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Comparative Survey of Dynamic Analyses of Free-Piston Stirling Engines

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ABSTRACT

This paper compares reported dynamic analyses for evaluating the steady-state response and stability of free-piston Stirling engine (FPSE) systems. Various analytical approaches are discussed to provide guidance on their salient features. Recommendations are made in the recommendations remarks for an approach which captures most of the inherent properties of the engine. Such an approach has the potential for yielding results which will closely match practical FPSE-load systems.

I. INTRODUCTION

Historically, the dynamic analyses of free-piston Stirling engine (FPSE) systems have focussed on two areas. Most of the analyses have emphasized the use of Laplace transformation and classical control techniques to determine the engine operating frequency and other dynamic parameters, notably, the piston-displacer oscillation amplitudes and their relative phase angles [1,2,3]. The other area of activity has been the determination of the conditions for engine stability by means of Laplace transformation [4] or the state-space technique [5,6]. In one instance [7], the operating frequency, piston and displacer amplitude, and other thermodynamic state variables are found by a linear harmonic analysis (LHA). This technique represents periodic variables with harmonic functions. The underlying reason for the bulk of the dynamic analyses is the prediction of the FPSE performance. The foregoing activities are essential to the design of a particular Stirling engine, and can enhance one's understanding of the behavior of FPSE's.

This paper reviews the existing literature on the dynamic analysis of FPSE systems. The purpose is to discuss the various analytical methods used, and provide guidance on their relative merits. The recommendations state the approach and salient features of the FPSE-load systems which will yield the most practical results, in a most expeditious manner.

II. ANALYTICAL FORMULATIONS

The discussions here pertain to the underlying assumptions and the equations of motion used for the dynamic analysis of FPSE's. For the papers reviewed and in this publication, the word "system" denotes a FPSE connected to a load. Where possible, the nature of the load is identified.

II-1. FPSE SYSTEMS

All the analyses found in the available literature on FPSE dynamics

emphasize the engine, with only allusions to its connected load. In some cases, the system consists of a single cylinder engine with a connected but unspecified load [1,5,6,7] modeled as a dashpot. In other instances, the system is a FPSE driving a linear alternator [2,3]. In reference [2], a connected load is implied but not explicitly stated. If the sole desire is the calculation of the engine dynamic parameters which impact the thermodynamic analysis, the system is usually confined to the engine itself [4].

Generally, the dynamic analysis of the FPSE system is expedited by assumptions which facilitate the solution of the system equations.

II-2. ASSUMPTIONS

The following simplifying assumptions are commonly found in the analyses:

- (1). Schmidt's thermodynamic analysis is assumed. Hence, (a) the piston and displacer motions are sinusoidal, as is the resulting working space pressure; (b) the working fluid obeys the ideal gas law and undergoes isothermal expansion and compression.
- (2). The system can be studied via linear analysis methods since (a) the working gas behaves like a linear spring and, (b) the connected load is assumed to be nearly linear.
- (3). The gas pressure in the working space (expansion and compression spaces, heater, regenerator and cooler volumes - Fig 1) is spatially constant but time variant.
- (4). The average working space pressure is balanced by the average pressure of the bounce space and other gas springs. Thus, the average force on the power piston, displacer and cylinder casing are zero. Consequently, the average positions of these elements are stationary.

II-3. EQUATIONS OF MOTION

A schematic of a single cylinder FPSE used by Rauch [1] is shown in Fig. 1. The dynamic equivalent model is illustrated in Fig. 2. The FPSE is represented as two masses, namely, the displacer (subscript 'D') and the power piston (subscript 'P'), which are coupled by gaseous springs (denoted 'K') and dampers (denoted 'C'). Unlike Fig. 2 in reference [1], an additional damping coefficient which couples the piston and displacer in the compression space is included here for completeness. Positive displacements of the piston and displacer are considered upward motions from their static equilibrium positions in Fig. 2.

The dynamics must be self-excited via the thermodynamics so as to induce the engine operation. That is, the dynamics of the piston and displacer motions must generate a pressure force which maintains their steady, periodic motions.

The piston, displacer and casing are three key elements of the FPSE. A sinusoidal steady oscillation is commonly assumed for these elements. Urieli and Berchowitz [9] show that operating the FPSE at a frequency in excess of the natural frequency of any of its elements will result in a reduced amplitude of oscillation for that element. One mode of FPSE operation is the removal of power out of the casing. For this mode the operating frequency is much greater than the natural frequency of the piston. Another mode is the removal of power from the piston. This mode is common for electrical output applications in which an alternator "load" is installed between the piston and casing. In this case, the operating frequency is greater than the natural frequency of the casing. This mode of operation is preferred for space power applications (and many others), due to the simplification obtained by a relatively heavier and, hence, stationary casing.

Based on the foregoing discussions, only two degrees of freedom are required to describe the dynamics of the FPSE. Hence, regarding Fig. 2, the general equations of motion are summarized in Eq. (1):

$$[M][\ddot{X}] + [C][\dot{X}] + [K][X] = [F(t)] \quad (1)$$

The matrices $[M]$, $[C]$ and $[K]$ represent the system masses, damping and stiffness coefficients external to the thermodynamic cycle. The forcing vector $[F(t)]$ is the sum of the forces due to thermodynamic cycle pressure, $[F_T(t)]$, and appropriate time dependent external forces, $[F_E(t)]$, on the piston and displacer. This is shown in Eq. 2:

$$[F(t)] = [F_T(t)] + [F_E(t)] \quad (2)$$

The thermodynamic forces can be represented as functions of the state variables $[X]$ and $[\dot{X}]$, which represent the displacements and their time derivatives, respectively, for the piston and displacer, as shown in Eq. 3:

$$[F_T(t)] = -[C_T][\dot{X}] - [K_T][X] \quad (3)$$

where $[C_T]$ and $[K_T]$ are, respectively, the thermodynamic damping and stiffness matrices which may be nonlinear. Negative signs associated with $[K_T]$ and $[C_T]$ indicate the restoring nature of $[F_T(t)]$.

The matrix $[C_T]$ is, in most analyses, given only passing attention. It represents the heat exchangers flow losses which, in many cases, are the dominant engine losses. Further, it may be shown that this damping is a non-linearity which stabilizes the amplitude of oscillation [4].

The $[K_T]$ matrix represents the dominant pressure forces acting on the piston and displacer. It may be written in terms of the piston area (A_P), displacer rod area (A_R) and the partials of pressure with respect to positions, as:

$$[K_T] = \begin{bmatrix} A_R \frac{\partial P}{\partial X_D} & A_R \frac{\partial P}{\partial X_P} \\ A_P \frac{\partial P}{\partial X_D} & A_P \frac{\partial P}{\partial X_P} \end{bmatrix} = \begin{bmatrix} K_{DDT} & K_{DPT} \\ K_{PDT} & K_{PPT} \end{bmatrix} \quad (4)$$

where K_{DDT} and K_{PPT} are the total thermodynamic stiffness coefficients of the displacer and piston, respectively. The term K_{DPT} is the thermodynamic stiffness on the displacer due to the piston motion. Similarly, K_{PDT} is the thermodynamic stiffness on the piston due to the displacer motion.

The terms in $[K_T]$ are functions of engine geometry, operating mean pressure and the expansion to compression space temperature ratio, T_e/T_c . Generally, these stiffness coefficients are uniquely different from each other. In particular, the coupling terms K_{DPT} and K_{PDT} are not equal, making $[K_T]$ asymmetric. This asymmetry is the source of self-excitation in the practical engine. However, when the working space temperatures are equal, that is T_e/T_c is unity, the coupling terms K_{DPT} and K_{PDT} are equal and $[K_T]$ becomes symmetric.

The thermodynamic stiffness matrix $[K_T]$ may be split into an isothermal term $[K_{T1}]$ and a temperature ratio dependent term $[K_{Tt}]$. Thus, Eq. (3) becomes:

$$[F_T(t)] = -[C_T][\dot{X}] - [K_{T1} + K_{Tt}][X] \quad (5)$$

The $[K_{T1}]$ matrix is symmetric and does not contribute to the self-excitation. The $[K_{Tt}]$ matrix is asymmetric, retains the temperature ratio dependence, and is the primary contributor to the self-excitation.

There are several ways of treating the thermodynamic force $[F_T(t)]$. One approach is to combine the thermodynamic matrices $[K_T]$ and $[C_T]$ with their external counterparts $[K]$ and $[C]$ on the left hand side (LHS) of Eq. (1), which becomes:

$$[M][\ddot{X}] + [C + C_T][\dot{X}] + [K + K_T][X] = [F_E(t)] \quad (6)$$

A second approach is to model the thermodynamic cycle as a forcing term in the right hand side (RHS) of Eq. (1). Combining Eqs. (1) and (2) yields Eq. (7):

$$[M][\ddot{X}] + [C][\dot{X}] + [K][X] = [F_T(t)] + [F_E(t)] \quad (7)$$

In this case Eq. (3) must also be satisfied for the complete solution.

A third method is to split the thermodynamic model, in which case $[C_T]$ and $[K_{T1}]$ terms of Eq. (5) are combined with $[C]$ and $[K]$, respectively, on the LHS of Eq. (1). The $[K_{Tt}]$ term remains on the RHS of (1) as a forcing term. Eq. (1) then becomes,

$$[M][\ddot{X}] + [C + C_T][\dot{X}] + [K + K_{T1}][X] = [F_{PT}(t)] + [F_E(t)] \quad (8)$$

where the force representing the relation between the thermodynamic cycle and piston and displacer motions, Eq. (9), must also be satisfied.

$$[F_{PT}(t)] = -[K_{Ti}][X] \quad (9)$$

Various forms of Eqs. (6), (7) or (8) are used in the literature. The equation used and unique assumptions of each reference are discussed below.

Rauch [1] splits the thermodynamic model and, thus, uses Eq. (8). The cylinder is assumed to be stationary. The engine modeled has no external springs. This implies that the elements in the expanded stiffness matrix of Eq. (10) are solely the isothermal counterparts of $[K_{Ti}]$.

$$[K+K_{Ti}] = \begin{bmatrix} K_D + K_C & -K_C \\ -K_C & K_P + K_C \end{bmatrix} \quad (10)$$

The common, off-diagonal term K_C represents the displacer-piston coupling spring stiffness. The terms K_D and K_P , respectively, denote the displacer and piston springs to ground. Note that the diagonal terms are the sum of K_D and K_C for the displacer, and K_P and K_C for the piston.

Reference [1] correctly points out that the elements of the total damping matrix $[C+C_T]$ are dependent on both the engine-connected load and internal windage. The damping dissipates the energy input to the piston and displacer. Maximization of system efficiency requires minimization of damping other than that due to the load. Reference [1] embeds the effects of connected load in the piston damping coefficient, C_P . Thus the damping matrix may be expanded to Eq. (11).

$$[C+C_T] = \begin{bmatrix} C_D + C_C & -C_C \\ -C_C & C_P + C_C \end{bmatrix} \quad (11)$$

To simplify the analysis, reference [1] neglects the coupling term C_C in the damping matrix. The forcing terms of (8) are assumed to be entirely due to the thermodynamics. Therefore, $[F_E(t)]$ is zero. The forcing vector $[F_{PT}(t)]$ is expressed in terms of a time dependent pseudo-pressure acting on the piston and displacer rod areas as shown in Eq. (12):

$$[F_{PT}(t)] = \begin{bmatrix} A_R \\ A_P \end{bmatrix} P_{PT}(t) \quad (12)$$

The constraint of Eq. (9) on the equations of motion requires equating the RHS of Eqs. (9) and (12). This is not explicitly evident in reference [1].

Unlike reference [1], Redlich and Berchowitz [2] incorporate the Schmidt thermodynamics into the total matrix. Thus, their equations of motion are similar to Eq. (6). The total stiffness matrix is expressed in Eq. (13),

$$[K+K_T] = \begin{bmatrix} K_d & \alpha_p \\ \alpha_T & K_p \end{bmatrix} \quad (13)$$

where K_d represents both the external and thermodynamic (K_{DDT}) effects. The K_p includes only the thermodynamic (K_{PPT}) effects. The term α_p is due to thermodynamic effects and represents the gaseous force exerted on the displacer rod area due to the piston motion. It is equivalent to K_{DPT} in Eq. (4). The term α_T , a thermodynamic coupling between the displacer motion and the piston force, denotes K_{PDT} in Eq. (4). Reference [2] shows that K_p , K_d and the stiffness coefficients α_p and α_T are functions of the displacer rod area, engine cylinder area, the gas pressure and the expansion and compression space temperatures. This observation is consistent with Eq. (4). Thus, values of these parameters may vary for Stirling engines with different geometric configurations and operating temperatures and pressures.

The damping matrix for reference [2] is shown in Eq. (14):

$$[C+C_T] = \begin{bmatrix} D_d & 0 \\ 0 & D_p \end{bmatrix} \quad (14)$$

The damping coefficient, D_d , of Eq. (14) embodies all viscous forces on the moving gas. The term, D_p , includes the effect of any piston-connected load. Both D_d and D_p include "incidental irreversibilities". An example of this is gas spring hysteresis. There is no explicit damping coupling term between piston and displacer in Eq. (14). Reference [2] equates $[F_E(t)]$ to zero since there are no external forces.

The mechanical analog representation of the Stirling engine in Fig. 2 is nearly identical to that used by Das and Bahrami [3]. Also, the underlying assumptions in references [1] and [3] are similar. Hence, Eq. (8) represents the system equations for reference [3]. Das and Bahrami quote reference [1] assertions that the piston-displacer coupling is weak and can be neglected in a well designed and efficient engine. They support this by noting the relatively heavier mass of the load compared to that of the piston, and the relatively dominant stiffness of the engine gas spring action. However, Das and Bahrami do not explicitly ignore the coupling terms in formulating the equations of motion.

The formulation of the thermodynamic stiffness matrix, $[K_T]$, and the dynamic equations by Benvenuto, et al., [4] is similar to that by Redlich and Berchowitz [2], namely, Eq. (6) with a few exceptions. Reference [4] more rigorously develops the thermodynamic damping matrix $[C_T]$. This matrix is shown to be a non-linear function of the piston and displacer velocities. Both the direct and coupling terms are included in the formulation.

Cichy, Carlini and Kucharski [5,6] use a formulation similar to Eq. (7). In their analysis, the $[K]$ and $[C]$ matrices are due entirely to components external to the thermodynamics. The displacer stiffness to ground is not explicitly shown by Carlini, et al. This is consistent with their dynamic representation of the Beale model 10B Stirling engine. Their system has no external forces. Thus, $[F_E(t)]$ is zero.

Their thermodynamic forcing term, $[F_T(t)]$, is similar to Eq. (15), which has been rewritten to be consistent with Fig. 2.,

$$[F_T(t)] = \begin{bmatrix} -A_R + A_P & A_P \\ 0 & -A_P \end{bmatrix} \begin{bmatrix} P_e(t) \\ P_c(t) \end{bmatrix} \quad (15)$$

where $P_e(t)$ and $P_c(t)$ are, respectively, the expansion and compression space pressures. Thus, the thermodynamic forces are modeled as non-homogeneous terms in the dynamic equations. The natural frequencies calculated with this model will differ from reality, since the "stiffness" effects of the working space are incorporated in the forcing terms $[F_T(t)]$. The constraint of Eq. (5) on the equations of motion is not explicitly recognized. This requires equating the RHS of Eqs. (5) and (15).

The analysis by Chen and Griffin [7] is an extension of their previous linear harmonic analysis work [8] which models the thermodynamics of the Stirling cycle. In reference [7] they have coupled the mechanical dynamic equations of motion with the differential equations describing the thermodynamic processes. Thus, Eq. (6) is most similar to their analysis. The major difference in their formulation is that the order of the matrices $[X]$, $[K_T]$ and $[C_T]$ in Eq. (3) are expanded to explicitly include thermodynamic and dynamic variables, as well as losses and adiabatic effects in the thermodynamic cycle.

Literature search reveals that the selected approach for solving the equations of motion depends on the objective of the author. The commonly used methods are discussed next.

III. METHODS OF SOLUTION

Analysts have their preferences for the technique employed in solving the dynamic equations. Time-domain analysis, either by direct solution of the differential equations or matrix formulation of the equations, is commonly used to evaluate the operating frequency, the displacer-piston displacements and relative phase angles. Other analysts opt for a combination of time- and frequency-domain analyses in establishing steady-state system stability and/or criterion for its occurrence.

Rauch [1] obtains the amplitudes of the piston and the displacer and their relative phase angles by evoking the Schmidt sinusoidal motion for the exerted force, and the piston and displacer responses. Substitution of the force and responses into the matrix equations of motion, and subsequent application of Gaussian elimination yields the solution for the basic equations at the desired frequency.

Redlich and Berchowitz [2] use Taylor series expansion to linearize the equations of motion about a steady-state operating point. The resulting equations are Laplace transformed. Nyquist criterion is applied to the characteristic equation to evaluate the necessary condition for system stability, and criteria for engine start-up, and maintenance of piston and displacer resonances.

Das and Bahrami [3] assume a sinusoidal forcing function and, hence, displacer and piston responses. These are substituted into the equations of motion which are rewritten in complex exponential form to determine the piston and displacer steady-state responses.

Dynamic stability of the FPSE is obtained from the system characteristic equation. Application of Laplace transformation and Routh's stability criterion gives the impact of parameter variations on engine stability.

Benvenuto, et al. [4] approach is to linearize the working gas pressure, pressures in the gas springs and pressure drop in the heat exchangers. The result is a system of linear and homogeneous equations of motion with constant coefficients. These equations are Laplace transformed into the complex domain. Solution of the polynomial characteristic equation yields the conditions for stable dynamic behavior of the FPSE.

Cichy and Carlini [5] and Carlini and Kucharski [6] employ the state-space formulation of the equations of motion for their analyses. Reference [5] uses a two-part approach for determining the thermodynamic parameters. Initially, a spatially uniform sinusoidal pressure variation is assumed for the working fluid. This is the so-called "zero-approximation" analysis. The pressure losses between the compression and expansion volume are included in a subsequent analysis. This is termed the "first approximation" analysis. For either type of analysis, the thermodynamic parameters and transient response are obtained for a unity step input and sinusoidal pressure input.

Carlini and Kucharski [6] apply the modal transformation and eigenvalue analysis to obtain the control requirements for, and parametric effects on the engine dynamic behavior, following both a step function and harmonic (or sinusoidal) input types of excitation.

Chen and Griffin [7] assume that all system variables are harmonic in nature. They then reduce the differential equations to a system of 18 homogeneous algebraic equations. These are solved for the operating frequency by means of a secant-bisection root-finding method. This method ensures stable convergence in a minimum time. The homogeneous set of equations is converted to non-homogeneous type by assuming a value for power piston amplitude. The equation representing the piston displacement is deleted in the set of homogeneous equations.

The influence of the piston motion on the remaining equations is replaced by a forcing term which is proportional to the piston displacement. Standard matrix algebra is used to solve for 17 of the 18 system variables, based on an assumed piston amplitude. A unique solution for the piston amplitude is found by matching the power produced by the engine to the load power requirement. The final solutions are obtained by successive iteration of the above solution techniques.

IV. CONCLUDING REMARKS

The primary objective of the reviewed dynamic analyses is to determine the dynamic behavior of FPSE systems. Several authors formulate their analyses such that system stability and parametric constraints for stable operation may be evaluated. However, with few exceptions, stability has been defined in the classic sense, i.e., with respect to static equilibrium, rather than "periodic" stability in which the system converges to a stable limit cycle.

Frequency domain analysis in the form of Laplace Transformation, with inverse transformation for time-domain response, has been effective in computing the basic thermodynamic parameters.

V. RECOMMENDATIONS

The dynamic behavior and stability of FPSEs depend on the total system, including the thermodynamics and load. Therefore, a complete analysis should incorporate detailed representations of both the thermodynamics, dynamics and load. In particular, for engines with a connected linear alternator, the model should include the electrical dynamics of the alternator and electrical load. Also, the thermodynamic model should account for both adiabatic effects and major losses directly affecting the working space pressure amplitude and phase.

The thermodynamic stiffness and damping coefficients, including the coupling terms, should be included in the homogeneous equation to yield a more accurate prediction of the system behavior. Thus, Eq (6) is recommended since all first-order dynamic effects are satisfied. Further, the use of Eqs (3) and (4), with non-linearities included in the matrices, for describing the thermodynamic system, is deemed sufficient. However, including the detailed thermodynamic equations with the dynamic analysis [7] is appropriate where thermodynamic performance is the primary concern.

The major source of self-excitation is contained in the stiffness matrix, while the major stabilizing influence is contained in the damping matrix. Both matrices can be shown to contain non-linear coefficients. Better understanding of the nature of various non-linear effects and their influence on system behavior is needed. Inclusion of the non-linearities should produce more realistic results. Although approximate, linearized solutions to the non-linear equations of motion can yield useful results, detailed solutions will require numerical analysis.

Frequency-domain analysis, using a state-space technique, facilitates a detailed sensitivity analysis of the effects of parameter variations on the engine dynamic behavior. This is useful, particularly during the engine design stage, in establishing acceptable stability margins.

Only a few of the analyses [2,7,9] have been compared to experimental results. Experimental validation of the dynamic analysis is recommended to enhance the analytic models and increase confidence in their predictive capability.

In contrast to dynamic stability, the transient response of the FPSE system will need to be evaluated by use of time domain analysis. Transient response, a potential operating mode of FPSE, has not yet received wide-spread attention.

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GLOSSARY:

Symbols:

A	- Area
C, D	- Damping coefficient
FPSE	- Free-Piston Stirling Engine
K	- Stiffness coefficient
P	- Pressure

Matrices/Vectors:

[C]	- Damping matrix
[F(t)]	- Forcing vector
[K]	- Stiffness matrix
[M]	- Mass matrix
[X],[\dot{X}],[\ddot{X}]	- State variable vectors: position, velocity, acceleration

Subscripts:

C	- Displacer - piston coupling
c	- Compression space
D,d	- Displacer
DDT	- Total thermodynamic, for displacer stiffness
DPT	- $\equiv \alpha_p$ = Thermodynamic stiffness on displacer due to piston motion
E	- External
e	- Expansion space
P	- Piston
PDT	- $\equiv \alpha_T$ = Thermodynamic stiffness on piston due to displacer motion
PPT	- Total thermodynamic, for piston stiffness

PT	- Between thermodynamic cycle and piston-displacer motions
R	- Displacer rod
T	- Thermodynamic
T _i	- Isothermal, thermodynamic
T _t	- Temperature ratio dependent, thermodynamic

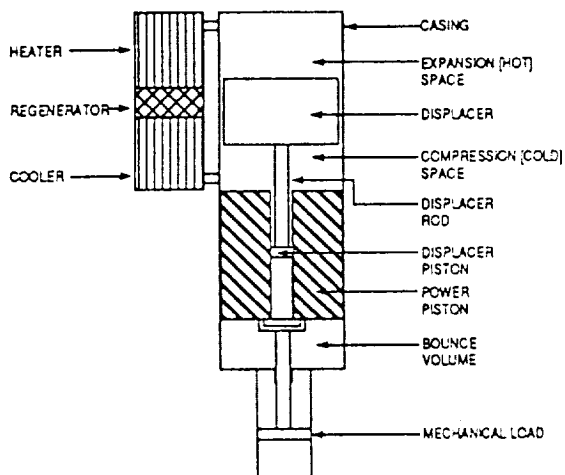


Figure 1. - Schematic of a typical single cylinder Free-Piston Stirling Engine.

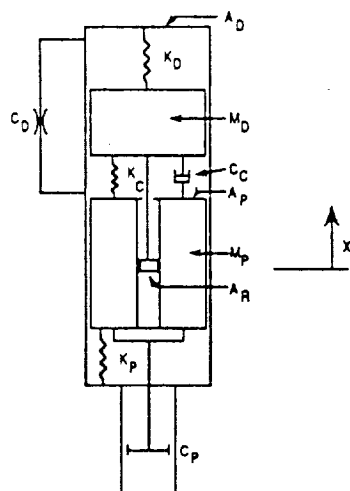


Figure 2. - Dynamic elements of a single cylinder Free-Piston Stirling Engine.



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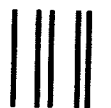
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