^4 \left(\frac{m}{m_\pi}\right)^4 \qquad |\mathcal{M}'_{\rm FM}({\rm np})|^2 = 3\cdot 2^{10} G_F^2 g_A^2 f^4 \left(\frac{m}{m_\pi}\right)^4,$$

where $G_F = 1.17\times 10^{-5}\,{\rm GeV}^{-2}$ is the Fermi constant, $g_A \simeq 1.26$ is the axial vector coupling constant, $f \sim 1.1$ is the pion-nucleon coupling, m is the nucleon mass, and $m_\pi \simeq 0.135\,{\rm GeV}$ is the pion mass. Note: (1) we have already factored out the neutrino-energy dependence from the matrix-element squared; and (2) as presented, the Friman-Maxwell matrix-element squared must be multiplied by a factor of $(2m)^4$ because of their nucleon-spinor normalization convention.

Now the phase-space volume integration. Brinkmann and Turner[9] have evaluated the five-particle phase-space volume element for nucleon-nucleon, axion bremsstrahlung for arbitrary nucleon degeneracy with the following assumptions: (1) nonrelativistic nucleons; (2) negligible axion three-momentum (compared to that of the nucleons); and (3) constant matrix element. As we will see very shortly the six-particle phase-space integral needed here can be reduced to the very same phase-space integral. For the neutrino bremsstrahlung case the analogous assumptions are: (1) nonrelativistic nucleons; and (2) negligible neutrino three momenta. In the axion case the integral over the axion's momentum in the expression for $\dot{\varepsilon}_a$ that is analogous to Eq. (6) can be reduced to a single integral over the axion's energy:

$$\int \frac{d^3q_a}{2E_a(2\pi)^3} E_a = \frac{1}{4\pi^2}\int E_a^2 dE_a.$$

In the present circumstance the integral over the momenta of the neutrino pair can be reduced to an integral over the sum of their energies:

$$\int \frac{d^3q_1}{2\omega_1(2\pi)^3}\frac{d^3q_2}{2\omega_2(2\pi)^3}\frac{\omega_1\omega_2}{\omega^2}\left(\frac{m_\nu}{2\omega_1}\right)^2 \omega_1 = \left(\frac{1}{4\pi^2}\right)^2 \frac{m_\nu^2}{48}\int \omega^2 d\omega.$$

Thus, by simply multiplying the results of Brinkmann and Turner[9] for the phase-space integrals over $p_1,\cdots,p_4$ and E_a by a factor of $m_\nu^2/192\pi^2$ we can obtain the phase-space integration needed here.

The axion phase-space integral can be expressed as

$$\int \frac{d^3p_1}{2E_1(2\pi)^3}\cdots\frac{d^3p_4}{2E_4(2\pi)^3}\frac{E_a^2}{4\pi^2}dE_a(2\pi)^4\delta^4(p_1+p_2-p_3-p_4-q_a)f_1f_2(1-f_3)(1-f_4)$$

$$= \begin{cases} m^{1/2}T^{13/2}\exp(y_1+y_2)/140\pi^{13/2} & \text{(nondegenerate limit)},\\ m^{1/2}T^{13/2}I(y_1,y_2) & \text{(in general)},\end{cases} \tag{9}$$

where $y_i \equiv (\mu_i - m)/T$, and $I(y_1, y_2)$ is a (different) dimensionless function that must be evaluated numerically (see below). In the nondegenerate limit,

$$n_i = 2\left(\frac{mT}{2\pi}\right)^{3/2} e^{y_i},$$

where n_i is the number density of species i (i = neutron or proton).

To begin, consider the nondegenerate limit, a reasonable approximation to the conditions that pertain.[9] In this circumstance the volume emissivity can be expressed as,

$$\dot{\varepsilon}_{\rm BREM} = \frac{16 g_A^2 f^4}{105\pi^{11/2}} \frac{G_F^2 m_\nu^2 m^{3/2} T^{7/2}}{m_\pi^4} f(X_n) n_N^2, \tag{10a}$$

$$\dot{\varepsilon}_{\rm BREM} \simeq 1.5 \times 10^{31}\, f(X_n) \rho_{14}^2 T_{10}^{7/2} m_{100}^2 \ {\rm erg\,cm^{-3}\,sec^{-1}}, \tag{10b}$$

where n_N is the total nucleon density, X_n is again the neutron fraction, the function $f(X_n) = 0.5 + 2X_n(1 - X_n)$ varies between 0.5 (for $X_n = 0, 1$) and 1.0 ($X_n = 1/2$), and we have included a factor of two to account for both the process where the neutrino has the wrong helicity and the one where the antineutrino has the wrong helicity. We can compare this energy-loss rate to that from neutrino-nucleon spin-flip scattering (taking the nondegenerate limit for both):

$$\frac{\dot{\varepsilon}_{\rm BREM}}{\dot{\varepsilon}_{\rm SF}} \simeq 0.14 \frac{\rho_{14}}{T_{10}^{1/2}} \frac{f(X_n)}{0.9 + 0.2X_n}. \tag{11}$$

At the core of the newly born neutron star where most of the emission of wrong-helicity neutrinos occurs $\rho_{14} \sim 4 - 10$ and $T_{10} \sim 3 - 10$, and thus the bremsstrahlung process should be of comparable importance.

In the general case the volume emissivity is

$$\dot{\varepsilon}_{\rm BREM} = \frac{160 f^4 g_A^2}{15\pi^2} \frac{G_F^2 m_\nu^2 m^{9/2} T^{13/2}}{m_\pi^4} \left[0.5\left\{I(y_1, y_1) + I(y_2, y_2)\right\} + 3I(y_1, y_2)\right], \tag{12a}$$

$$\simeq 2.4 \times 10^{35}\, (m/0.94\,{\rm GeV})^{1/2}\, m_{100}^2\, T_{10}^{13/2}$$
$$\times \left[0.5\left\{I(y_1, y_1) + I(y_2, y_2)\right\} + 3I(y_1, y_2)\right] \ {\rm erg\,cm^{-3}\,sec^{-1}}, \tag{12b}$$

where we have displayed explicitly the kinematical dependence upon the nucleon mass (i.e., we have not pulled out the m^4 factor associated with the pion-nucleon coupling, m^4/m_π^4). On the basis of the nonlinear-sigma model it has been argued that the ratio of the nucleon mass to the pion mass should not change significantly with density.[10] In the nondegenerate limit, $\dot{\varepsilon}_{\rm BREM}$ varies as $m^{-5/2}$ and would increase by a significant factor if the effective nucleon mass is half its vacuum value. In the degenerate limit $\dot{\varepsilon}_{\rm BREM}$ is independent of the effective nucleon mass.

Since the nucleons in a newly born, hot neutron star are closer to being nondegenerate than degenerate, $\dot{\varepsilon}_{\rm BREM} \propto m^{-5/2}$ will increase by a factor of $\mathcal{O}(6)$ if the effective nucleon mass is half its vacuum value, while $\dot{\varepsilon}_{\rm SF} \propto m^0$ does not change. Thus, if the effective nucleon mass is substantially smaller than its vacuum value, the numerical factor in Eq. (11) is closer to unity, implying that the bremsstrahlung process dominates the spin-flip scattering process.

Brinkmann and Turner[9] give a simple fit to $I(y_1, y_2)$ that is accurate to better than 25% for all values of y_1 and y_2:

$$I_{\rm fit}(y_1, y_2)^{-1} = 2.39 \times 10^5 \left(e^{-y_1-y_2} + 0.25e^{-y_1} + 0.25e^{-y_2}\right)$$

$$+1.73 \times 10^4(1+|\bar{y}|)^{-1/2} + 6.92 \times 10^4(1+|\bar{y}|)^{-3/2} + 1.73 \times 10^4(1+|\bar{y}|)^{-5/2}, \qquad (13)$$

where $\bar{y} \equiv (y_1 + y_2)/2$.

If one is interested in producing helicity-flipped electron neutrinos, the URCA process can be very important ($n + p \to n + n + e^+ + \nu_+$ and $p + p \to n + p + e^+ + \nu_+$). [Note the process where an electron rather than a positron is produced in the final state is highly suppressed because of electron degeneracy: $\mu_e \sim 300\,\text{MeV}$.] The matrix element for this process is four times larger than that for the neutron-neutron or proton-proton process. However, we are interested in the production of wrong-handed μ and τ neutrinos.

Finally, Grifols and Masso[11] have also calculated the volume emissivity due to the bremsstrahlung process in the nondegenerate limit. Our results in this limit are larger than theirs by about a factor of five. The difference traces to a number of factors. First, they forgot to take in account the exchange diagrams (about a factor of two); second, they forgot to account for both wrong-helicity neutrino and antineutrino emission; and third, they did not take into account the *pp* bremsstrahlung process, which is important since during the early cooling phases $X_n \sim X_p \sim 1/2$.

Neutrino degeneracy

In computing the rate for the bremsstrahlung process we have assumed that proper-helicity neutrinos are nondegenerate. In light of our discussion of the important effect of neutrino degeneracy upon the spin-flip-scattering process we should re-examine that assumption.

The effect here is far less pronounced. If the neutrino seas are degenerate, then there will be a significant blocking factor that suppresses the emission a proper-helicity neutrino, but not a proper-helicity antineutrino. In the nondegenerate limit the two processes, $N+N \to N+N+\nu_+ + \bar{\nu}_+$ and $N+N \to N+N+\nu_- + \bar{\nu}_-$, contribute equally to $\dot{\varepsilon}_{\rm BREM}$; in the highly degenerate limit only the first of these will contribute, the second being suppressed by the degenerate sea of ν_-'s. The net effect is a reduction of the volume emissivity by a factor of two. However, in this limit the spin-flip-scattering process is enhanced so much that the bremsstrahlung process becomes subdominant and unimportant.

Pion-nucleon, neutrino-pair production

The conditions at the core of a neutron star are very close to those where a pion condensate should form. Because of this, the abundance of negative pions may well be comparable or greater than that of nucleons.[12] Needless to say, the abundance of pions in the core depends critically upon the equation of state. If the pion abundance is large, then the process $\pi^- + p \to n + \nu_+ + \bar{\nu}_+$ may also be important—in fact it may be dominant.[13]

The matrix element squared for this process is

$$|\mathcal{M}|^2 = \frac{32G_F^2 f^2 m^2 m_\nu^2 \omega_2}{m_\pi^2 \omega_1} \left\{ g_A^2 (1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}_1)(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}_2) + 0.5(1 - 0.5\hat{\mathbf{k}} \cdot \hat{\mathbf{q}}_1 - 0.5\hat{\mathbf{k}} \cdot \hat{\mathbf{q}}_2) \right\},$$

where we have assumed that the nucleons are nonrelativistic, that the pions are relativistic, and summed over initial and final nucleon spins. The three momenta of the pion, wrong-helicity neutrino, and proper-helicity antineutrino are $\mathbf{k}$, $\mathbf{q}_1$, and $\mathbf{q}_2$ respectively, and the energies of these particles are ω $(= \omega_1 + \omega_2)$, ω_1, and ω_2 respectively. The volume emissivity for this process is given by

$$\begin{aligned}
\dot{\varepsilon}_{\pi N} &= 2 \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3k}{2\omega(2\pi)^3} \frac{d^3q_1}{2\omega_1(2\pi)^3} \frac{d^3q_2}{2\omega_2(2\pi)^3} \\
&\quad \times (2\pi)^4 |\mathcal{M}|^2 \delta^4(p_1 + k - p_2 - q_1 - q_2) f_1 f_\pi \omega_1, \\
&= \frac{5(g_A^2 + 0.5) f^2}{\pi^3} \frac{G_F^2 m_\nu^2 (m/m_\pi)^2}{m^2} T^3 X_p n_\pi n_N, \qquad (14a) \\
&\simeq 2.1 \times 10^{33}\, m_{100}^2 (m/0.94\,\mathrm{GeV})^{-2} (n_\pi/n_N) X_p \rho_{14}^2 T_{10}^3 \ \mathrm{erg\,cm^{-3}\,sec^{-1}}, \qquad (14b)
\end{aligned}$$

where we have assumed that nucleons are nondegenerate, that the pion phase-space distribution function $f_\pi = \exp(\alpha - k/T)$, and included a factor of 2 to account for both the process where the neutrino has the wrong helicity and the one where the antineutrino has the wrong helicity. Note that $\dot{\varepsilon}_{\pi N} \propto m^{-2}$, so that it is a factor of four larger if the effective nucleon mass is half its vacuum value.

Now compare this production process to the bremsstrahlung process; taking the non-degenerate limit for $\dot{\varepsilon}_{\mathrm{BREM}}$, cf. Eq. (10b), we find

$$\frac{\dot{\varepsilon}_{\pi N}}{\dot{\varepsilon}_{\mathrm{BREM}}} \simeq 140 \left(\frac{n_\pi}{n_N}\right) \left(\frac{X_p}{f(X_p)}\right) T_{10}^{-1/2}. \qquad (15)$$

Thus, if the number density of negative pions is comparable to that of nucleons, pion-nucleon, neutrino-pair production is even more important than the bremsstrahlung process. In this case, unless the massive Dirac neutrino is degenerate, the pion-nucleon process will dominate.

Axions

In passing we note that it has been assumed that the dominant axion emission process for a hot young neutron is nucleon-nucleon, axion bremsstrahlung.[1] If there are lots of

negative pions in the core then pion-axion conversion $\pi^- + p \to n + a$, the analog of the process just discussed, can likewise dominate the bremsstrahlung process. In this case the axion volume emissivity is

$$\dot{\varepsilon}_a = \frac{30 f^2 \bar{g}_{aN}^2 T^3}{\pi\, m^2 m_\pi^2} X_p n_\pi n_N, \tag{16a}$$

$$\simeq 4.9 \times 10^{49}\, X_p \left(\frac{n_\pi}{n_N}\right) \rho_{14}^2 T_{10}^3 \ \mathrm{erg\,cm^{-3}\,sec^{-1}}, \tag{16b}$$

where $\bar{g}_{aN}$ is a combination of axion-proton and axion-neutron couplings which is of order $m/(f_a/N)$. The ratio of this process to the usual axion bremsstrahlung process is about $50(n_\pi/n_N)T_{10}^{-1/2}$, and if the abundance of negative pions is comparable to nucleons this process will be the dominant one. If this is the case, then the upper limit to the axion mass derived from SN 1987A improves by almost an order of magnitude: from about 10^{-3} eV to almost 10^{-4} eV.[1,14]

IV. Discussion

We have computed the volume emissivity of wrong-helicity neutrinos due to the spin-flip scattering process off nucleons and nucleon-nucleon bremsstrahlung, both for arbitrary nucleon degeneracy and nondegenerate neutrinos. The two processes are found to be of comparable importance at the core of a newly born hot neutron star. Relative to the volume emissivity used by Gandhi and Burrows[3] (corrected for the errant factor of four) the total volume emissivity is larger by about a factor of four, which should restore their original, very conservative mass limit of 14 keV.

We have also calculated the production of wrong-helicity neutrinos (and antineutrinos) due to the process $\pi^- + p \to n + \nu\bar{\nu}$, and find that if the number density of negative pions is comparable to that of nucleons (as could occur for a core on the verge of pion condensation), this process dominates both spin-flip scatterings and bremsstrahlung by a large factor. If this were the case the mass limit would improve to of order a few keV. Because the pion abundance is very sensitive to the equation of state, it is difficult to argue convincingly that such a bound is rigorous.

Perhaps the most important effect is that of neutrino degeneracy. Electron neutrinos are certainly degenerate at the core of a neutron star (with chemical potential $\mu_e \sim 300$ MeV); for masses in the keV range and mixing with the electron-neutrino of order 1% the massive Dirac neutrino should also become degenerate. This has the effect of increasing the emission of wrong-helicity neutrinos due to the spin-flip-scattering process by a factor of order $(\mu/T)^4 \sim 10^4$ relative to the nondegenerate rates previously used. Provided that the massive Dirac neutrino mixes sufficiently with the electron neutrino this should improve the mass limit to around 1 keV. Of course, the massive Dirac neutrino need not mix with the electron neutrino at all.

A precise limit to a Dirac-neutrino mass based upon SN 1987A awaits incorporation of the effects discussed here into detailed numerical cooling models, work which is currently

in progress.[15] However, it seems clear that the limit obtained will be more stringent than 10 keV, and if the massive Dirac neutrino mixes with the electron neutrino at the 1% level or more probably as stringent as 1 keV.

I thank Jim Wilson for suggesting the importance of the pion-nucleon, neutrino-pair production process. I thank G.G. Raffelt for constructive criticism of an early version of this manuscript. I also thank A. Burrows, J. Harvey, J. Rosner, and D.N. Schramm for useful discussions. This work was supported in part by the NASA (through grant NAGW-1340) and DOE at Fermilab.

References

1. See e.g., M.S. Turner, *Phys. Rep.* **197**, 67 (1990); or G.G. Raffelt, *ibid* **198**, 1 (1991).
2. G.G. Raffelt and D. Seckel, *Phys. Rev. Lett.* **60**, 1793 (1988); K. Gaemers, R. Gandhi, and J. Lattimer, *Phys. Rev. D* **40**, 309 (1989); R. Gandhi and A. Burrows, *Phys. Lett. B* **246**, 149 (1990) and *Erratum* to be published in *Phys. Lett. B.*; J.A. Grifols and E. Masso, *Nucl. Phys.* **B331**, 244 (1990); *Phys. Lett. B* **242**, 77 (1990); A. Perez and R. Gandhi, *ibid* **41**, 2374 (1990).
3. R. Gandhi and A. Burrows, *Phys. Lett. B* **246**, 149 (1990) and *Erratum* to be published in *Phys. Lett. B.*.
4. For evidence of a 17 keV mass eigenstate see, J.J. Simpson, *Phys. Rev. Lett.* **54**, 1891 (1985); A. Hime and J.J. Simpson, *Phys. Rev. D* **39**, 1837 (1989); A. Hime and N.A. Jelley, Oxford Univ. preprint OUNP-91-01 (1990); J.J. Simpson and A. Hime, *Phys. Rev. D* **39**, 1825 (1989); J.J. Simpson, *ibid* **174B**, 113 (1986); B. Sur et al., LBL preprint 30109 (1990). However, there are other β-decay experiments that do not see evidence for a 17 keV mass eigenstate; see J. Markey and F. Boehm, *Phys. Rev. C* **32**, 2215 (1985); T. Altzitzoglou et al., *Phys. Rev. Lett.* **55**, 799 (1985); T. Ohi et al., *Phys. Lett.* **160B**, 322 (1985); D.W. Hetherington et al., *Phys. Rev. C* **36**, 1504 (1987); I. Zlimen, S. Kauciuc, and A. Ljubicic, *Physica Scripta* **38**, 539 (1988). For a discussion of the astrophysical and cosmological constraints that apply to a 17 keV neutrino see, E.W. Kolb and M.S. Turner, *Phys. Rev. Lett.*, in press (1991); L. Krauss, *Phys. Lett. B*, in press (1991); J. Cline and T.P. Walker, Ohio State Univ. preprint (1991).
5. K. Gaemers, R. Gandhi, and J. Lattimer, *Phys. Rev. D* **40**, 309 (1989); A. Perez and R. Gandhi, *ibid* **41**, 2374 (1990).
6. For a discussion of the weak, neutral hadronic current, see e.g., J.E. Kim, P. Langacker, and S. Sarkar, *Phys. Rev. D* **18**, 123 (1978); E. Fischbach et al., *Phys. Rev. Lett.* **37**, 582 (1976); L.A. Ahrens et al., *Phys. Rev. D* **35**, 785 (1987).

7. See e.g., L. Wolfenstein, *Phys. Rev. D* **17**, 2369 (1978); *ibid* **20**, 2634 (1979); R. Barbieri and A.D. Dolgov, *Phys. Lett. B* **237**, 440 (1990); K. Kainulainen, *ibid* **244**, 191 (1990); K. Kainulainen, J. Maalampi, and J.T. Peltoniemi, NORDITA preprint (1990).
8. B.L. Friman and O.V. Maxwell, *Astrophys. J.* **232**, 541 (1979).
9. R.P. Brinkmann and M.S. Turner, *Phys. Rev. D* **38**, 2338 (1988).
10. M.S. Turner, H.-S. Kang, and G. Steigman, *Phys. Rev. D* **40**, 299 (1989).
11. J.A. Grifols and E. Masso, *Nucl. Phys.* **B331**, 244 (1990); *Phys. Lett. B* **242**, 77 (1990).
12. In J. Wilson's numerical models of early neutron star cooling the pion number density is comparable to the nucleon number density (J. Wilson, private communication).
13. The non spin-flip, charged-current, neutrino production process $\pi + n \rightarrow n + e^- + \bar{\nu}_e$ has been discussed previously in the context of neutron-star cooling; see e.g., G. Baym et al., *Phys. Lett. B* **58**, 304 (1975); O. Maxwell et al., *Astrophys. J.* **216**, 77 (1977).
14. R. Mayle, D.N. Schramm, M.S. Turner, and J. Wilson, work in progress (1991); A. Burrows and R. Gandhi, work in progress (1991).
15. M.S. Turner, work in preparation (1991).