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DESIGN OF OPTIMAL CORRELATION FILTERS FOR HYBRID VISION SYSTEMS

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ABSTRACT

Research is underway at the NASA Johnson Space Center on the development of vision systems that recognize objects and estimate their position by processing their images. This is a crucial task in many space applications such as autonomous landing on Mars sites, satellite inspection and repair, and docking of space shuttle and space station. Currently available algorithms and hardware are too slow to be suitable for these tasks. Electronic digital hardware exhibits superior performance in computing and control; however, they take too much time to carry out important signal processing operations such as Fourier transformation of image data and calculation of correlation between two images. Fortunately, because of the inherent parallelism, optical devices can carry out these operations very fast, though they are not quite suitable for computation and control type of operations. Hence, investigations are currently being conducted on the development of hybrid vision systems that utilize both optical techniques and digital processing jointly to carry out the object recognition tasks in real time.

The author of this report, during his tenure as a summer faculty fellow at the Johnson Space Center studied the various aspects of this research. He collaborated with Dr. Richard juday and his colleagues on developing algorithms for the design of optimal filters for use in hybrid vision systems. Specifically, an algorithm was developed for the design of real-valued frequency plane correlation filters. Further, research was also conducted on designing correlation filters optimal in the sense of providing maximum signal-to-noise ratio when noise is present in the detectors in the correlation plane. Algorithms were developed for the design of different types of optimal filters: complex filters, real-valued filters, phase-only filters, ternary-valued filters, coupled filters. This report presents some of these algorithms in detail along with their derivations.

INTRODUCTION

Recognition of objects by machines is a vital task in many applications related to NASA missions. Docking of space shuttle and space station, extra-vehicular activities by robots, and autonomous landing on Mars are some examples of such applications. To carry out the tasks efficiently and accurately in real time, suitable technologies are being investigated. Most of these tasks involve some form of processing of images acquired using a camera. A straightforward approach is to use electronic digital hardware to carry out the necessary processing involved in the recognition of objects. Researchers in many universities and research laboratories around the world are working on the development and evaluation of algorithms for such processing and pattern recognition. One major drawback of the electronic processing technique is that at the current technology, the algorithms take too much time to be suitable for real time applications.

An alternate technology that is under investigation by many researchers is the use of optical processing. Since through optical techniques processing can be done in a parallel mode, very high speed can be achieved. Taking Fourier transformation, multiplication of a signal by a filter, and obtaining correlation function of two images are some of the operations that optical processing can efficiently perform. However, some operations such as storage and retrieval of data, arithmetic operations, etc., cannot so easily be done using optical techniques. Hence, a hybrid system employing optical techniques for correlation filtering and electronic digital techniques for image storage, retrieval, correlation plane processing, etc., is being investigated for development and use in vision systems.

One major area of investigation in the development of hybrid vision systems is the design of optimal filters that yield high signal to noise ratio (SNR) facilitating accurate detection of objects in the presence of noise and other extraneous objects. During the summer fellowship period, the author collaborated with his JSC colleague Dr. Richard Juday and Dr. B.V.K. Vijaya Kumar of Carnegie Mellon University and developed algorithms for the design of several classes of optimal filters: complex filters, real-valued filters, ternary valued filters, phase only filters, and coupled filters. Some of these algorithms and their derivations will be presented in the following sections.

CORRELATION FILTERS

The theory of correlation filters for the recognition of known objects has been well discussed in the literature [1]. It basically involves the following steps. From a knowledge of the reference image s(x) (we will use 1-D notation for the sake of simplicity), a filter function h(x) is designed. When an image t(x) is to be tested to find whether the reference image is present in the test image or not, the image t(x) is convolved with the filter h(x) to yield c(x). If c(x) has a well pronounced peak above a preassigned threshold, then it is concluded that the reference image is present in the test image; otherwise it is not. If a peak is present, from the position of the peak, one may also estimate the location of the reference image in the test image. When h(x) is equal to s(-x), the matched filter of the reference image, it can be shown that c(x) will yield the cross correlation between the test image and the reference image. Hence, these filters are called correlation filters.

The implementation of correlation filters in the space domain is quite time consuming. An alternate approach is to transform the test and reference images into the Fourier frequency domain, multiply the transforms and then perform the inverse transform to obtain the filter output c(x). If one uses digital techniques using electronic hardware, the Fourier transformation is also equally time consuming. However, Fourier transformation can easily and almost instantaneously be carried out using optical techniques. As a result, considerable research is being conducted on the design of optical correlators for real time pattern recognition purposes. Since the invention of the holographic matched filter for optical correlations by VanderLugt [2] in 1964, many other filters with improved performance have been proposed. The accuracy of detection of these correlation filters depends to a large extent on the absence of noise in an image. It may be noted that the nonreference image components present in an image may also be considered as noise. To account for the presence of noise and to measure the performance of a filter in the presence of noise, signal-to-noise ratio (SNR) has been introduced and procedures have been devised to design optimal filters that maximize the SNR and thus improve the accuracy of identification.

Derivation of maximum SNR filters of different types (complex, phase only, ternary and coupled filters) in the presence of image-plane noise but no detector noise has been presented in literature. For the case of real filters, derivation of optimal filters starting from the fundamental expression for SNR has not been done so far. We first present such a derivation obtained during the fellowship period. This result has been submitted for publication [3].

OPTIMAL REAL CORRELATION FILTERS

Let s(x) denote the reference image and S(f) its Fourier transform. Let H(f) denote the filter transform. In the absence of input noise the resulting correlator output at the origin is given by

$$c(0) = \int S(f)H(f)df \tag{1}$$

where the limits of integration are those implied by the bandwidths of S(f) and H(f) (whichever has a smaller bandwidth.) A model for possible uncertainties in the input is the additive noise n(x). We model this as a sample realization with mean μ_n and power spectral density $P_n(f)$. The additive noise n(x) in the input leads to randomness in the output c(0). We can show that

$$E\{c(0)\} = \mu_n H(0) + \int S(f)H(f)df \tag{2}$$

and

$$Var\{c(0)\} = \int P_n(f)|H(f)|^2 df \tag{3}$$

When the input to the correlator is only noise n(x), the output c(0) will be a random variable with mean μ_n H(0) and variance as given in Eqn. (3). For good detection, we need to separate the two means as much as possible while keeping the variance small. A convenient measure for this is the signal-to-noise ratio (SNR) defined as below:

$$SNR = \frac{\left|\int S(f)H(f)\,df\right|^2}{\int P_n(f)|H(f)|^2\,df} \tag{4}$$

We will next derive the optimum real filter H(·) that maximizes the SNR. The derivation can be done in more than one way. In the following we use discrete arguments and partial differential equations. For the continuous argument case one may employ variational calculus and obtain identical results. One may also employ Cauchy-Schwartz inequality to obtain the same result. We use such a technique in the next section where detector noise is taken into account.

Let us sample the frequency domain quantities at intervals Δf to produce amplitudes A_k and phases ϕ_k :

$$
\begin{aligned}
A_k \exp(j\phi_k) &= S(k\,\Delta f) \\
H_k &= H(k\,\Delta f) \\
P_{nk} &= P_n(k\,\Delta f)
\end{aligned}
\tag{5}
$$

Then

$$
\mathrm{SNR} = \frac{\left[\sum_{\ell} A_\ell H_\ell \exp(+j\phi_\ell)\right]\left[\sum_{k} A_k H_k \exp(-j\phi_k)\right]}{\sum_k H_k^2 P_{nk}}
\tag{6}
$$

where ℓ and k are summed over values appropriate to the filter bandwidth. For an extremum SNR, each filter value H_m is chosen so that the SNR is stationary; i.e., subject to the constraint

$$
\frac{\partial}{\partial H_m}(\mathrm{SNR}) = 0 \quad .
\tag{7}
$$

Defining β and B real ≥ 0 such that

$$
B\exp(j\beta) = \sum_k A_k H_k \exp(j\phi_k)
\tag{8}
$$

and taking the partial derivative in (7) we get

$$
H_m P_{nm} B^2 = A_m \left[\sum_k H_k^2 P_{nk}\right] B\cos(\phi_m - \beta) \quad .
\tag{9}
$$

We have, then,

$$
H_m \propto \frac{A_m}{P_{nm}} \cos(\phi_m - \beta)
\tag{10}
$$

with proportionality constant independent of m. The SNR is independent of a constant multiplier of H_k, so we may as well make Eqn. (10) an equality. Then we need only solve for β. Eqn.(10) indicates several interesting things. As might have been expected, the optimum-SNR real filter value is directly proportional to the local amplitude of the reference signal transform and inversely proportional to the local noise power. The novelty in this result is that the filter is also weighted according to how well the signal's local phase (ϕ_m) lines up with the phase (β) of the filtered result.

Now, to determine the filter completely, we need to find the value of β. For this we need to solve the nonlinear equations

$$H_m = \frac{A_m}{P_{nm}} \cos(\phi_m - \beta)$$

where (11)

$$\beta = \mathrm{Arg}\left\{ \sum_k H_k A_k \exp(j\phi_k) \right\}$$

A traditional method of solving these equations is as follows: Choose a trial value for $\beta = \beta_1$. Solve for H_m and see if the result is consistent. That is, if the β calculated using the second expression is equal to β_1, then we have a stationary SNR. This is similar to the previous [4] approaches where calculating a filter giving an extremum in correlation intensity involved a search over a single parameter. Relating to the present case, we would require a search over β. As a significant advance, we have been able to separate β in Eqn. (11). The search for β is over for real filters. Substituting for H_m in the expression for β in Eqn. (11) we have

$$\beta = \mathrm{Arg}\left\{ \sum_k \frac{A_k}{P_{nk}} \cos(\phi_k - \beta) A_k \exp(j\phi_k) \right\}$$

$$= \mathrm{Arg}\left\{ \sum_k \frac{A_k}{P_{nk}} [\exp(j\phi_k)\exp(-j\beta) + \exp(-j\phi_k)\exp(j\beta)] A_k \exp(j\phi_k) \right\} \tag{12}$$

$$= \mathrm{Arg}\left\{ \exp(j\beta) \left[\left(\sum_k \frac{A_k^2}{P_{nk}} \exp(j2\phi_k) \right) \exp(-j2\beta) + \sum_k \frac{A_k^2}{P_{nk}} \right] \right\}$$

The above will be true if and only if

$$\beta = \left(\frac{1}{2}\right) \mathrm{Arg}\left\{ \sum_k \frac{(A_k \exp(j\phi_k))^2}{P_{nk}} \right\} + n\frac{\pi}{2}; \text{ where } n \text{ is an integer} \tag{13}$$

By substituting for β in the expression for SNR, we can show that n should be equal to zero for maximum SNR.

We now give an abbreviated discussion of the derivation of the optimal real filter in the continuous domain. Define u and v as follows.

$$u = \left\{ \int H(f)A(f)\exp[+j\phi(f)]df \right\} \left\{ \int H(f)A(f)\exp[-j\phi(f)]df \right\}$$

$$v = \int H^2(f) P_n(f) df \tag{14}$$

$$\mathrm{SNR} = \frac{u}{v}$$

(The region of integration, ordinarily a single diffraction order, is implicit.) Following the variational calculus approach given in more detail elsewhere [4], we denote the arbitrary disturbance function as $\mu(\cdot)$, so that the deviated value of the optimum real filtering function is $H(\cdot) + \alpha\mu(\cdot)$ for a (presumed small) scalar, α. The deviated value of the correlation field is obtained by replacing $H(\cdot)$ by $H(\cdot) + \alpha\mu(\cdot)$. Let u_α be u evaluated with $H(\cdot) + \alpha\mu(\cdot)$ replacing $H(\cdot)$ and similarly for v_α. The first-order variation in SNR induced by $\alpha\mu(\cdot)$ is

$$\delta(\mathrm{SNR}) = \alpha\left[\frac{\partial}{\partial\alpha}\left(\frac{u_\alpha}{v_\alpha}\right)\right]_{\alpha=0} \quad . \tag{15}$$

That a given function $H(\cdot)$ produces an extremum of SNR is stated as

$$\delta(\mathrm{SNR}) = 0 \text{ , implying } v\ \delta u = u\ \delta v \quad . \tag{16}$$

A relatively straightforward extension of the method shown earlier leads to a form very similar to the discrete case.

$$\begin{aligned} H(f) &= \frac{A(f)}{P_n(f)}\cos[\phi(f) - \beta_1] \\ \beta_2 &= \mathrm{Arg}\int H(f)A(f)\exp[j\phi(f)]df \end{aligned} \tag{17}$$

We have consistency if $\beta_1 = \beta_2$. These expressions are of the same form as those obtained for the discrete case with the replacement of integration in place of summation. Then applying the same development as in Eqns. (12) - (13), we get

$$\begin{aligned} \beta &= \left(\frac{1}{2}\right)\mathrm{Arg}\left\{\int \frac{[S(f)]^2}{P_n(f)}df\right\} \\ &= \left(\frac{1}{2}\right)\mathrm{Arg}\left\{\int \frac{[S_R(f)]^2 - [S_I(f)]^2}{P_n(f)}df + j2\int \frac{S_R(f)\ S_I(f)}{P_n(f)}df\right\} \end{aligned} \tag{18}$$

We will now consider some special cases . The discussion in the following is restricted to the continuous case. Results for the discrete case are exactly analogous, as is apparent from the congruence between Eqns. (11) and (17).

i) s(x) is real. Then $\int \frac{S_R(f)\ S_I(f)}{P_n(f)}df = 0 \; ; \quad \therefore\ \beta = 0 \text{ or } \frac{\pi}{2}.$ (19)

ii) s(x) is real and $\int \frac{[S_R(f)]^2}{P_n(f)}df = \int \frac{[S_I(f)]^2}{P_n(f)}df$. Then β is arbitrary. (20)

iii) s(x) is complex and $\int \frac{[S_R(f)]^2}{P_n(f)} df = \int \frac{[S_I(f)]^2}{P_n(f)} df$. Then $\beta = \frac{\pi}{4}$ (21)

Cases i) and ii) correspond to the cases considered by Kumar [5] and case iii) is a new result.

The SNR of the real filter, obtained by substituting the optimal value of H(f) and simplifying the expression, is given by

$$SNR_{RMF} = \frac{1}{2}\int \frac{|S(f)|^2}{P_n(f)} df + \frac{1}{2}\left|\int \frac{[S(f)]^2}{P_n(f)} df\right| \tag{22}$$

where SNR_{RMF} represents the SNR of the real matched filter. A relative measure of performance of the real filter in comparison with the complex matched filter is obtained as

$$\frac{SNR_{RMF}}{SNR_{CMF}} = \frac{1}{2} + \frac{1}{2}\frac{\left|\int \frac{[S(f)]^2}{P_n(f)} df\right|}{\int \frac{|S(f)|^2}{P_n(f)} df} \tag{23}$$

where SNR_{CMF} represents the SNR of the complex matched filter. In the above expression we see that the SNR of the optimal real filter is less than or equal to the optimal complex filter. However, the maximum loss that results from the use of our optimal real filter is 3 dB.

Figure 1 in the earlier paper [5] shows the loss in SNR (relative to the matched filter) when using the optimal real filter. That is valid here also. Only what is meant by "normalized even-part energy" changes depending on $P_n(f)$. As shown before, we lose at most 3 dB in SNR (compared with the matched filter) when we use the optimum real filter. This discussion also shows that the optimal real filter is not the amplitude part of the complex matched filter.

DETECTOR NOISE AND SNR

When the correlation output c(0) is detected by a photodetector, several things happen. Detectors respond only to |c(0)|, thus ignoring all phase information. Also, the detectors introduce a gain and some noise. An accurate model for detector noise is complicated and must include the signal dependent nature of detector noise. Instead, we will use the following simple model for y, the detector output:

$$y = c(0) + n_d. \tag{24}$$

In this detector noise model, we assume (without loss of generality) that the detector gain is unity and that the detector noise n_d is additive. The noise n_d is assumed to have a mean μ_d and variance σ_d^2. The additive assumption is somewhat questionable. However, it makes the analysis tractable and it helps us to illustrate the main point we want to emphasize (i.e., we must trade-off input noise tolerance for detector noise tolerance). We will now find the mean and variance of y in Eqn. (24) for the two possible input cases.

When the input contains only noise n(x) let the output be y_0. Then the mean and variance of y_0 are given as follows:

$$E\{y_0\} = \mu_d + \mu_n \tag{25}$$

and

$$Var\{y_0\} = \sigma_d^2 + \int P_n(f)|H(f)|^2 df \tag{26}$$

When the input contains signal s(x) corrupted by additive noise n(x), let the output be y_1. Then the mean and variance of y_1 are given as below:

$$E\{y_1\} = \mu_d + \mu_n + \int S(f)H(f)df \tag{27}$$

and

$$Var\{y_1\} = \sigma_d^2 + \int P_n(f)|H(f)|^2 df \tag{28}$$

Using the statistics in Eqns. (25)-(28), the SNR in the presence of detector noise can be expressed as

$$\begin{aligned} \text{SNR} &\triangleq \frac{|E\{y_1\} - E\{y_0\}|^2}{\frac{1}{2}(Var\{y_1\} + Var\{y_0\})} \\ &= \frac{|\int S(f)H(f)df|^2}{\sigma_d^2 + \int P_n(f)|H(f)|^2 df} \end{aligned} \tag{29}$$

Note that the only difference between the SNR expressions in Eqn. (4) and Eqn. (29) is the extra σ_d^2 in the denominator of Eqn. (29). However this makes the optimal filter choices for the two SNR's different. When σ_d^2 is very small (compared to the input noise term), the two SNRs are identical and the previous optimal filters will still be optimal. However, when σ_d^2 is very large (such that the input noise term can be ignored) SNR is simply proportional to $|E\{c(0)\}|^2$ and we must simply maximize the correlation value at the center. In the next section we derive expressions for H(f) that maximize the SNR in Eqn. (29).

OPTIMAL FILTERS WHEN DETECTOR NOISE IS PRESENT

In this section we will determine H(f) that maximize the SNR in Eqn. (29). There are five different cases: (1) Complex filters, (2) real filters, (3) phase only filters (4) binary phase only filters and (5) coupled filters. In each case while determining these filters, we must use the condition

$$|H(f)| \leq 1 \tag{30}$$

since the transmittance of an optical filter can never exceed unity. Also, without this constraint, the detector noise variance σ_d^2 will not be meaningful.

Complex Filters

Let us allow H(f) to be complex. Let the filter energy E_h be defined as follows:

$$E_h = \int |H(f)|^2 df \tag{31}$$

Then the SNR in Eqn. (29) can be written as below:

$$\mathrm{SNR} = \frac{|\int S(f)H(f)\,df|^2}{\int [\sigma_{id}^2 + P_n(f)]\,|H(f)|^2 df} \tag{32}$$

where

$$\sigma_{id}^2 = \frac{\sigma_d^2}{E_h} \tag{33}$$

is the variance of an equivalent white noise at the input which gives the same effect at the origin of the correlation plane as the detector noise with variance σ_d^2 for the given filter H(f).

To find the optimal choice of H(f) we now apply the Cauchy-Schwartz inequality to the numerator to get the following:

$$\begin{aligned} |\int S(f)H(f)\,df|^2 &= |\int \left[\frac{S(f)}{\sqrt{\sigma_{id}^2 + P_n(f)}}\right]\left[H(f)\sqrt{\sigma_{id}^2 + P_n(f)}\right] df|^2 \\ &\leq \left(\int \frac{|S(f)|^2}{[\sigma_{id}^2 + P_n(f)]}\,df\right)\left(\int |H(f)|^2\left[\sigma_{id}^2 + P_n(f)\right] df\right) \end{aligned} \tag{34}$$

Substituting Eqn. (34) in Eqn. (32), we obtain the following result:

$$SNR \leq \int \frac{|S(f)|^2}{\sigma_{id}^2 + P_n(f)}\,df = SNR_{max}(\sigma_{id}^2) \tag{35}$$

with equality occurring if and only if

$$H(f) = \alpha \frac{S^*(f)}{\sigma_{id}^2 + P_n(f)} \tag{36}$$

0.2

where α is chosen to satisfy the constraint that the maximum magnitude of the filter is unity. Then the optimal filter H(f) can be written as

$$H_{opt}(f) = \frac{S^*(f)/[\sigma_{id}^2 + P_n(f)]}{|S^*(f)/[\sigma_{id}^2 + P_n(f)]|_{max}} \tag{37}$$

As σ_{id}^2 is related to the detector noise σ_d^2, this filter will also be optimal for a detector noise $\sigma_d^2 = \sigma_{id}^2 E_h$ where

$$E_h = \int |H(f)|^2 \, df \tag{38}$$

Thus starting with σ_{id}^2, we can design an optimal filter and find the corresponding detector noise for which this filter is the optimal one. To design a filter for a given σ_d^2, we may use either an optimization scheme or do the following: Obtain a graph relating σ_{id}^2 and σ_d^2 by designing a number of filters with different values of σ_{id}^2. Then from the graph, a suitable σ_{id}^2 is picked for a given σ_d^2. Once we know σ_{id}^2, we can design the optimal filter. It may be verified that for the special case of $P_n(f)$ = a constant, the classical matched filter is still the optimal filter.

Real Filters

Let us now consider the case when H(f) is real. Then using the same notation for E_h as above, we have

$$SNR = \frac{|\int S(f)H(f)\,df|^2}{\sigma_d^2 + \int P_n(f)|H(f)|^2 df} = \frac{|\int S(f)H(f)\,df|^2}{\int (\frac{\sigma_d^2}{E_h} + P_n(f))|H(f)|^2 df} = \frac{cc^*}{\int P_{nt}(f)\,|H(f)|^2\,df} \tag{39}$$

where

$$c = \int S(f)H(f)\,df, \quad \text{and} \quad P_{nt} = \frac{\sigma_d^2}{E_h} + P_n. \tag{40}$$

Expressing cc^* as

$$cc^* = \left(\frac{cc^* + c^*c}{2|c|}\right)^2 \tag{41}$$

we get

$$SNR = \frac{\left(\int \frac{cS^*(f)H^*(f) + c^*S(f)H(f)}{2|c|} df\right)^2}{\int P_{nt}(f)\,|H(f)|^2\,df}. \tag{42}$$

As for real filters $H^* = H$, we can write the expression for SNR as

$$SNR = \frac{\left(\int \frac{(cS^*(f) + c^*S(f))}{2|c|} H(f)\,df\right)^2}{\int P_{nt}(f)\,|H(f)|^2\,df} \tag{43}$$

Applying Cauchy-Schwartz inequality we get

$$SNR = \frac{\left(\int \frac{cS^*(f) + c^*S(f)}{2|c|\sqrt{P_{nt}(f)}} \sqrt{P_{nt}(f)}\; H(f)\,df\right)^2}{\int P_{nt}(f)|H(f)|^2\,df} \leq \int \left(\frac{cS^*(f) + c^*S(f)}{2|c|\sqrt{P_{nt}(f)}}\right)^2 df \tag{44}$$

Equality (and hence maximum SNR) is achieved iff

$$H(f) = \alpha\, \frac{cS^*(f) + c^*S(f)}{2|c|P_{nt}(f)} = \alpha\, \frac{|S(f)|}{P_{nt}(f)} \cos(\phi_s(f) - \beta) \tag{45}$$

where $\phi_s(f) = Arg\{S(f)\}$, $\beta = Arg\{c\}$ and α is a real constant such that the maximum response condition is satisfied. To determine β, substitute the optimal H(f) in the expression for c (Eqn. (40)) and simplify as

$$|c|e^{j\beta} = \frac{\alpha}{2}\left(\int \frac{|S(f)|^2}{P_{nt}(f)} df + e^{-j2\beta} \int \frac{[S(f)]^2}{P_{nt}(f)} df\right) e^{j\beta} \tag{46}$$

The above condition will be satisfied if and only if

$$\beta = \frac{1}{2}\left(Arg \int \frac{[S(f)]^2}{P_{nt}(f)} df + n\pi\right) \tag{47}$$

where n is an integer. Further we can show that SNR will be larger if n = 0 (or any even integer) than when n is odd. Thus β value is given by

$$\beta = \frac{1}{2} Arg\left(\int \frac{[S(f)]^2}{P_{nt}(f)} df\right) \tag{48}$$

With this β value the maximum SNR value can be shown to be

$$SNR_{max} = \frac{1}{2}\int \frac{|S(f)|^2}{P_{nt}(f)} df + \frac{1}{2}\left|\int \frac{[S(f)]^2}{P_{nt}} df\right| \tag{49}$$

The optimal H(f) satisfying the maximum condition is given by

$$H_{opt}(f) = \frac{\dfrac{|S(f)|}{P_{nt}(f)} \cos(\phi_s(f) - \beta)}{\left| \dfrac{|S(f)|}{P_{nt}(f)} \cos(\phi_s(f) - \beta) \right|_{max}} \tag{50}$$

The above H(f) is optimum for a given input noise. To determine H(f) for a given detector noise one can follow a procedure similar to that used in the complex filter case of either using an optimization scheme or setting up a graph between σ_{id}^2 and σ_d^2 and selecting a suitable σ_{id}^2 for a given σ_d^2.

Phase Only Filter

Let H(f) be a phase-only filter (POF) with a region of support R defined below:

$$H(f) = \begin{cases} e^{j\phi_h(f)} & \text{for} \quad f \in R \\ 0 & \text{for} \quad f \notin R \end{cases} \tag{51}$$

Substituting this in Eqn. (29) we obtain the following expression for SNR:

$$SNR = \frac{\left| \int_R |S(f)| e^{j[\phi_s(f) + \phi_h(f)]} df \right|^2}{\sigma_d^2 + \int_R P_n(f) df} \tag{52}$$

The denominator in Eqn. (52) does not depend on $\phi_h(f)$ and the numerator is maximized by choosing $\phi_h(f) = -\phi_s(f)$, i.e., the conventional POF maximizes the SNR in the presence of detector noise. The next task is to find the optimal region of support R. Let A_R denote the area of the region R. Then the SNR in Eqn. (52) can be rewritten as follows:

$$SNR = \frac{\left(\int_R |S(f)| df \right)^2}{\int_R \left(\dfrac{\sigma_d^2}{A_R} + P_n(f) \right) df} \tag{53}$$

The SNR expression in Eqn. (53) is identical to the SNR expression derived elsewhere [6] for the case of no detector noise and input colored noise with spectral density $[P_n(f) + \sigma_d^2 / A_R\}$. It is shown there that the region of support R that maximizes the SNR must be of the form:

$$R = \left\{ f : \frac{|S(f)|}{\left(\dfrac{\sigma_d^2}{A_R} + P_n(f) \right)} \geq T \right\} \tag{54}$$

where T is an unknown threshold. Since A_R depends on R , Eqn. (54) is an implicit equation. An algorithmic procedure for determining R is given below:

To illustrate the workings of the algorithm, let us assume that S(f) is represented by an array of 64 X 64 = 4096 pixels. Each frequency in this array may or may not be included in R. Thus there are 2^{4096} possible choices of R from which we must select the one that maximizes the SNR. Exhaustive search is obviously impossible, but the characterization in Eqn. (54) proves useful. For example, there are 4096 different choices for R with $A_R = 1$. Among all of them, the best R is the one that includes the highest value of $|S(f)|/[\sigma_d^2 + P_n(f)]$. Extending this to other A_R values, we get the following algorithm:

Step 0: Start with $A_R = 1$.
Step 1: Compute

$$G(f) = \frac{|S(f)|}{\frac{\sigma_d^2}{A_R} + P_n(f)} \tag{55}$$

Step 2: Arrange sampled values of G(f) in the descending order as $G_1 > G_2 > ... > G_N > 0$.
Step 3: Construct the optimal $R^*(A_R)$ of area A_R by including pixels corresponding to $G_1, G_2, ... , G_{AR}$.
Step 4: Compute the $SNR(A_R)$ obtained by using $R^*(A_R)$. If A_R is greater than or equal to N, go to Step 5. Otherwise go to step 2.
Step 5: Determine the largest $SNR(A_R)$. This determines the optimal region of support.

The above algorithm involves N sorts where each sort is of N values. Since A_R changes by only 1 from one sort step to the next, we do not expect that the sorting order will change significantly from one step to the next. This can be used to speed up the algorithm still further.

Ternary-Valued and Coupled Filters

Optimal ternary-valued filters (with values -1, +1, and 0) in the presence input and detector noises have also been derived during the fellowship period. However due to lack of space, they are not discussed in this report. Similarly, the design of optimal coupled filters in the presence of input and detector noises have also been investigated. These results will be sent for publication in due course.

SUMMARY

In this report, the derivation of optimal real correlation filters when detector noise is not present is first presented. Then a model for the detector noise is presented. Using this model, the optimal filter design problem when detector noise is present is formulated. Derivations for the optimal filter when the filter is an unconstrained complex filter, real filter, and phase only filter are presented. Algorithms for their design are also presented. Implementation and testing of these algorithms will be taken up during the follow-up grant period.

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