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NASA Technical Memorandum 104453

Balancing Reliability and Cost to Choose the Best Power Subsystem

Ronald C. Suich
California State University
Fullerton, California

and

Richard L. Patterson
Lewis Research Center
Cleveland, Ohio

Prepared for the
26th Intersociety Energy Conversion Engineering Conference
cosponsored by the ANS, SAE, ACS, AIAA, ASME, IEEE, and AIChE
Boston, Massachusetts, August 4-9, 1991

NASA

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Ronald C. Suich
California State University
Fullerton, CA 92634

Richard L. Patterson
NASA Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

How does a design engineer or manager choose between a power subsystem with .990 reliability and a more costly power subsystem with .995 reliability? When is the increased cost of a more reliable power subsystem justified?

A mathematical model is presented for computing total (spacecraft) subsystem cost including both the basic subsystem cost and the expected cost due to the failure of the subsystem. This model is then used to determine power subsystem cost as a function of reliability and redundancy. Minimum cost and maximum reliability and/or redundancy are not generally equivalent. Two example cases are presented. One is a small satellite, and the other is an interplanetary spacecraft.

INTRODUCTION

The methods described here can be applied to power subsystems in launch vehicles, satellites, and on earth. In addition, they can be utilized in many other types of applications which do not necessarily involve power.

High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the statistically expected cost due to a subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We cannot consider either the cost of the subsystem or the expected cost due to subsystem failure separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.

We will be looking at subsystems which are part of a larger main system, such as a power subsystem which is a part of a main satellite system. In development of our analyses, we will talk about subsystems and main systems. In the examples, we will bring out power subsystems and larger aerospace-related systems.

Expected value is an important ingredient in our quest for finding the best power subsystem. Therefore, we'll first consider the expected cost due to subsystem failure, which is written as E{cost due to subsystem failure}. As with all expected values, this number depends upon both the dollar cost and the probability of its occurrence. Let c_1 be the dollar cost due to failure of the subsystem, including all costs incurred by subsystem failure (but not the cost of the subsystem itself). This number could be the entire cost of the main system (even greater in some circumstances) if failure of the subsystem resulted in complete failure of the main system. In other instances c_1 would be less than the cost of the main system, e.g., failure of the subsystem resulted in only a partial failure of the main system.

Now the expected cost due to subsystem failure is c_1 times the probability that this cost will be experienced. Subsystem failure for us can only occur when the main system is good. If the main system fails (for other than failure of the subsystem), we'll not experience cost due to subsystem failure. So, we discount the E{cost due to subsystem failure} by multiplying by the reliability of the main system. For example, let's consider a power subsystem in a rocket. The rocket may explode on the launch pad due to a fuel problem. Even if the power subsystem would have failed in flight, we would not experience this failure. Let r_M be the reliability of the main system (for other than failure of the subsystem), and let r_S be the reliability of the subsystem.

Then E{cost due to subsystem failure}

$= c_1 \Pr\{\text{subsystem failure} \mid \text{mainsystem good}\} \times \Pr\{\text{main system good}\}$

$= c_1(1-r_S)r_M = r_M c_1(1-r_S)$.

We can minimize this expected cost by building a subsystem with an extremely low probability of failure, i.e., a subsystem with extremely high reliability. In this situation it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected cost due to subsystem failure but does not consider the cost of building the subsystem itself. To make this decision, we should not consider the two costs separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure. The total cost to be minimized is given by

C = cost of the subsystem + E{cost due to subsystem failure}

= cost of subsystem + $r_M c_1(1-r_S)$.

In minimizing cost C we see that we are balancing the cost of the subsystem against the expected cost due to subsystem failure.

NON-REDUNDANT POWER SUBSYSTEMS: SELECTING THE BETTER OF TWO ALTERNATIVES

Let's look at a microsatellite example. Suppose that we have two possible power subsystems under consideration for the microsatellite. Power subsystem 1, which costs \$200,000, has a .97 reliability. Power subsystem 2, with a cost of \$100,000, has a .94 reliability. Without further information and analysis, there is no clear "best" power subsystem, and the choice is often based upon the amount budgeted for the power subsystem.

For further analysis, let us say that the main microsatellite system has a reliability (exclusive of the subsystem under consideration) of $r_M = .96$. We'll further assume that failure of the power subsystem will result in a cost of $c_1 = \$10{,}000{,}000$. Let us first compare the E{cost due to subsystem failure} for each of the two power subsystems. For power subsystem 1,

E{cost due to subsystem failure}

$= r_M c_1 \Pr\{\text{subsystem failure}\}$

$= r_M c_1(1-r_{S1}) = .96 \times \$10{,}000{,}000 \times .03 = \$288{,}000$.

For power subsystem 2,

E{cost due to subsystem failure}

$= r_M c_1(1-r_{S2}) = .96 \times \$10{,}000{,}000 \times .06 = \$576{,}000$.

Since power subsystem 2 is less reliable than power subsystem 1, it has a higher expected cost of failure. However, since 2 is also less expensive to build, we need to compare the overall cost, C, for 1 and for 2. For power subsystem 1,

$$C_{S1} = \$200{,}000 + \$288{,}000 = \$488{,}000.$$

For power subsystem 2,

$$C_{S2} = \$100{,}000 + \$576{,}000 = \$676{,}000.$$

Since $C_{S1} < C_{S2}$, we select power subsystem 1 over power subsystem 2.

For further information on expected values or on selecting the best subsystem in simple situations as above, you may refer to [2]. We also note that the methods contained in this paper do not consider time-related functions, such as the cost of failure as a function of mission time. Time-related functions are covered in considerable depth in [2].

THE EFFECT OF REDUNDANCY: K OUT-OF-N:G SUBSYSTEMS

In this article we'll direct our attention to a specific type of subsystem, called a k-out-of-n:G subsystem. Such a subsystem has n modules, of which k are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called k. For example, the engineer may decide that k = 4. This means that each module is 1/4 of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called n. For example, an n = 6 and k = 4 subsystem would have 6 modules each of 1/4 th power and thus would have the output capability of 1.5 times the required power. The engineer is free to choose n and k. Selection of the different values of n and k results in different subsystems, each with different costs and reliabilities. Since each n and k yields a different subsystem with different costs, we can therefore choose the subsystem, i.e., the n and k, which will minimize cost C.

Here we'll assume perfect switching devices (if needed) of negligible cost and independence of the modules of the subsystem.

Although there are many variations of the k-out-of-n: G model, we'll present a few of these possibilities to give some idea of its uses and potentialities.

<u>MODEL 1</u> (<u>k fixed and n variable</u>)

The simplest k-out-of-n: G model is one where the modules are independent and all have common probability p of being good and common probability of failure q = 1-p. Let X count the number of good modules. Now E{cost due to subsystem failure}

$$= r_M c_1 \Pr\{\text{subsystem failure}\}$$

$$= r_M c_1 \Pr\{X < k\} = r_M c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x} \qquad (1)$$

Recall that C = cost of subsystem + E{cost due to subsystem failure}. We therefore need also to consider the cost of the subsystem. First consider a simple situation where k is fixed. Here we are free to choose n. Then n-k will be the redundancy or number of spares in the subsystem. If each module costs c_4 then the cost of subsystem = nc_4. Using this with (1) we obtain

$$C = \text{cost of subsystem} + E\{\text{cost due to subsystem failure}\}$$

$$= nc_4 + r_M c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.$$

We wish to find the n which minimizes cost C.

As an example, let's look at an interplanetary spacecraft. Consider the situation where k = 1, i.e., only one power module is required to be operational for the power subsystem to be operational. Suppose that the reliability of this <u>single module</u> is .95, i.e., p = .95. Let the reliability of the main spacecraft system for other than failure of the subsystem be .9, i.e., $r_M = .9$. Suppose that the cost of one power module is 1 (hundred million dollars, for example) i.e., $c_4 = 1$, and that the cost due to failure of the power subsystem is 10 (hundred million), i.e., $c_1 = 10$.

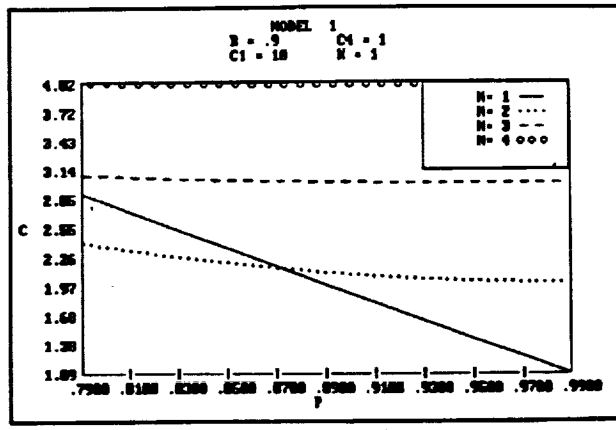

Figure 1 Model 1 Example where $c_1 = 10$.

Figure 1 shows a plot of C for p ranging from .79 to .99 and n's of 1 through 4. If the reliability of a single power module is .95, i.e., p = .95, note that n = 1 has the lowest value of C. Therefore the best power subsystem in this case is one with no spares. As a matter of fact we can see from Figure 1 that the subsystem of n = 1 has the lowest value of C for any p > .87. Therefore, as long as the reliability of a single power module is greater than .87, the best power subsystem is one with no spares. If p < .87, then n = 2 has the lowest value of C. Therefore, if the individual power module has reliability less than .87 (but greater than .79), then the best power subsystem is one with one spare. For values of p < .79, we should view the graph over this range to find the best subsystem.

Now suppose that c_1 (cost due to failure of the subsystem) increases to 50 (plot in Figure 2).

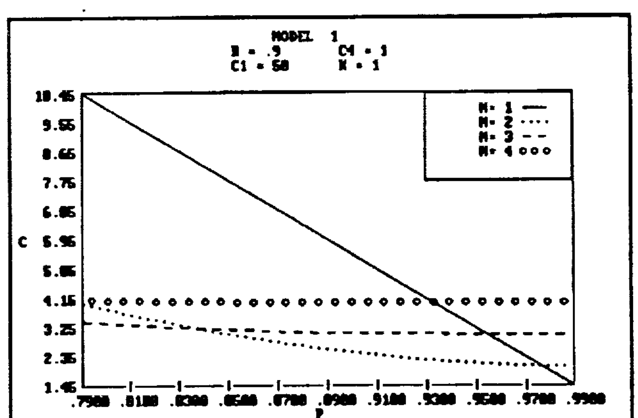

Figure 2 Model 1 Example where $c_1 = 50$.

We first note that if p = .95, then the n = 2 power subsystem is the best. If we compare Figures 1 and 2 (at $p = .95$) we see that the larger value of c_1 (in Figure 2) requires a larger value of n. This principle holds in general and makes sense. If the cost of subsystem failure increases, then more redundancy is required. Figure 2 reveals that if $.83 < p < .98$, then the $n = 2$ power subsystem is the best. If p falls below .83, then more redundancy is required ($n = 3$). If $p > .98$, then no redundancy is required ($n = 1$).

MODEL 2 (both k and n variable)

Suppose in model 1 that we are also free to choose k in our subsystem. If k is free to vary, then we'll call this model 2. Let c_3 be the cost of a subsystem consisting of exactly one module. Further suppose that the cost of a subsystem with exactly k modules is $c_3\, g(k)$. Here g(k) is the factor which measures the (generally) increased cost of building a subsystem consisting of k smaller modules rather than one large module. If $g(k) = 1$ for all k, then a subsystem of k modules costs the same as a subsystem consisting of a single module. Any g(k) may be used. For example, if a subsystem of 2 smaller modules costs 4 times as much as a single module subsystem then $g(2) = 4$. Therefore this subsystem would cost $c_3\, g(k) = c_3\, g(k) = 4c_3$. If a subsystem of 3 smaller modules costs 7 times as much as a single module subsystem then $g(3) = 7$. Other values for g(k) may be defined in a similar manner. Therefore, in the above example, $g(1) = 1$, $g(2) = 4$, $g(3) = 7$, etc. We also assume that each module in the subsystem costs $c_3 g(k)/k$, which is 1/k th of the total cost for k modules. Since we have a total of n modules in the subsystem, then the cost of the subsystem = $nc_3 g(k)/k$. Using this with (1) we obtain

$$C = \text{cost of subsystem} + E\{\text{loss due to subsystem failure}\}$$

$$= n\, c_3\, g(k)/k + r_M c_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.$$

For any particular situation with given values of c_1, c_3, r_M, p and g(k) we select the n and k to minimize C as given above. The n and k thus selected will be the optimal subsystem.

Consider the example of a space electrical power subsystem. A rough rule of thumb says that the cost of smaller modules for a space electrical power subsystem is proportional to the electrical power raised to the .7. Thus, for this example $g(k) = k(1/k)^{.7}$. Therefore, a subsystem consisting of a single module capable of full power would cost $c_3 g(1) = c_3 1(1/1)^{.7} = 1.0c_3$, a subsystem consisting of 2 modules, each of 1/2 power, would cost $c_3 g(2) = c_3 2(1/2)^{.7} = 1.23c_3$ to build, etc. An $n = 3$ and $k = 2$ subsystem, i.e., one having 3 modules each of 1/2 power, would cost $nc_3\, g(k)/k = 3 \times 1.23c_3/2 = 1.85c_3$ to build.

Suppose for a small satellite that the cost due to power subsystem failure, c_1, is 100 (million dollars). Let the reliability of the satellite (for other than failure of the subsystem) be .99, i.e., $r_M = .99$. Furthermore, the cost of building a single module capable of full power is .5 million, i.e., $c_3 = .5$. And last, let's say that each power module has a reliability of .95.

From Figure 3 we see, at $p = .95$, that the $n = 2$, $k = 1$ power subsystem is the best (has the lowest value of C). Note however, if $p < .948$, that the $n = 4$, $k = 2$ subsystem is the best. Additionally, note that this is a much flatter curve. Unless we're fairly sure that p is close to .95, we might choose the $n = 4$, $k = 2$ subsystem, since it gives us a relatively low value for C over a wide range of p.

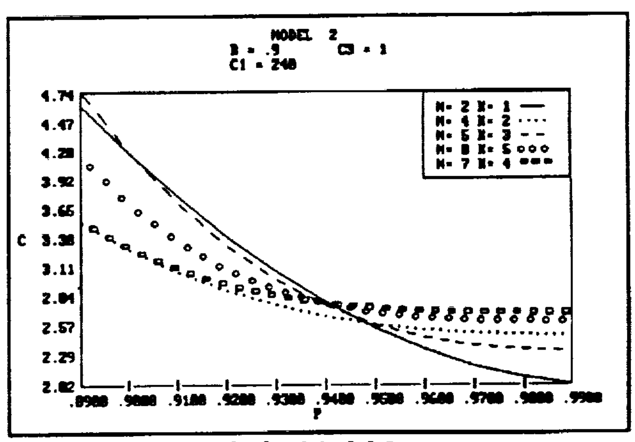

Figure 3 Example for Model 2.

BASIC PROGRAMS

The authors will provide, upon request, copies of BASIC programs (Quickbasic 4.5) to both evaluate C and also to search for an n and k which minimize C. These programs are also appropriate for models other than the two we've covered here (see [1] or [2] for more detailed information on other models). If you wish a copy on disk, please send a formatted disk floppy with your request. We also note that all models may be used when k is fixed by replacing $nc_3 g(k)/k$ by nc_4.

Additionally, the cost of launching the subsystem may easily be considered merely by including this cost in C for the various models.

CONCLUSIONS

The methods brought forward in this paper can be used to make very definitive decisions in choosing the best power (or other) subsystem from a number of alternatives. Essentially we minimize the total of two costs, i.e., the total of the cost of the (power) subsystem itself plus the expected cost due to the failure of the (power) subsystem. A computer program which utilizes the methods is available. Its output plots can yield very clear, obvious, and straightforward decisions.

ACKNOWLEDGEMENTS

This work was supported by the NASA Office of Aeronautics and Exploration Technology under Grant Number NAG3-1100 from the NASA Lewis Research Center. This manuscript represents U.S. Government work and is not subject to copyright.

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Report Documentation Page

NASA
National Aeronautics and Space Administration

1. Report No. NASA TM-104453	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Balancing Reliability and Cost to Choose the Best Power Subsystem		5. Report Date
		6. Performing Organization Code
7. Author(s) Ronald C. Suich and Richard L. Patterson		8. Performing Organization Report No. E-6294
		10. Work Unit No. 506-41-41
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes
Prepared for the 26th Intersociety Energy Conversion Engineering Conference cosponsored by ANS, SAE, ACS, AIAA, ASME, IEEE, and AIChE, Boston, Massachusetts, August 4-9, 1991. Ronald C. Suich, California State University, Fullerton, California 92634; Richard L. Patterson, NASA Lewis Research Center. Responsible person, Richard L. Patterson, (216) 433-8166.

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17. Key Words (Suggested by Author(s)) Cost analysis; Reliability; Reundancy; Power; Space	18. Distribution Statement Unclassified - Unlimited Subject Category 83

19. Security Classif. (of the report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	6	A02

*For sale by the National Technical Information Service, Springfield, Virginia 22161