



# Fermi National Accelerator Laboratory

FERMILAB-Pub-90/181-A

April 1991

## Energetic Neutrinos from Heavy-Neutralino Annihilation in the Sun

MARC KAMIONKOWSKI

*Department of Physics and Enrico Fermi Institute,  
The University of Chicago, Chicago, IL 60637-1433*

and

*NASA/Fermilab Astrophysics Center, Fermi  
National Accelerator Laboratory, Batavia, IL 60510-0500*

### ABSTRACT

Neutralinos may be captured in the Sun and annihilate therein producing high-energy neutrinos. Present limits on the flux of such neutrinos from underground detectors such as IMB and Kamiokande II may be used to rule out certain supersymmetric dark-matter candidates, while in many other supersymmetric models the rates are large enough that if neutralinos *do* reside in the galactic halo, observation of a neutrino signal may be possible in the near future. Neutralinos that are either nearly pure Higgsino or a Higgsino/gaugino combination are generally captured in the Sun by coherent scattering off nuclei via exchange of the lightest Higgs boson. If the squark mass is not much greater than the neutralino mass then capture of neutralinos that are primarily gaugino occurs predominantly by spin-dependent scattering off hydrogen in the Sun. The neutrino signal from annihilation of WIMPs with masses in the range 80-1000 GeV in the Sun should generally be stronger than that from WIMP annihilation in the Earth, and detection rates for mixed-state neutralinos are generally higher than those for Higgsinos or gauginos.

NAGW-1340  
IN-72-CR  
19357  
P/59

N91-24866

Unclas  
0019357

G3/72

(NASA-CR-188447) ENERGETIC NEUTRINOS FROM  
HEAVY-NEUTRALINO ANNIHILATION IN THE SUN  
Ph.D. Thesis (Fermi National Accelerator  
Lab.) 59 p CSCI 20H



## I. INTRODUCTION

The idea that stable weakly-interacting massive particles (WIMPs) make up the bulk of the dark matter in the Universe and in the galactic halo has been the focus of much theoretical and experimental research recently.<sup>1</sup> Now that the original WIMP, the Dirac neutrino, has been ruled out,<sup>2</sup> the neutralino<sup>3</sup>—a linear combination of the supersymmetric partners of the photon,  $Z^0$ , and Higgs bosons—has become the preferred thermal relic. Although the original treatises considered only neutralinos lighter than the  $W^\pm$ ,<sup>4,5</sup> heavy neutralinos—those more massive than the  $W$ —may also be suitable dark-matter candidates.<sup>6,7</sup> Although “extremely” massive neutralinos are not favored theoretically,<sup>8</sup> neutralinos in the 100-GeV range may still solve the naturalness problem and become increasingly attractive as unsuccessful accelerator searches push the mass scale for supersymmetry upward.

Since many neutralinos are not yet accessible in accelerators and are such compelling dark-matter candidates, a variety of complementary experiments to detect neutralinos in our galactic halo are currently being pursued. Some seek to observe neutralinos by detecting the energy deposited in an ultra-low background detector when a neutralino elastically scatters off of a nucleus therein.<sup>9</sup> Alternatively, neutralino dark matter in the galactic halo may be indirectly detected by its annihilation products. A continuum spectrum of cosmic-ray antiprotons,<sup>10</sup>  $\gamma$  rays,<sup>11</sup> and positrons,<sup>12</sup> are produced in the cascade resulting from the annihilation products of the neutralinos; however, astrophysical uncertainties involving the propagation of cosmic rays from conventional sources are so great that it seems unlikely that WIMP-induced continuum cosmic rays could ever be distinguished from those from standard sources. Some authors have boldly suggested that annihilation of WIMPs in the galactic halo could produce either  $\gamma$ -ray<sup>13</sup> or positron<sup>14,15</sup> line radiation which could be readily distinguished from background. While such a signal would provide unambiguous evidence for particle dark matter, because of astrophysical uncertainties an observable signal of this kind is not guaranteed even if suitable WIMPs *do* reside in the galactic halo.

In this paper we address the possibility of indirect detection of heavy neutrali-

nos by observation of yet another annihilation product: high-energy neutrinos. WIMPs in the galactic halo will be captured in the body of the Sun or the Earth<sup>16,17,18</sup> and annihilate therein producing high-energy neutrinos that may be observable in underground neutrino detectors. This method of detection has several advantages over cosmic-ray signatures: First of all, whereas cosmic rays are expected to be isotropically distributed, the neutrino signal comes from a fixed direction and is therefore much more easily distinguished from background. The number density  $n_{\tilde{\chi}}$  of neutralinos in the halo is inversely proportional to the neutralino mass and, as we shall see, the annihilation rate in the Sun is  $\propto n_{\tilde{\chi}}$  while the annihilation rate in the halo is  $\propto n_{\tilde{\chi}}^2$ , making the neutrino signal favored for higher neutralino masses. In addition, the uncertainties in the predicted rates for neutrino events are smaller than those in the predicted cosmic-ray fluxes (roughly factors of about two for neutrino events and orders of magnitude for cosmic-ray fluxes). Basically this is because the local halo density is known better than the dark-matter distribution throughout the galaxy, and propagation of neutrinos through the Sun is more easily modeled than cosmic-ray propagation through the galaxy. It should also be noted that neutrino searches and cosmic-ray searches are mutually complementary: For example, the neutralinos that may be discovered through distinctive cosmic-ray positron signatures are primarily Higgsinos,<sup>15</sup> whereas neutrino signals are strongest for neutralinos that are a mixed Higgsino/gaugino state.

Unlike Dirac neutrinos which annihilate directly into light (i.e.,  $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ ) neutrinos, neutralinos are Majorana particles and therefore do *not* produce prompt neutrinos; the neutrinos from neutralino-neutralino annihilations come from the decays of the annihilation products, so the neutrino spectrum is considerably softer. Detailed neutrino spectra from energetic quarks and leptons injected into the core of the Sun were calculated by Ritz and Seckel (RS).<sup>19</sup> The analysis for light neutralinos was originally carried out by Giudice and Roulet<sup>20</sup> who considered only annihilation into fermion-antifermion pairs and more completely by Gelmini, Gondolo, and Roulet<sup>21</sup> who considered annihilation into pairs of Higgs bosons as well. Here we extend this work to heavy neutralinos by considering the effect of the gauge-boson, Higgs-boson, and top-quark annihila-

tion channels which open up for heavy neutralinos. We also consider the effect of interactions of the annihilation products and resulting high-energy neutrinos in the Sun which become important at higher energies.

First let us briefly review the minimal supersymmetric extension of the Standard Model (MSSM) and the properties of the neutralino. For more details we refer the reader to Ref. 3 and Griest, Kamionkowski, and Turner (GKT)<sup>6</sup> whose notation we use throughout. There are actually four neutralinos, and the lightest (the  $n$ th) is assumed to be the lightest supersymmetric particle (LSP) and stable and is denoted as *the* neutralino,

$$\tilde{\chi} = Z_{n1}\tilde{B} + Z_{n2}\tilde{W}^3 + Z_{n3}\tilde{H}_1 + Z_{n4}\tilde{H}_2, \quad (1)$$

where  $(Z)_{ij}$  is a real orthogonal matrix that diagonalizes the neutralino mass matrix [Eq. (C38) in Ref. 3] and depends only on the gaugino mass parameter  $M$ , Higgsino mass parameter  $\mu$ , and the ratio of Higgs vacuum expectation values  $\tan\beta$ . In Fig. 1 we plot neutralino mass contours (broken curves) and contours of  $Z_{n1}^2 + Z_{n2}^2$  (solid curves), the gaugino fraction, for  $\tan\beta = 2$  (plots for other values of  $\tan\beta$  are similar). As noted originally by Olive and Srednicki<sup>6</sup> in much of parameter space where the neutralino is heavier than the  $W$ , the gaugino fraction is greater than 0.99 and the neutralino is almost pure  $B$ -ino. In much of parameter space, the gaugino fraction is less than 0.01 and the neutralino is almost pure Higgsino. Near the 0.5 gaugino fraction curve, a curve that asymptotes to  $\mu = \frac{5}{3}M \tan^2\theta_W$  at high neutralino mass, the neutralino is a mixed state, half gaugino and half Higgsino.

In the MSSM there are three neutral Higgs bosons.<sup>22</sup> The mass of the lightest,  $H_2^0$ —which must be less than  $m_Z \cos 2\beta$  (provided the top quark is not unusually heavy; see Ref. 23)—and  $\tan\beta$  determine the masses of the other two,  $H_1^0$ —which must be heavier than the  $Z$ —and  $H_3^0$ —whose mass falls between  $m_{H_2^0}$  and  $m_{H_1^0}$ . There are also charged Higgs bosons  $H^\pm$  which are always heavier than the  $W$  and two charginos, linear combinations of the supersymmetric partners of the  $W$  and charged Higgs bosons. The masses of the superpartners of the quarks and leptons, which we will collectively refer to as squarks, are all undetermined, but

for simplicity we give them all the same mass  $M_{\tilde{q}}$  which, assuming the neutralino is the LSP, is greater than  $m_{\tilde{\chi}}$ .

Although the MSSM has many undetermined parameters<sup>3</sup> ( $\tan\beta$ ,  $M$ ,  $\mu$ ,  $m_{H_2^0}$ ,  $M_{\tilde{q}}$ , and the top-quark mass  $m_t$ ) the parameters are not entirely unconstrained, and by studying several “corners” of parameter space we can get an understanding of the dependence of detection rates on the different parameters of the model. Although  $m_t$  is constrained only to be greater than 80 GeV<sup>24</sup> (from unsuccessful accelerator searches) and less than about 200 GeV<sup>25</sup> (from limits on radiative corrections to  $\sin^2\theta_W$ ), we will assume  $m_t = 120$  GeV throughout; as we will discuss later, varying the top-quark mass should have little effect on our results. Recent searches for neutral Higgs bosons at LEP have constrained regions of  $m_{H_2^0}$ - $\tan\beta$  space.<sup>26</sup> In addition, we will only consider  $\tan\beta > 1$ , since radiative corrections drive  $\tan\beta$  to values greater than one when  $m_t \gg m_b$ , and  $\tan\beta < m_t/m_b \simeq 25$ , required for electroweak symmetry breaking in many supergravity models.<sup>27</sup> To see the range of possible capture and detection rates due to the range of all possible values for the squark mass we will present results assuming the squark mass is infinite and then show results assuming the squark mass is slightly heavier than the neutralino mass.

Although determination of the event rate is relatively straightforward, it is quite lengthy and depends on a variety of input physics such as solar physics, neutrino physics, hadronization of quarks, underground detectors, and, of course, the interactions of neutralinos with ordinary matter. The flux of high-energy neutrinos of type  $i$  (*e.g.*,  $i = \nu_\mu, \bar{\nu}_\mu$ , etc.) from neutralino annihilation in the Sun is simply

$$\left(\frac{d\phi}{dE}\right)_i = \frac{\Gamma_A}{4\pi R^2} \sum_F B_F \left(\frac{dN}{dE}\right)_{Fi} \quad (2)$$

The quantity  $\Gamma_A$  is the rate of neutralino-neutralino annihilations in the Sun, and  $R$  is simply the distance of the Earth from the Sun. Neutralinos from the galactic halo are accreted onto the Sun and their number in the Sun is depleted by annihilation. In most cases of interest these two processes come to equilibrium on a time scale much shorter than the solar age in which case  $\Gamma_A = C/2$  where

$C$  is the rate for capture of neutralinos from the halo. As one might imagine, the capture rate is basically determined by the flux of neutralinos incident on the Sun and a probability for capture which in turn depends on kinematic factors and the cross sections for elastic scattering of the neutralino off of the elements in the Sun. The sum is over all annihilation channels  $F$  (e.g., pairs of gauge or Higgs bosons or fermion-antifermion pairs),  $B_F$  is the annihilation branch for channel  $F$ , and  $(dN/dE)_{Fi}$  is the differential energy flux of neutrino type  $i$  at the *surface* of the Sun expected from injection of the particles in channel  $F$  in the *core* of the Sun. The flux  $(dN/dE)_{Fi}$  is a function of the energy of the neutrino and of the energy of the injected particles. Determination of these fluxes is quite complicated as it involves hadronization of the annihilation products, interaction of the particles in the resulting cascade with the solar medium and the subsequent interaction of high-energy neutrinos with the solar medium as they propagate from the core to the surface of the Sun.<sup>19</sup> Neutralinos may also be captured in the Earth; however, for a number of reasons which we will discuss below, the rates for neutrino events from neutralino annihilation in the Earth will generally be smaller than those from the Sun if the neutralino is heavy.

The experimental signature on which we will eventually focus will be the number of upward-moving muons induced by high-energy neutrinos from the Sun that are observed in underground detectors. Given the fluxes  $(d\phi/dE)_i$ , the final result for the rate (per unit detector area) for neutrino-induced upward moving muons may be written simply as

$$\Gamma_{\text{detector}} = \sum_i D_i \int \left( \frac{d\phi}{dE} \right)_i E^2 dE, \quad (3)$$

where the sum is over  $\nu_\mu$ , which produce muons, and  $\bar{\nu}_\mu$ , which produce antimuons. Since the cross section for the neutrino to produce a muon in the rock below the detector is proportional to the neutrino energy  $E$  and the range of the muon is roughly proportional to its energy, the probability a neutrino of energy  $E$  produces a muon which traverses the detector is  $E^2$  times a constant  $D_i$ ; hence the integral in Eq. (3). Neutrinos may also be detected by contained events in which a charged lepton is produced within the detector, but because this process

is proportional only to the neutrino energy  $E$  (as opposed to  $E^2$  for throughgoing events), the throughgoing muons should provide a more promising signature for heavy neutralinos.

In the next Section we discuss the rate  $\Gamma_A$  of neutralino-neutralino annihilation in the Sun and in the Earth. The annihilation rate is proportional to the square of the number of neutralinos in the Sun or Earth, and this number is increased by capture of neutralinos from the halo while neutralinos are depleted by annihilation. Capture occurs by elastic scattering of neutralinos in the galactic halo off of nuclei in the Sun. We show the regions of parameter space in which capture occurs predominantly by scattering off of heavy nuclei via a coherent scalar ("spin-independent") interaction involving exchange of the lightest Higgs boson and the regions where capture occurs primarily by scattering via an axial ("spin-dependent") interaction involving squark exchange off of hydrogen. We also show the regions of parameter space where the capture and annihilation rates are large enough that the annihilation rate is half the capture rate and the neutrino flux is at "full signal."

In Section III we discuss the neutrino spectra  $(dN/dE)_{Fi}$  from products of neutralino-neutralino annihilation in the Sun and Earth. We describe the hadronization and decays of the annihilation products and the interaction of the annihilation products and high-energy neutrinos with the Sun. In Section IV we discuss detection of high-energy neutrinos from the Sun (and Earth) and argue that for heavy WIMPs the neutrino signal from the Sun should be stronger than that from the Earth. We then point out that the most promising method of detection is via observation of upward-moving throughgoing muons induced by high-energy neutrinos in the rock below the detector and discuss the calculation of the event rate.

In Section V we present our results, discuss which supersymmetric candidates for the primary component of the galactic halo are already ruled out by current neutrino-flux limits and which may be observable in the near future. Most of the models that are inconsistent with current limits from IMB<sup>28</sup> and Kamiokande<sup>29</sup> on high-energy neutrino fluxes are those where the neutralino is a mixed gaug-

ino/Higgsino state and the mass of the lightest Higgs boson is near the current lower limits imposed by LEP.<sup>26</sup> We find that if observational neutrino-flux limits are improved by a factor of ten, say, many more supersymmetric models will become detectable by these methods. The neutrino signal from neutralinos that are primarily gaugino is greater for models where the squark mass is smaller, while the neutrino rates from neutralinos that are Higgsinos or mixed gaugino/Higgsino states are relatively insensitive to the squark mass. In the last Section we discuss our results, briefly discuss backgrounds and detection strategies, and make some concluding remarks. In Appendix A we display the cross section for elastic scattering of a neutralino off of nuclei, and Appendix B contains new results for cross sections for annihilation of neutralinos into mixed Higgs/gauge boson final states.

## II. RATE OF ANNIHILATION IN THE SUN

The first step in calculating the rate for WIMP-induced neutrino events from the Sun is the determination of the rate at which neutralinos annihilate in the Sun. As mentioned previously, neutralinos accumulate in the Sun or Earth by capture from the galactic halo and are depleted by annihilation. If  $N$  is the number of neutralinos in the Sun then the differential equation governing the time evolution of  $N$  is

$$\dot{N} = C - C_A N^2, \quad (4)$$

where the dot denotes differentiation with respect to time. Here,  $C$  is the rate of accretion of neutralinos onto the Sun (or Earth). The determination of  $C$  is straightforward and will be discussed in detail below, and if the halo density of neutralinos remains constant in time,  $C$  is of course time-independent.

The second term on the right-hand side is twice the annihilation rate in the Sun (or Earth),  $\Gamma_A = C_A N^2/2$ , and accounts for depletion of neutralinos. The quantity  $C_A$  depends on the cross section for neutralino-neutralino annihilation and the distribution of neutralinos in the Sun (or Earth),<sup>30</sup>

$$C_A = \frac{\langle \sigma v \rangle_A V_2}{V_1^2}, \quad (5)$$



where  $\langle\sigma v\rangle_A$  is the spin-averaged total annihilation cross section times relative velocity in the limit of zero relative velocity (since captured neutralinos move *very* slowly), and can be evaluated using the formulas in GKT and Appendix B, and the quantities  $V_j$  are effective volumes for the Sun or Earth:<sup>30,17</sup>

$$V_j = \left( \frac{3m_{\text{Pl}}^2 T}{2jm_{\tilde{\chi}}\rho} \right), \quad (6)$$

where  $T$  is the temperature of the Sun or Earth,  $m_{\text{Pl}}$  is the Planck mass, and  $\rho$  is the core density of the Sun or Earth. In Ref. 30 it is found that  $V_j = 6.5 \times 10^{28} (jm_{\tilde{\chi}}^{10})^{-3/2} \text{ cm}^3$ , where  $m_{\tilde{\chi}}^{10}$  is the neutralino mass in units of 10 GeV, for the Sun, and in Ref. 17 it is found that  $V_j = 2.0 \times 10^{25} (jm_{\tilde{\chi}}^{10})^{-3/2} \text{ cm}^3$  for the Earth.

Solving Eq. (4) for  $N$ , we find that the annihilation rate at any given time is

$$\Gamma_A = \frac{C}{2} \tanh^2(t/\tau_A), \quad (7)$$

where  $\tau_A = (CC_A)^{-1/2}$  is the time scale for capture and annihilation to equilibrate. Therefore, if the age of the Sun is much greater than the equilibration time scale ( $t_{\odot} = 1.5 \times 10^{17} \text{ s} \gg \tau_A$ ) then the neutrino flux is at “full signal” ( $\Gamma_A = C/2$ ), but if  $\tau_A \gg t_{\odot}$  then the annihilation rate is smaller and the neutrino signal is diluted accordingly. As we shall see, the capture rate in the Earth is generally  $\lesssim 10^{-9}$  that in the Sun while the value of  $V_j$  in the Earth is only about  $3 \times 10^{-4}$  that in the Sun, so the value of  $\tau_A$  is always larger in the Earth than in the Sun; consequently, the fraction of full signal in the Earth can never be greater than that in the Sun.

Although the calculation of the rate of accretion of WIMPs onto an astrophysical object is quite involved the basic idea is simple.<sup>17</sup> Suppose a halo WIMP which has a velocity  $v_{\infty}$  far away from the object has a trajectory that passes through the object. At a point within the body where the escape velocity is  $v_{\text{esc}}$  the WIMP velocity will then be  $(v_{\infty}^2 + v_{\text{esc}}^2)^{1/2}$ . If the WIMP elastically scatters off of a nucleus of mass  $m_i$  to a velocity less than  $v_{\text{esc}}$  the WIMP will be captured. Kinematics tells us that the fractional energy loss ( $\Delta E/E$ ) of the WIMP in the

collision must lie in the range

$$0 \leq \frac{\Delta E}{E} \leq \frac{4m_{\tilde{\chi}}m_i}{(m_{\tilde{\chi}} + m_i)^2}, \quad (8)$$

and in the simplest case the cross section  $\sigma_{\text{SD}}^i$  for elastic scattering of the neutralino off of nucleus  $i$  is isotropic so the probability for a given energy loss is flat in this interval. [As will be discussed below, if the neutralino interacts coherently with the entire nucleus, at high momentum transfer there will be a form-factor suppression to the cross section so the probability for a given energy loss will no longer be flat in the interval given by Eq. (8).] The rate of capture of the WIMP by scattering off of nucleus  $i$  at this point in the Sun is then the rate of elastic scattering  $\sigma_{\text{SD}}^i n^i (v_{\text{esc}}^2 + v_{\infty}^2)^{1/2}$  (where  $n^i$  is the number density of nucleus  $i$ ) times the consitional probability that the WIMP is scattered to a velocity less than  $v_{\text{esc}}$ :

$$\begin{aligned} \frac{1}{\chi_+} \left( \chi_+ - \frac{v_{\infty}^2}{v_{\infty}^2 + v_{\text{esc}}^2} \right) \theta \left( \chi_+ - \frac{v_{\infty}^2}{v_{\infty}^2 + v_{\text{esc}}^2} \right) \\ = \frac{1}{v_{\text{esc}}^2 + v_{\infty}^2} \left( v_{\text{esc}}^2 - \frac{v_{\infty}^2}{\chi_-} \right) \theta \left( v_{\text{esc}}^2 - \frac{v_{\infty}^2}{\chi_-} \right), \end{aligned} \quad (9)$$

where  $\chi_{\pm} = 4m_{\tilde{\chi}}m_i/(m_{\tilde{\chi}} \pm m_i)^2$  and  $\theta$  is the Heaviside step function.

The essence of Gould's resonant enhancement in the capture of WIMPs [and the kinematic suppression factor  $S_i(m_{\tilde{\chi}})$  discussed below] is contained in Eq. (9): The conditional probability that a WIMP will be captured in a scattering event is greatest when  $\chi_-$  is maximized which occurs when the neutralino mass closely matches the mass of the nucleus off of which it scatters. Furthermore, this resonance effect is much sharper in the Earth than in the Sun: The velocities of the WIMP have a Maxwell-Boltzmann distribution with velocity dispersion of  $\bar{v} = 300 \text{ km s}^{-1}$  and the escape velocity from the Earth ranges from  $11.2 \text{ km s}^{-1}$  (at the surface) to  $14.8 \text{ km s}^{-1}$  (at the center), so the probability is nonzero only for the very slow WIMPs on the Boltzmann tail or for WIMPs with masses that very nearly match  $m_i$ . In a detailed analysis Gould<sup>17</sup> finds that WIMPs in the "resonance range" 10-75 GeV have masses which are sufficiently close to

the mass of an element with a significant abundance in the Earth so that their capture is not kinematically suppressed. On the other hand, the escape velocity just at the surface of the Sun is  $618 \text{ km s}^{-1}$  (and  $v_{\text{esc}}$  is much greater at the center), so capture is not kinematically suppressed unless  $\chi_-$  is quite small (*i.e.*, the neutralino and nuclear masses are *very* mismatched) and the resonance range for capture in the Sun is much larger than in the Earth.

The neutralino scatters off of nuclei with spin (which for the purpose of capture in the Sun or Earth includes only the hydrogen in the Sun) via an axial or “spin-dependent” interaction characteristic of Majorana particles. In addition, the neutralino may scatter off of any nucleus via a scalar interaction in which the neutralino interacts coherently with the entire nucleus; for heavy neutralinos, the scalar cross section  $\sigma_{\text{SC}}$  is proportional to the fourth power of the nuclear mass. For the elastic scattering cross section we use the results of Griest,<sup>5,31</sup> which include both a spin-dependent and a scalar term due to the exchange of a squark and the  $Z$  boson, and of Barbieri, Frigeni, and Roulet,<sup>32</sup> which includes a coherent scattering term due to the exchange of the lightest Higgs boson. We also include the effect of the exchange of  $H_1^0$ , the heavier scalar Higgs boson (which increases the elastic scattering cross section only slightly). As recently pointed out by Gelmini, Gondolo, and Roulet,<sup>21</sup> the cross section for scalar interactions of neutralinos with nuclei is larger than that given in Refs. 5 and 32 when one takes into account the substantial strange-quark content in the nucleus as implied by the pion-nucleon sigma term.<sup>33</sup> For the convenience of the reader the complete formulas for the elastic scattering cross section are listed in Appendix A.

Until now we have assumed that the elastic scattering cross section is isotropic and the conditional probability for a given energy loss in the range given by Eq. (8) is uniform; however, if the neutralino interacts coherently with the nucleus and the momentum transfer  $q$  is not small compared to the inverse of the nuclear radius  $R$  this assumption is not necessarily true as the neutralino does not “see” the entire nucleus and the cross section for scattering of neutralinos off of nuclei is form-factor suppressed (like that for electromagnetic elastic scattering of electrons from nuclei). In terms of the energy loss  $\Delta E$  the form factor suppression may be written as<sup>34</sup>

$$|F(q^2)|^2 = \exp(-\Delta E/E_0) \quad (10)$$

where  $E_0 = 3/(2m_i R^2)$ .

Now let us consider the relevance of a form-factor suppression on the capture of heavy neutralinos in the Sun and Earth. First of all, for a WIMP with a kinetic energy  $E_\infty = m_{\tilde{\chi}} v_\infty^2/2$  in the halo to be captured it must have an energy loss in the range

$$E_\infty \leq \Delta E \leq \chi_+(E_\infty + E_{\text{esc}}), \quad (11)$$

where  $E_{\text{esc}} = m_{\tilde{\chi}} v_{\text{esc}}^2/2$  is the WIMP escape energy at the point of collision in the Sun. The lower limit comes from the condition that the WIMP scatter from a velocity  $(v_\infty^2 + v_{\text{esc}}^2)^{1/2}$  to a velocity less than  $v_{\text{esc}}$ , and the upper limit is the kinematic limit. This implies that in order to be captured the WIMP energy in the halo must be  $E_\infty \leq \chi_- E_{\text{esc}}$ , which in turn implies that the largest energy loss involved in capture of WIMPs from the halo is  $\Delta E_{\text{max}} = \chi_- E_{\text{esc}}$ . The value of  $E_0$  for iron, the heaviest element important for capture in both the Sun and Earth, is  $8 \times 10^{-5}$  GeV. Because of the factor of  $(m_{\tilde{\chi}} - m_i)^2$  in the denominator of  $\chi_-$  the energy loss is largest for the lightest WIMP we consider, one with a mass of 80 GeV. For capture in Earth, the largest energy loss occurs at the center of the Earth and is roughly  $2 \times 10^{-6}$  GeV, so the form-factor suppression is negligible for capture of heavy WIMPs in the Earth.<sup>17</sup> On the other hand, the maximum energy loss for capture off of iron in the Sun is  $8.1 \times 10^{-3}$ , which implies that a proper calculation of capture in the Sun must include the effects of form-factor suppression of the coherent scalar interaction.

The full capture rate calculation assumes the astrophysical object moves through a homogeneous Maxwell-Boltzmann distribution of WIMPs and requires information about the elemental composition of the object and the distribution of elements in the object. One must integrate over the trajectories of the WIMP through the Sun and over the velocity distribution of the WIMPs. The final result for the capture rate, adapted from Gould,<sup>17</sup> is

$$C = c \frac{\rho_{0.4}^{\tilde{\chi}}}{m_{\tilde{\chi}} \bar{v}_{300}} \sum_i \left[ \sigma_{\text{SD}}^{i(40)} + F_i(m_{\tilde{\chi}}) \sigma_{\text{SC}}^{i(40)} \right] f_i \phi_i S_i(m_{\tilde{\chi}})/m_i, \quad (12)$$

where  $c = 5.8 \times 10^{24} \text{ s}^{-1}$  for the Sun and  $c = 5.7 \times 10^{15} \text{ s}^{-1}$  for the Earth,  $\rho_\chi^{0.4}$  is the mass density of neutralinos in the galactic halo in units of  $0.4 \text{ GeV cm}^{-3}$ ,  $m_\chi$  is the neutralino mass in units of GeV, and  $\bar{v}_{300}$  is the velocity dispersion of the neutralinos in the galactic halo in units of  $300 \text{ km s}^{-1}$ . The sum is over all species of nuclei in the astrophysical object (here the Earth or Sun),  $m_i$  is the mass of the  $i$ th nuclear species in GeV,  $f_i$  is the mass fraction of element  $i$ ,  $\sigma_{\text{SD}}^{i(40)}$  is the cross section for elastic scattering off of nucleus  $i$  via an axial interaction (given in Appendix A) in units of  $10^{-40} \text{ cm}^2$ , and  $\sigma_{\text{SC}}^{i(40)}$  is the cross section for elastic scattering of the neutralino off of nucleus  $i$  via a coherent scalar interaction (given in Appendix A) in units of  $10^{-40} \text{ cm}^2$ . The quantities  $\phi_i$  describe the velocity distribution of element  $i$  in the Sun or Earth and are given in the Appendix of Ref. 21 as are the quantities  $f_i$ .

The quantity  $S_i(m_\chi)$  is the kinematic suppression factor for capture of a WIMP of mass  $m_\chi$  off of nucleus  $i$ . We use an approximation that interpolates between the two limiting cases of the suppression factor given by Gould:

$$S_i(m_\chi) = \left[ \frac{A^b}{1 + A^b} \right]^{1/b}, \quad (13)$$

where

$$A = \frac{3}{8} \frac{m_\chi m_i}{(m_\chi - m_i)^2} \left( \frac{v_{\text{esc}}^2}{\bar{v}^2} \right) \phi_i, \quad (14)$$

and  $b = 1.5$ . We obtain this expression from the RS expression which approximates Gould's kinematic factor to 5% for scattering off of protons by noting that the neutralino and nuclear masses enter into Gould's kinematic formula only in the combination  $m_\chi m_i / (m_\chi - m_i)^2$ . The quantity  $v_{\text{esc}}$  is the escape velocity at the surface of the Sun or the Earth ( $618 \text{ km s}^{-1}$  for the Sun and  $11.2 \text{ km s}^{-1}$  for the Earth). To check our approximation for  $S_i$ , we calculate the capture rate in the Earth for neutralinos with masses between 10 GeV and 80 GeV using an elastic scattering cross section due only to neutral-Higgs and  $Z$  exchange and note that our results reproduce the resonance structure found in Ref. 20. Note also that Eq. (12) reduces to the simple expression for capture by the Earth when the mass of the neutralino is far from the resonance range (Ref. 18).

Although heavy neutralinos are outside the resonance range  $10 \text{ GeV} \lesssim m_{\tilde{\chi}} \lesssim 75 \text{ GeV}$  for capture by the Earth so that only the form of  $S_i$  in the limit  $A \ll 1$  is important for capture by the Earth,  $v_{\text{esc}}$  is much larger for the Sun and  $\phi_i$  is typically 2-3 times larger for the Sun, so as mentioned above, the resonance range for capture in the Sun is much wider than in the Earth. For example, in the Sun  $A = 1$  when the neutralino mass is roughly seven times that of the nucleus off of which it scatters.

The form-factor suppression  $F_i(m_{\tilde{\chi}})$  of the capture of a WIMP of mass  $m_{\tilde{\chi}}$  from nuclei  $i$  is obtained by comparing the results of integrating Gould's differential capture rate [Eq. (A10) in Ref. 17] over the mass of the Sun including form-factor suppression and comparing it with the integral of the analogous expression [Eq. (2.24) in Ref. 17] in which full coherence is assumed. In doing so, the density of the Sun as a function of radius  $r$  was taken to be

$$\rho(r) = \rho_0 \exp(-7.7r/R_{\odot})(1 - r/R_{\odot})^{1.6}, \quad (15)$$

where  $\rho_0$  is the density at the center of the Sun, and  $R_{\odot}$  is the solar radius. This form approximates the solar density in Ref. 35 and yields the correct gravitational potential at the center of the Sun (5.1 times as large as the potential at the surface of the Sun) and the average gravitational potential for heavy nuclei (3.4 times that at the surface of the Sun). The resulting  $F_i$  are plotted in Fig. 2. From Fig. 2 we see that the form-factor suppression for capture from scattering off of hydrogen and helium is negligible, capture from scattering off of elements with atomic masses 12-32 is moderately suppressed, while capture from scattering off of iron is suppressed by several orders of magnitude for WIMPs in the several hundred GeV range. If there were no form-factor suppression, owing to the factor of  $m_i^4$  in the scalar cross section one would expect scattering from iron nuclei to dominate the capture of WIMPs in the Sun; however, because of the form-factor suppression, capture of heavy WIMPs in the Sun occurs primarily by scattering off of oxygen.<sup>17</sup> Even so, capture from scattering off of iron nuclei is still significant. When considering the complete capture rate due to scalar interaction of WIMPs off of nuclei in the Sun, one finds that the form-factor suppression of the scalar elastic scattering cross section decreases the capture

rate by a factor of about 0.3 for WIMPs of mass 80 GeV and about 0.07 for TeV-mass WIMPs.

The relative importance of the capture rates due to spin-dependent scattering opposed to coherent scattering due to squark and Higgs exchange depends on the supersymmetric model. Coherent scattering vanishes as the neutralino becomes a pure  $B$ -ino or Higgsino as does spin-dependent scattering due to  $Z$  exchange. To study the effect of Higgs-exchange scattering on the capture rate we set the squark mass to infinity. Doing so we find that the capture rate due to Higgs exchange is generally more important than that due to  $Z$  exchange when the neutralino is heavier than the  $W$ . In Fig. 3 we show contour plots in the  $M$ - $\mu$  plane of the rate of capture of neutralinos in the Sun for (a)  $\tan\beta = 2$ ,  $m_{H_2^0} = 20$  GeV, and  $\mu > 0$ ; (b)  $\tan\beta = 2$ ,  $m_{H_2^0} = 20$  GeV, and  $\mu < 0$ ; (c)  $\tan\beta = 2$ ,  $m_{H_2^0} = 35$  GeV and  $\mu > 0$ ; and (d)  $\tan\beta = 25$ ,  $m_{H_2^0} = 35$ , and  $\mu > 0$  assuming the squark mass is infinite. As expected, when squark exchange is negligible mixed-state neutralinos are captured far more readily than pure  $B$ -inos or pure Higgsinos. For fixed masses the capture rate decreases with increasing purity. For heavy neutralinos of fixed gaugino/higgsino composition that are heavy enough to be outside the Sun's resonance range, the capture rate is roughly proportional to  $m_{\tilde{\chi}}^{-5/2}$ ; one factor of  $m_{\tilde{\chi}}^{-1}$  is due to the number density in the galactic, one factor of  $m_{\tilde{\chi}}^{-1}$  is due to the kinematic suppression [*cf.*, Eqs. (13) and (14)], while the other factor of roughly  $m_{\tilde{\chi}}^{-1/2}$  comes from form-factor suppression. [The cross section for scattering due to exchange of the lightest Higgs boson does *not* decrease as the neutralino mass is increased far past the mass of the nucleus off of which it scatters; see Eq. (2.5) in Ref. 20.] Incidentally, as the neutralino mass is increased past a TeV, the form-factor suppression ceases to decrease with increasing WIMP mass; the reason is that if the nuclear mass is negligible compared to the WIMP mass, the momentum transfer does not depend on the WIMP mass.

From Fig 3 we also find that if  $\tan\beta$  is held fixed, the capture rate generally decreases with increasing  $m_{H_2^0}$  due to the propagator suppression, and if we hold  $m_{H_2^0}$  fixed, the capture rate generally increases with increasing  $\tan\beta$ ; this is simply because the Higgs couplings contain terms inversely proportional to  $\cos\beta$ .

To see the effect of the squark mass on the capture rate we show in Fig. 4 the rate of capture of neutralinos in the Sun when we take the squark mass to be 20 GeV heavier than the neutralino mass. Doing so, we find that the capture rate for Higgsinos and mixed-state neutralinos is similar to that when the squark mass is infinite; this implies that capture of Higgsinos and mixed-state neutralinos occurs primarily by Higgs-exchange scattering and the capture rate is insensitive to the squark mass. On the other hand, for models where the neutralino is mostly  $B$ -ino and the squark is taken to be 20 GeV heavier than the neutralino, capture occurs primarily by spin-dependent scattering of the neutralino off of the hydrogen in the Sun. This is illustrated in Fig. 5 where we show contours of the fraction of the capture rate that occurs due to spin-dependent scattering. Scattering that occurs via spin-dependent exchange of the squark depends only very weakly on  $\tan\beta$  and does not depend on  $m_{H_2^0}$  at all; therefore, if the squark mass is small enough so that capture of the neutralino occurs primarily by squark-exchange scattering, the capture rate depends primarily on the squark mass. We should also mention that in computing the spin-dependent cross section we used the (still controversial) EMC<sup>36</sup> results for the spin content of the proton. As discussed in Appendix A, if instead we used the naive flavor-SU(3) quark model for the proton the spin-dependent cross section due to squark exchange would be roughly 3 times larger.

Now that we have results for the capture rate we can see where the annihilation rate is at full signal  $\Gamma_A = C/2$  and where the time scale for equilibration of the number of WIMPs  $N$  is so large that  $\Gamma_A \ll C$ . In Fig. 6 we show the regions of parameter space where energetic neutrinos are not at full signal because neutralinos have not had sufficient time to collect in the Sun. In the dark shaded regions the signal is less than 10% of the full signal ( $t_\odot/\tau_A < 0.33$ ) and in the light shaded region the signal is less than 90% of the full signal ( $t_\odot/\tau_A < 1.82$ ); elsewhere, capture and annihilation of neutralinos occurs rapidly enough so that the neutrino rates are at full signal ( $t_\odot/\tau_A > 1.82$ ). In Fig. 6(a)  $\tan\beta = 2$ ,  $m_{H_2^0} = 20$ , the squark mass is taken to be infinite, and  $\mu > 0$ ; Fig. 6(b) is similar but  $\mu < 0$  is shown; and Fig. 6(c) is similar to Fig. 6(a) but the squark mass is taken to be 20 GeV heavier than the neutralino mass. Note that in most models



where the neutralino is lighter than a TeV the neutrino flux is at full signal. Later we will find that in regions of parameter space where the neutrino flux is large enough to be near current observational limits, the flux is at full signal. We will also see that  $\tau_A$  generally stays small enough so that the rates remain at full signal even for most models with a neutrino flux several orders of magnitude weaker than the current observational limits.

### III. NEUTRINO SPECTRA FROM NEUTRALINO ANNIHILATION

Given the annihilation rate  $\Gamma_A = C \tanh^2(t_\odot/\tau_A)/2$ , the differential flux of neutrino type  $i$  (e.g.,  $\nu_e, \nu_\mu, \bar{\nu}_\mu$ , etc.) produced by the annihilation of neutralinos in the Sun or Earth at a distance  $R$  from the source is

$$\left(\frac{d\phi}{dE}\right)_i = \frac{\Gamma_A}{4\pi R^2} \sum_F B_F \left(\frac{dN}{dE}\right)_{Fi}, \quad (16)$$

where the sum is over all annihilation channels.

The quantities  $B_F$  are the branching ratios for annihilation into final state  $F$ . Since the neutralinos are moving nonrelativistically in the Sun or Earth,  $B_F$  may be determined by the relative magnitude of the cross sections for annihilation into channel  $F$  at zero velocity given in Ref. 6 [Eqs. (A10), (B7), (C11), and (D6)] and in Appendix B. The final states  $F$  into which neutralinos may annihilate at zero relative velocity are  $f\bar{f}$  where  $f$  is a quark or charged lepton,  $W^+W^-$ ,  $Z^0Z^0$ ,  $H_1^0H_3^0$ ,  $Z^0H_1^0$ ,  $Z^0H_2^0$ ,  $W^+H^-$ , and  $W^-H^+$ .<sup>37</sup> The cross sections for annihilation into other combinations of gauge and Higgs bosons vanish as the relative velocity approaches zero. The calculation of the cross sections for annihilation into the mixed gauge- and Higgs-boson final states  $Z^0H_1^0$ ,  $Z^0H_2^0$ ,  $W^+H^-$ , and  $W^-H^+$  at zero relative velocity are new and the results are presented in Appendix B. As noted by Olive and Srednicki,<sup>7</sup> annihilation into the mixed gauge/Higgs boson final states is generally small for pure  $B$ -inos and Higgsinos but may be important for mixed-state neutralinos. For models where the neutralino is a pure  $B$ -ino and the squark masses are much larger than all other masses involved, annihilation into the mixed gauge/Higgs boson states may be comparable to annihilation into Higgs-boson states; in this case, neutralinos annihilate predominantly into these

states, but the total rate for annihilation is very small and the neutralinos are generally very weakly interacting.

The  $(dN/dE)_{Fi}$  are the differential energy fluxes of neutrino type  $i$  at the surface of the Sun (or Earth) that result from the injection of particles in final state  $F$  at the center of the Sun (or Earth). These fluxes are functions of the neutrino energy  $E$  and the energy  $E_i$  of the injected particles. Calculation of the fluxes requires information about the cascade following the decay of the annihilation products, the hadronization of heavy quarks in the cascade, and the interactions of particles in the cascade with the medium at the core of the Sun or Earth. Since the  $(dN/dE)_{Fi}$  are the neutrino fluxes at the surface of the astrophysical object while neutralino annihilation occurs at the center and the Sun is not transparent to neutrinos with energies in the 100-GeV range, absorption and energy loss of neutrinos by the solar medium must also be included in the calculation. Since the density and thickness of the Earth are different from those in the Sun, the  $(dN/dE)_{Fi}$  from particles injected in the Earth will be different than those from the Sun.

A detailed calculation of the neutrino spectrum from injected quarks and leptons was performed by the authors of Ref. 19 using the Lund Monte Carlo. Their calculation includes hadronization of quarks and interactions of the fermions and neutrinos with the solar medium. Electrons, muons, and light ( $u, d, s$ ) quarks are stopped in the Sun before they decay and therefore do not produce high-energy neutrinos. The top quark is expected to hadronize and then decay far before it can lose a substantial amount of energy, and the  $\tau$  will also decay immediately. Bottom and charm quarks hadronize and due to the high density of the core of the Sun, the heavy hadron may subsequently lose a significant fraction of its energy before decaying. RS estimate that if  $E_0$  is the initial heavy hadron energy in the Sun, the mean energy of the hadron when it decays will be

$$\langle E \rangle = E_c e^{E_c/E_0} \int_{E_c/E_0}^{\infty} \frac{e^{-x}}{x} dx, \quad (17)$$

and they estimate that  $E_c \approx 250$  GeV for charmed hadrons, and  $E_c \approx 470$  GeV

for bottom hadrons. Evaluating the integral, one finds that  $\langle E \rangle \simeq E_0(1 - E_0/E_c)$  for  $E_0 \ll E_c$  and  $\langle E \rangle \simeq E_c[\ln(E_0/E_c) - \gamma_E]$  for  $E_0 \gg E_c$ , where  $\gamma_E = 0.577\dots$  is Euler's constant, so the mean energy of the decaying hadron never grows much larger than  $E_c$ .

At high energies the Sun is no longer transparent to neutrinos and interactions of neutrinos with the solar medium may significantly alter the energy spectrum. For  $\tau$ 's injected at energies above several hundred GeV, the flux of muon neutrinos may be significantly enhanced by the decay of additional  $\tau$ 's produced by charged-current interactions of tau neutrinos with the solar medium. Electron and muon neutrinos are absorbed by charged-current interactions: The probability that a neutrino of initial energy  $E_i$  will escape from the Sun is  $\exp(-E_i/E_{\text{Abs}})$ , where  $E_{\text{Abs}} = 198$  GeV for neutrinos and  $E_{\text{Abs}} = 296$  GeV for anti-neutrinos. Furthermore, at high energies neutral-current interactions degrade the neutrino energy.

Since the density of the core of the Earth is about 1/12 that of the core of the Sun, muons and light quarks are still stopped before they decay, while stopping of heavy hadrons may be ignored until several TeV. Moreover, the optical depth of the Earth is much smaller than that of the Sun, so interactions of neutrinos with the Earth may be ignored for neutrino energies less than several TeV. As a result, Ritz and Seckel's non-interacting results may be used for the neutrino spectra from the Earth.

The results presented by RS are for neutrino spectra from fermions injected into the core of the Sun at 60 GeV and at 1000 GeV; however, we need to obtain information about the spectra for fermions injected at any energy up to a TeV. For reasons to be discussed below, we will eventually focus on detection of neutrinos via neutrino-induced upward-moving muons. Since the cross section for a neutrino to produce a muon in the rock below the detector is proportional to the neutrino energy and the range of the muon is roughly proportional to the energy, the probability for a neutrino to produce a throughgoing muon is proportional to the energy squared. Therefore, to obtain event rates we need only the second moments  $\langle N z^2 \rangle_{F_i} m_{\tilde{\chi}}^2$ , where

$$\langle Nz^2 \rangle_{Fi} \equiv \frac{1}{m_{\tilde{\chi}}^2} \int \left( \frac{dN}{dE} \right)_{Fi} E^2 dE. \quad (18)$$

The functional forms of the spectra are not required.

For fermions injected into the core of the Earth, interactions are negligible and the moments of the neutrino spectra are easily obtained from Ritz and Seckel's non-interacting results. In this case,<sup>19</sup>

$$\langle Nz^2 \rangle = \frac{1}{3} \langle N \rangle \langle y^2 \rangle \left( \langle z_f^2 \rangle - \frac{m_f^2}{4E_i^2} \right), \quad (19)$$

where  $E_i$  is the fermion injection energy,  $\langle N \rangle$ ,  $\langle y^2 \rangle$ , are the rest-frame yield and second moments listed in Table 2 of RS,  $\langle z_f^2 \rangle$  is the second moment of the fragmentation function listed in Table 3 of RS, and  $m_f$  is the mass of the injected fermion.

For fermions injected into the core of the Sun, the calculation is much more difficult since one must take interactions into account. RS outline a procedure for analytically estimating the effect of interactions which reproduces the Monte Carlo results reliably for injection energies  $\lesssim 200$  GeV. An effort to modify and apply the corrections to describe interactions at higher energies resulted in moments of the neutrino spectra that reproduced those obtained from the Monte Carlo only to within  $\sim 50\%$ ; however, in doing so one finds that for injected  $b$  and  $c$  quarks, the most important effect is the stopping of heavy hadrons. Therefore, for the scaled second moment of the neutrino spectra for  $b$  and  $c$  quarks we assumed that

$$\langle Nz^2 \rangle = \left[ ax_0 e^{x_0} \int_{x_0}^{\infty} \frac{e^{-x}}{x} dx \right]^2, \quad (20)$$

where  $x_0 \equiv E_c/E_i$ , and fitted  $a$  and  $E_c$  to match the interacting results of RS at 60 GeV and 1000 GeV. (Actually, since RS did not present interacting results at 60 GeV for anti-neutrinos or, in the case of the  $b$  quark, for neutrinos, we obtained these numbers using the corrections for interactions described in their paper.) We found that for neutrinos from  $c$  quarks,  $a = 0.056$  and  $E_c = 155$  GeV;

for anti-neutrinos from  $c$  quarks,  $a = 0.052$  and  $E_c = 275$  GeV; for neutrinos from  $b$  quarks,  $a = 0.086$  and  $E_c = 185$  GeV; and for anti-neutrinos from  $b$  quarks,  $a = 0.082$  and  $E_c = 275$  GeV.

Since  $\tau$  leptons are not stopped and do not hadronize, absorption of muon neutrinos is the most important interaction effect for the spectra from  $\tau$  leptons; production of muon neutrinos from interactions of  $\tau$  neutrinos is also significant at high energies, but these neutrinos are predominantly low energy and do not contribute significantly to the second moment. Thus, we take the second moment of the neutrino spectrum from injected  $\tau$  leptons to be

$$\langle Nz^2 \rangle = ae^{-E_i/E_{\text{Abs}}}, \quad (21)$$

and fit  $a$  and  $E_{\text{Abs}}$  to reproduce the RS results at 60 GeV and 1000 GeV. For neutrinos,  $a = 0.0204$  and  $E_{\text{Abs}} = 476$  GeV, and for anti-neutrinos,  $a = 0.0223$  and  $E_{\text{Abs}} = 599$  GeV.

Our estimates for the spectra from the top quark are far more uncertain. RS used a top-quark mass of 40 GeV, and here we have assumed that it is 120 GeV. Since even 40 GeV is so much heavier than all other lighter particle masses, we assumed that the scaled rest-frame neutrino spectra would be the same for a top quark of 120 GeV as it would for a top quark of 40 GeV. We then estimated the effect of interactions for a top quark injected into the solar core at 120 GeV and assumed that the RS interacting results at 1000 GeV would also be valid for a 120 GeV top quark. At injection energies just above threshold the moments of the neutrino distribution have a strong dependence on the fragmentation function, and at higher energies, absorption of neutrinos determines the behavior of the spectral moments. Therefore, neither of the expressions in Eq. (20) or Eq. (21) really describe the injection-energy dependence of  $\langle Nz^2 \rangle$ . Nevertheless, the effect of interactions, which we can reliably estimate at low energies, is better described by Eq. (20) than by Eq. (21), so we use the form of Eq. (20) with  $a = 0.18$  and  $E_c = 110$  GeV for neutrinos, and  $a = 0.14$  and  $E_c = 380$  GeV for anti-neutrinos.

Although these estimates of  $\langle Nz^2 \rangle$  are somewhat *ad hoc* and admittedly crude for arbitrary injection energies between 60 GeV and 1000 GeV, they should

be relatively accurate for neutrino spectra from annihilation of neutralinos not much heavier than the  $W$  or  $Z$ ; at higher energies, our approximations are far from pinpoint accuracy, but they should still be good enough to indicate the effect of interactions of the decay products and neutrinos with the solar medium.

Since Higgs and vector bosons decay into pairs of quarks and leptons immediately, it is easy to obtain  $\langle Nz^2 \rangle$  for injected bosons from our previous results for the neutrino spectra from injected fermions.<sup>38</sup> Suppose boson  $B$  undergoes 2-body decays into fermions  $f$ , and  $N_f^B$  is the number of fermions of type  $f$  produced on average per  $B$  decay (i.e.,  $\sum_f N_f^B = 2$ ) and can be obtained from the branching ratios for decay of  $B$  into the various final states and the contents of those channels. If  $E_i$  is the injected boson energy, then the energy of the fermion in the rest frame of the  $B$  is  $m_B/2$ , where  $m_B$  is the  $B$  mass, and in the moving frame it is  $E_f = E_i(1 + \beta \cos \theta)/2$  where  $\beta$  is the velocity of  $B$  in units of the speed of light and  $\theta$  is the angle between the direction of motion of the decay product and the direction of motion of  $B$ . For Higgs bosons and unpolarized vector bosons (which are produced by the annihilation of neutralinos provided the interactions of the neutralinos are  $CP$ -conserving, which is assumed throughout here) the decay is isotropic which means that the laboratory-frame energies of the fermions from the decay of  $B$  are evenly distributed from  $E_i(1 - \beta)/2$  to  $E_i(1 + \beta)/2$ . Therefore,

$$\langle Nz^2 \rangle_{Bi} = \sum_f \frac{N_f^B}{\beta E_i} \int_{E_i(1-\beta)/2}^{E_i(1+\beta)/2} \langle Nz^2 \rangle_{fi}(E) dE, \quad (22)$$

where  $\langle Nz^2 \rangle_{fi}(E)$  are the second moments of the neutrino spectra presented above as a function of the injected fermion energy  $E$ .

The three neutral Higgs bosons of the minimal extension of the supersymmetric standard model decay into fermion-antifermion pairs. The branching ratios for the decays of  $H_2^0$  and  $H_3^0$ , from which the  $N_f^B$  are obtained are given in the Appendix of Ref. 21 and are proportional to the fermion mass squared (so the Higgs bosons do not decay directly into energetic neutrinos), and the branching

ratios for the decay of  $H_1^0$  may be obtained from those for  $H_2^0$  decay by switching  $\cos \alpha$  and  $\sin \alpha$ .

If the neutralinos annihilate into  $\tau$  leptons or  $b$ ,  $c$ , or  $t$  quarks, and an energetic neutrino is produced in the decay of these fermions, then the typical neutrino energy is  $1/3$  the mass of the neutralino. If the neutralinos annihilate into Higgs bosons there is another step in the decay chain before energetic neutrinos are produced, so their energies would typically be  $1/6$  the mass of the neutralino. This is partially compensated by the fact that each Higgs boson produces two fermions, but since the detection rates are proportional to the energy squared, the net effect is that if the neutralinos annihilate into Higgs bosons the detection rate is roughly half the rate if they annihilated into fermions (assuming, of course, that the branching ratio for the various fermions from Higgs-boson decays is nearly the same as the branching ratios for the various fermions from neutralino annihilation if only fermion final states are considered). Although  $H_2^0$  must be lighter than  $m_Z \cos 2\beta$ , and most certainly decays only into quarks and leptons, the other Higgs bosons may be much heavier and may include other exotic decay channels as well which may also produce energetic neutrinos which would most likely have a much softer spectrum. If this is the case, then by assuming that they decay only into quarks and leptons we are overestimating the neutrino yields.

It turns out that the most favorable annihilation channel for observing high-energy neutrinos is the gauge-boson final state. The reason is that  $W$  and  $Z$  bosons decay directly into neutrinos with appreciable branching ratios. Compared with the event rate from these "semi-prompt" neutrinos, the event rate for neutrinos which come from the quark and charged-lepton decay products of the gauge bosons is negligible. A  $W$  decays to a muon and a muon neutrino about 11% of the time,<sup>39</sup> so neglecting interactions,  $\langle Nz^2 \rangle$  is roughly 0.025 for slow  $W$ 's and 0.033 for relativistic  $W$ 's. This is larger than all the values expected from fermion-antifermion pairs (see Table 1 of RS), although  $\tau^\pm$  final states come close. Furthermore, at higher energies, no energy is lost from hadronization or stopping of the vector bosons. (At higher energies, the value of  $\langle Nz^2 \rangle$  for gauge-boson final states becomes smaller than that from  $\tau^\pm$  final states; this is because the energies of neutrinos from gauge-boson decays are generally larger than those

from  $\tau$  decays so absorption of neutrinos in the Sun from gauge-boson decays is stronger than absorption of neutrinos from  $\tau$  decays. Even so, if the neutralino annihilates to  $\tau^\pm$  pairs, it will also have a significant and usually larger annihilation branch to  $b\bar{b}$ ,  $c\bar{c}$ , and if kinematically accessible,  $t\bar{t}$  pairs, so the total neutrino yield from gauge-boson final states will be greater than the total yield from fermion-antifermion states.) The branching ratio for  $Z^0 \rightarrow \nu\bar{\nu}$  is slightly smaller than the branching ratio for  $W \rightarrow \mu\bar{\nu}_\mu$ , but two neutrinos are produced so  $\langle Nz^2 \rangle$  is a little larger.

For  $W$  bosons injected in the core of the Earth with velocity  $\beta$  we can ignore interactions of the neutrinos with the Earth, and

$$\langle Nz^2 \rangle_{Wi} = \Gamma_{W \rightarrow \mu\bar{\nu}_\mu} (3 + \beta^2) / 12, \quad (23)$$

where  $i$  is a neutrino or anti-neutrino;  $\langle Nz^2 \rangle_{Zi}$  may be obtained by multiplying by two and replacing  $\Gamma_{W \rightarrow \mu\bar{\nu}_\mu}$  by  $\Gamma_{Z \rightarrow \nu\bar{\nu}}$ . To account for interactions of the neutrinos with the solar medium for vector bosons injected into the core of the Sun we use the estimate of RS that a neutrino injected with an energy  $E$  leaves the Sun with energy

$$E_f = \frac{E}{1 + E\tau_i}, \quad (24)$$

where  $\tau_\nu = 1.01 \times 10^{-3} \text{ GeV}^{-1}$  and  $\tau_{\bar{\nu}} = 3.8 \times 10^{-4} \text{ GeV}^{-1}$ , and probability

$$P_f = \left( \frac{1}{1 + E\tau_i} \right)^{\alpha_i}, \quad (25)$$

where  $\alpha_\nu = 5.1$  and  $\alpha_{\bar{\nu}} = 9.0$  for anti-neutrinos. Doing so we find that

$$\langle Nz^2 \rangle_{Wi} = \frac{\Gamma_{W \rightarrow \mu\bar{\nu}_\mu}}{\beta E_i^3} \frac{2 + 2E\tau_i(1 + \alpha_i) + E^2\tau_i^2\alpha_i(1 + \alpha_i)}{\alpha_i\tau_i^3(\alpha_i^2 - 1)(1 + E\tau_i)^{\alpha_i+1}} \bigg|_{E=E_i(1-\beta)/2}^{E=E_i(1+\beta)/2}, \quad (26)$$

for  $W$ 's injected into the core of the sun with energy  $E_i$ .

In Fig. 7 we show the second moments  $m_{\tilde{\chi}}^2 \langle Nz^2 \rangle$  of the neutrino yield from the Sun for the  $c\bar{c}$ ,  $b\bar{b}$ ,  $t\bar{t}$ ,  $\tau^\pm$ ,  $W^\pm$ , and  $H_2^0 H_3^0$  (using  $\tan\beta = 2$  and  $m_{H_2^0} = 20$ ) final states as a function of the neutralino mass. The neutrino yields from  $Z^0$



pairs (not shown) is similar to, but slightly smaller, than the yields from  $W^\pm$  pairs and the yield from the  $H_1^0 H_3^0$  (when it is kinematically accessible) final state is similar to that from the  $H_2^0 H_3^0$  final state. We remind the reader that although the yield from  $\tau^\pm$  pairs surpasses that from gauge-boson pairs for neutralinos heavier than about 200 GeV, if the neutralino annihilates to lepton pairs, then it also has a significant annihilation branch into quark-antiquark pairs and the yield from gauge-boson pairs is still larger than the *total* yield from fermion-antifermion final states.

#### IV. RATES FOR DETECTION IN UNDERGROUND DETECTORS

Generally, neutrinos are detected either by contained events where the neutrino undergoes a charged-current interaction and produces a lepton in the detector or by upward-moving throughgoing muons in which a muon neutrino undergoes a charged-current interaction in the rock below the detector and produces a muon which then passes through the detector. Since the cross section for a charged-current interaction is proportional to the neutrino energy and the effective range of a muon is proportional to the muon energy, the rate for contained events is roughly proportional to the neutrino energy and the rate for neutrino-induced throughgoing muons is proportional to the square of the neutrino energy. Therefore, at sufficiently high energies the rate for throughgoing muons should be greater than that for contained events. In Ref. 21 the regions of parameter space ruled out by searches for contained events from Frejus<sup>40</sup> very nearly matches those regions ruled out by searches for throughgoing muons from IMB<sup>28</sup> for neutralinos less massive than the  $W$ . (In addition, NUSEX<sup>41</sup> reports that limits on muons produced by neutrino interactions in the rock below the detector that stop inside the fiducial volume of the detector are in agreement with those from contained or throughgoing events from IMB, Frejus, and Kamiokande.) Furthermore, Ref. 20 indicates that the rate for detecting high-energy neutrinos from the Sun via throughgoing muons per 100 m<sup>2</sup> becomes larger than that for contained events per kiloton for neutralinos heavier than roughly 60 GeV while the rate for observing throughgoing muons is greater than that for contained events from neutrinos from the Earth for neutralinos heavier than roughly 20 GeV. Therefore,

since neutralinos heavier than the  $W$  are considered here, we will concentrate on detection of neutrinos via throughgoing muon events.

After taking the cross section for muon production in the rock and the effective range of the muons into account but ignoring detector thresholds (which are near 2 GeV—far lower than the average neutrino energies considered here), the rate (per unit detector area) for neutrino-induced throughgoing muon events is<sup>19</sup>

$$\Gamma_{\text{detect}} = 1.27 \times 10^{-29} C m_{\tilde{\chi}}^2 \sum_i a_i b_i \sum_F B_F \langle N z^2 \rangle_{Fi} \text{ m}^{-2} \text{ yr}^{-1}, \quad (27)$$

for neutrinos from the Sun; the same expression multiplied by  $5.6 \times 10^8$  (the square of the ratio of the Earth-Sun distance to the Earth's radius) gives the rate for neutrino events from the Earth. Here,  $C$  is the capture rate in units of  $\text{s}^{-1}$ , the sum on  $i$  is over muon neutrinos and anti-neutrinos, the  $a_i$  are neutrino scattering coefficients,  $a_\nu = 6.8$  and  $a_{\bar{\nu}} = 3.1$ , the  $b_i$  are muon range coefficients,  $b_\nu = 0.51$  and  $b_{\bar{\nu}} = 0.67$ , and  $\langle N z^2 \rangle_{Fi}$  is the second moment of the spectrum of neutrino type  $i$  from final state  $F$  scaled by the neutralino mass squared.

Given the expressions for  $\langle N z^2 \rangle$  for neutrino spectra from the Sun and the Earth it turns out that for heavy neutralinos with masses not much greater than a TeV, the neutrino signal from the Sun should be larger than that from the Earth. To see this, first note that the difference in the prefactors  $c$  for the Sun and the Earth in the capture-rate equation Eq. (12) is roughly compensated by the geometric factor  $5.6 \times 10^8$  accounting for the difference in the distances between us and the Sun or the center of the Earth ( $5.8 \times 10^{24} \text{ s}^{-1}$  for the Sun opposed to  $3.2 \times 10^{24} \text{ s}^{-1}$  for the Earth). Therefore, for dark-matter candidates with masses in the Earth's resonance range  $10 \text{ GeV} \lesssim m_{\tilde{\chi}} \lesssim 75 \text{ GeV}$  the kinematic suppression factor  $S_i$  is nearly unity and since the fraction of the Earth's mass due to heavy elements is higher than that in the Sun, the neutrino flux from the Earth may well be comparable to or greater than that from the Sun (if the WIMP in question has spin-independent interactions).

In contrast, the heavy neutralinos considered here have masses outside the Earth's resonance range, so capture by the Earth is strongly suppressed due to the factor of  $(v_{\text{esc}}/\bar{v})^2 \approx 1.4 \times 10^{-3}$ . Even if the neutralino mass is large enough

to fall outside the Sun's resonance range, capture of WIMPs by the Earth is still suppressed relative to capture in the Sun because  $v_{\text{esc}}$  is so much smaller in the Earth than in the Sun. Although form-factor suppression does not occur for capture of heavy WIMPs in the Earth, the form-factor suppression of the capture rate in the Sun never falls far below  $10^{-1}$ , whereas the kinematic suppression of capture of WIMPs in the Earth is of order  $10^{-4}$  that in the Sun. In addition, if the capture and annihilation rates for the neutralino in question are small then the neutrino signal from the Earth may be further weakened relative to that from the Sun as the time  $\tau_A$  for the number of neutralinos to reach equilibrium in the Earth is generally smaller than that in the Sun. When considering heavy WIMPs the calculation of the capture rate in the Earth is also far more uncertain than that for capture in the Sun. The reason is that only heavy WIMPs that are moving very slowly may be captured in the Earth and an isotropic Maxwell-Boltzmann distribution does not necessarily give a good approximation to the phase-space density of such WIMPs.<sup>18</sup> At this point we should also remind the reader that if the WIMP in question has only an axial interaction with nuclei (such as a  $B$ -ino in models with a relatively light squark) it may be captured in the Sun by scattering off of hydrogen, but it will *not* be captured in the Earth.

Although the rate of capture of WIMPs by the Earth remains small relative to that by the Sun at higher WIMP masses, above some large mass the neutrino signal from the Earth might become comparable to or larger than that from the Sun because of interactions of decay products and neutrinos with the Sun. We can estimate this mass scale by taking the following simplified model: Assume capture occurs only by scattering off of iron and neglect form-factor suppression of capture in the Sun; in the Earth this provides a good estimate of the capture rate, and if anything, this should underestimate the capture rate in the Sun. Doing so, the capture rates differ only in the prefactors  $c$ , the factors  $f_i$ ,  $\phi_i$ , and the factor of  $v_{\text{esc}}^2$  in  $S_i$ . To include the effect of form-factor suppression of capture in the Sun we multiply the capture rate by 0.07, the value of the suppression factor for a WIMP of mass 1 TeV. (At smaller WIMP masses the suppression is not as severe, while the suppression does not become significantly stronger for WIMP masses greater than a TeV.) Doing so we find that the capture rate of

very heavy neutralinos in the Sun is roughly 20 times that in the Earth when scaled by the difference in the geometric factor. It turns out that the values of  $\langle Nz^2 \rangle$  for particles injected into the core of the Sun at 1 TeV are of order 1/10 of those for particles injected into the core of the Earth at 1 TeV; therefore, the neutrino signal from heavy neutralino annihilation in the Sun should remain much larger than that from the Earth for neutralino masses below 1 TeV and become comparable at a WIMP mass near 1 TeV. For WIMPs heavier than a TeV, the neutrino signal from WIMP annihilation in the Earth should become stronger than that from annihilation in the Sun because of absorption of neutrinos in the Sun. Here we have ignored the fact that the number of very heavy neutralinos in the Sun or Earth may not have reached equilibrium; correcting for this would only increase the mass scale at which the neutrino signal from the Earth might become comparable to that from the Sun.

So, the neutrino signal from the Sun should be much stronger than that from the Earth for neutralinos just heavier than the  $W$ , and the strength of the signal from the Sun relative to that from the Earth should decrease as the neutralino mass is increased until a WIMP mass of order a TeV when the signal from the Earth becomes comparable to that from the Sun. Since the signal from the Earth should be small compared to that from the Sun in the range 80-1000 TeV, in the following we will focus our attention on the neutrino signal from WIMP annihilations in the Sun only. We should also point out that these results imply that observation of a neutrino signal from the Sun and the absence of one from the Earth would be a signature of particle dark matter in the mass range 80-1000 GeV.

## V. RESULTS

Since the MSSM has many undetermined parameters we will show results in the  $M$ - $\mu$  plane for several values of  $\tan \beta$  and  $m_{H_2^0}$  allowed by null results from searches for neutral Higgs bosons at LEP<sup>26</sup>. Again, we will first take the squark masses to be infinite; this minimizes the capture rate and emphasizes gauge- and Higgs-boson final states. Then we will consider squark masses 20 GeV higher than the neutralino mass; this will emphasize capture by spin-dependent scattering and

fermion final states for neutralinos where such effects are important.

When the neutralino is mostly Higgsino, it annihilates primarily into gauge bosons, and the effects of the squark, Higgs-boson, and top-quark masses are relatively unimportant.<sup>6</sup> When the neutralino is mostly  $B$ -ino, it annihilates primarily into fermions (provided the squark mass is not too large), and when the top-quark channel is open, it annihilates predominantly into the top quark. Mixed-state neutralinos generally annihilate into gauge bosons, fermions, and Higgs bosons as well with comparable magnitudes.

In Fig. 8 we plot contours of the fraction of the neutrino signal that comes from gauge bosons. When the squark mass is taken to be infinite [Fig. 8(a)], the neutralino does not annihilate into fermions and since gauge bosons yield a much harder spectrum of neutrinos than Higgs bosons, virtually all of the neutrino signal from heavy neutralinos comes from gauge-boson final states. When the squark mass is 20 GeV heavier than the neutralino mass [Fig. 8(b)], fermions are the dominant annihilation products from  $B$ -inos, so the neutrino signal is not always dominated by neutrinos from gauge bosons. Still, neutrinos from gauge-boson final states dominate the signal for Higgsinos and contribute a signal comparable to that from fermions in many regions of parameter space with mixed-state neutralinos and  $B$ -inos.

The IMB collaboration has found an upper limit on the flux of upward-moving muons induced by neutrinos from the Sun with energy larger than 2 GeV of  $2.65 \times 10^{-2} \text{ m}^{-2} \text{ yr}^{-1}$ ,<sup>28</sup> (and similar, though slightly weaker limits have been found by Kamiokande II<sup>29</sup>). Therefore, supersymmetric models in which the capture and annihilation of the neutralino yields larger neutrino fluxes are inconsistent candidates for the primary component of the galactic halo. (To be precise, we do not implement the 2 GeV cutoff in our calculation, but since we are primarily interested in heavy neutralinos here the fraction of our signal from lower energy neutrinos should be insignificant.) In Fig. 9 the dark shading denotes the regions of parameter space excluded by this constraint. The light shaded regions are those that would be excluded if the observational flux limits were to be improved by a factor of 100. The curve inside the light shaded areas encloses

regions of parameter space that would be excluded if current observational limits were improved by a factor of 10. To indicate the sensitivity of these results to uncertainties in the calculation, the dashed curve inside the excluded region indicates the region excluded if the true neutrino rate is only 1/5 as large as our calculations indicate. In (a)  $\tan \beta = 2$ ,  $m_{H_2^0} = 20$  GeV, the squark mass is taken to be infinite and  $\mu > 0$ , and (b) is similar except that  $\mu < 0$ . In (c)  $\tan \beta = 2$  and  $m_{H_2^0} = 20$  GeV, in (d)  $\tan \beta = 2$  and  $m_{H_2^0} = 35$  GeV, and in (e)  $\tan \beta = 25$  and  $m_{H_2^0} = 35$  GeV. In (c), (d), and (e), the squark mass is assumed to be 20 GeV greater than the neutralino mass and only regions of positive  $\mu$  are shown.

From Fig. 9, we see that limits on energetic neutrino fluxes from the Sun already exclude many supersymmetric models with heavy mixed-state neutralinos lighter than about a TeV when the lightest Higgs is light and  $\tan \beta$  is small [Fig. 9(a), (b), and (c)], or when  $\tan \beta$  is large [Fig. 9(e)], independent of the squark mass. Unfortunately, the region of  $m_{H_2^0}$ - $\tan \beta$  parameter space in which current neutrino limits might exclude neutralinos as dark matter candidates is similar to that excluded by current LEP results;<sup>26</sup> the rates for neutrino events from models with larger values of  $m_{H_2^0}$  [Fig. 9(d)] are much smaller. Also, current neutrino-flux bounds are ineffective in ruling out neutralinos that are almost pure Higgsino or  $B$ -ino; however, if the observational bounds are improved by a factor of ten, far more supersymmetric dark-matter candidates would be observable. For values of  $\tan \beta$  and  $m_{H_2^0}$  near the current observational limits [Fig. 9(a), (b), and (e)], most heavy Higgsinos would be observable, independent of the squark mass, should they be the primary component of the galactic halo; for larger  $m_{H_2^0}$ , the rates are smaller [Fig. 9(d)]. The rates from heavy  $B$ -inos are sensitive to the squark mass as may be seen by comparing Fig. 9(a) and Fig. 9(c). If the squark mass is much greater than the neutralino mass [Fig. 9(a)], then  $B$ -inos that are extremely pure will not be observable, but if the squark mass is near the neutralino mass [Fig. 9(c)], the event rates are much greater. Also, note that the event rates are much smaller from supersymmetric models with negative  $\mu$ . This is because the elastic-scattering cross sections are generally smaller<sup>42</sup> which leads to a smaller capture rate.

Throughout we have taken the top-quark mass to be 120 GeV; however, our

results are generally insensitive to this assumption. This is because the event rates are determined primarily by the capture rates in the Sun which do not depend on the top-quark mass. Increasing the top-quark mass would increase the fraction of annihilation products that are top quarks relative to the fraction that are gauge or Higgs bosons, and the neutrino spectrum from top quarks is generally softer than that from gauge bosons. Therefore, an increase in the top-quark mass would result in a slightly lower event rate for models where the number of top-quark final states is comparable to the number of gauge-boson final states.

By comparing Fig. 9 with Fig. 6 we find that in the excluded regions the capture and annihilation rates are large enough that the number of neutralinos in the Sun has reached equilibrium ( $t_{\odot} > \tau_A$ ). Generally, we find that current observational limits on energetic neutrino fluxes would have to be increased by about 2 orders of magnitude until neutralinos that have not yet reached their equilibrium in the Sun are detected.

## VI. CONCLUDING REMARKS

One of the most important questions facing particle physics and cosmology is the nature of the dark matter known to exist throughout the Universe and in our galactic halo. A well-motivated extension of the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  model of particle interactions is the minimal supersymmetric standard model. If low-energy supersymmetry exists in Nature then it is likely that the neutralino is the lightest supersymmetric particle. Although the neutralino was originally taken to be light, its mass could also lie in the 100-GeV range, and as unsuccessful accelerator searches push the mass scale for supersymmetry upward this possibility becomes more attractive. Calculations<sup>6</sup> show that in much of parameter space the neutralino has a relic abundance suitable for solving the dark matter problem. Given this result, it remains to be seen experimentally whether neutralinos do indeed populate our halo.

In this paper we have proposed that the presence of heavy neutralino dark matter be inferred through the observation of energetic neutrinos produced by neutralino annihilation in the Sun. Neutralinos that are primarily Higgsinos or a

mixed Higgsino/gaugino state are captured in the Sun by coherent elastic scattering due to light-Higgs-boson exchange off of nuclei in the Sun, and for mixed-state neutralinos the capture is quite efficient. If the squark is not much heavier than the neutralino, gauginos are captured via spin-dependent squark-exchange scattering off of hydrogen in the Sun. Since the masses of heavy neutralinos lie outside the Earth's resonance range, capture in the Earth is relatively inefficient.

Neutralinos that have been captured in the Sun will annihilate therein and high-energy neutrinos will be produced by the decays of the annihilation products. Calculation of the energy spectrum of neutrinos from such a source as they emerge from the Sun is quite involved as the cascade from the annihilation products must be modeled considering, amongst other things, the effect of the solar medium on the shower. In addition, since the neutrinos have very high energies, absorption and energy loss of the neutrinos as they pass through the Sun must be included in the calculation.

The most promising method of detection of these neutrinos is through observation in underground detectors of upward-moving muons produced by the neutrinos in the rock below the detector. Current limits from IMB on the number of such throughgoing muons may already be used to constrain regions of heavy-neutralino parameter space where the neutralino is a mixed Higgsino/gaugino state and with a mass less than about 300 GeV. Furthermore, in other regions of parameter space, where the neutralino is either slightly heavier (though still in the sub-TeV range) or closer to being a pure Higgsino or gaugino state, the predicted event rates are large enough that energetic neutrino signals may be observable in the near future with increased observing time or larger detectors. Given the enormous importance of such a discovery and the promise of observation of such a signal from many supersymmetric dark-matter candidates, the search for energetic neutrinos from the Sun should be pursued.

The final result of our calculation that was compared with experiment was the flux of neutrino-induced upward-moving muons; therefore, the strongest limits should eventually come from detectors with the largest surface area or longest exposure time. The current IMB<sup>28</sup> limits come from a detector of area roughly



400 m<sup>2</sup> and an exposure time of about a year, and the limits from Kamiokande II<sup>29</sup> come from a slightly smaller exposure. The next improvement should come from MACRO<sup>43</sup> which will have an area more than twice as large as IMB, and in the more distant future there may be a factor of 10 improvement in the collection area with a deep-sea detector.<sup>44</sup> There is also the intriguing possibility of an increase in detector area of several orders of magnitude by looking for Cherenkov radiation from energetic muons in deep antarctic ice.<sup>45</sup>

To see the prospects for discovery of dark-matter candidates via observation of muons induced by neutrinos from WIMP annihilation in the Sun let us consider the background of throughgoing muons induced by atmospheric neutrinos. The flux of such muons (with energies larger than 2 GeV) is<sup>28</sup>

$$\Phi_\mu(E > 2 \text{ GeV}) = 0.075 \text{ m}^{-2} \text{ yr}^{-1} \text{ sr}^{-1}. \quad (28)$$

Now although the angular size of the Sun in the sky is quite small and the detector resolution may be quite good, the angle between the muon direction and the direction of the parent neutrino has an intrinsic distribution with average of roughly  $\bar{\theta}_{\mu\nu} \simeq 15^\circ / [E_\mu / (2 \text{ GeV})]^{1/2}$ , so muon tracks from within  $15^\circ$  of the Sun need to be accepted. We see that the background from an angular window of this size is comparable to the IMB limit of  $0.0265 \text{ m}^{-2} \text{ yr}^{-1}$ . So additional exposure will improve this flux limit by providing the statistics needed to distinguish excess signal from background.

Another strategy for improving the signal to noise ratio is to raise the muon-energy cutoff  $E_\mu^{\text{cut}}$ . Since the atmospheric neutrino flux decreases roughly as  $E_\nu^{-3}$  (to be conservative) and the probability for detection of a neutrino of energy  $E_\nu$  is proportional to  $E_\nu^2$ , the background event rate decreases only logarithmically with increasing cutoff energy; of course, this is not the whole story. Since the mean muon-production angle  $\bar{\theta}_{\mu\nu} \propto E_\mu^{-1/2}$  the size of the angular window around the Sun from which muon tracks must be accepted is accordingly smaller; consequently, the background event rate is proportional to  $(E_\mu^{\text{cut}})^{-1}$ . On the other hand, most of the neutrinos from WIMPs with masses of 100-1000 GeV should have energies well above 10 GeV; furthermore, the detectability of energetic neutrinos is proportional to the neutrino energy. So by accepting muons with energies

greater than 10 GeV, for example, the background is decreased by a factor of five while the dark-matter signal should be reduced only slightly. Of course, if such a cutoff is to be implemented the neutrino spectra from heavy-WIMP annihilation in the Sun should be more carefully determined, either through Monte Carlo or more detailed analytic modeling of interactions of decay products and neutrinos with the solar medium to determine exactly how much of the signal is lost by rejecting muon events with energies lower than the cutoff.

We should mention that throughout we have assumed that neutralinos are the primary component of the galactic halo. Of course, if neutralinos constitute only a fraction of the dark matter, then the rates for detection will be lowered accordingly. There is also the question of whether the relic abundance of the LSP associated with a given supersymmetric model can be suitable to account for the dark matter in galactic halos. Generally, it is assumed that if the fraction of critical density contributed by neutralinos today is  $0.025 \lesssim \Omega_{\tilde{\chi}} h^2 \lesssim 1$ , where  $h$  is the present Hubble parameter in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , then the neutralino is a good dark-matter candidate. If  $\Omega_{\tilde{\chi}} h^2 \gtrsim 1$  the relic density is too large to be consistent with the observed age of the Universe and if  $\Omega_{\tilde{\chi}} h^2 \lesssim 0.025$ , the relic abundance is too small to make up the primary component of the galactic halo.

Here we assume that all of the heavy neutralinos we consider *are* candidates for the primary component of the galactic halo. The relic abundance of a WIMP depends on its abundance in the early Universe at “freeze out”, when the annihilation rate of the WIMP falls below the expansion rate. The annihilation rate at any given time depends on the temperature of the Universe and the cross section for annihilation of the WIMP which is determined by the particle-physics model. On the other hand, since we have little familiarity with the conditions in the Universe before big bang nucleosynthesis, the expansion rate at freeze out cannot be reliably predicted. If one makes the simplest—and standard—assumption, that the early Universe was radiation dominated, then it is found that the relic abundance of heavy neutralinos is generally greater than 0.001.<sup>46</sup> However, many nonstandard scenarios accommodate an expansion rate at freeze out larger than that in the radiation-dominated Universe,<sup>47</sup> so if the standard calculations find a relic abundance greater than 0.001, nonstandard scenarios

allow for a relic abundance greater than 0.025. Conversely, if standard calculations yield  $\Omega_{\tilde{\chi}} h^2 > 1$ , a value of  $\Omega_{\tilde{\chi}} h^2 < 1$  is possible if the abundance was diluted by some entropy-producing process such as inflation, a quark/hadron or electroweak phase transition, or out-of-equilibrium decay of a massive particle. Therefore, since the standard calculations yield relic abundances for LSPs within a few orders of magnitude of the dark matter window,  $0.025 \lesssim \Omega_{\tilde{\chi}} h^2 \lesssim 1$ , and the abundance of a thermal relic in nonstandard cosmological models may differ from that in the standard radiation-dominated Universe by a few orders of magnitude, almost all heavy neutralinos should be considered dark-matter candidates.

Given that energetic neutrinos from heavy neutralino annihilation in the Sun may be observable, we speculate that neutrinos from annihilation of other heavy dark-matter candidates (such as Majorana neutrinos) may also be observable. Such a heavy WIMP would have to be captured readily in the Sun, either by a coherent interaction with heavy nuclei or by a sizable spin-dependent elastic scattering cross section that could result from the exchange of another particle not much heavier than the WIMP (*e.g.*, a heavy lepton in the case of a Majorana neutrino), or maybe by a strong coupling to the  $Z$ . Even if the dark matter consists of some heavy WIMP other than the MSSM neutralino, the MSSM provides a good example of a particle-physics model with a well-determined phenomenology that is consistent with current laboratory results and contains an excellent dark-matter candidate. This example shows that the idea that galactic halos are populated by (possibly detectable) WIMPs is alive and well and that the quest for their discovery should be pursued vigorously.

To conclude, we note that the properties of the heavy neutralino in many models are such that their capture and annihilation in the Sun yields an observable flux of energetic neutrinos. We also point out that in many models, a heavy neutralino may easily make up the primary component of the galactic halo while remaining invisible to neutrino detectors, so null results from energetic neutrino searches are not likely to rule out supersymmetric dark matter. Nevertheless, given the present uncertainty as to the nature of the dark matter, the popularity of supersymmetry in particle physics, and the interesting “coincidence” that the relic abundance of the LSP in most supersymmetric models falls near the dark-

matter window, it is clear that the search for energetic neutrinos from the Sun holds considerable promise for discovery, should neutralinos reside in the galactic halo.

## ACKNOWLEDGEMENTS

Presented as a thesis to the Department of Physics, The University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree. It is a pleasure to thank David Seckel, Kim Griest, Michael Turner, Francis Halzen, and Stuart Mufson for useful discussions. This research was supported in part by the DoE (at Chicago and at Fermilab), by NASA (grant NGW-1340 at Fermilab), and by the NASA Graduate Student Researchers Program.

## APPENDIX A: ELASTIC SCATTERING CROSS SECTION

The neutralino may elastically scatter off of a nucleus via a scalar interaction where the WIMP interacts coherently with the entire nucleus, and if the nucleus has spin the neutralino may also scatter via an axial interaction. The cross section for scattering of a neutralino off of nucleus  $i$  via an axial interaction (the "spin-dependent cross section") is<sup>5</sup>

$$\sigma_{SD} = \frac{24m_{\tilde{\chi}}^2 m_i^2 G_F^2}{\pi(m_{\tilde{\chi}} + m_i)^2} \frac{4}{3} \lambda^2 J(J+1) \left[ \sum_{u,d,s} A'_q \Delta q \right]^2, \quad (A1)$$

where

$$A'_q = \frac{1}{2} T_{3L}^q (Z_{i3}^2 - Z_{i4}^2) - x_q^2 \left\{ \frac{2m_q^2 d_q^2}{4m_W^2} + [T_{3L}^q Z_{i2} - \tan \theta_W (T_{3L}^q - e_q) Z_{i1}]^2 + \tan^2 \theta_W e_q^2 Z_{i1}^2 \right\}, \quad (A2)$$

and

$$x_q^2 = \frac{m_W^2}{(m_{\tilde{\chi}} + m + i)^2 - (M_{\tilde{q}} - m_i)^2}, \quad (A3)$$

is the squark-exchange suppression factor,<sup>31</sup> and

$$\lambda = \frac{1}{2} \{1 + [s_p(s_p + 1) - l(l + 1)]/[J(J + 1)]\}, \quad (\text{A4})$$

is the Lande factor from the one-particle nuclear shell model for a nucleus with spin  $J$  and an unpaired nucleon with spin  $s_p$  and orbital angular momentum  $l$ . Here,  $m_q$  is the (current) quark mass,  $d_q = -Z_{i3}/\cos\beta$  for down-type quarks,  $d_q = Z_{i4}/\sin\beta$  for up-type quarks,  $T_{3L}^q$  is the weak isospin of the quark,  $e_q$  is its charge, and  $\theta_W$  is the Weinberg angle. The quantity  $\Delta q$  measures the fraction of the nucleon spin carried by the quark. In the naive flavor-SU(3) quark model  $\Delta u = 0.97$ ,  $\Delta d = -0.28$  and  $\Delta s = 0$ ; however, the EMC collaboration reports  $\Delta u = 0.746$ ,  $\Delta d = -0.508$  and  $\Delta s = -0.226$ .<sup>36</sup>

For the capture-rate calculation the spin-dependent cross section and the sum that appears in Eq. (A1) may be simplified considerably. The only element with spin in the Sun found in abundance is hydrogen. For hydrogen  $(4/3)\lambda^2 J(J + 1) = 1$  and in the EMC model

$$\begin{aligned} \sum A'_q \Delta q = & 0.37(Z_{i3}^2 - Z_{i4}^2) - x_q^2 \left[ -3.98 \times 10^{-7} \frac{Z_{i3}^2}{\cos^2 \beta} + 2.86 \times 10^{-9} \frac{Z_{i4}^2}{\sin^2 \beta} \right. \\ & \left. + 0.003 Z_{i2}^2 + 0.133 Z_{i2} Z_{i1} + 0.073 Z_{i1}^2 \right], \end{aligned} \quad (\text{A5})$$

while in the flavor-SU(3) model

$$\begin{aligned} \sum A'_q \Delta q = & 0.3125(Z_{i3}^2 - Z_{i4}^2) - x_q^2 \left[ -3.5 \times 10^{-10} \frac{Z_{i3}^2}{\cos^2 \beta} + 3.72 \times 10^{-9} \frac{Z_{i4}^2}{\sin^2 \beta} \right. \\ & \left. + 0.173 Z_{i2}^2 + 0.1125 Z_{i2} Z_{i1} + 0.122 Z_{i1}^2 \right]. \end{aligned} \quad (\text{A6})$$

The term proportional to  $(Z_{i3}^2 - Z_{i4}^2)$  arises from  $Z$  exchange, and the second term arises from squark exchange. For heavy  $B$ -inos  $Z_{i3} \simeq Z_{i4} \simeq 0$ , for heavy Higgsinos  $Z_{i3}^2 \simeq Z_{i4}^2$ , and as we will see below, for heavy mixed-state neutralinos the axial interaction is much weaker than the coherent interaction; therefore,

scattering of heavy neutralinos via  $Z$  exchange is essentially negligible. In addition, from Eqs. (A5) and (A6) one can see that if the neutralino is pure Higgsino spin-dependent scattering due to squark exchange is also negligible, but if the neutralino is pure  $B$ -ino ( $Z_{i1} \simeq 1$  and  $Z_{i2} \simeq Z_{i3} \simeq Z_{i4} \simeq 0$ ) and the squark is not much heavier than the neutralino then spin-dependent scattering due to squark exchange may be significant. By comparing Eqs. (A5) and (A6) we also see that had we used the flavor-SU(3) quark model the capture rates would be roughly 3 times as large as those obtained using the EMC results which we used in this work.

The cross section for scattering via a scalar interaction is obtained from Refs. 5, 32, 33, and 21. Griest<sup>5</sup> obtained the results for a coherent scalar interaction via exchange of a virtual squark, and Barbieri, Frigeni, and Giudice<sup>32</sup> obtained results for a coherent scalar interaction in which a Higgs boson is exchanged; however, in both of these papers it was assumed that the nucleon mass is due to gluons.<sup>48</sup> Recent measurements of the pion-nucleon sigma term imply that a significant fraction of the nuclear mass is due to a sea of strange quarks.<sup>33</sup> When applied to coherent neutralino-nucleus scattering it is found that although the component of squark- and Higgs-nucleon coupling due to gluons is reduced, there is an additional component due to squark and Higgs coupling to the strange-quark sea and the net effect is a significant increase in the squark- and Higgs-nucleon coupling.<sup>21</sup>

The scalar cross section may be derived from the effective Lagrangian<sup>5,32</sup>

$$\mathcal{L}_{\text{eff}} = \sqrt{2}G_F(Z_{i2} - Z_{i1}\tan\theta_W) \times \sum_q \left[ \frac{m_W}{m_{H_2^0}^2} g_{H_2} k_q^{(2)} + \frac{m_W}{m_{H_1^0}^2} g_{H_1} k_q^{(1)} + \frac{\epsilon d_q x_q^2}{m_W} \right] m_q \tilde{\chi}\tilde{\chi}\bar{q}q, \quad (\text{A7})$$

where  $k_q^{(1)} = \sin\alpha/\sin\beta$  and  $k_q^{(2)} = \cos\alpha/\sin\beta$  for up-type quarks,  $k_q^{(1)} = \cos\alpha/\cos\beta$  and  $k_q^{(2)} = -\sin\alpha/\cos\beta$  for down-type quarks,  $g_{H_2} = (Z_{i3}\sin\alpha + Z_{i4}\cos\alpha)$ , and  $g_{H_1} = (Z_{i3}\cos\alpha + Z_{i4}\sin\alpha)$ . In addition to terms due to exchange of the lightest Higgs boson<sup>32</sup> and the squark,<sup>5</sup> to be complete we have included a term due to exchange of the heaviest Higgs boson although it should generally

be smaller than that due to exchange of the lightest Higgs boson.

The scattering cross section is obtained from the square of the matrix element  $\langle f | \mathcal{L}_{\text{eff}} | i \rangle$  of this effective Lagrangian between the initial and final neutralino-nuclear states. In Ref. 48 (as modified by Ref. 33) it is shown that the coupling of a scalar field to the gluons in the nucleus occurs via a heavy-quark ( $c$ ,  $b$ , and  $t$  quarks) loop so that

$$\langle N | m_h \bar{h} h | N \rangle = \frac{2}{27} m_i (0.56), \quad (\text{A8})$$

where  $h$  is a heavy-quark field,  $m_h$  is the heavy-quark mass, and  $|N\rangle$  is the nuclear wave function. In addition, measurements of the pion-nucleon sigma term imply that<sup>33</sup>

$$\langle N | m_s \bar{s} s | N \rangle = \frac{2}{27} m_i (5.94), \quad (\text{A9})$$

where  $s$  is the strange-quark field and  $m_s$  is its mass. The matrix elements of  $m_q \bar{q} q$  for the  $u$  and  $d$  quarks are much smaller. With these results it is easy to find that the matrix element is

$$\begin{aligned} \langle f | \mathcal{L}_{\text{eff}} | i \rangle = & \sqrt{2} G_F \frac{2}{27} m_i (Z_{i2} - Z_{i1} \tan \theta_W) \\ & \times \left[ \frac{m_W}{m_{H_2^0}^2} g_{H_2} \left( 1.12 \frac{\cos \alpha}{\sin \beta} - 6.5 \frac{\sin \alpha}{\cos \beta} \right) \right. \\ & + \frac{m_W}{m_{H_1^0}^2} g_{H_1} \left( 1.12 \frac{\sin \alpha}{\sin \beta} + 6.5 \frac{\cos \alpha}{\cos \beta} \right) \\ & \left. + \frac{\epsilon x_q^2}{m_W} \left( 1.12 \frac{Z_{i4}}{\sin \beta} - 6.5 \frac{Z_{i3}}{\cos \beta} \right) \right], \end{aligned} \quad (\text{A10})$$

and the cross section for scattering off of nucleus  $i$  via a coherent scalar interaction is

$$\sigma_{\text{SC}} = \frac{4m_{\tilde{\chi}}^2 m_i^2}{\pi(m_{\tilde{\chi}} + m_i)^2} |\langle f | \mathcal{L}_{\text{eff}} | i \rangle|^2. \quad (\text{A11})$$

We should clarify that this is the cross section that would be measured only if the neutralino interacted coherently with the entire nucleus. If the inverse

of the momentum transfer  $1/q$  in the scattering event is small compared with the nuclear radius  $R$  then the neutralino does not interact coherently with the entire nucleus and the actual cross section is momentum-transfer dependent (or equivalently, scattering-angle dependent) and is given by Eq. (A11) times  $|F(q^2)|$ , the form-factor suppression. The effect of the form-factor suppression on capture in the Sun and Earth is discussed in Section II.

## APPENDIX B: MIXED GAUGE/HIGGS BOSON FINAL STATES

In addition to the gauge-boson and Higgs-boson final states considered in Ref. 6, neutralinos may annihilate into mixed Higgs/gauge boson final states when the mass of the neutralino exceeds the average of the gauge- and Higgs-boson masses. At zero relative velocity the available channels are  $ZH_1^0$ ,  $ZH_2^0$ ,  $W^+H^-$ , and  $W^-H^+$ . Annihilation into  $ZH_3^0$  is possible in general, but does not occur at zero relative velocity for  $CP$ -conserving theories. The reason is that at zero relative velocity, neutralino-neutralino annihilation occurs via an  $s$ -wave and due to Fermi statistics, the initial state has  $CP = -1$ . Since the  $Z$  has spin 1 and the Higgs is a scalar, the orbital wave function of the outgoing state must have  $l = 1$ , and since the  $Z$  is  $CP$ -even and the  $H_3^0$  is  $CP$ -odd, the final state must have  $CP = 1$  and is therefore inaccessible from the initial state.

Since the  $ZH_2^0$  final state is the first mixed channel to open up as the neutralino mass is increased, we will consider it first. [Incidentally, since  $(m_{H_2^0} + m_Z)/2$  may be less than  $m_W$ , this channel may be open for neutralinos that are lighter than the  $W$ , a possibility that was not considered in previous work.] Throughout this Appendix we will use the notation of Griest, Kamionkowski, and Turner (GKT),<sup>6</sup> and some of the couplings we will use here are defined there.

Annihilation of two neutralinos into  $ZH_2^0$  occurs via  $s$ -channel exchange of a  $Z$  and a  $H_3^0$  and by  $t$ - and  $u$ -channel exchange of all four neutralinos. The cross section  $\sigma_{ZH_2^0}$  for this process as relative velocity  $v_{\text{rel}} \rightarrow 0$  is

$$\sigma_{ZH_2^0} v_{\text{rel}} = \frac{k X_{ZH_2^0}}{32\pi m_{\tilde{\chi}}^3}, \quad (\text{B1})$$



where

$$k = \left[ m_{\tilde{\chi}}^2 - \frac{1}{2}(m_Z^2 + m_{H_2^0}^2) + \frac{(m_Z^2 - m_{H_2^0}^2)^2}{16m_{\tilde{\chi}}^2} \right]^{1/2}, \quad (\text{B2})$$

is the momentum of the outgoing particles and

$$X_{ZH_2^0} = 2k^2 \frac{m_{\tilde{\chi}}^2}{m_Z^2} \left[ \frac{zF_{nn}}{m_Z^2} + \frac{4M_{3nn}hm_{\tilde{\chi}}}{s - m_{H_3^0}^2} + \sum_k \frac{2gM_{2nk}F_{nk}(m_{\tilde{\chi}_k^0} - m_{\tilde{\chi}})}{t - m_{\tilde{\chi}_k^0}^2} \right]^2. \quad (\text{B3})$$

Here,  $z = m_Z \sin(\beta - \alpha)/\cos\theta_W$  is the coupling at the  $H_2^0 ZZ$  vertex,  $F_{ij} = (Z_{i3}Z_{j3} - Z_{i4}Z_{j4})/2\cos\theta_W$  is the coupling at the  $Z\tilde{\chi}_i^0\tilde{\chi}_j^0$  vertex,  $M_{ijk}$  is the  $H_i^0\tilde{\chi}_j^0\tilde{\chi}_k^0$  coupling and is given in Eq. (C9) of GKT,  $h = \cos(\alpha - \beta)/2\cos\theta_W$  is the  $ZH_2^0H_3^0$  coupling, the sum is over all four neutralinos, and  $t = [(m_Z^2 + m_{H_2^0}^2)/2] - m_{\tilde{\chi}}^2$ .

For larger neutralino masses the  $ZH_1^0$  channel opens up. (Recall that the  $H_1^0$  is always heavier than the  $Z$ .) The cross section for annihilation into  $ZH_1^0$  may be obtained from that for annihilation into  $ZH_2^0$  by simply replacing  $m_{H_2^0}$  by  $m_{H_1^0}$ ,  $M_{2ij}$  by  $M_{1ij}$ , and using  $z = m_Z \cos(\beta - \alpha)/2\cos\theta_W$  and  $h = \sin(\alpha - \beta)/2\cos\theta_W$ .

Annihilation into  $WH^\pm$  final states occurs through  $s$ -channel exchange of the  $H_3^0$  and  $t$ - and  $u$ -channel exchange of the two charginos. The cross section for this process as relative velocity  $v_{\text{rel}} \rightarrow 0$  is

$$\sigma_{WH^\pm v_{\text{rel}}} = \frac{kX_{WH^\pm}}{32\pi m_{\tilde{\chi}}^3}, \quad (\text{B4})$$

where

$$X_{WH^\pm} = \left( \frac{gm_{\tilde{\chi}}k}{m_W} \right)^2 \left\{ \frac{4M_{3nn}m_{\tilde{\chi}}}{s - m_{H_3^0}^2} + \sum_i \frac{[m_{\tilde{\chi}_i^+}(e_i Q_R^i - f_i Q_L^i) + m_{\tilde{\chi}}(f_i Q_R^i - e_i Q_L^i)]}{t - m_{\tilde{\chi}_i^+}^2} \right\}^2. \quad (\text{B5})$$

The sum is over the two charginos, and the quantities  $e_i$  and  $f_i$  are given in GKT

Eq. (A2), and

$$\begin{aligned} \begin{pmatrix} Q_L^1 \\ Q_L^2 \end{pmatrix} &= g \cos \beta \left[ Z_{n4} \begin{pmatrix} \cos \phi_+ \\ \sin \phi_+ \end{pmatrix} + \frac{1}{\sqrt{2}} (Z_{n2} + Z_{n1} \tan \theta_W) \begin{pmatrix} -\epsilon \sin \phi_+ \\ \epsilon \cos \phi_+ \end{pmatrix} \right], \\ \begin{pmatrix} Q_R^1 \\ Q_R^2 \end{pmatrix} &= g \sin \beta \left[ Z_{n3} \begin{pmatrix} \cos \phi_- \\ \sin \phi_- \end{pmatrix} - \frac{1}{\sqrt{2}} (Z_{n2} + Z_{n1} \tan \theta_W) \begin{pmatrix} -\sin \phi_- \\ \cos \phi_- \end{pmatrix} \right], \end{aligned} \quad (\text{B6})$$

where the angles  $\phi_+$  and  $\phi_-$  are related to the diagonalization of the chargino mass matrix and are given in Ref. 49, and  $\epsilon = \det X / |\det X|$  and  $X$  is the matrix defined in Eq. (C9) of Ref. 3. Here,

$$k = \left[ m_{\tilde{\chi}}^2 - \frac{1}{2}(m_W^2 + m_{H^+}^2) + \frac{(m_W^2 - m_{H^+}^2)^2}{16m_{\tilde{\chi}}^2} \right]^{1/2}, \quad (\text{B7})$$

and  $t = [(m_W^2 + m_{H^+}^2)/2] - m_{\tilde{\chi}}^2$ .

## REFERENCES

1. For recent reviews of dark matter and its detection, see V. Trimble, *Ann. Rev. Astron. Astrophys.* **25**, 425 (1989); J. R. Primack, B. Sadoulet, and D. Seckel, *Ann. Rev. Nucl. Part. Sci.* **B38**, 751 (1988); *Dark Matter in the Universe*, eds. J. Kormendy and G. Knapp (Reidel, Dordrecht, 1989).
2. K. Griest and J. Silk, *Nature* **343**, 26 (1990); L. Krauss, *Phys. Rev. Lett.* **64**, 999 (1990).
3. H. E. Haber and G. L. Kane, *Phys. Rep.* **117**, 75 (1985).
4. J. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, *Nucl. Phys.* **B238**, 453 (1984).
5. K. Griest, *Phys. Rev.* **D38**, 2357 (1988); FERMILAB-Pub-89/139-A (E); *Phys. Rev. Lett.* **61**, 666 (1988).
6. K. A. Olive and M. Srednicki, *Phys. Lett.* **B230**, 78 (1989); K. Griest, M. Kamionkowski, and M. S. Turner, *Phys. Rev. D* **41**, 3565 (1990).
7. K. A. Olive and M. Srednicki, University of Minnesota Report No. UMN-TH-805/90.
8. R. A. Barbieri and G. F. Giudice, *Nucl. Phys.* **B306**, 63 (1988).
9. See, e.g., J. Primack *et al.* in Ref. 1, or *Particle Astrophysics: Forefront Experimental Issues*, ed. E.B. Norman (WSPC, Singapore, 1989).
10. J. Silk and M. Srednicki, *Phys. Rev. Lett.* **53**, 624 (1984); J. Ellis *et al.*, *Phys. Lett. B* **214**, 403 (1989); F. Stecker, S. Rudaz, and T. Walsh, *Phys. Rev. Lett.* **55**, 2622 (1985); F. Stecker and A. Tylka, *Astrophys. J.* **336**, L51 (1989).
11. J. Silk and M. Srednicki, *Phys. Rev. Lett.* **53**, 624 (1984); F. Stecker and A. Tylka, *Astrophys. J.* **336**, L51 (1989); S. Rudaz and F. Stecker, *Astrophys. J.* **325**, 16 (1988); F. Stecker, *Phys. Lett. B* **201**, 529 (1988); J. Ellis *et al.*, *Phys. Lett. B* **214**, 403 (1989).
12. J. Silk and M. Srednicki, *Phys. Rev. Lett.* **53**, 624 (1984); S. Rudaz and F. Stecker, *Astrophys. J.* **325**, 16 (1988); J. Ellis *et al.*, *Phys. Lett. B* **214**,

- 403 (1989); F. Stecker and A. Tylka, *Astrophys. J.* **336**, L51 (1989).
13. M. Srednicki, S. Thiessen, and J. Silk, *Phys. Rev. Lett.* **56**, 263 (1986); S. Rudaz, *Phys. Rev. Lett.* **56**, 2128 (1986); L. Bergstrom and H. Snellman, *Phys. Rev. D* **37**, 3737 (1988); L. Bergstrom, *Nucl. Phys.* **B325**, 647 (1989); L. Bergstrom, in *Particle Astrophysics*, ed. M. Norman (WSPC, Singapore, 1989); L. Bergstrom, *Phys. Lett. B* **225**, 372 (1989); G.F. Giudice and K. Griest, *Phys. Rev. D* **40**, 2549 (1989); S. Rudaz, *Phys. Rev. D* **39**, 3549 (1989); A. Bouquet, P. Salati, and J. Silk, *Phys. Rev. D* **40**, 3168 (1989).
  14. M. S. Turner and F. Wilczek, *Phys. Rev. D* **42**, 1001 (1990); A. J. Tylka, *Phys. Rev. Lett.* **63**, 840 (1989).
  15. If the neutralino is heavier than the  $W$  the energy spectrum of cosmic-ray positrons may be easily distinguished from background even though the spectrum is not a line; see M. Kamionkowski and M. S. Turner, *Phys. Rev. D* **43**, 1774 (1991).
  16. W. H. Press and D. N. Spergel, *Astrophys. J.* **296**, 679 (1985).
  17. A. Gould, *Astrophys. J.* **321**, 571 (1987).
  18. A. Gould, *Astrophys. J.* **368**, 610 (1991).
  19. S. Ritz and D. Seckel, *Nucl. Phys.* **B304**, 877 (1988).
  20. G. F. Giudice and E. Roulet, *Nucl. Phys.* **B316**, 429 (1989).
  21. G. Gelmini, P. Gondolo, and E. Roulet, SISSA Report No. SISSA-88/89-EP.
  22. J. F. Gunion and H. E. Haber, *Nucl. Phys.* **B272**, 1 (1986); J. F. Gunion and H. E. Haber, *Nucl. Phys.* **B278**, 449 (1986).
  23. J. Ellis, G. Ridolfi, and F. Zwirner, CERN Report No. CERN-TH-5946-90; H. E. Haber and R. Hempfling, UC Santa Cruz Report No. SCIPP-90-42.
  24. CDF Collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* **64**, 142 (1990).
  25. U. Amaldi *et al.*, *Phys. Rev. D* **36**, 1385 (1987); B. W. Lynn, M. E. Peskin, and R. G. Stewart, in *Physics at LEP*, CERN report CERN 86-02 (1986).

The Almaldi *et al.* top quark mass limit is for the Standard Model, but Lynn *et al.* show that the corrections due to heavy squarks or sleptons go in the same direction as corrections due to a heavy top quark, and so the mass limit should not be weaker for the supersymmetric models we consider.

26. The Delphi Collaboration: P. Abreu *et al.*, *Phys. Lett. B* **245**, 276 (1990); The Opal Collaboration, CERN-EP/90-100.
27. G. F. Giudice and G. Ridolfi, *Z. Phys. C* **41**, 447 (1988).
28. IMB Collaboration: J. M. LoSecco *et al.*, *Phys. Lett. B* **188**, 388 (1987); R. Svoboda *et al.*, *Astrophys. J.* **315**, 420 (1987); D. Casper, Talk given at XXIIIrd Rencontre de Moriond, Mar. 8-15, 1988. We thank E. Roulet for bringing this last reference to our attention.
29. Totsuka, Y., Institute for Cosmic Ray Research Report No. ICR-Report-192-89-9; N. Sato *et al.*, KEK Report No. 90-166.
30. See, for example, K. Griest and D. Seckel, *Nucl. Phys. B* **283**, 681 (1987).
31. The evaluation of the squark-exchange propagator differs here from that in Ref. 5. Since the neutralino interacts with a quark in the nucleus and the squark is exchanged in the  $s$ -channel, Griest used  $[(m_{\tilde{\chi}} + m_q)^2 - M_{\tilde{q}}^2]^{-1} \simeq [m_{\tilde{\chi}}^2 - M_{\tilde{q}}^2]^{-1}$  for the squark-exchange propagator; however, the intermediate state is most likely an exotic nuclear resonance with mass near  $M_{\tilde{q}} + m_i$  where the quark is replaced by the squark, and the center-of-mass energy is  $m_{\tilde{\chi}} + m_i$ , so we take the propagator to be  $[(m_{\tilde{\chi}} + m_i)^2 - (M_{\tilde{q}} + m_i)^2]^{-1}$ . This differs only slightly from Griest's expression, especially for heavy superpartners and light nuclei. Also, note that the resonance near  $M_{\tilde{q}} \simeq m_{\tilde{\chi}}$  found in Ref. 5 still occurs.
32. R. Barbieri, M. Frigeni, and G. F. Giudice, *Nucl. Phys. B* **313**, 725 (1989).
33. T. P. Cheng, *Phys. Rev. D* **38**, 2869 (1988); H.-Y. Cheng, *Phys. Lett. B* **219**, 347 (1989); J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*, (Addison-Wesley, Redwood City, 1990).
34. A. Gould, Ref. 17. The exponential form factor is not necessarily a good

approximation to the actual form factor at large momentum transfers; however, since most capture occurs at small momentum transfer (since the correct form factor also becomes very small at large momentum transfers) where the exponential form is a good approximation, the results obtained for the capture rate using the exponential form should be accurate. However, the rates for direct detection of heavy neutralinos in the laboratory may not be determined accurately using the exponential form factor (M. Kamionkowski and D. Seckel, work in progress).

35. J. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, 1989).
36. J. Ashman *et al.*, *Phys. Lett. B* **206**, 364 (1988). For further discussion see Griest (Ref. 5) and J. Ellis and R. A. Flores, *Nucl. Phys. B* **307**, 883 (1988).
37. In addition to the fermion and gauge- and Higgs-boson final states, the two-gluon final state which, although suppressed by the fourth power of the strong coupling constant  $\alpha_s^2$ , may have an annihilation branch comparable to that into light fermions since light-fermion final states are helicity suppressed (see, *e.g.*, Rudaz and Bergstrom, Ref. 13). Therefore, if in the analysis here the neutralino annihilates predominantly into light fermions (*e.g.*, a pure  $B$ -ino less massive than the top quark in a model where the squark is not much heavier than the neutralino) then there may be a comparable annihilation branch into gluons and the neutrino signal may be diluted accordingly. In the majority of the models considered here the neutralino is either a pure Higgsino or is heavier than the top quark and the dominant final states are gauge bosons or top quarks, so the gluon final states have little or no effect.
38. The authors of Ref. 21 explain how to get the neutrino spectra from injected Higgs bosons given the fermionic neutrino spectra; however, the expressions they give are only valid if the Higgs boson is moving relativistically, which is not always the case. In addition, if one is only interested in  $\langle Nz^2 \rangle$ , the complete apparatus for obtaining the full neutrino spectrum is not needed.

39. Particle Data Group, *Physics Letters*, **B239**, 1 (1990).
40. Frejus Collaboration, presented by H. J. Daum, Topical Seminar on Astrophysics and Particle Physics, San Miniato, Italy, 1989.
41. M. Aglietta *et al.*, unpublished.
42. R. Flores, K. A. Olive, and M. Srednicki, *Phys. Lett. B* **237**, 72 (1991).
43. B. Barish, talk given at UCLA International Conference on Trends in Astroparticle Physics, Santa Monica, Nov. 1990.
44. T. Aoki *et al.*, U. of Washington Report No. UWSEA-PUB-90-22.
45. S. Barwick, D. Lowder, T. Miller, P. B. Price, A. Westphal, F. Halzen, and B. Morse, U. of Wisconsin Report No. MAD/PH/629.
46. See Figs. 15-17 in Griest, Kamionkowski, and Turner (Ref. 6).
47. M. Kamionkowski and M. S. Turner, *Phys. Rev. D* *in press* (1990); J. D. Barrow, *Nucl. Phys.* **B208**, 501 (1982).
48. M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Phys. Lett. B* **78**, 443 (1978); A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, *Usp. Fiz. Nauk.* **130**, 537 (1980) [*Sov. Phys. Usp.* **23**, 429 (1980)].
49. R. M. Barnett and H. E. Haber, *Phys. Rev.* **D31**, 85 (1985).

## FIGURE CAPTIONS

1. Lightest neutralino composition and mass for  $\tan \beta = 2$ . The broken curves are contours of constant neutralino mass  $m_{\tilde{\chi}}$ , and the solid curves are contours of constant gaugino fraction  $(Z_{n1}^2 + Z_{n2}^2)$ ; in (a)  $\mu > 0$  and in (b)  $\mu < 0$ .
2. Form-factor suppression of the rate of accretion of heavy WIMPs onto the Sun from scattering off of nuclei with atomic masses 4, 12, 16, 24, 32, and 56 as a function of the neutralino mass.
3. Contour plots of the capture rate of neutralinos in the Sun assuming neutralinos make up the primary component of the dark matter and that the squark mass is infinite. The double curve indicates a capture rate of  $10^{24} \text{ s}^{-1}$ ; the spacing between other curves are decades, the capture rate decreasing toward higher masses. In (a)  $\tan \beta = 2$ ,  $m_{H_2^0} = 20 \text{ GeV}$ , and  $\mu > 0$  and (b) is the same except  $\mu < 0$ . In (c)  $\tan \beta = 2$  and  $m_{H_2^0} = 35$ , and in (d)  $\tan \beta = 25$  and  $m_{H_2^0} = 35$ . In (c) and (d) only regions of positive  $\mu$  are shown; the plots for negative  $\mu$  are similar. For convenience, the mass and composition contours are also shown.
4. Same as Fig. 3(a) but here the squark mass is assumed to be 20 GeV heavier than the neutralino mass.
5. Contours of the fraction of the capture rate due to spin-dependent scattering when the squark is assumed to be 20 GeV heavier than the neutralino, and  $\tan \beta = 2$  and  $m_{H_2^0} = 20$ . In the shaded regions the fraction is greater than 0.5, and the contours indicate where the fraction is 0.01, 0.5, and 0.99. Again, mass and composition contours are also shown, and plots for other values of  $\tan \beta$  and  $m_{H_2^0}$  are qualitatively similar.
6. Contours of  $t_{\odot}/\tau_A$ . In the dark shaded regions  $t_{\odot}/\tau_A < 0.33$  and in the light shaded region  $t_{\odot}/\tau_A < 1.82$ ; elsewhere,  $t_{\odot}/\tau_A > 1.82$ . In (a)  $\tan \beta = 2$ ,  $m_{H_2^0} = 20$ , the squark mass is taken to be infinite, and  $\mu > 0$ ; (b) is similar but  $\mu < 0$  is shown; and (c) is similar to (a) but the squark mass is taken to be 20 GeV heavier than the neutralino mass. Plots for other values of



$\tan \beta$  and  $m_{H_2^0}$  are similar.

7. Second moments of the neutrino yields from the Sun from the  $c\bar{c}$ ,  $b\bar{b}$ ,  $t\bar{t}$ ,  $\tau^\pm$ ,  $W^\pm$ , and  $H_2^0 H_3^0$  (where  $\tan \beta = 2$  and  $m_{H_2^0} = 20$  GeV) annihilation channels as a function of the neutralino mass.
8. Contours of the fraction of the neutrino signal that comes from gauge-boson final states. In the shaded regions the fraction is greater than 0.5, and the contours indicate where the fraction is 0.01, 0.5, and 0.99. In (a) the squark mass is taken to be infinite, and in (b) the squark mass is assumed to be 20 GeV heavier than the neutralino mass. In both,  $\tan \beta = 2$  and  $m_{H_2^0} = 20$  GeV and  $\mu > 0$ . Plots for other values of  $\tan \beta$  and  $m_{H_2^0}$  and for negative  $\mu$  are qualitatively similar.
9. Regions where the neutralino is excluded as the primary component of the galactic halo by limits on the flux of upward-moving neutrino-induced muons from the Sun. The dark shaded regions are those excluded by current IMB limits. The light shaded regions are those that would be excluded if current observational limits were improved by a factor of 100. The curve inside the excluded region encloses the region that would be excluded if the true neutrino flux was 1/5 of the results of the calculation here, and the curve inside the light shaded region encloses regions that would be excluded if the current observational limits were improved by a factor of 10. In (a)  $\tan \beta = 2$ ,  $m_{H_2^0} = 20$  GeV,  $\mu > 0$ , and the squark mass is taken to be infinite and (b) is the same except  $\mu < 0$ . In (c)  $\tan \beta = 2$  and  $m_{H_2^0} = 20$  GeV, in (d)  $\tan \beta = 2$  and  $m_{H_2^0} = 35$  GeV, and in (e)  $\tan \beta = 25$  and  $m_{H_2^0} = 35$  GeV. In (c), (d), and (e), the squark mass is assumed to be 20 GeV greater than the neutralino mass and only regions of positive  $\mu$  are shown. Plots for negative  $\mu$  are similar, but excluded regions are smaller.

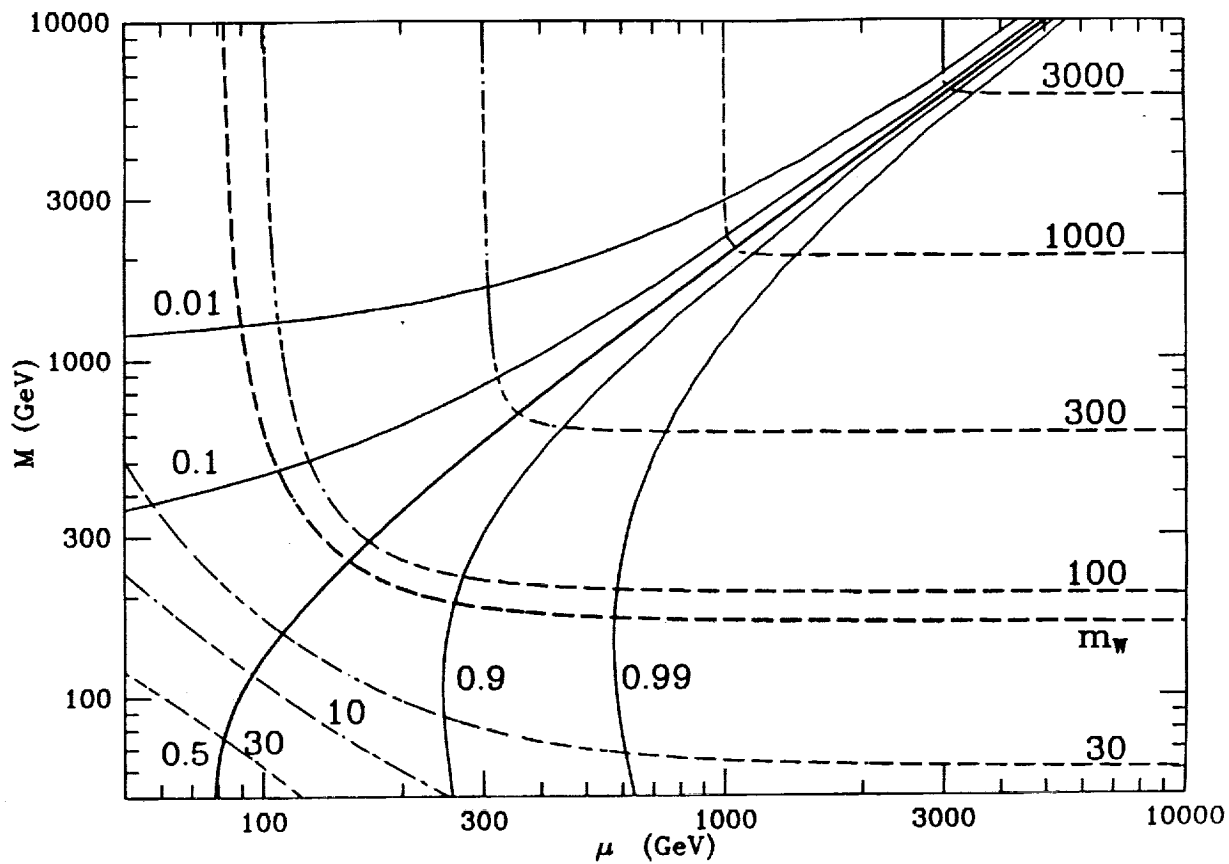


Fig. 1(a)

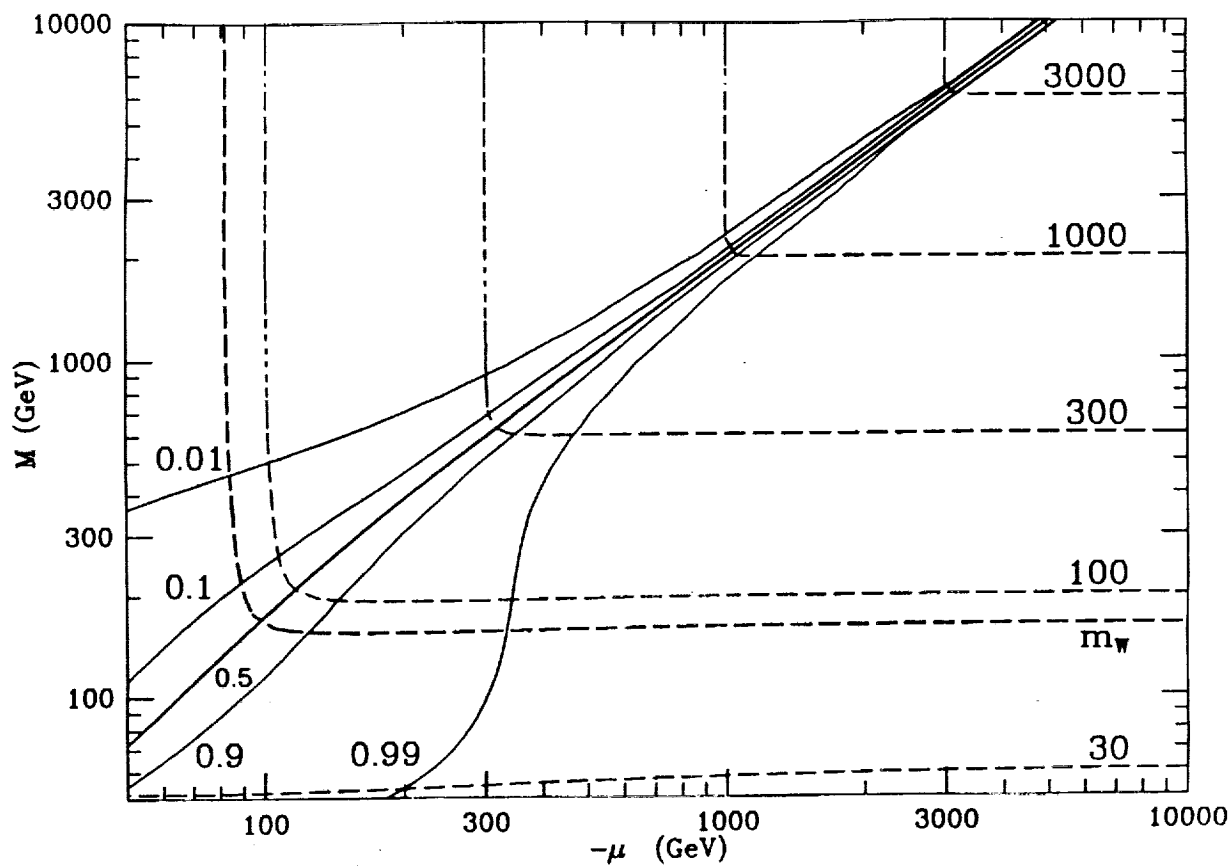


Fig. 1(b)

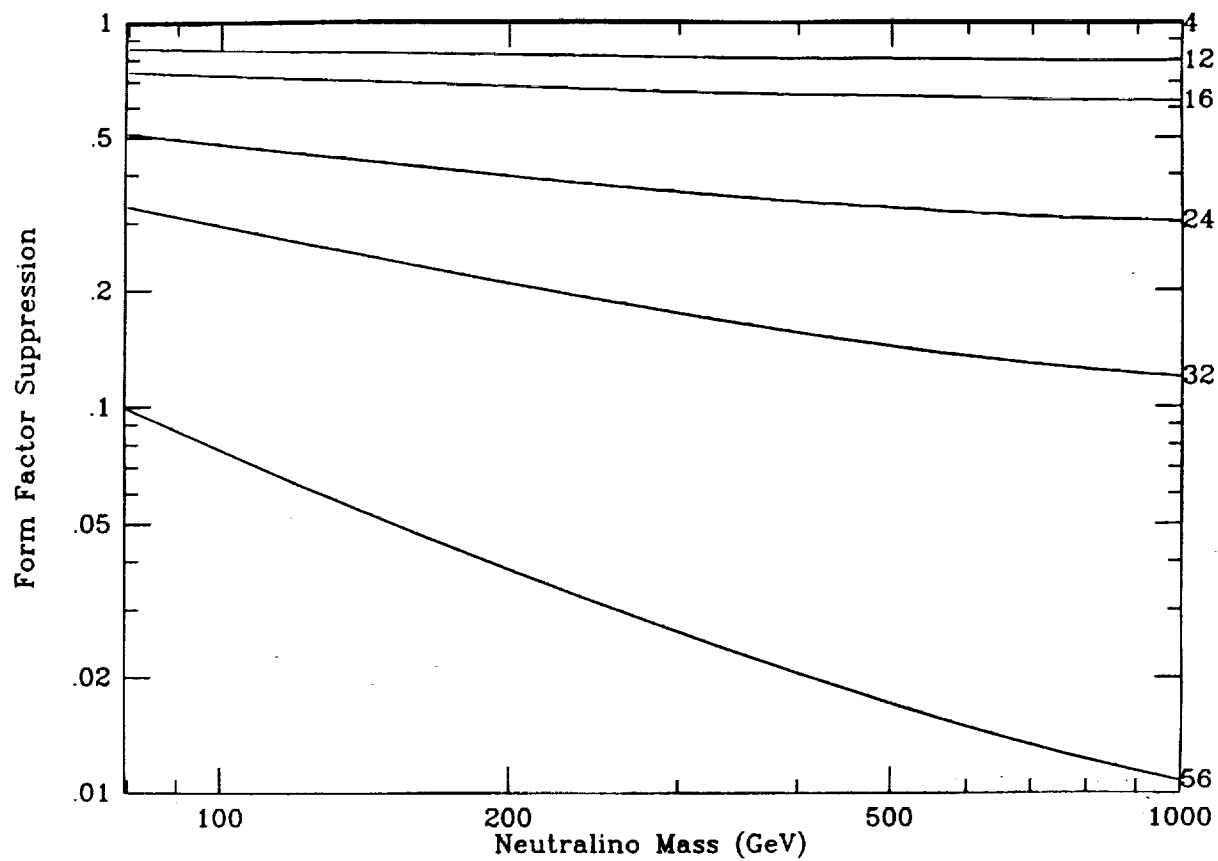


Fig. 2

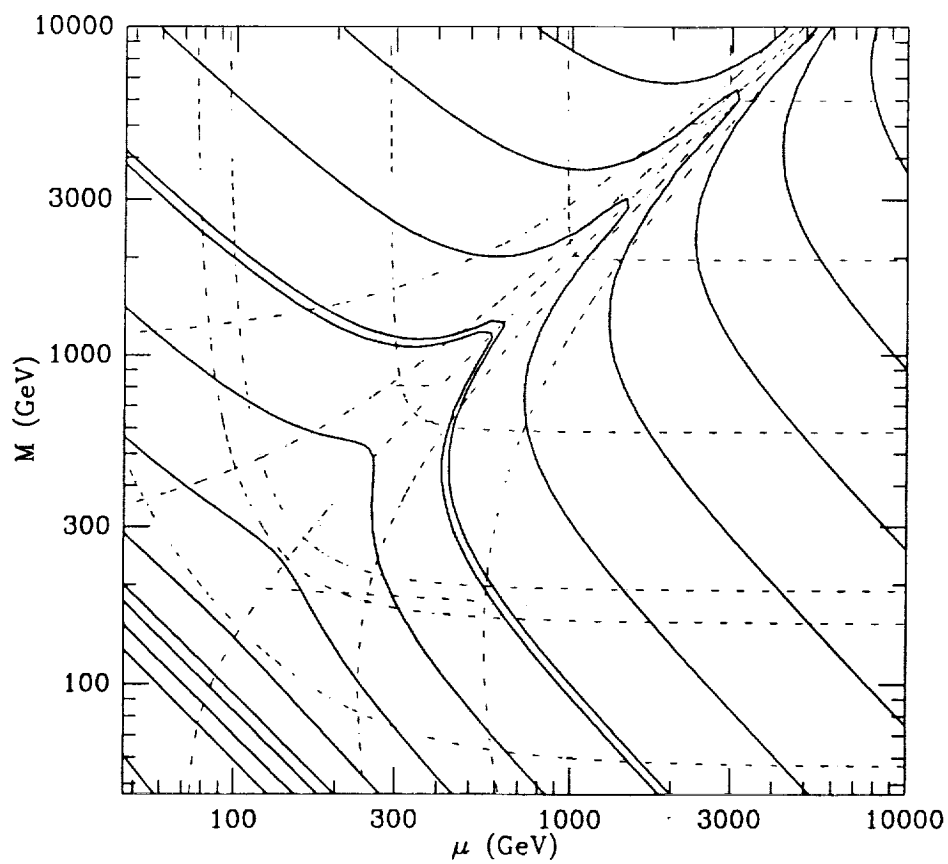


Fig. 3(a)

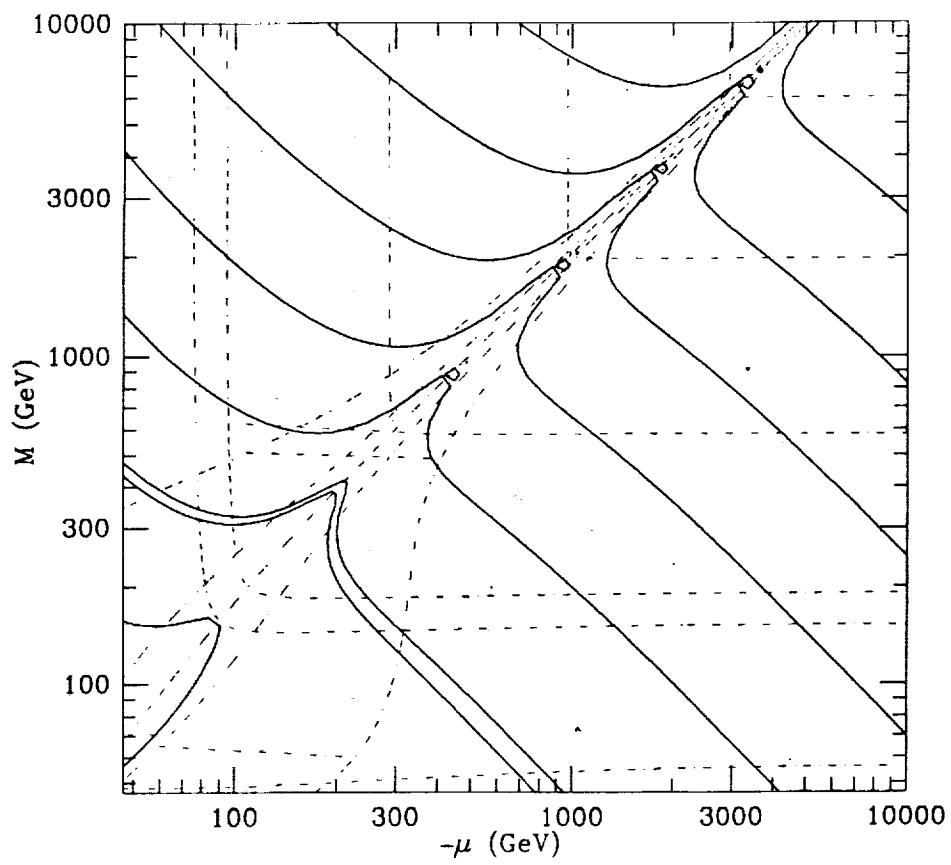


Fig. 3(b)

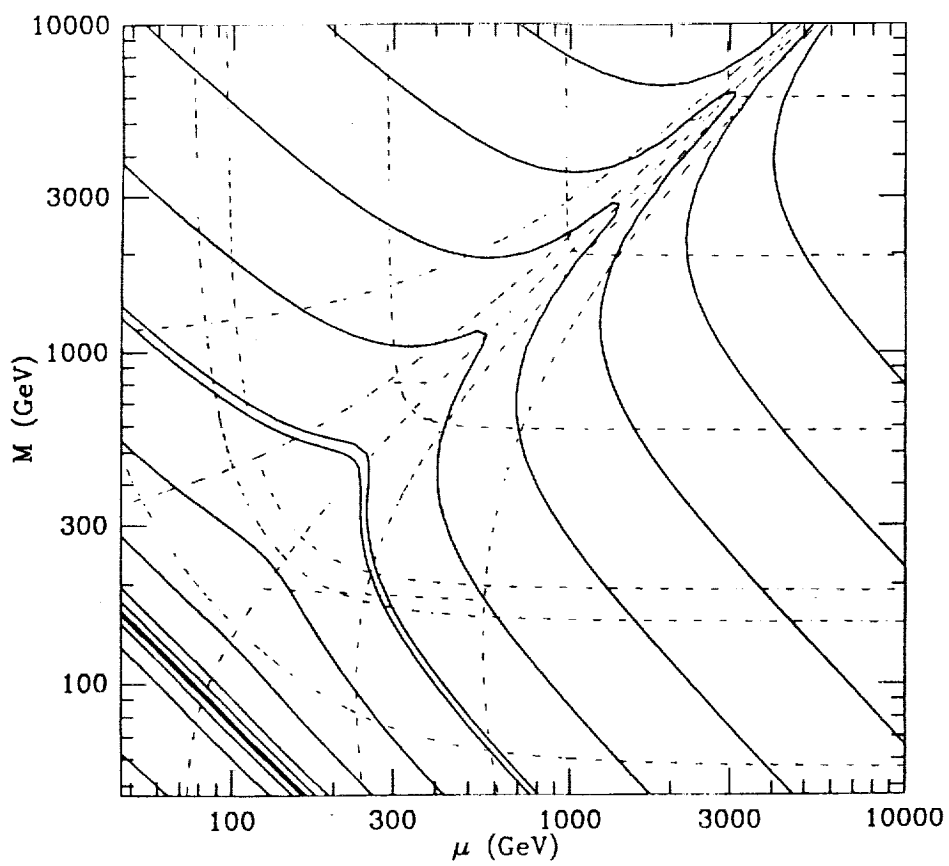


Fig. 3(c)

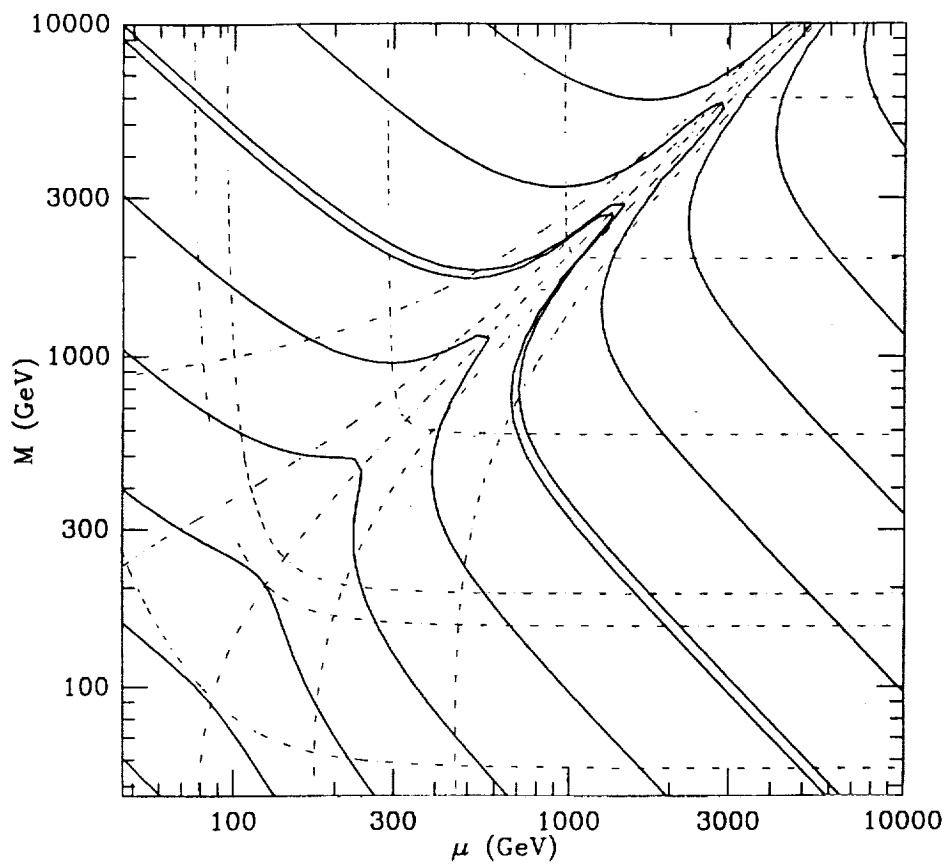


Fig. 3(d)

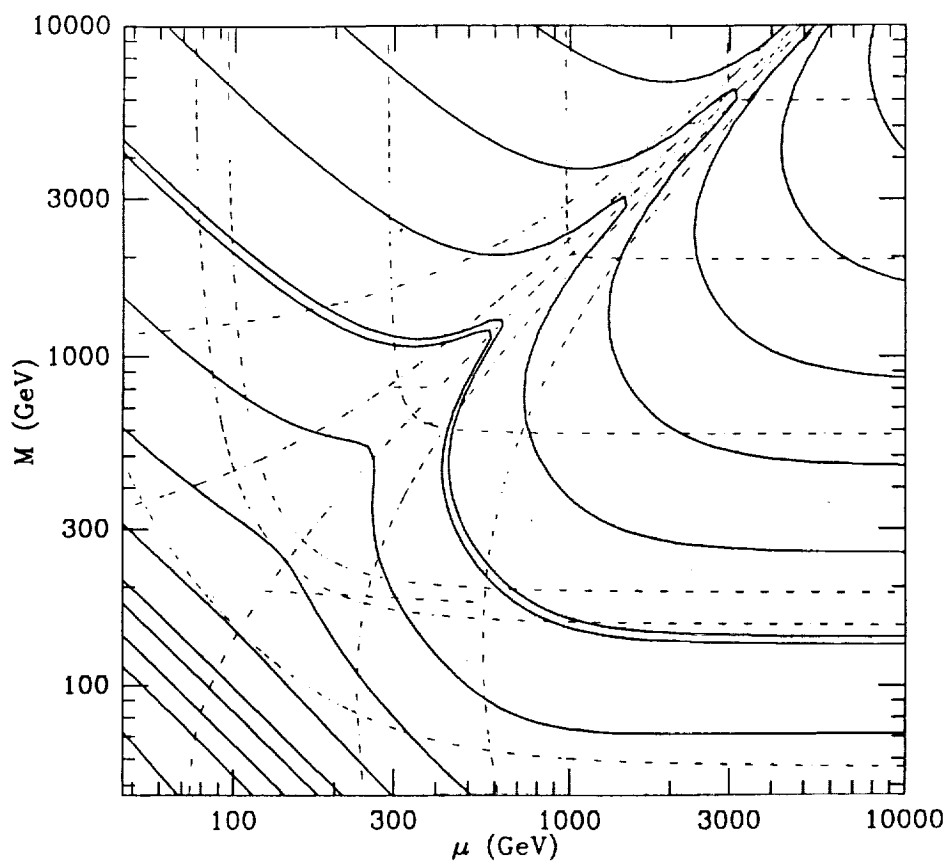


Fig. 4

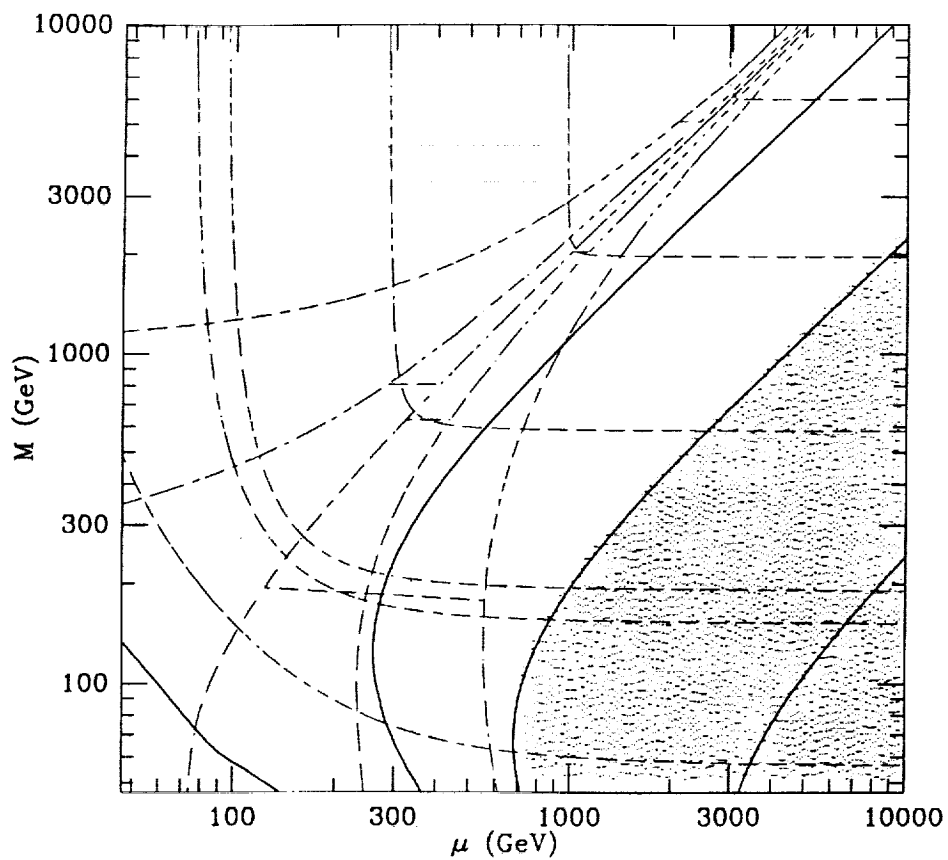


Fig. 9.5

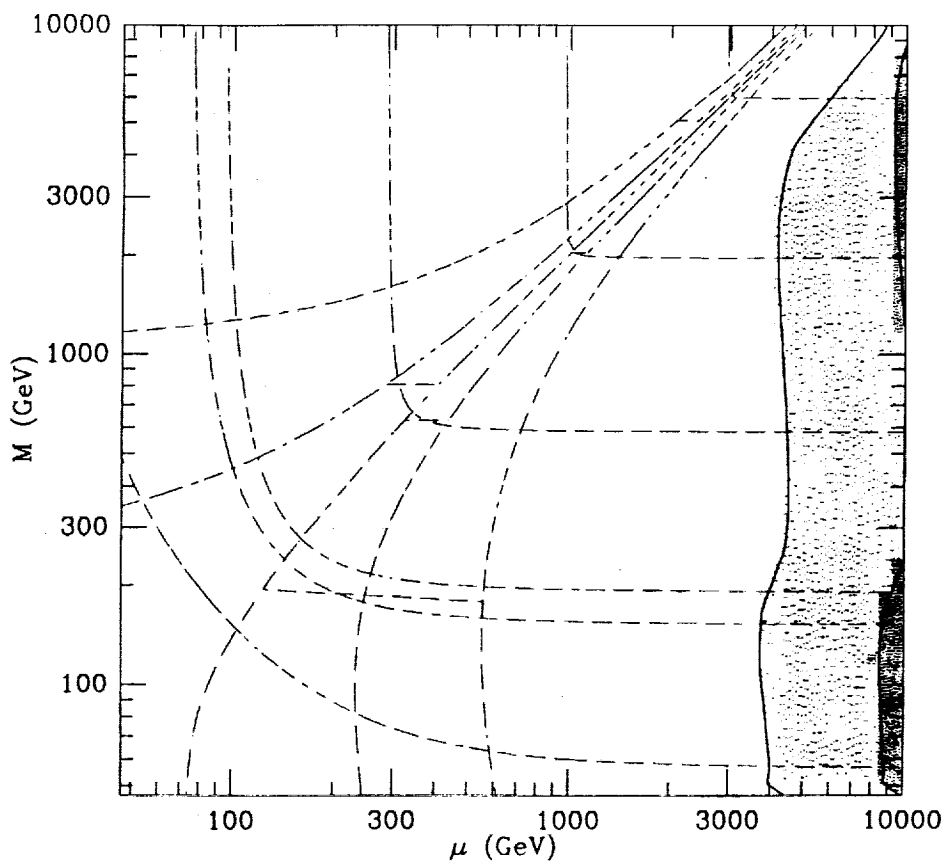


Fig. 6(a)

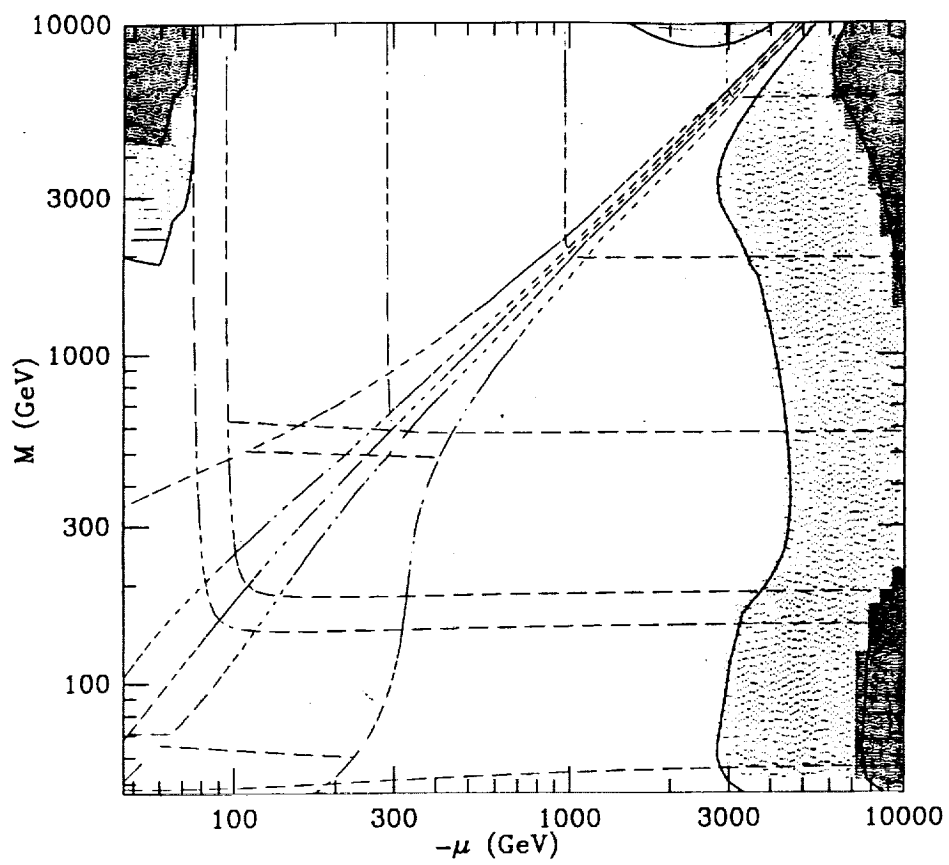


Fig. 6(b)

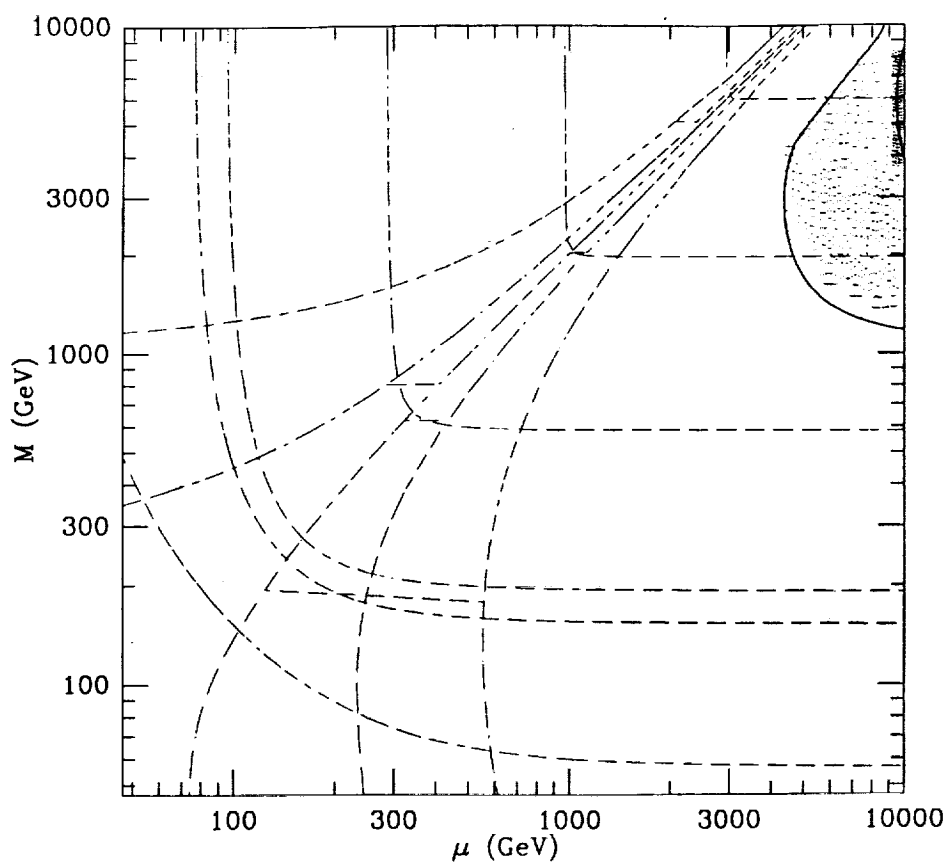


Fig. 6(c)

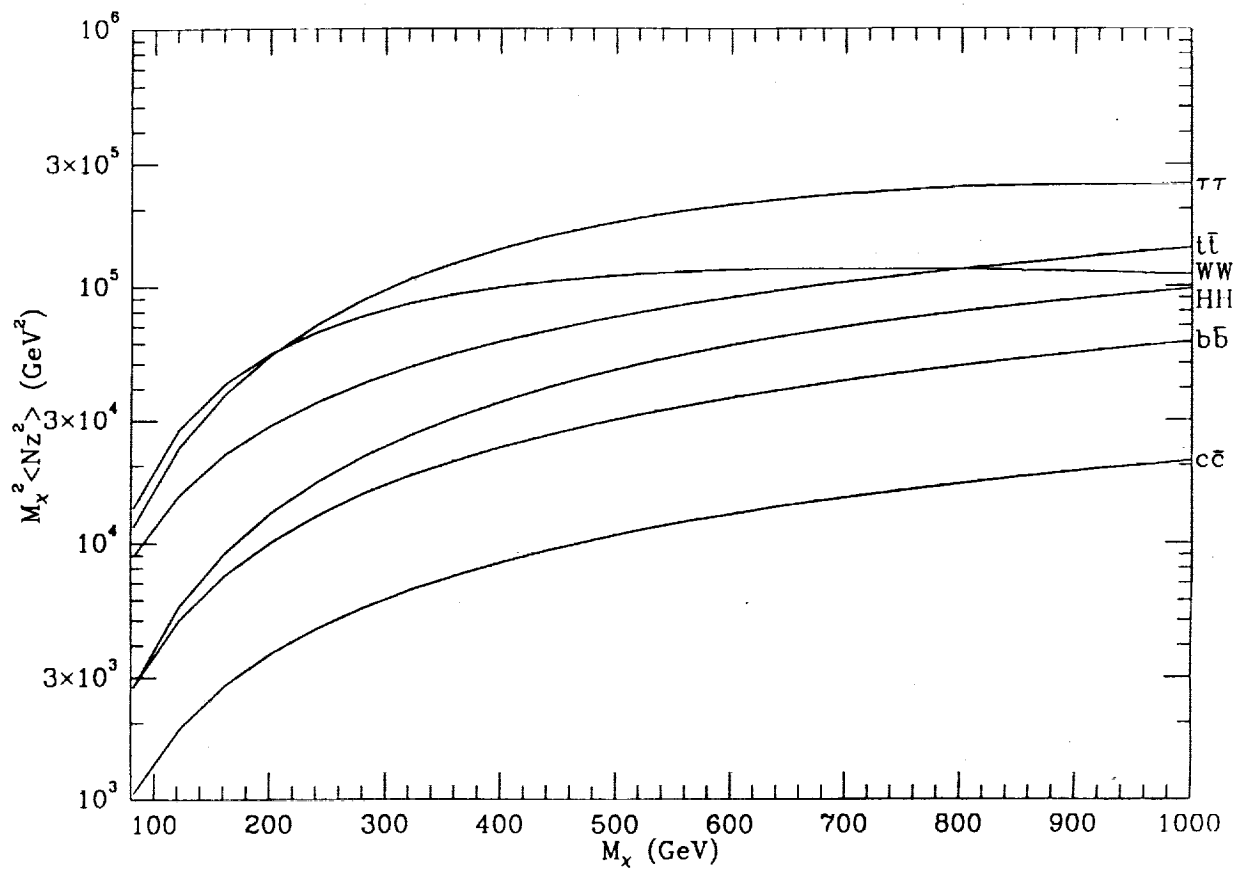


Fig. 7

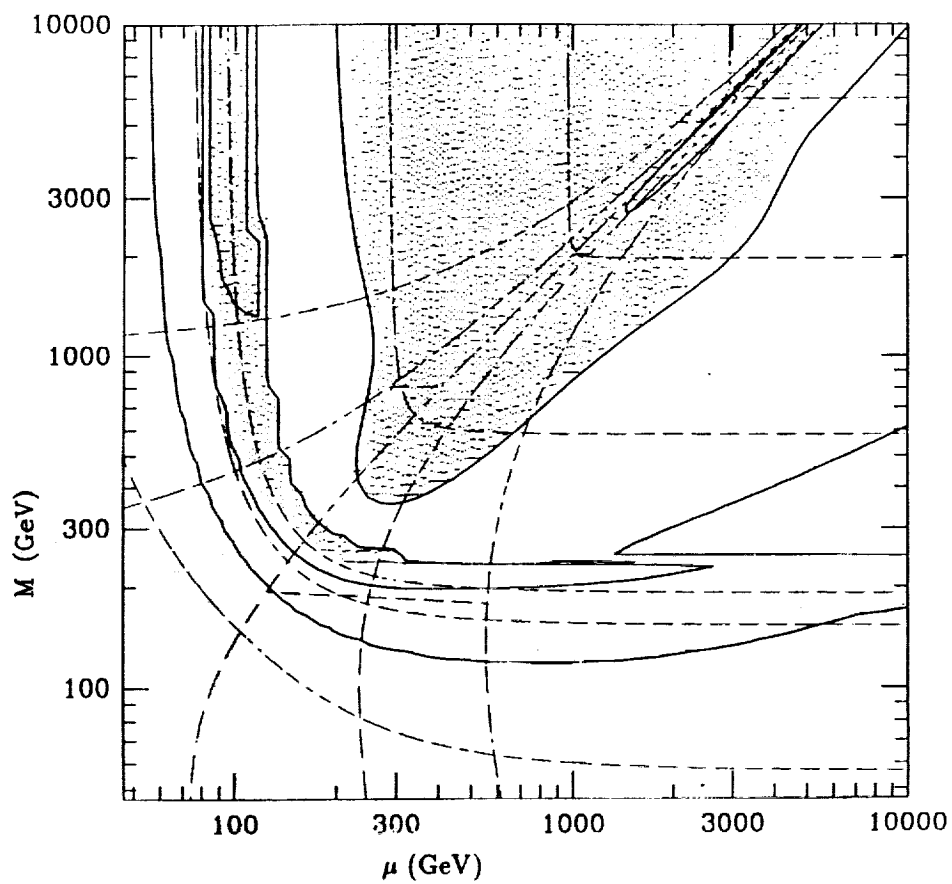


Fig. 8(a)



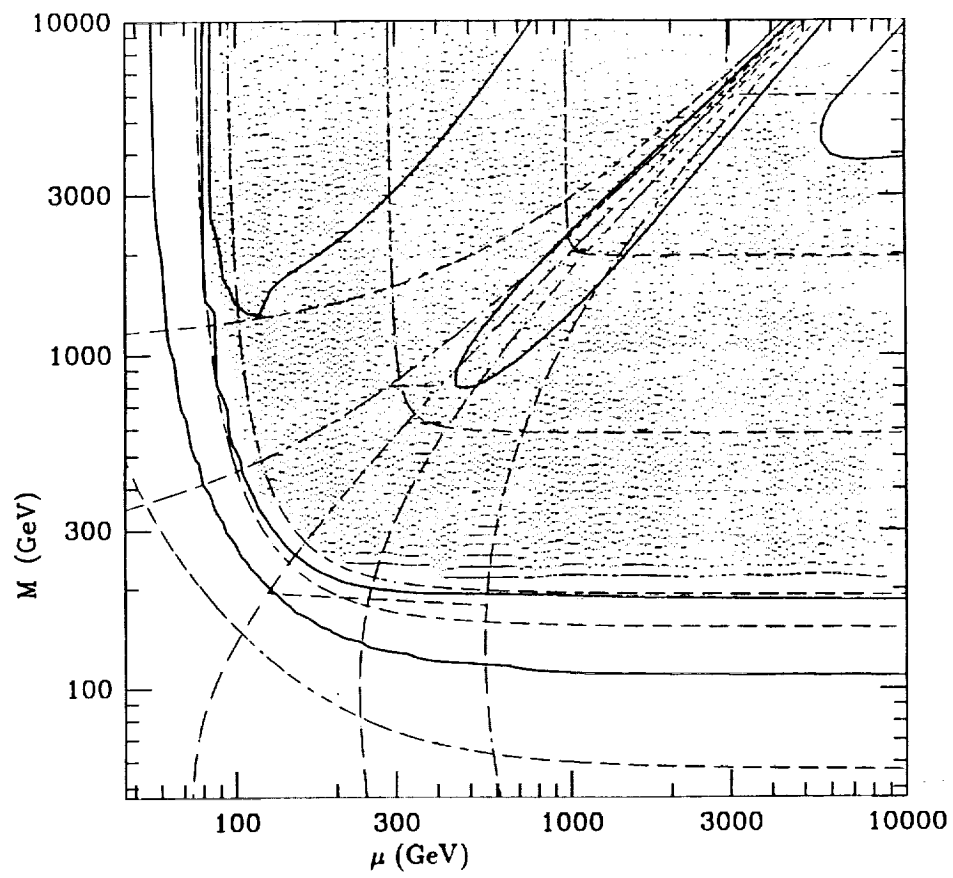


Fig. 8(b)

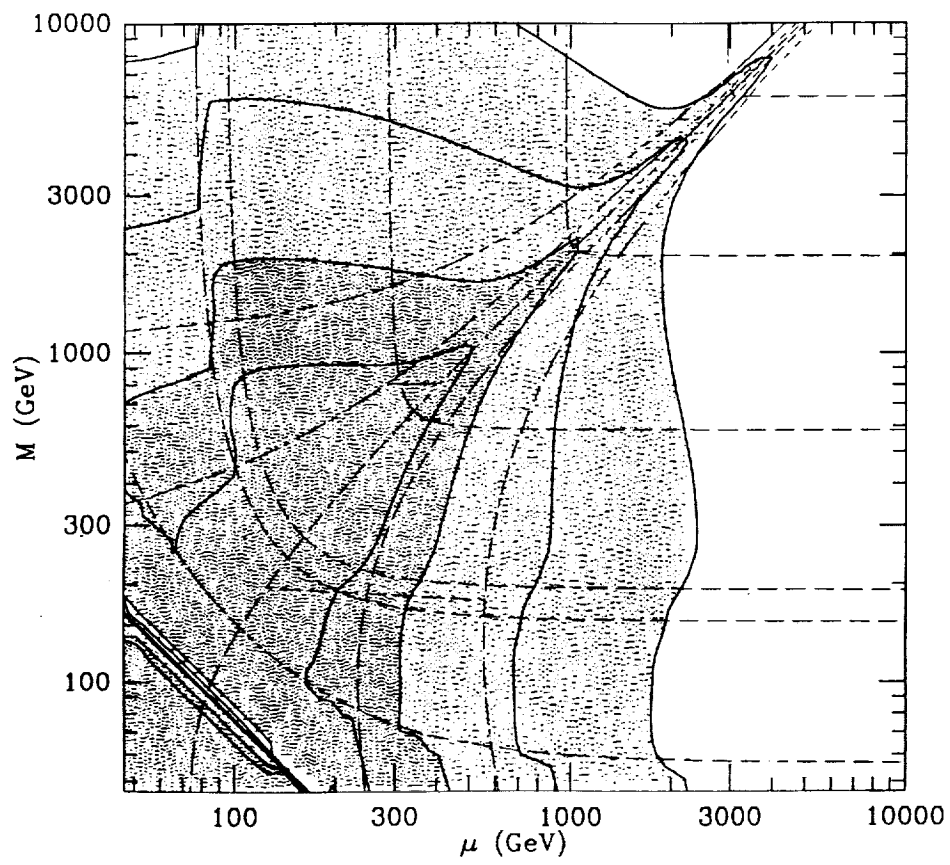
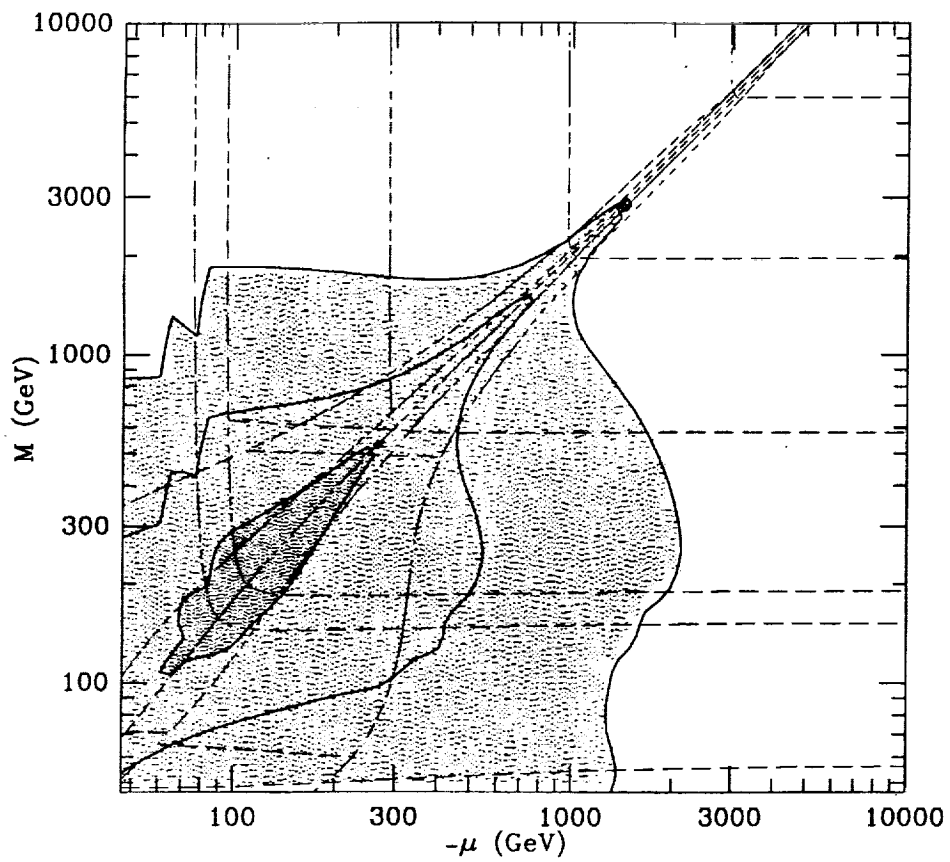
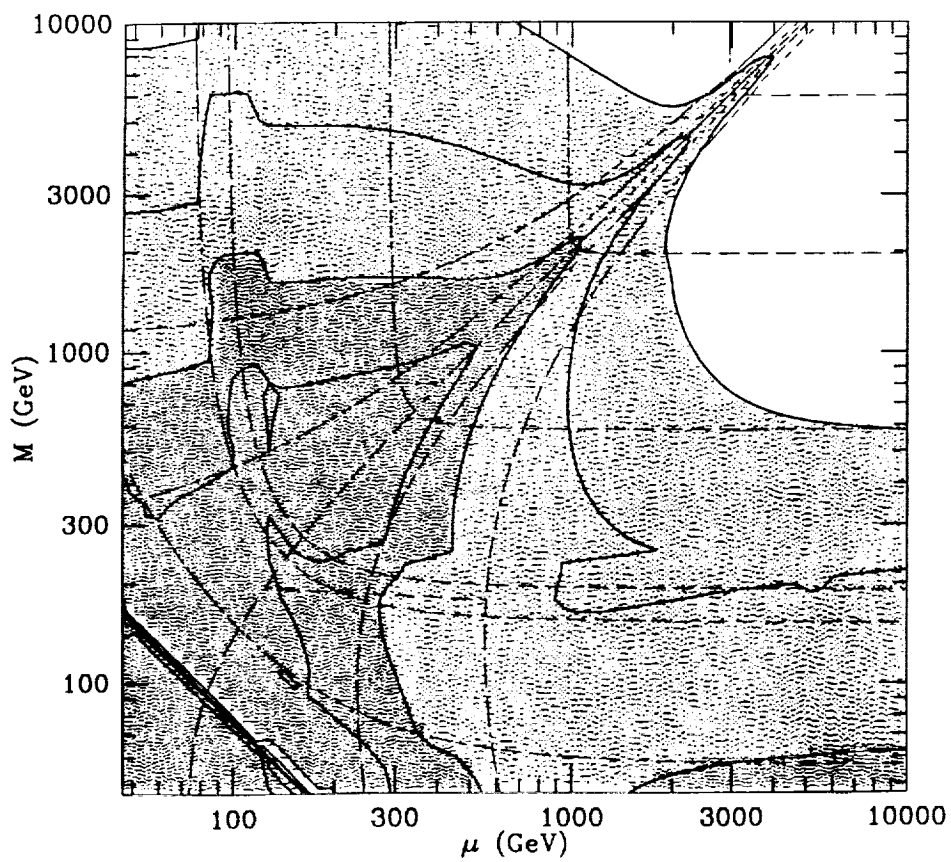


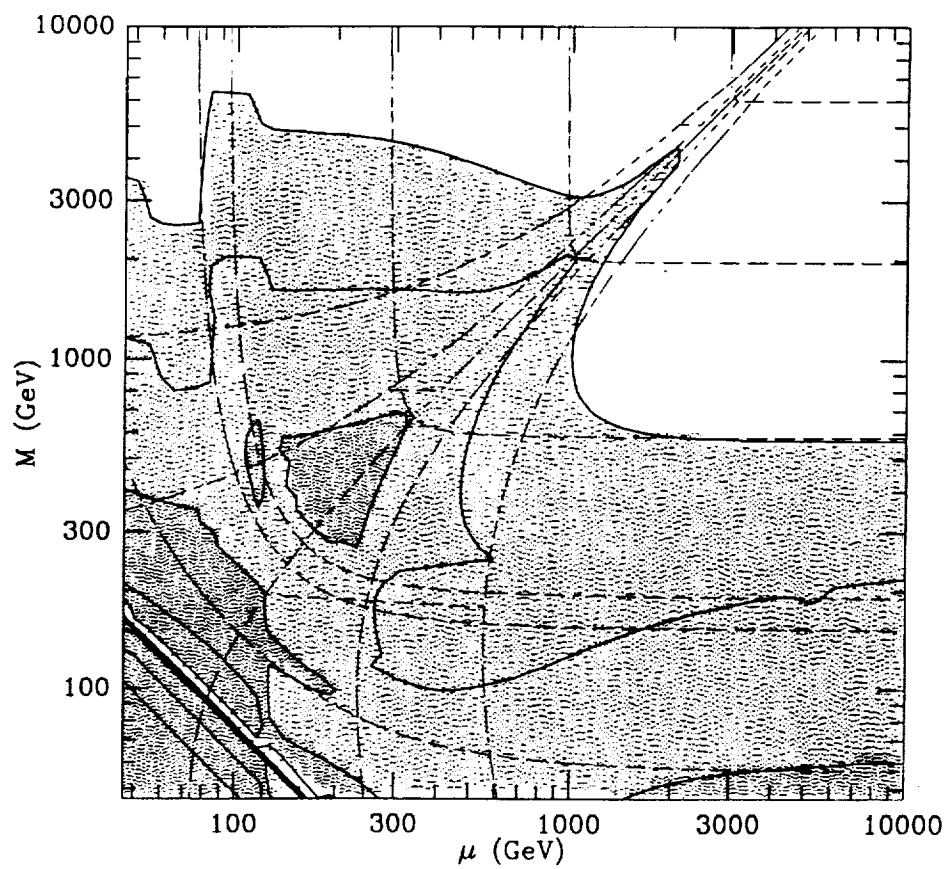
Fig. 9(a)



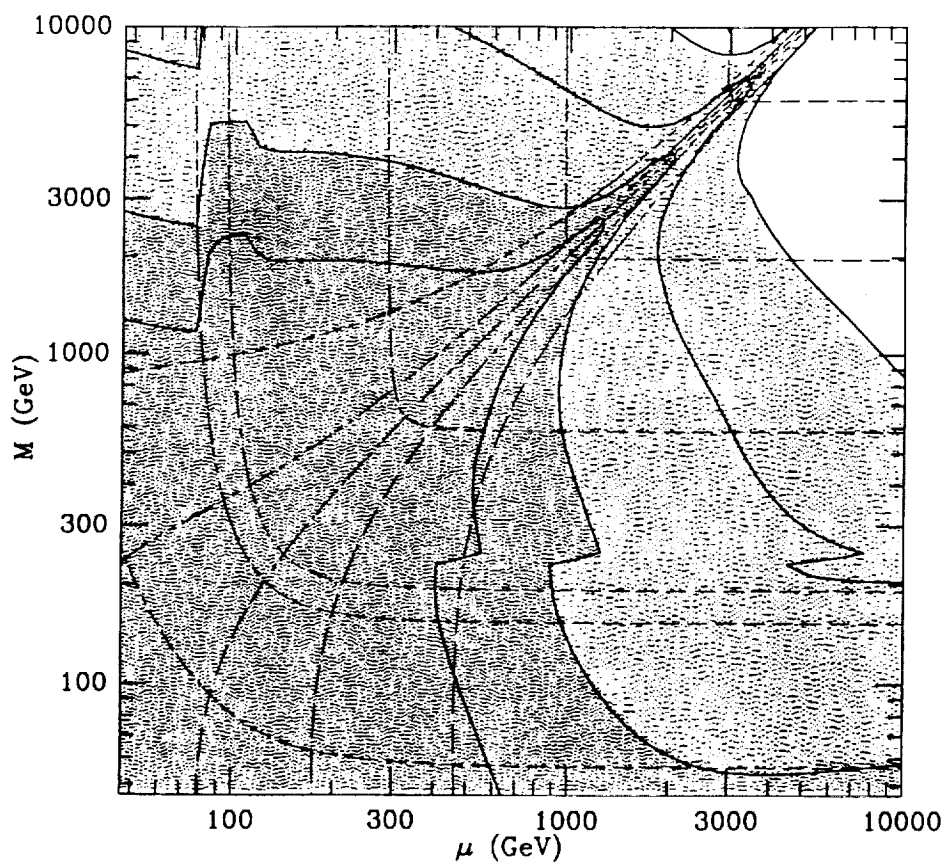
*Fig. 9(b)*



*Fig. 9(c)*



*Fig. 9(d)*



*Fig. 9(e)*

