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Plausible double inflation

Richard Holman,[1] Edward W. Kolb,[2,3] Sharon L. Vadas,[2,4] Yun Wang[1]

[1] *Physics Department, Carnegie Mellon University, Pittsburgh, PA 15213*

[2] *NASA/Fermilab Astrophysics Center*
Fermi National Accelerator Laboratory, Batavia, IL 60510

[3] *Department of Astronomy and Astrophysics and Enrico Fermi Institute*
The University of Chicago, Chicago, IL 60637

[4] *Physics Department, and the Enrico Fermi Institute*
The University of Chicago, Chicago, IL 60637

Abstract

It is likely that extended inflation is followed by an epoch of slowroll inflation. Such a sequence of events may lead to a very interesting perturbation spectrum with significant power on the scale of the transition between the extended and slowroll phase, superimposed upon a power-law spectrum with deviations from the Harrison–Zel'dovich slope. Normalization of the spectra above and below the transition scale is expected to differ.

The cold dark matter (CDM) scenario[1] is the most detailed, the most predictive, as well as the best motivated theory ever proposed for structure formation. There are two crucial ingredients in the CDM model: dark matter that is cold (i.e., small velocity at the time of matter domination), and a spectrum of fluctuations in the dark matter.

The spectrum of perturbations is usually assumed to be the one predicted by the original slowroll inflation models;[2] namely, curvature fluctuations with a featureless power-law spectrum and the "Harrison-Zel'dovich" slope, which results in perturbations that have a magnitude independent of scale as they come within the horizon. Recent observations of the Universe on large scales suggests (but by no means proves) that a CDM model with such a spectrum may not be able to explain (very) large-scale structure. The purpose of this *Letter* is to discuss an inflation model that predicts a spectrum that is not scale free (i.e., there is a feature corresponding to a particular scale) and does not have the Harrison-Zel'dovich slope. Although spectra with such features have been discussed previously,[3,9] we feel that our model is more plausible than previous models in the sense that the feature in the spectrum occurs at an interesting length scale for reasonable parameters of the model, and is not overly sensitive to parameter choices. We suggest that this result may be generic to many extended inflation models.

In the model examined here, there are two episodes of inflation: first a period of extended inflation,[4] followed by a period of slowroll inflation. The power-law fluctuations produced during each epoch may be traced to the same origin—quantum fluctuations in *(the same)* minimally coupled scalar field. We will denote the amplidude of the perturbation when a length scale crosses back within the horizon after inflation as $(\delta\rho/\rho)_{\rm HOR}$. If σ is a scalar field evolving under the influence of a potential $V(\sigma)$, and $H = \sqrt{8\pi V(\sigma)/3M_{PL}^2}$ is the expansion rate during inflation, then $(\delta\rho/\rho)_{\rm HOR} = H^2/\dot{\sigma}$. In this expression, H and $\dot{\sigma}$ are evaluated at the time the perturbation left the horizon during inflation. Now, $\dot{\sigma}$ can be expressed in terms of H and $V'(\sigma) = dV(\sigma)/d\sigma$ through the slowroll equations of motion: $3H\dot{\sigma} = -V'(\sigma)$.

In a previous paper[5] (denoted as I), we proposed a model of extended inflation which made use of the non-linear realization of scale invariance involving a dilaton coupled to a scalar field whose potential admits a metastable ground state. The model resembles the Jordan–Brans–Dicke (JBD) theory, with the exception that quantum effects in the form of the conformal anomaly generate a mass for the dilaton (equivalent to the JBD scalar), thus allowing our model to evade the problems of the original version of extended inflation. There are two fields in such a model: the dilaton (JBD scalar field) denoted as σ, and some scalar field ϕ that undergoes a first-order phase transition. We assume that the ϕ field is associated with some Grand Unified Theory (GUT).

In I we discussed the choices and motivation of incorporation of scale invariance; the reader is referred there for details. Here we simply write the effective low-energy ($E \ll M_{PL}$) action for the resulting theory:

$$\begin{aligned} S[g,\phi,\sigma] \;=\; \int d^4x\,\sqrt{-g}\,\Big\{ &-\frac{R}{16\pi G_N} + \frac{1}{2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{1}{2}\exp(2\sigma/f) g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &-\exp(4\sigma/f)\left[V(\phi) + \Delta(\phi)\frac{\sigma}{f}\right] + \Lambda \Big\}. \end{aligned} \tag{1}$$

Several features of the action require explanation: The mass scale f is the dilaton decay constant, which will be taken near the Planck scale. $V(\phi)$ is the 1-loop potential for ϕ, assumed to have the requisite form for a first-order phase transition. During extended inflation ϕ is trapped in some false-vacuum state (assumed to be $\phi = 0$) and $V(0) = \rho_V$. At the end of extended inflation ϕ is in its true-vacuum state ϕ_T. The term $\Delta(\phi) = [2/(8\pi)^2][V_0''(\phi)]^2$ arises from the 1-loop corrections to the tree-level potential $V_0(\phi)$. It is this term that breaks the classical scale invariance of the theory and gives a minimum to the potential for σ. It is during the evolution of σ to its minimum that a period of slowroll inflation ensues. Finally, $\Lambda = e^{-1}\Delta(\phi_T)/4$ is a cosmological constant which is adjusted to cancel the contribution of the potential at the minimum.

With $\Delta(\phi) = 0$, the above action is exactly the JBD action expressed in the Einstein conformal frame with $\omega = 2\pi f^2/M_{PL}^2 - 3/2$. The key feature for successful extended inflation is that with $\Delta(\phi_T) \neq 0$, the JBD field is massive today, which implies that the usual limit $\omega \gtrsim 500$ does not apply, and it is easy to arrange for ω to be small enough to avoid the problems with large bubbles that plagued the original extended inflation model. Since $\Delta(\phi)$ arises at the 1-loop level, it is suppressed by a factor of $32\pi^2$ relative to tree-level quantities such as ρ_V. Thus for values of $\sigma/f \lesssim 32\pi^2$, it will not affect the extended inflation scenario. Although we have described a particular model, it has features generic to many approaches to extended inflation; i.e., a mass for the JBD field that for one reason or another is small.

The calculation of the perturbation spectrum is as in the standard slowroll analysis. In the first phase the scalar potential is $V_E(\sigma) = \exp(4\sigma/f)\rho_V$. In the second phase of inflation the scalar potential is $V_S(\sigma) = \exp(4\sigma/f)\Delta(\phi_T)\sigma/f$. Although we refer to the first phase as "extended" and the second phase as "slowroll," both phases actually obey the slowroll approximations; this will be used in the calculation of $(\delta\rho/\rho)_{\rm HOR}$.[7]

The spectrum of perturbations produced during extended inflation by quantum fluctuations in the σ field has been analyzed by Kolb, Salopek, and Turner.[7] During extended inflation the σ field slowly rolls down the potential $V_E(\sigma)$. When scale λ reenters the horizon after inflation $(\delta\rho/\rho)_{\rm HOR} \propto \lambda^{1/(\alpha-1)}$, where $\alpha \equiv \pi f^2/M_{PL}^2$. If we take $f = 3M_{PL}/2$ for instance, then $(\delta\rho/\rho)_{\rm HOR} \propto \lambda^{0.165}$. We will normalize the spectrum later. Since horizon crossing is "first out—last in," extended inflation produces the perturbations on large scales. If extended inflation is not followed by a slowroll phase, then the single power-law spectrum would obtain. However, there can be a second phase of slowroll inflation.

Once ϕ tunnels to its true minimum (where $V(\phi_T) = 0$ is assumed for convenience), σ will evolve under the influence of $V_S(\sigma)$. In the tunnelling process there is nothing to force the dilaton σ to be at the minimum of its potential $\langle\sigma\rangle = -f/4$. In *I* we calculated the value of σ at the end of extended inflation: $2\sigma(t_{\rm end})/f = \{\ln[(3\alpha - 1)/(8\pi\alpha)] + \ln(M_{PL}^2/\rho_V^{1/2}) - S_B/2\}$, where S_B is the bounce action. Using $\rho_V \sim M^4$ with $M \sim 10^{14}$GeV, $\sigma(t_{\rm end})/f \sim 10.4 - S_B/4$.

Before turning to the second phase of inflation, we remark that there are other sources of perturbations during extended inflation, including perturbations caused in the mess of bubble collisions and reheating. If extended inflation is not followed by another episode of inflation, these perturbations, which are the size of the horizon at the end of extended inflation, are much too small to be of interest today. However if slowroll inflation closely follows on the heels of extended inflation, these perturbations will be inflated to a much larger size and might be an interesting scale today. Even estimating the amplitude of perturbations from bubble-wall collisions is difficult. It cannot even be stated whether they are in the linear regime.

Now consider the second phase. Examination of $V_S(\sigma)$ shows that the potential becomes quite flat for $\sigma \lesssim \langle\sigma\rangle = -f/4$. It turns out, though, that the slowroll conditions hold for both positive and negative values of σ. In fact, for negative values the potential is *too* flat, leading to a large number of e-folds of slowroll inflation, pushing scales that left the horizon during extended inflation beyond our horizon. Thus, we will only consider $\sigma(t_{\rm end})/f > 0$, which will be guaranteed at GUT scales if $S_B < 41.6$. There are three calculations to be done: First, we must calculate the slope of the power-law perturbation spectrum. Then, we must calculate the total number of e-folds of slowroll inflation. Finally, we must match the magnitude of perturbations at the beginning of slowroll inflation to the perturbations at the end of extended inflation.

The potentials for the two phases are very similar. The only difference between the slowroll potential and the extended inflation potential, which are proportional to $\sigma\exp(4\sigma/f)$ and $\exp(4\sigma/f)$ respectively, is the multiplicative factor σ. Since $\exp(2\sigma/f)$ is an exponentially steeper function than $\sigma^{1/2}$, we can ignore the time-dependence of $\sigma^{1/2}$

relative to $\exp(2\sigma/f)$ in calculating $(\delta\rho/\rho)_{\rm HOR}$ in slowroll inflation. Thus, the slope of the spectrum will be very nearly the same in extended inflation and slowroll inflation: $(\delta\rho/\rho)_{\rm HOR} \propto \lambda^{1/(\alpha-1)}$. Numerical integrations confirm this.[5]

Now we turn to the question of the number of e-folds of slowroll inflation. Slowroll inflation will occur so long as the slowroll conditions, $|V_S''(\sigma)/V_S(\sigma)| \ll 24\pi/M_{PL}^2$ and $|V_S'(\sigma)/V_S(\sigma)| \ll \sqrt{48\pi}/M_{PL}$ are satisfied.[8] If the subscript i denotes the initial values at the onset of slowroll inflation and the subscript f denotes the final values, then the number of e-folds is

$$N_{SL} = \int_{t_i}^{t_f} dt H(t) = -\frac{8\pi}{M_{PL}^2}\int_{\sigma_i}^{\sigma_f} d\sigma\,\frac{V_S(\sigma)}{V_S'(\sigma)} = -2\alpha\int_{x_i}^{x_f} dx\frac{4x+\exp(-1-4x)}{1+4x}, \quad (2)$$

where $x = \sigma/f$. The value of σ_i depends on quantities such as S_B, ρ_V, and f. We take $f = 3M_{PL}/2$ as an example. First we find the value of σ/f for which the slowroll conditions break down for $\sigma/f > -0.25$: $x_f \simeq -0.10$, and then we use $x_i \equiv \sigma_i/f \simeq -1.09 + \ln(M_{PL}/M) - S_B/4$. Taking $M \simeq 10^{14}$ GeV, $x_i \simeq 10.4 - S_B/4$. If $S_B = 25$, we find that $N_{SL} = 50$. Thus, by varying the bounce action we can obtain differing amounts of e-folds during slowroll inflation.

If the number of e-folds of slowroll inflation is larger than 65, then scales that left the horizon during extended inflation are still outside of our horizon today, and the only effect of the extended-inflation phase is to provide the initial conditions for slowroll inflation. If however the number of e-folds of slowroll inflation is smaller than this, then scales larger than some comoving transition length λ_T went outside the horizon during extended inflation, and scales smaller than λ_T went outside the horizon during slowroll inflation; this is known as double inflation.[9] Furthermore, any large perturbations from reheating on the scale of the horizon at the end of extended inflation will correspond to a present scale λ_T. We now calculate λ_T.

The present size of the scale that went outside the horizon at the end of extended inflation is $\lambda_T = H^{-1}(t_{\rm end})a_0/a(t_{\rm end})$, where a_0 is the value of the scale factor today. From the end of extended inflation to today, a increases by the factors $\exp(N_{SL})$ during slowroll inflation, $V_S^{1/3}(\sigma_f)/T_{RH}^{4/3}$ between the end of slowroll inflation and reheating, and T_{RH}/T_0 between reheating and today. Here T_{RH} is the reheat temperature (estimated in I to be 10^6GeV) and $T_0 = 2.735\text{K} = (2.7 \times 10^{-26}\text{Mpc})^{-1}$ is the present temperature. During the extended era $a(t) \propto (1 + Bt)^\alpha$, so $H^{-1}(t) = (1 + Bt)/\alpha B$, where $B^2 = 8\pi M^4 \exp(4\sigma_0/f)/\alpha(3\alpha - 1)M_{PL}^2$ and σ_0 is the value of σ at the start of the extended phase. Now $H^{-1}(t_{\rm end})$ can be found by the usual requirement that $\epsilon = \Gamma/H^4 \sim 1$ where Γ is the bubble nucleation probability per unit 4 volume: $H^{-1}(t_{\rm end}) = [8\pi\alpha/(3\alpha - 1)]^{1/2} \exp(S_B/2)/M_{PL}$. Putting together all the factors, λ_T (in Mpc) is

$$\lambda_T = \sqrt{\frac{8\pi\alpha}{3\alpha - 1}} \exp(-59 + N_{SL} + S_B/2) \left(\frac{\Delta(\phi_T)}{M_{PL}^3 T_{RH}}\right)^{1/3} \left(\exp(4\sigma_f/f)\frac{\sigma_f}{f} + \frac{e^{-1}}{4}\right)^{1/3}.$$

Thus, given S_B, ρ_V and f, we determine σ_f/f from the slowroll conditions, calculate N_{SL} from Eq. (2), and use this equation to find λ_T. Using the value $T_{RH} = 10^6$GeV from I and taking $M = 10^{14}$GeV, $f = 1.5M_{PL}$, and $S_B = 25$, we find that $N_{SL} = 50$. Therefore, for $\Delta(\phi_T) = M^4/(32\pi^2)$, a scale that left the horizon during the transition corresponds to a length scale of 0.025 Mpc today.

We now have all the information necessary to specify the perturbation spectrum. Using $(\delta\rho/\rho)_{\rm HOR} = 3H^2/V'(\sigma)$ during the two inflation epochs with the fluctuations set by extended inflation for $\lambda > \lambda_T$ and by the slow-roll epoch for $\lambda < \lambda_T$:

$$\begin{aligned}
(\delta\rho/\rho)_{\rm HOR} &= \frac{3\alpha - 1}{\sqrt{6\alpha}} \exp(-S_B/2) \left(\frac{\lambda}{\lambda_T}\right)^{1/(\alpha-1)} && (\lambda > \lambda_T) \\
(\delta\rho/\rho)_{\rm HOR} &= \frac{3\alpha - 1}{\sqrt{6\alpha}} \exp(-S_B/2) \left[\frac{A_S \Delta^{1/2}(\phi_T)}{2M^2}\right] \left(\frac{\lambda}{\lambda_T}\right)^{1/(\alpha-1)} && (\lambda < \lambda_T), \quad (3)
\end{aligned}$$

where $A_S = |2\ln[(3\alpha - 1)/(8\pi\alpha)] + 4\ln(M_{PL}/M) - S_B|^{1/2}$. In addition, a "feature" in the spectrum at $\lambda = \lambda_T$ is expected.

What can we say about f (or equivalently, ω)? If the model is to make sense, we expect $f = \mathcal{O}(M_{PL})$. In the original JBD extended inflation model, in order to suppress the production of large bubbles, it was necessary that $\omega \lesssim 20$ (or $f < 1.85 M_{PL}$).[6] Here, however, this constraint is evaded. At the end of slowroll inflation the energy density comes from reheating, and the extra energy density outside the voids (created by large true-vacuum bubbles) is inflated away during slowroll inflation. Thus, by the time reheating occurs there is no appreciable difference in energy density between the regions that contained true-vacuum bubbles and the surrounding regions.

In the model where there was only one period of inflation, if the fluctuations produced in inflation had anything to do with galaxy formation, then $\omega \gtrsim 6$ (or $f > 1.19 M_{PL}$).[7] This limit arose from demanding that the slope of the power-law perturbation spectrum not be steep enough to violate the quadrapole anisotropy limits. Here, also, these limits are lessened. Assume for the moment that $3000h^{-1}\mathrm{Mpc} > \lambda_T > 8h^{-1}\mathrm{Mpc}$, and that $(\delta\rho/\rho)_{8h^{-1}\mathrm{Mpc}}$ is normalized to approximately 10^{-4} to seed galaxy formation. On scales larger than λ_T, $(\delta\rho/\rho)_{\mathrm{HOR}} = (\delta\rho/\rho)_{8h^{-1}\mathrm{Mpc}}(\lambda/8h^{-1}\mathrm{Mpc})^{1/(\alpha-1)}\,(2M^2/\Delta^{1/2}A_S)$. The new feature in our model is the final factor of $2M^2/(\Delta^{1/2}A_S)$. Current limits to the quadrapole anisotropy, $\delta T/T \simeq (1/15) \times (\delta\rho/\rho)_{3000h^{-1}\mathrm{Mpc}} \lesssim 3 \times 10^{-5}$, can be accommodated for $\alpha > 1$ by making $2M^2 < \Delta^{1/2}A_S$. So, the "jump" avoids problems with the quadrapole anisotropy, and f/M_{PL} can be small. In fact, for large enough Δ, the only constraint is that $f > M_{PL}/\sqrt{\pi}$, which must be satisfied in order that inflation occur.

Another way to avoid the second constraint is through the "bump". As mentioned previously, there will be a bump in the density spectrum at λ_T due to bubble collisions. Thus, if this "bump" is responsible for galaxy formation, then the constraint on f/M_{PL} is lessened because the slope for $(\delta\rho/\rho)_{\mathrm{HOR}}$ can be steeper because it is no longer necessary to normalize the power-law spectrum at galaxy scales.

In Fig. 1 we show $(\delta\rho/\rho)_{\mathrm{HOR}}$ and λ_T as a function of S_B and f/M_{PL}. Here we have calculated $(\delta\rho/\rho)_{\mathrm{HOR}}$ at the end of extended inflation. Inspection of Eq. (3) shows that

$(\delta\rho/\rho)_{\rm HOR}$ at the beginning of slowroll inflation may be greater or less than that shown in Fig. 1. Also shown are contours for different values of λ_T. Of particular interest is the possibility that λ_T, which is expected to correspond to a feature in the spectrum from reheating, might be at a scale of cosmological interest (e.g., 50 or 120 Mpc). As discussed above, the slope of the spectrum is $(\delta\rho/\rho)_{\rm HOR} \propto \lambda^{1/(\alpha-1)}$; a most interesting slope, which can differ from the "Harrison-Zel'dovich" flat spectrum, as discussed above, when f/M_{PL} is near 1.

In conclusion, we have shown that in our model double inflation occurs automatically. It seems reasonable to speculate that other models of extended inflation with a small mass for the JBD field will have similar properties. Unlike some other models of double inflation, it is quite reasonable to expect the second phase of inflation to last just long enough for the boundary between the two phases to be at a scale of interest for large-scale structure. We also demonstrated that no unreasonably small numbers need be introduced, and that results are relatively insensitive to variations of model parameter.

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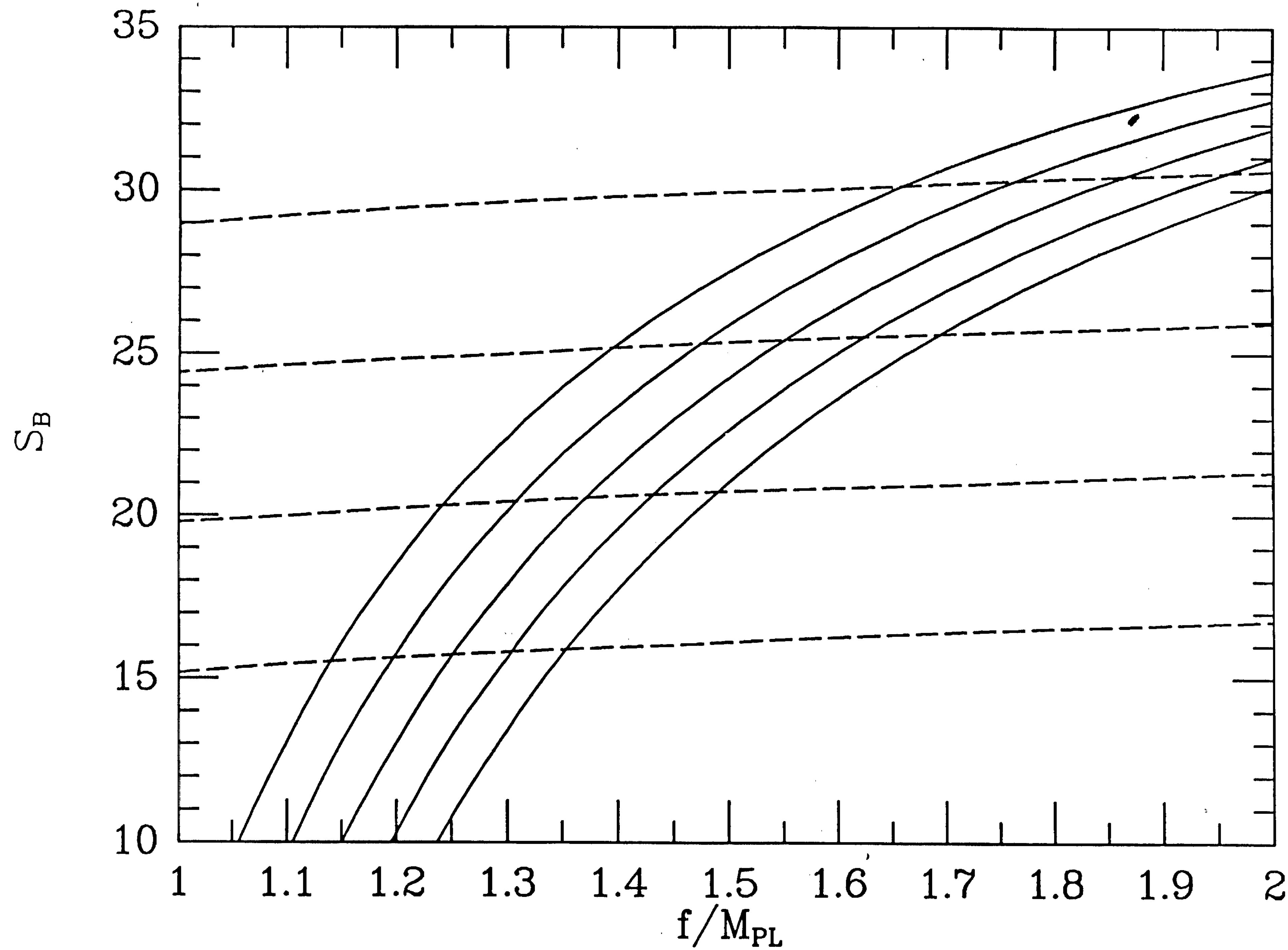
FIGURE CAPTION

Fig. 1: Contours of $(\delta\rho/\rho)_{\rm HOR}$ (at the end of extended inflation) and λ_T (in Mpc) as a function of f/M_{PL} and S_B. We have taken $M = 10^{14}$ GeV and $\Delta(\phi_T) = M^4/(32\pi^2)$. The contour lines for $(\delta\rho/\rho)_{\rm HOR}$ are dashed, and have values 10^{-6}, 10^{-5}, 10^{-4}, and 10^{-3} from top to bottom, while those for λ_T are the solid lines at the values 10^{-4}, 10^{-2}, 10^{0}, 10^{2} and 10^{4} from left to right. Note that the comoving mass contained within a scale λ is $M(\lambda) = 1.5 \times 10^{11}(\Omega_0 h^2)(\lambda/{\rm Mpc})^3 M_{\odot}$.



35
30
25
20
15
10
S_B
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
f/M_{PL}