NASA Technical Memorandum 104083

SUGGESTIONS FOR CAP-TSD MESH AND TIME-STEP INPUT PARAMETERS

SAMUEL R. BLAND

JUNE 1991

SUGGESTIONS FOR CAP-TSD MESH AND TIME STEP
INPUT PARAMETERS

Samuel R. Bland

Unsteady Aerodynamics Branch
NASA Langley Research Center
Hampton, Virginia 23665-5225

June 1991

Abstract

This paper gives suggested values for some of the input parameters used in the CAP-TSD computer code. These parameters include those associated with the mesh design and time step. The guidelines are based principally on experience with a one-dimensional model problem used to study wave propagation in the vertical direction.

Introduction

Wave propagation through the finite-difference meshes employed in CFD may be simulated most simply by one-dimensional model problems. For TSD theory, an appropriate model is the second order wave equation, which describes the vertical waves in the CAP-TSD code (ref. 1). The use of this equation provides significant advantages: the exact solution of the partial differential equation is well known for any initial/boundary conditions; the numerical solution of the finite-difference version of the continuous equation is very inexpensive to compute; analysis of the difference equations may be used to aid in interpreting the numerical results; and computer graphics animation of the wave motion is easy to produce and clearly demonstrates wave propagation, reflection, and distortion effects.

In the transonic small disturbance code CAP-TSD independent one-dimensional meshes are used in each of the computational coordinates ξ, η, and ζ. Typically, the streamwise ξ and vertical ζ meshes extend 10 to 40 chord lengths from the configuration and the spanwise η mesh extends 2 to 4 span lengths. The choice of number, extent, and spacing of mesh points is a difficult problem. Once these choices have been made, the interactive computer code described in reference 2 may be used to aid the mesh designer.

Results obtained with the simple model problem were reported in reference 3. The algorithm employed was that for the vertical sweep in the CAP-TSD code. Wave reflection and time step effects similar to those experienced with two-dimensional TSD codes were observed. Analysis of the finite-difference equations supported the numerical results. Additional calculations for this model problem are reported herein.

The guidelines presented in this paper are based on three levels of experience: extensive calculation and analysis for the simple model problem; use of the CAP-TSD code, especially for two-dimensional flow; and physically plausible requirements for modelling the transonic flow. All calculations and figures were produced using a SUN workstation.

Symbols

h	local mesh spacing
$h_{far-field}$	mesh spacing at the far-field boundary
h_{max}	maximum mesh spacing
$h_{surface}$	mesh spacing near the wing surface
k	reduced frequency
k_{max}	maximum reduced frequency for the mesh
M	Mach number
N_{cy}	number of cycles of harmonic motion
N_{pc}	number of time steps per cycle
N_{pw}	number of mesh points per wave length
$N_{Z_{max}}$	total number of mesh points
P	velocity potential
t	time
t_c	time at pulse center
T	temporal period
T_{max}	total time for converged calculation
x	streamwise coordinate
z	vertical coordinate
Z_{max}	maximum coordinate (far-field boundary location)
β	$\sqrt{1 - M^2}$
Δt	time step size
λ	spatial wave length
ξ, η, ζ	computational coordinates

All quantities are normalized — lengths by chord, time by chord over stream speed, and frequency by stream speed over semichord.

Wave Propagation

In two-dimensional linear theory the wave fronts are circles which propagate outward with sonic speed as their centers are convected downstream at the stream speed. These wave patterns for four subsonic Mach numbers are shown in figure 1. In each case the mesh extends 20 chord lengths from the origin both vertically and streamwise. The circles represent the wave crests created by a harmonically oscillating disturbance located at the origin. Several comments on these results will be helpful in understanding the mesh and time step sizes needed to describe wave propagation using a TSD code.

Since time is normalized by stream speed, in one cycle the centers of the circles are convected downstream a distance that is independent of the Mach number. Attention is drawn to the smallest circle in each figure to illustrate this conclusion. The center of this circle lies at

$$x = T = \pi/k = 2\pi$$

in which the first equality results from the normalization of t by stream speed and chord, the second from normalization of k by semichord, and the third from the choice of $k = 0.5$ for this case.
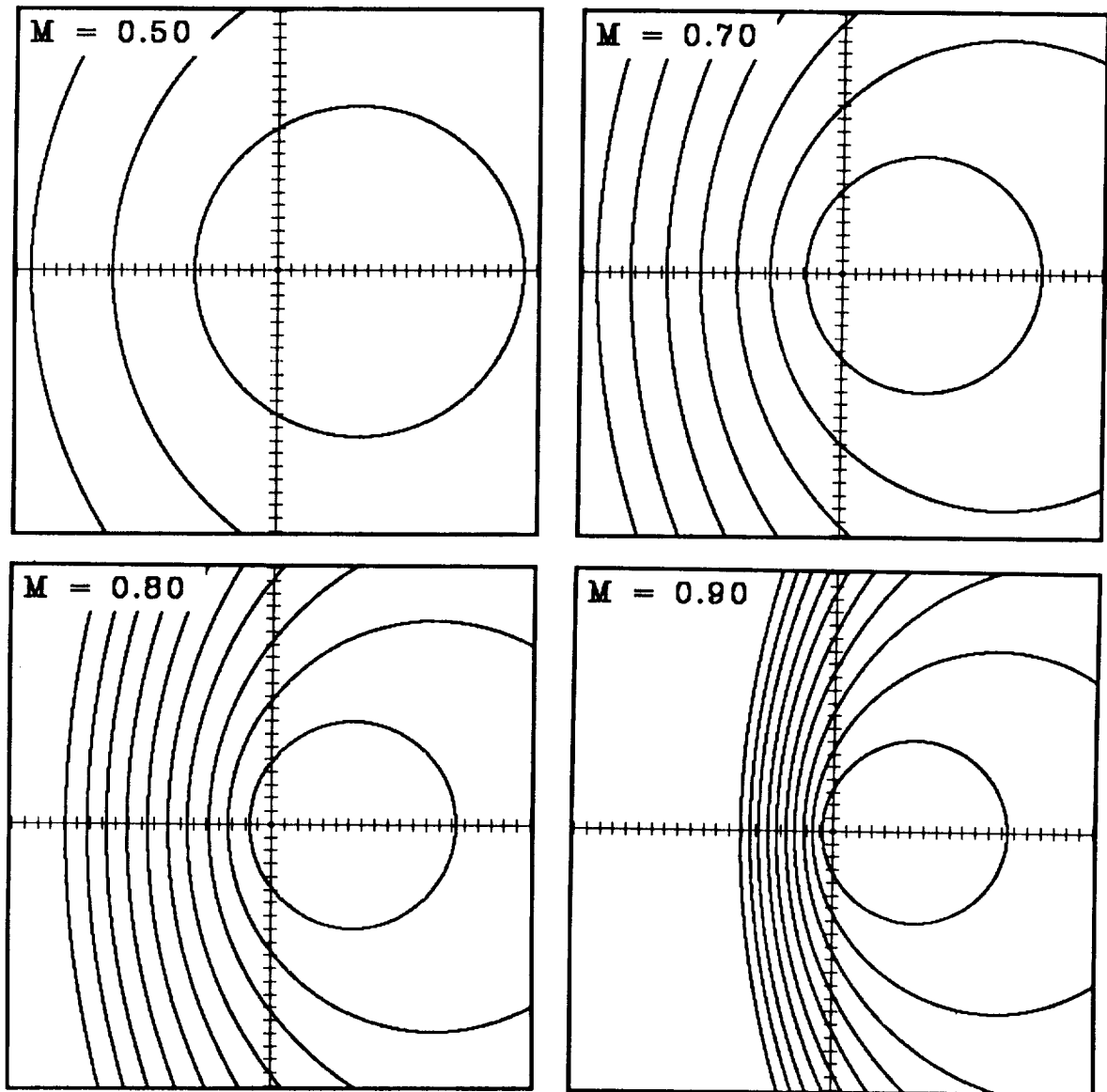

Figure 1.– Wave patterns for two-dimensional linear theory.

The radii of the circles depend on the Mach number. The wave length as measured from the center of the circle is

$$\lambda = T/M = \pi/Mk$$

which decreases with increasing Mach number. With time normalized by stream speed, waves propagate more quickly through the mesh as the Mach number is decreased. Therefore, for a fixed total calculation time T_{max}, disturbances travel a greater distance, which leads to a greater opportunity for reflections from the boundary to affect the solution.

On the other hand, as the Mach number is increased the wave pattern as viewed from the origin becomes more and more distorted. Waves propagate slowly upstream and the effective wave lengths that must be resolved on the mesh become shorter, implying a need for very fine mesh spacing to track these disturbances accurately, or conversely, leading to increasingly large errors on a fixed mesh due to inadequate resolution. Specifically, wave lengths for disturbances propagating along the axes are

$$\lambda = \frac{1-M}{M}\frac{\pi}{k} \qquad \text{upstream}$$

$$\lambda = \frac{1+M}{M}\frac{\pi}{k} \qquad \text{downstream}$$

$$\lambda = \frac{\beta}{M}\frac{\pi}{k} \qquad \text{vertically}$$

Since propagation speed is directly proportional to wave length, the time required for a disturbance to reach a far-field boundary will be inversely proportional to the wave length.

The remainder of the paper contains three main sections. The first section presents results for a pulse disturbance which illustrate the effects of time step and mesh spacing on frequency response. The next section contains the recommendations for time step and mesh properties. The final section shows results for harmonic disturbances which illustrate the efficacy of the guidelines.

Pulse Results

All pulse calculations were made on the same mesh. This mesh has a spacing which is fine near the origin, stretches smoothly to a maximum, and then shrinks somewhat at the far-field boundary. The mesh is defined by z as a fifth degree polynomial function of the mesh index and was designed using the program described in reference 2. The mesh properties are

$$Z_{max} = 20$$
$$N_{Z_{max}} = 32$$
$$h_{surface} = 0.01$$
$$h_{far-field} = 0.5$$
$$h_{max} = 1.0123$$

where the mesh spacing h would equal Δz for the vertical mesh in TSD theory.

A Gaussian pulse in angle of attack is routinely used to produce the frequency response function in applications of the TSD code. Such a pulse contains energy at all frequencies; however,

4

most of the excitation occurs at low frequency and the effective range depends on the pulse width. The pulse responses for four time step sizes are presented in this section. The calculations give the solution for the wave equation with unit speed

$$\frac{\partial^2}{\partial t^2} P = \frac{\partial^2}{\partial z^2} P$$

for an input in downwash centered at t_c given by

$$\frac{\partial}{\partial z} P(0, t) = 4(t - t_c) e^{-2(t-t_c)^2}$$

applied at $z = 0$. The function P represents the potential in TSD theory. The exact solution is

$$P(z, t) = e^{-2(t-t_c-z)^2}$$

It may be remarked that for a vertical wave the unit propagation speed assumed here occurs for

$$M = \beta = \sqrt{0.5} \approx 0.7$$

Figure 2 gives the results for a large time step size. The lines with symbols are the numerical results and the plain lines are the exact solution. The discussion of the results for this figure will be given after some general comments on its organization.

The lower part of the figure shows the time history of the solution at $z = 0$. In the TSD code this solution would correspond to the pulse in potential (or pressure, or lift) which results from the input pulse in angle of attack. The heavy tic-mark near $t = 42$ marks the time for the initial disturbance, centered at $t_c = 2.36$ (the third time step for this case), to travel to the far-field boundary and return.

The figure in the upper right gives the shape of the wave in space at a time shortly before the propagating pulse reaches the far-field boundary located at $Z_{max} = 20$. The tic-marks show the mesh points, with fine spacing at $z = 0$, moderate spacing at $z = 20$, and coarse spacing near $z = 10$.

The figure in the upper left gives the Fourier transform of the time history shown at the bottom. The real and imaginary parts of the transform are plotted against reduced frequency for k up to 2.0. Most of the energy for a pulse of the width used is contained in this frequency range.

Details of the parameters for this case are printed in the upper right of the figure. In this list dt is used for Δt and CN is the ratio of time step size to mesh spacing.

Figure 2 gives the pulse response for $\Delta t = \pi/4$. This time step is too large, with fewer than four steps contained in the pulse, as may be seen in the time history at the bottom. The resulting severe broadening and loss in amplitude of the propagating pulse are seen at the upper right. This dissipation error is a consequence of the large time step. The frequency response function at the upper left is very smooth, but is accurate only at the lowest frequencies. Without the exact solution for reference the errors at higher frequency would not be detected.

MODEL EQUATION FOR CAP-TSD
Quintic 32 pts, dt = 0.785

Implicit, 2nd order dt
dt = 0.78540   tc = 2.36
Newton = 1
0.0100 < dz < 1.0123
0.7758 < CN < 78.5398
sig = 2.0
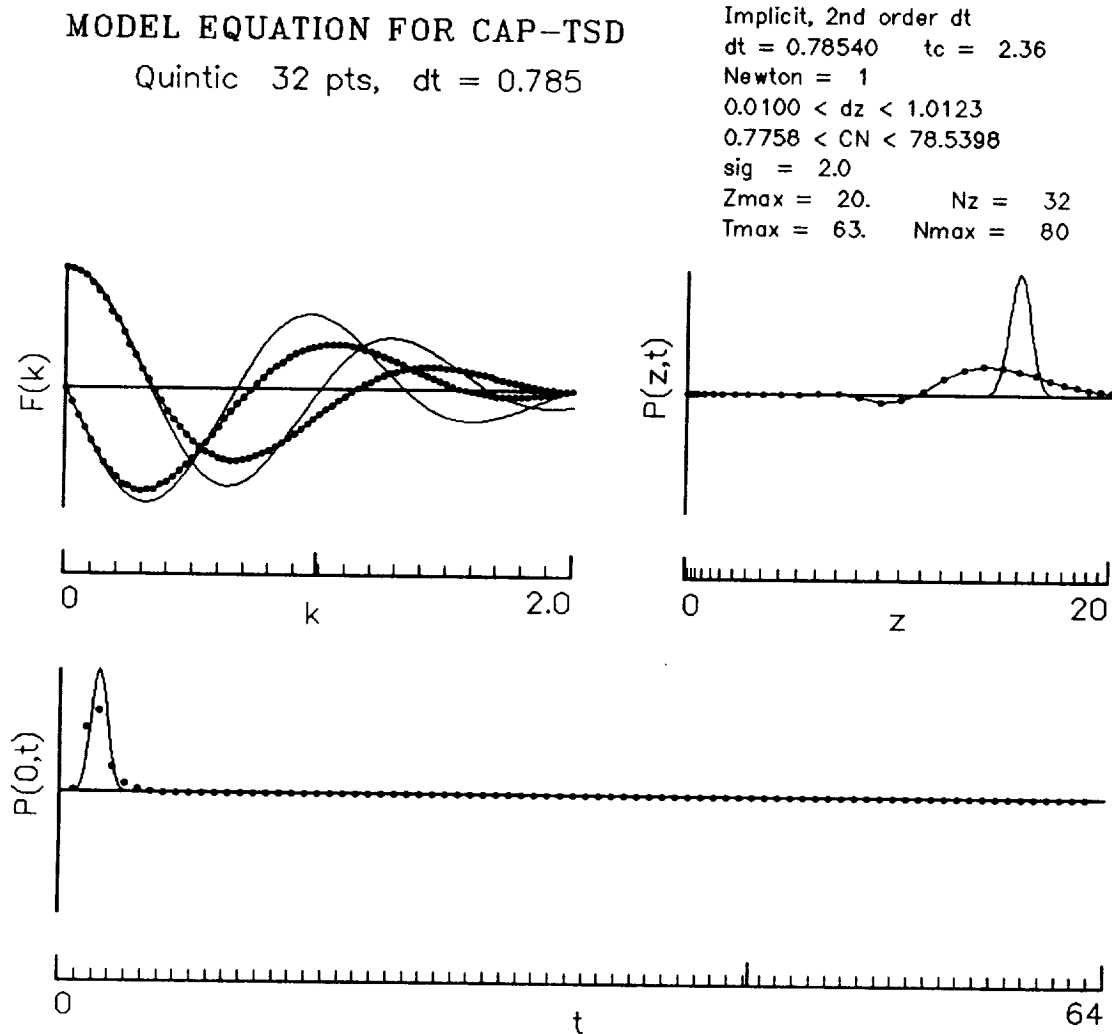Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 80

Figure 2.– Pulse response for $\Delta t = 0.785$.

Figure 3 gives the pulse response for $\Delta t = \pi/8$, half of the value used in the preceding figure. The frequency response results are fairly good up to about $k = 0.4$, but the propagating wave still suffers from dissipation errors. Dispersion errors, the spreading out of the frequency components as the wave propagates through the mesh, are also present. As will be apparent shortly, the latter errors become more severe as the time step is reduced, that is, as the spatially discretized equations are solved more accurately.

MODEL EQUATION FOR CAP-TSD

Quintic 32 pts, dt = 0.393

Implicit, 2nd order dt
dt = 0.39270   tc = 2.75
Newton = 1
0.0100 < dz < 1.0123
0.3879 < CN < 39.2699
sig = 2.0
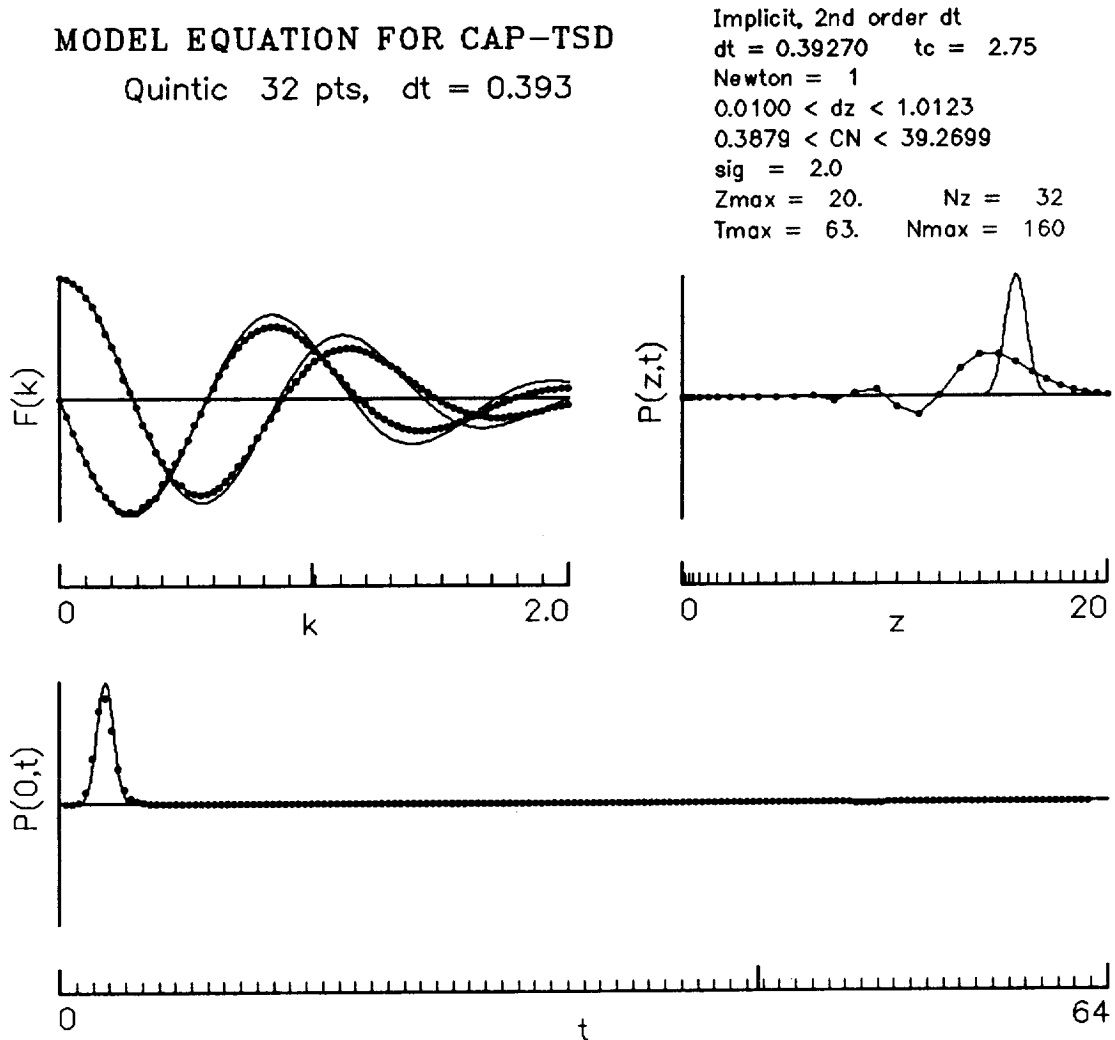Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 160

Figure 3.— Pulse response for $\Delta t = 0.393$.

7

Results for a further reduction in time step to $\Delta t = \pi/16$ are given in figure 4. The dissipation errors in the upper right part of the figure are smaller but the dispersion errors are larger as compared with those in figure 3. These errors result from too few mesh points to represent adequately all of the wave lengths present in the solution and lead to shorter waves (high frequency components) propagating more slowly than longer waves (low frequencies). This spreading out of frequencies as the wave propagates is a consequence of the spatial discretization inherent in the finite-difference method and limits the frequency resolution attainable. The shortest waves that may be described on the mesh have two mesh points per period for which $\lambda = 2/k$. These dispersion errors also affect both the time history and the frequency response. There are only about three points contained within the width of the pulse in the coarsest part of the mesh.

Oscillations in the time history occur as the wave propagates into the region of increased mesh spacing and consequently a decreased number of points per wave length. These errors reflect back to the left and contaminate the solution at the origin.

MODEL EQUATION FOR CAP-TSD
Quintic   32 pts,   dt = 0.196

Implicit, 2nd order dt
dt = 0.19635   tc = 2.95
Newton = 1
0.0100 < dz < 1.0123
0.1940 < CN < 19.6350
sig = 2.0
Zmax = 20.   Nz =   32
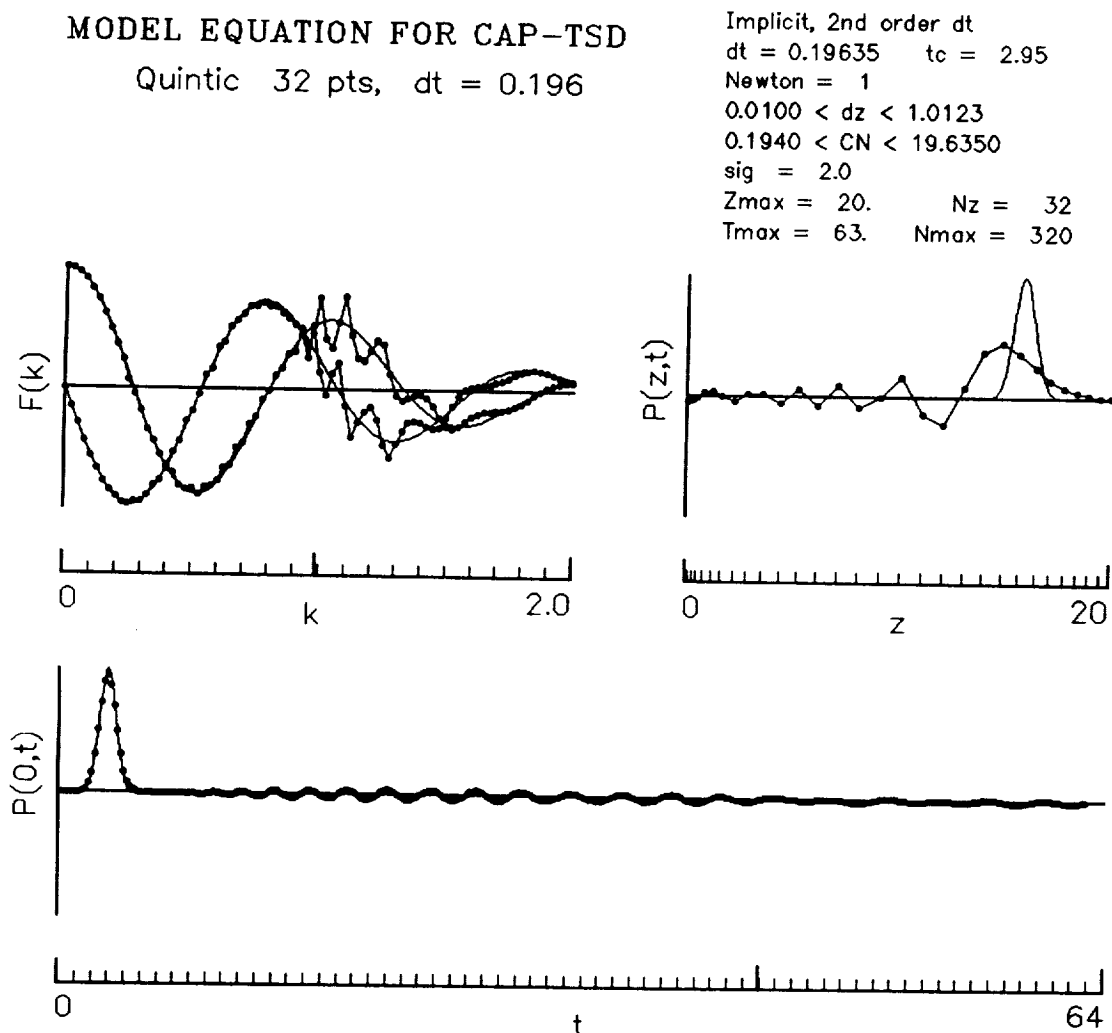Tmax = 63.   Nmax = 320

Figure 4.– Pulse response for $\Delta t = 0.196$.

8

The frequency response is reasonably accurate up to about $k = 1.0$, although some small errors may be detected earlier. The catastropic breakdown in the solution above $k = 1.0$ arises because this mesh has a maximum spacing $h_{max} = 1.0123$ for which two points per wave length implies $\lambda = 2.0246$. Consequently, the highest frequency which may be represented is

$$k_{max} = 2/\lambda \approx 1.0$$

This value k_{max} is indicated with a heavy tic-mark on the figure.

Figure 5 gives the pulse response for $\Delta t = \pi/32$, the value recommended in the guidelines below. The time history of the response at the origin (airfoil) shows the severe oscillations that result from trying to propagate frequencies that are too high for the maximum mesh spacing used. However, these large errors do not affect adversely the low frequency response. Although not shown here, further reduction in Δt leads to little change in the quality of the solution.

MODEL EQUATION FOR CAP−TSD
Quintic 32 pts, dt = 0.098

```
Implicit, 2nd order dt
dt = 0.09817      tc =   2.95
Newton =  1
0.0100 < dz < 1.0123
0.0970 < CN <  9.8175
sig  =  2.0
Zmax =  20.          Nz =   32
Tmax =  63.        Nmax =  640
```
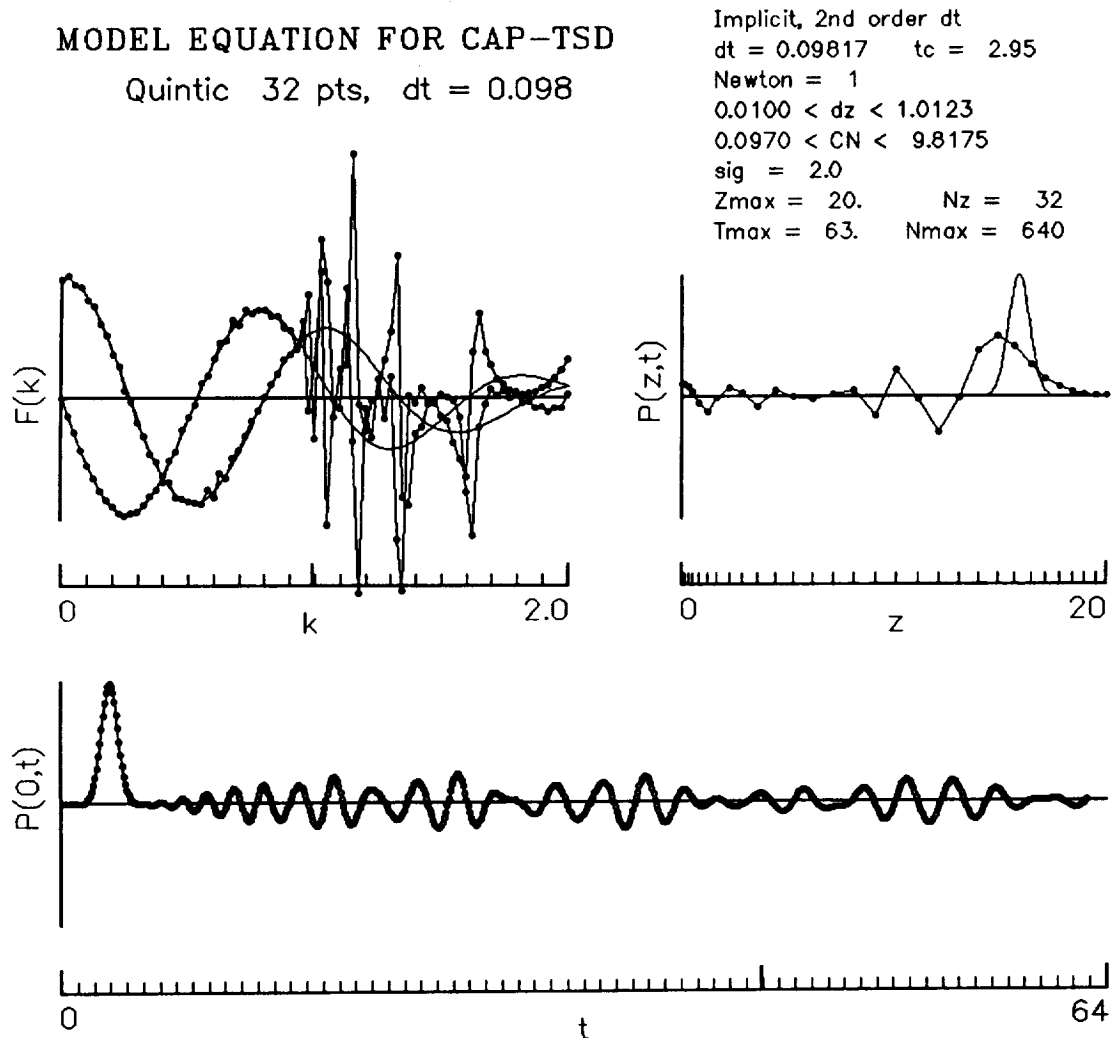
Figure 5.– Pulse response for $\Delta t = 0.098$.

9

Guidelines

Guidelines for mesh and time step parameters, with an indication of the reasons for their selection, are presented in this section. These choices are motivated by the results of the previous sections. Although the study leading to these recommendations strictly applies only to the vertical mesh, the guidelines should be reasonable for the streamwise mesh as well. As the transonic Mach number approaches one, the shrinking wave lengths illustrated in figure 1 for both the upstream and vertical waves would suggest that no mesh could be fine enough to properly treat any but the lowest frequencies. In the guidelines which follow these features are not addressed, and therefore the recommendations strictly apply only to the vertical mesh at $M \approx 0.7$.

Mesh parameters

Choice of the extent and spacing of the vertical ζ-mesh is based on the following considerations:

(a) the far-field boundary should lie at least 20 chord lengths from the wing;

(b) to provide accuracy in both the near-field solution and the downwash boundary condition, the first mesh point should lie at about 0.00001 and the mesh spacing at the wing surface should be comparable to that used chordwise of 0.002 to .02 (for unit chord);

(c) to avoid boundary reflections, the spacing at the far-field boundary should allow at least 8 mesh points per wave length at the maximum frequency of interest; and

(d) to avoid internal mesh reflections (i.e., to reduce dispersion errors), the maximum spacing should provide at least 4 points per wave length at the maximum frequency. These considerations lead to the following constraints on the mesh spacing h:

$$h_{\text{surface}} \approx 0.01$$
$$h_{\text{far-field}} = \pi/8k$$
$$h_{\text{max}} = \pi/4k$$

in which h_{surface} is the spacing between the first two mesh points at the surface. The last two requirements result from the approximate relation (for a wave of unit speed) between the spatial wave length λ and the number of mesh points per wave length N_{pw}

$$\lambda = h N_{\text{pw}} \approx \pi/k$$

from which

$$h \approx \pi/N_{\text{pw}} k$$

Time parameters

Selection of the time step size Δt (DT in CAP-TSD) and number of time steps N_{max} (NSTEP) is based on the following considerations. The time step must be small enough to resolve the frequencies of interest and to capture significant transients. The total time T_{max} must be great enough to allow for transients to be convected away from the configuration and for the steady-state

10

solution to develop. For an oscillatory case, at least two cycles of motion N_{cy} should be used with at least 64 time steps per cycle N_{pc}. For a reduced frequency k the period is

$$T = \pi/k$$

The time parameters are selected such that

$$\Delta t < 0.1$$
$$T_{max} \geq 20\pi$$
$$N_{cy} \geq 2$$
$$N_{pc} \geq 64$$

For a given frequency k the following algorithm may be used to meet these criteria:

1. $N_{pc} = 32/k$
2. if $N_{pc} < 64$ then $N_{pc} = 64$
3. $\Delta t = \pi/kN_{pc}$
4. $N_{cy} = 20k$
5. if $N_{cy} < 2$ then $N_{cy} = 2$
6. $N_{max} = N_{cy}N_{pc}$
7. $T_{max} = N_{max}\Delta t$

This algorithm produces the following input values for different frequency ranges

For $0.0 < k < 0.1$	$\Delta t = \pi/32$	and $N_{max} = 64/k$
For $0.1 \leq k \leq 0.5$	$\Delta t = \pi/32$	and $N_{max} = 640$
For $0.5 < k$	$\Delta t = \pi/64k$	and $N_{max} = 1280k$

It is noteworthy that N_{max} attains its minimum of 640 in the frequency range of greatest interest.

Harmonic Oscillation Results

The results presented here for the model problem with harmonic input illustrate the effectiveness of the guidelines given above. Calculations are first presented for a range of frequencies using the mesh described previously. According to the guidelines, this mesh should be adequate for frequencies up to about $k = 0.75$. Finally, a new mesh is designed for use at the maximum frequency of $k = 1.5$ to verify the adequacy of the recommendations.

A sequence of eight values of frequency is presented for k ranging from 0.025 to 1.5. The calculations give the solution for the wave equation with unit speed ($M = \beta \approx 0.7$)

$$\frac{\partial^2}{\partial t^2}P = \frac{\partial^2}{\partial z^2}P$$

for an input in downwash given by

$$\frac{\partial}{\partial z}P(0,t) = 2k\sin 2kt$$

11

applied at $z = 0$. The exact solution is

$$P(z,t) = \cos 2k(t - z)$$

The layout of the remaining figures is very similar to that used earlier, with two exceptions (refer to figure 6). Now, the Fourier components are computed from the last cycle of the time history shown in the bottom figure. The mean and first four harmonics are plotted with symbols and are listed in the table. For an exact result, all entries would be 0.0000 except for a 1.0000 in the real part at the input frequency ($k = 0.025$ for this case). The exact transform shown is the continuous result obtained by integrating the cosine response over one cycle. Also, the picture of the wave as it propagates across the mesh at the upper right is now shown for $t = T_{max}$.

We now turn to a discussion of the results in the order of increasing frequency. In each case the time parameters were chosen using the guidelines given above. The mesh is held fixed until the final figure.

The results for $k = 0.025$ are shown in figure 6. As is typical for low frequencies, the time step is at its upper limit, which requires a large number of steps per cycle and, consequently, a large number of time steps to obtain two cycles of oscillation. These large numbers are not required for accuracy in the model problem, but are appropriate for a TSD calculation. Notice that only a small portion of one wave is present across the spatial mesh, and the mesh is much finer than is required to represent the wave accurately. This mesh fineness leads to very small truncation errors and the Fourier components are essentially exact. The small differences shown may be round-off errors due to the 32-bit word length of the SUN workstation used.

MODEL EQUATION FOR CAP—TSD

Quintic 32 pts, k = 0.025

k	ReF	ImF
0.000	−0.0006	0.0000
0.025	1.0003	0.0001
0.050	0.0000	0.0000
0.075	0.0000	0.0000
0.100	0.0000	0.0000

Implicit, 2nd order dt
dt = 0.09817   k = 0.025
Newton = 1
0.0100 < dz < 1.0123
0.0970 < CN < 9.8175
Npc = 1280
Zmax = 20.   Nz = 32
Tmax = 251.   Nmax = 2560

Figure 6.– Harmonic response for $k = 0.025$.

13

The results for $k = 0.05$ are shown in figure 7. The quality of the calculation is again quite high. For this case the number of steps per cycle is $N_{pc} = 640$.

MODEL EQUATION FOR CAP–TSD

Quintic 32 pts, k = 0.050

k	ReF	ImF
0.000	0.0003	0.0000
0.050	1.0015	−0.0004
0.100	0.0000	0.0000
0.150	0.0000	0.0000
0.200	0.0000	0.0000

Implicit, 2nd order dt
dt = 0.09817   k = 0.050
Newton = 1
0.0100 < dz < 1.0123
0.0970 < CN < 9.8175
Npc = 640
Zmax = 20.   Nz = 32
Tmax = 126.   Nmax = 1280

Figure 7.– Harmonic response for $k = 0.05$.

14

The results for $k = 0.1$ are shown in figure 8. The errors are quite small for this case with $N_{pc} = 320$ and wave length $\lambda > 20$. The total time $T_{max} = 20\pi$, the minimum allowed by the guidelines.

MODEL EQUATION FOR CAP-TSD
Quintic 32 pts, k = 0.100

k	ReF	ImF
0.000	−0.0001	0.0000
0.100	0.9983	−0.0045
0.200	0.0003	0.0000
0.300	0.0002	−0.0003
0.400	−0.0001	−0.0002

Implicit, 2nd order dt
dt = 0.09817   k = 0.100
Newton = 1
0.0100 < dz < 1.0123
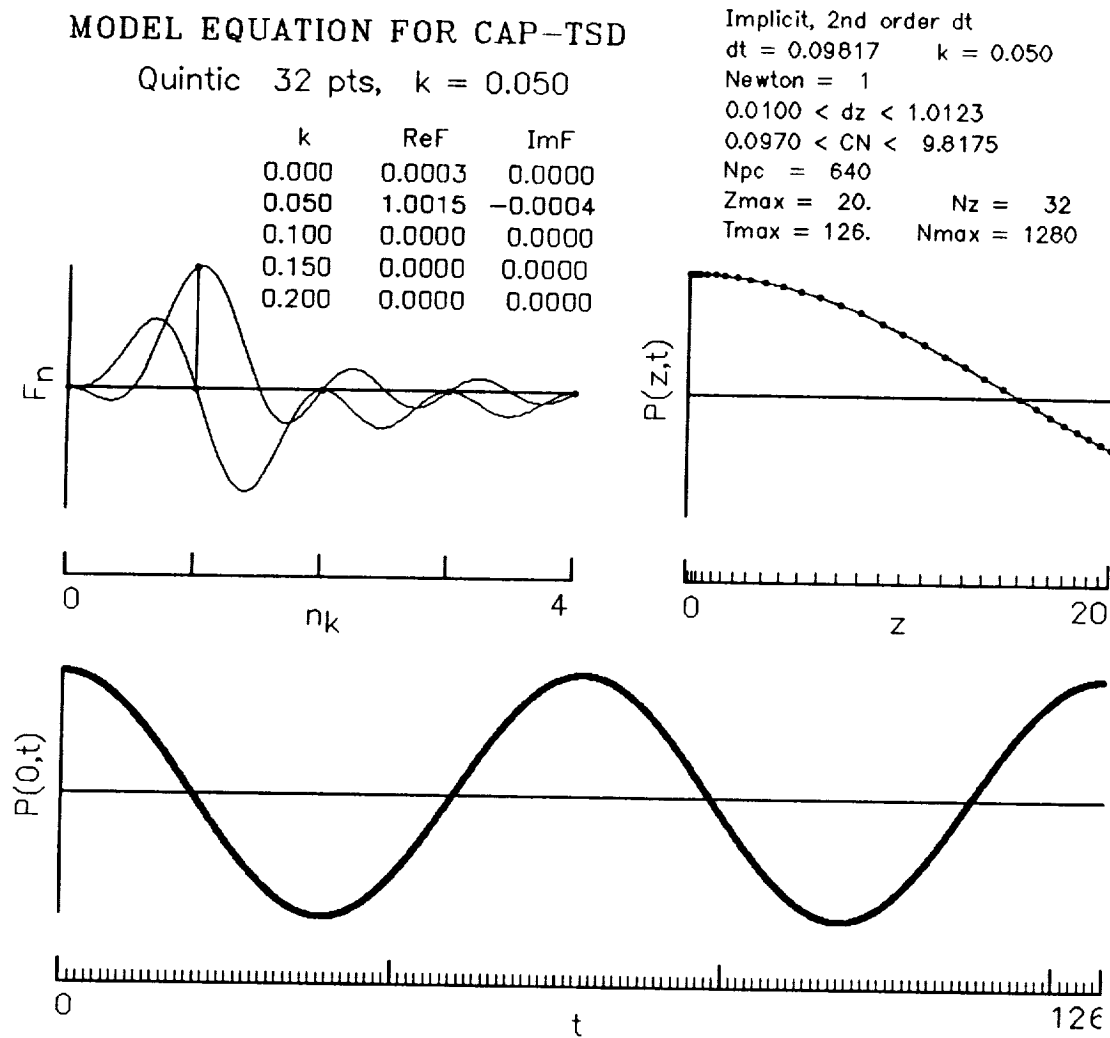0.0970 < CN < 9.8175
Npc = 320
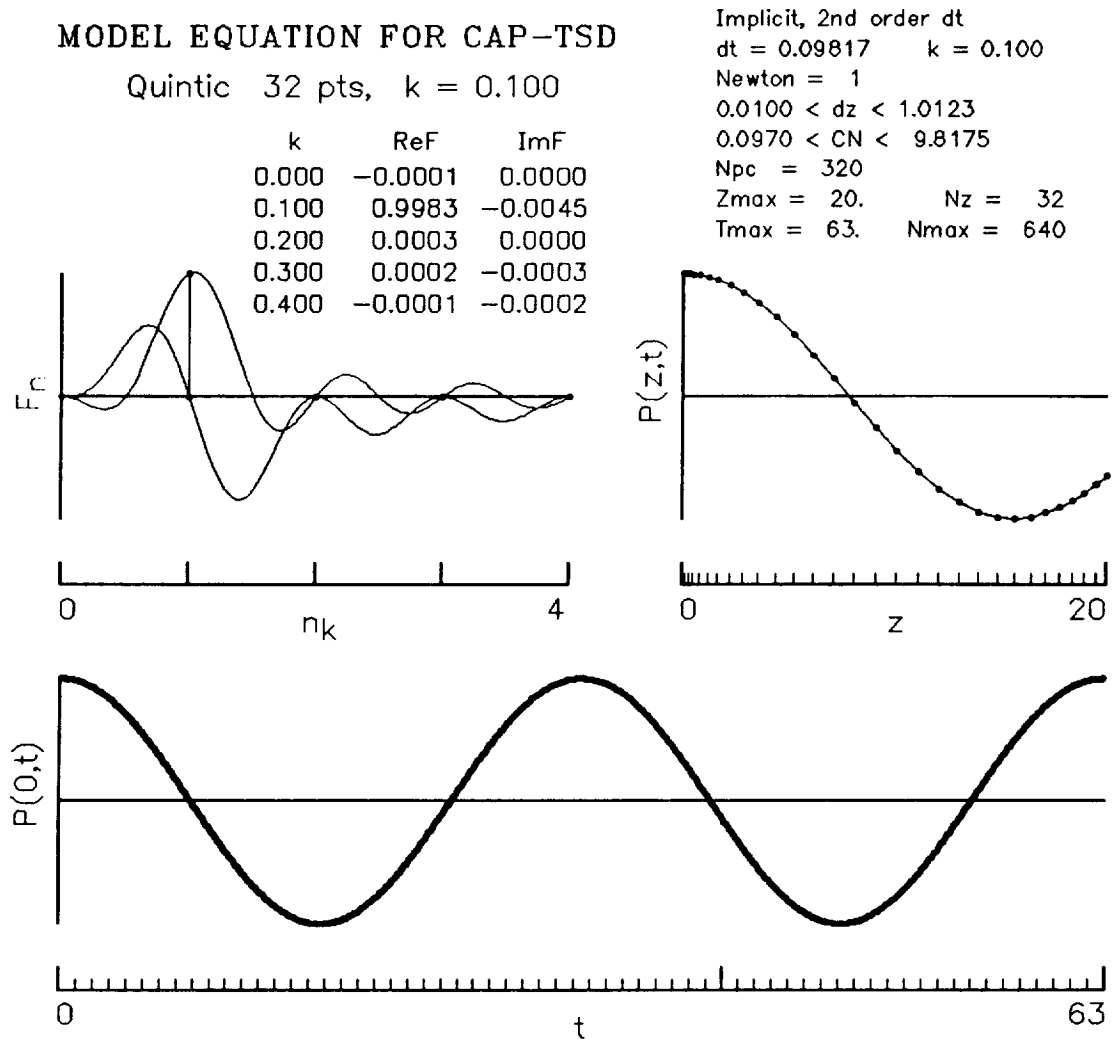Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 640

Figure 8.– Harmonic response for $k = 0.1$.

The results for $k = 0.25$ are shown in figure 9. The time step $\Delta t = 0.09817$ is still at its upper limit and $N_{pc} = 128$, which is twice the minimum suggested. Five cycles of oscillation are required to meet the minimum T_{max} requirement. More than one and one-half wave lengths are present across the mesh, which results in about 20 mesh points per wave length.

The Fourier components are still quite accurate with the exception of the imaginary part of the first harmonic which is in error by about 1.5 percent. This term indicates a small phase error in the time history during the last cycle.
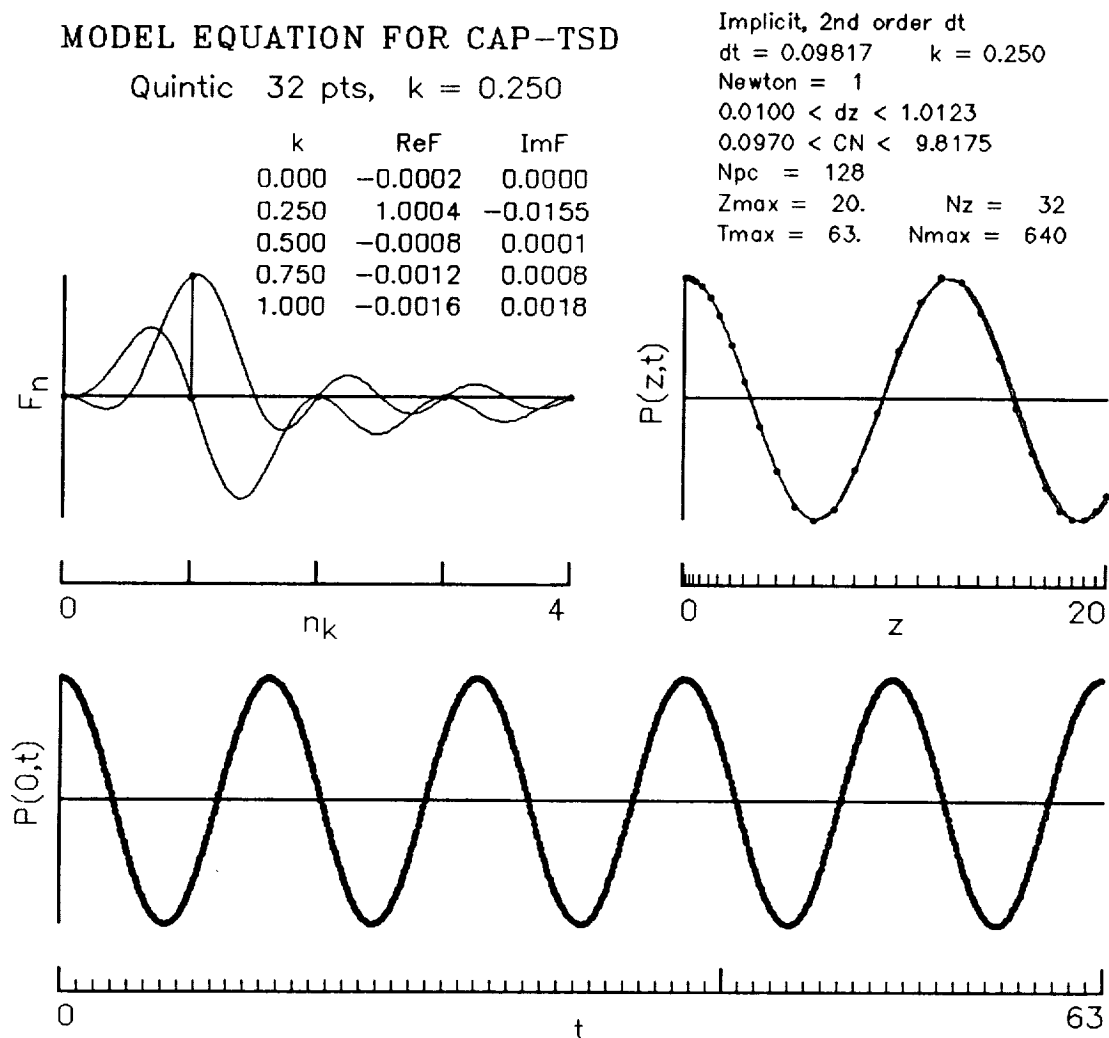

Figure 9.– Harmonic response for $k = 0.25$.

16

The results for $k = 0.5$ are shown in figure 10. For this case, N_{pc} has attained its minimum value of 64. The dispersion error is revealed in the slower propagation of the numerical wave shown at the upper right. This slowdown leads to a significant difference between the numerical and exact spatial wave lengths. Here, there are approximately 6 mesh points per wave length in the center of the mesh, too few to maintain spatial accuracy. This inaccuracy leads to internal mesh reflections which result in an error of over 4 percent in the fundamental Fourier coefficent.

MODEL EQUATION FOR CAP–TSD

Quintic 32 pts, k = 0.500

k	ReF	ImF
0.000	0.0013	0.0000
0.500	0.9581	−0.0039
1.000	−0.0073	0.0003
1.500	0.0018	−0.0014
2.000	0.0000	−0.0001

Implicit, 2nd order dt
dt = 0.09817   k = 0.500
Newton = 1
0.0100 < dz < 1.0123
0.0970 < CN < 9.8175
Npc = 64
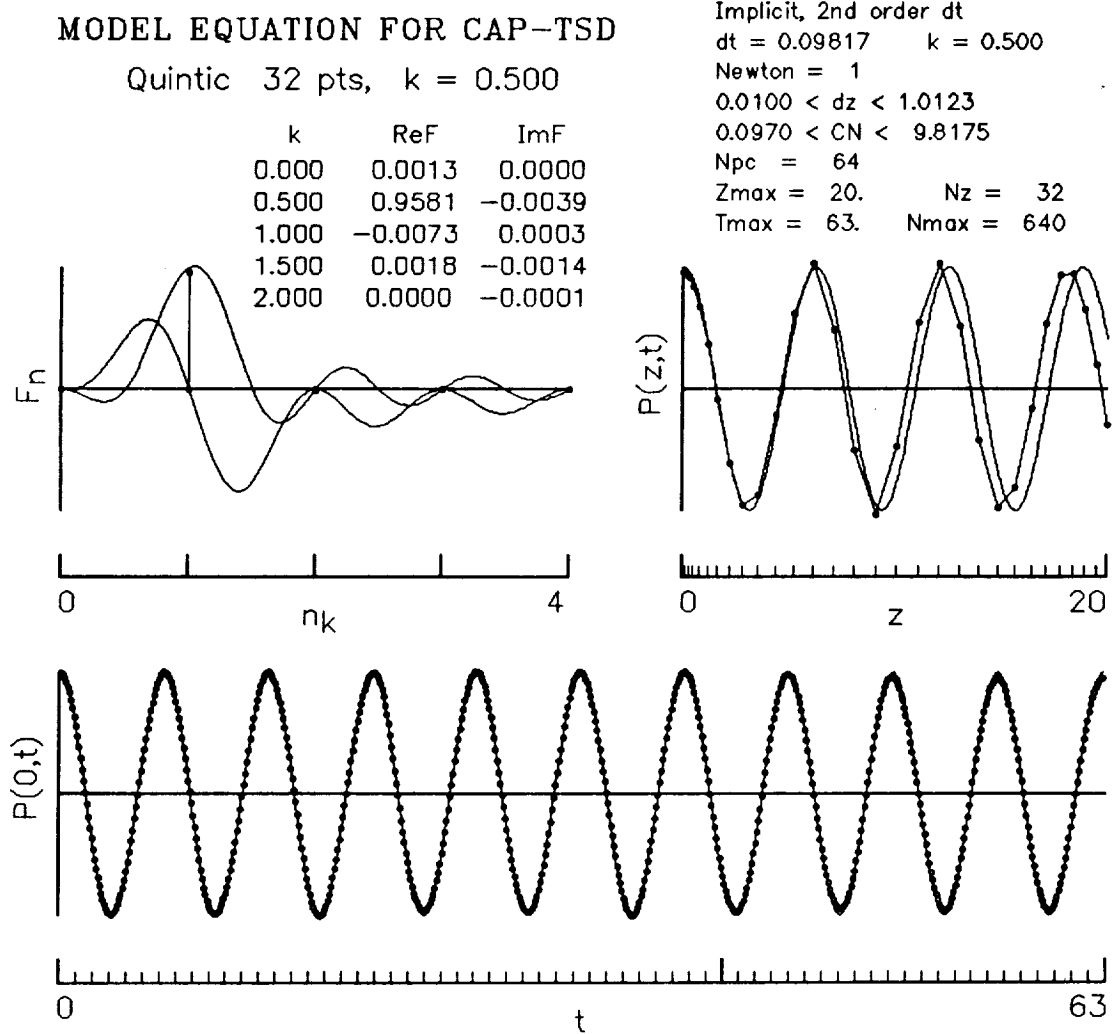Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 640

Figure 10.– Harmonic response for $k = 0.5$.

17

The results for $k = 0.75$ are shown in figure 11. For this case Δt has been reduced to maintain N_{cy} at its minimum value of 64. The mesh has approximately 8 points per wave length at $z = 20$ and 4 points near $z = 10$. These are the minimum values recommended above. Therefore, this frequency is the highest one for which the mesh fineness might be considered adequate. This relative mesh coarseness results in noticeable inaccuracy in the time history, probably due to both internal mesh reflections (e.g., for $t \approx 30$) and boundary reflections (for $t > 40$). There is a significant error in the wave propagation speed across the mesh and the error in the fundamental Fourier coefficent now exceeds 8 percent. Although these errors are larger than those which would be acceptable for this model problem, the corresponding errors for a TSD application would not be expected to be as large because of the reduction in amplitude which occurs for multidimensional wave propagation.

MODEL EQUATION FOR CAP-TSD

Quintic 32 pts, k = 0.750

k	ReF	ImF
0.000	−0.0098	0.0000
0.750	0.9543	−0.0871
1.500	0.0173	0.0170
2.250	0.0052	0.0106
3.000	0.0018	0.0057

Implicit, 2nd order dt
dt = 0.06545   k = 0.750
Newton = 1
0.0100 < dz < 1.0123
0.0647 < CN < 6.5450
Npc = 64
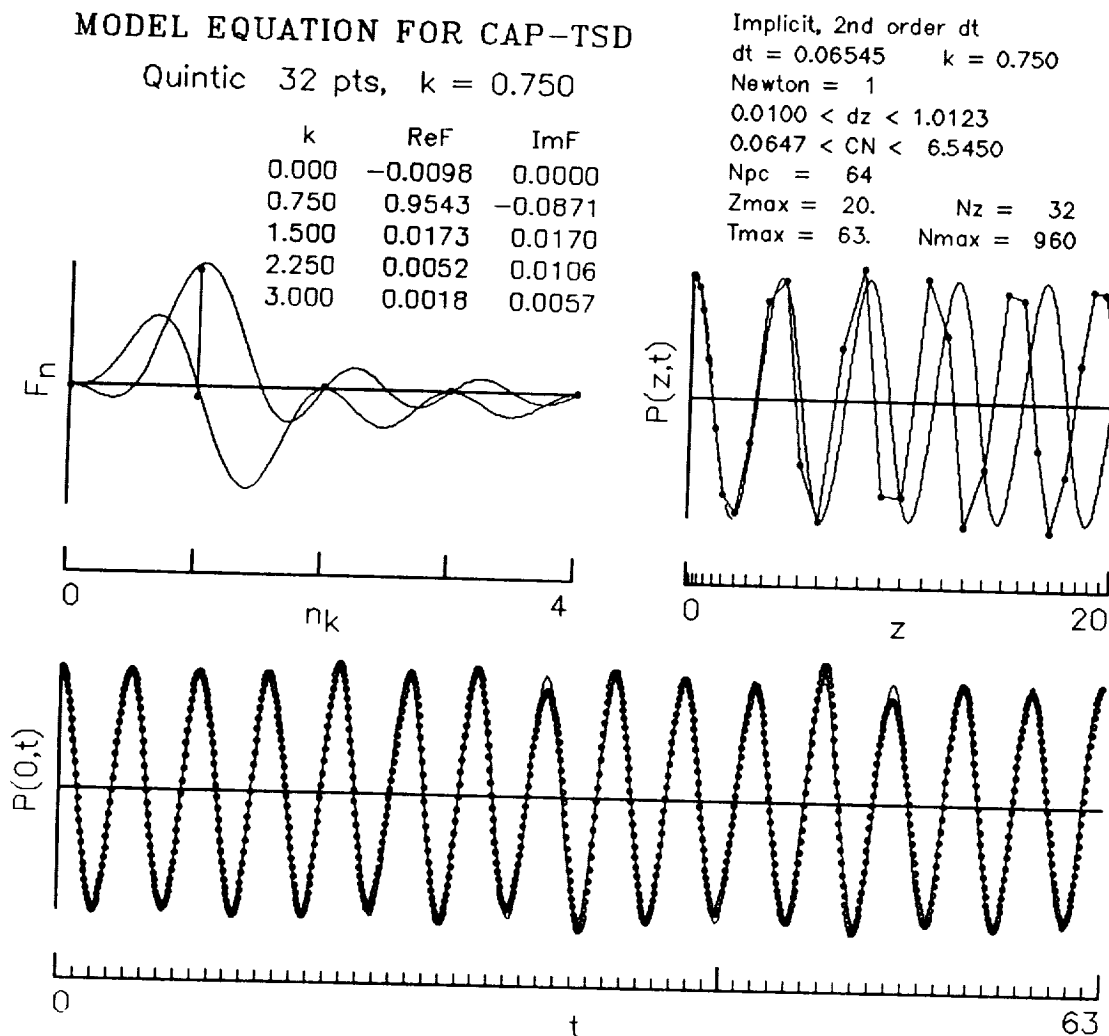Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 960

Figure 11.— Harmonic response for $k = 0.75$.

18

The results for $k = 1.0$ are shown in figure 12. For this case, the mesh is obviously too coarse for the wave to propagate correctly and the spatial plot shows the typical saw-tooth wave with about 2 points per wave length which results. As was pointed out in the discussion of figure 4 above, this mesh cannot propagate a wave of this frequency with any accuracy. The amplitude is much too small (at this instant of time when $t = T_{max}$). The time history shows large errors, including a phase error of about one-quarter cycle at T_{max}. This error is present even though the boundary condition is forcing the downwash to continue its sinusoidal oscillation.

MODEL EQUATION FOR CAP-TSD

Quintic 32 pts, k = 1.000

Implicit, 2nd order dt
dt = 0.04909   k = 1.000
Newton = 1
0.0100 < dz < 1.0123
0.0485 < CN < 4.9087
Npc = 64
Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 1280

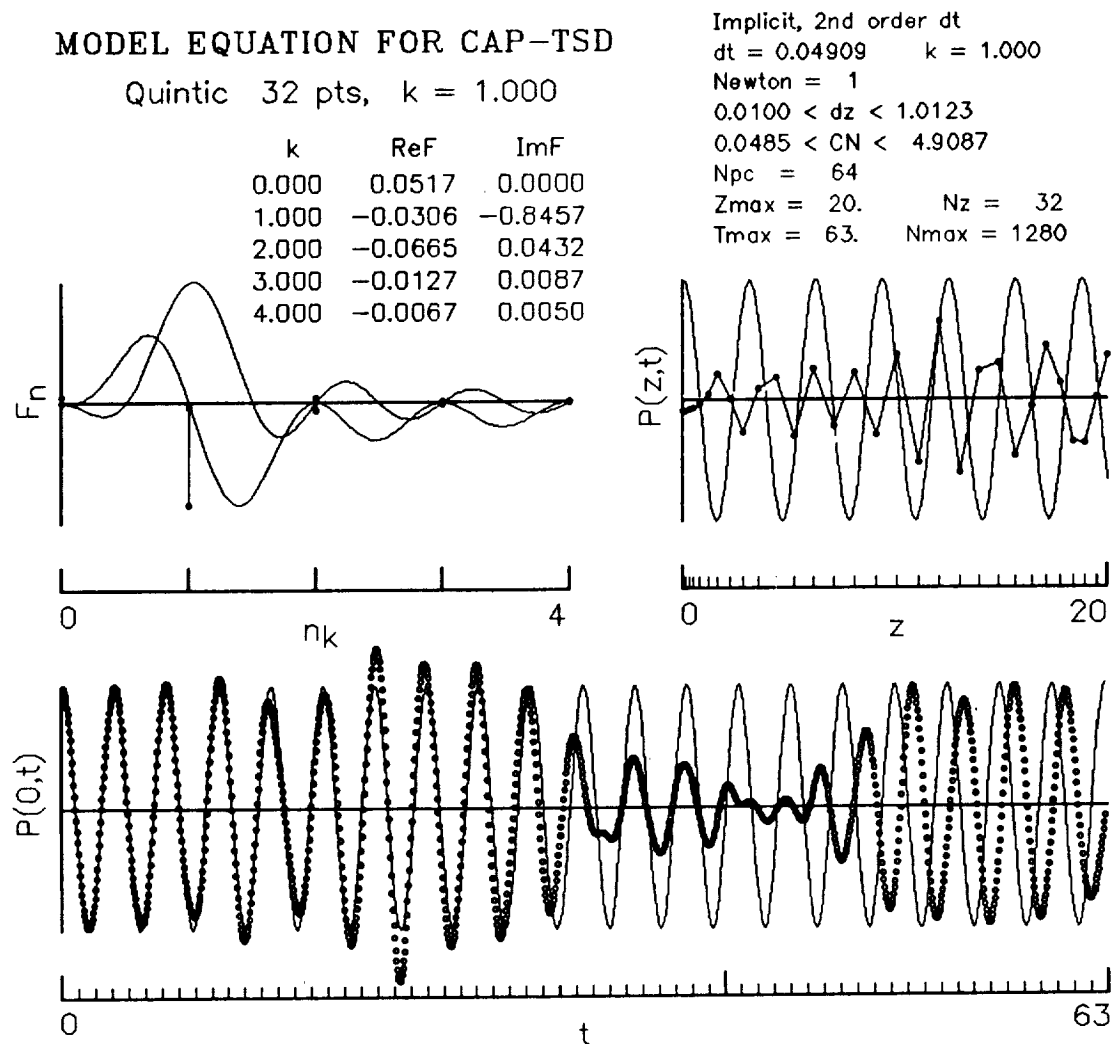| k	ReF	ImF
0.000	0.0517	0.0000
1.000	-0.0306	-0.8457
2.000	-0.0665	0.0432
3.000	-0.0127	0.0087
4.000	-0.0067	0.0050

Figure 12.– Harmonic response for $k = 1.0$.

The results for $k = 1.5$ are shown in figure 13. For this high frequency case (i.e., high for the mesh used), the wave cannot propagate across the mesh, the disturbances remain trapped near the origin, and the time history contains large apparently random errors.
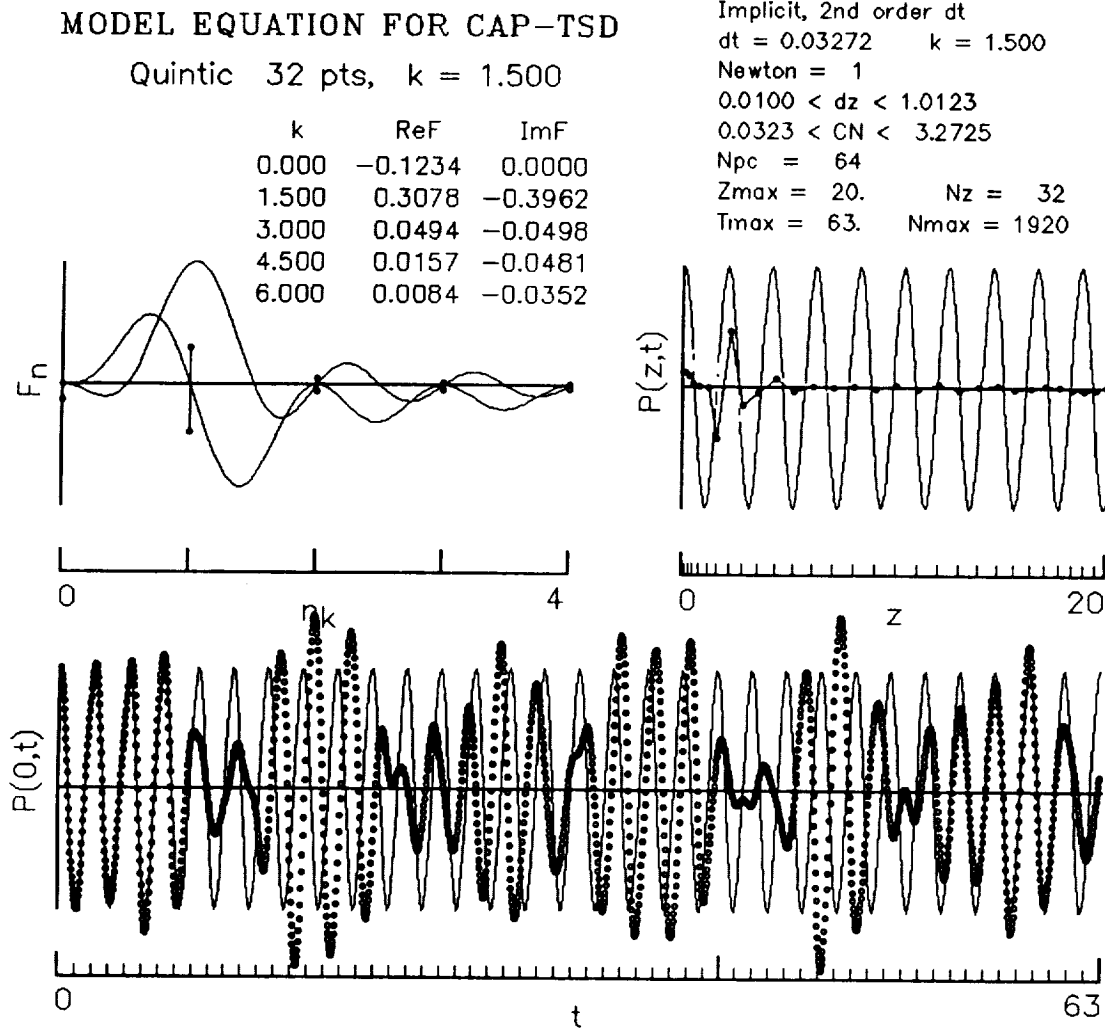
MODEL EQUATION FOR CAP–TSD

Quintic   32 pts,   k = 1.500

k	ReF	ImF
0.000	−0.1234	0.0000
1.500	0.3078	−0.3962
3.000	0.0494	−0.0498
4.500	0.0157	−0.0481
6.000	0.0084	−0.0352

Implicit, 2nd order dt
dt = 0.03272   k = 1.500
Newton = 1
0.0100 < dz < 1.0123
0.0323 < CN < 3.2725
Npc = 64
Zmax = 20.   Nz = 32
Tmax = 63.   Nmax = 1920

Figure 13.– Harmonic response for $k = 1.5$.

In order to verify the usefullness of the guidelines for mesh parameter selection, a new mesh was designed to correct the complete breakdown in the solution for $k = 1.5$ seen in figure 13. The mesh design program of reference 2 was used with far-field and maximum spacings chosen to meet the criteria

$$h_{far-field} = \pi/8k = 0.26$$

$$h_{max} = \pi/4k = 0.52$$

for $k = 1.5$. This new mesh required $N_{z_{max}} = 61$ points, nearly twice as many as the 32 points used previously.

20

The results for $k = 1.5$ on the finer mesh are shown in figure 14. The maximum error in the fundamental Fourier coefficient is about 9 percent and the overall quality of the calculation is comparable to that obtained on the original mesh at $k = 0.75$, as seen in figure 11. As compared with the coarse mesh calculation in figure 13, the wave can now propagate across the mesh and the severe internal mesh reflections have been eliminated. Although not shown herein, the results on the 61 point mesh were improved over those on the 32 point mesh at all frequencies.
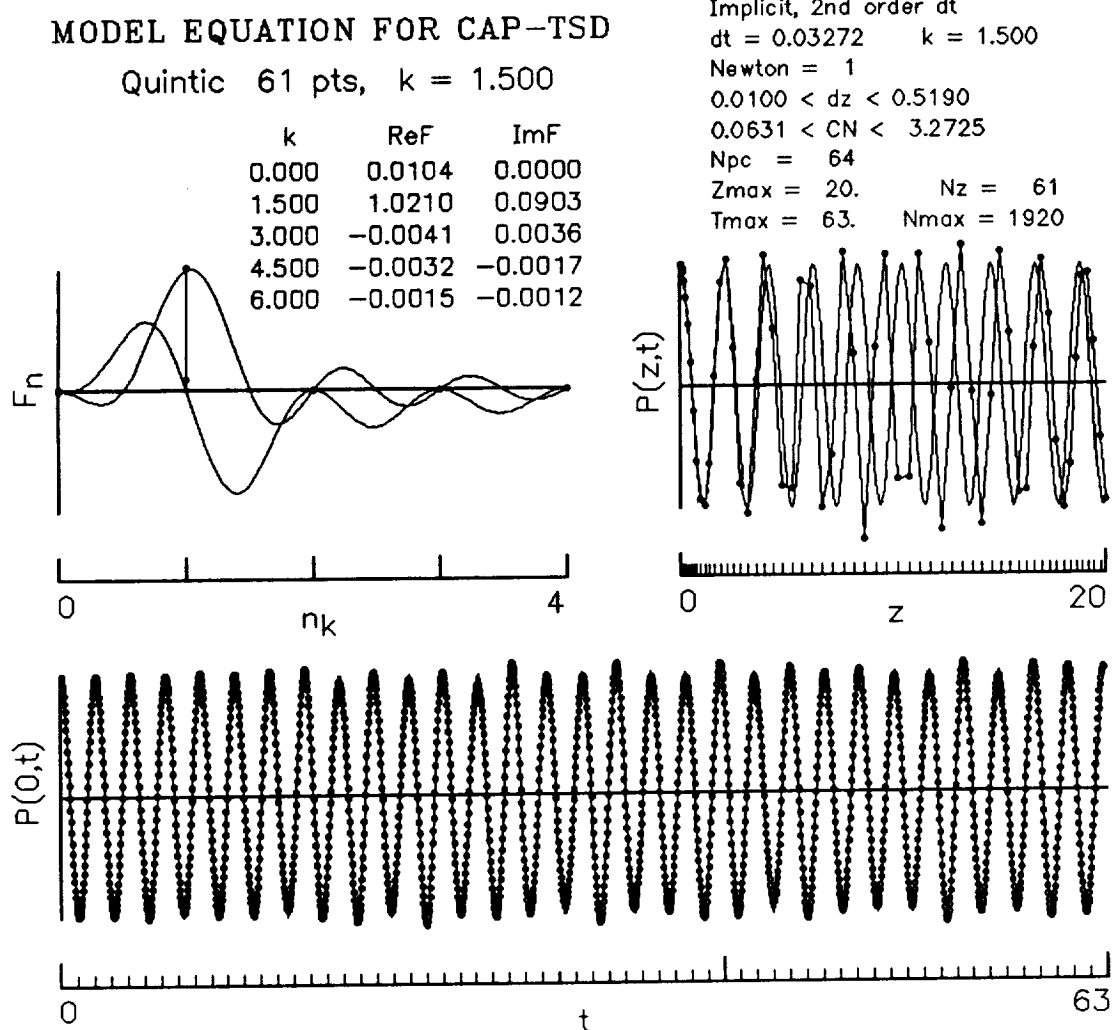
MODEL EQUATION FOR CAP−TSD

Quintic 61 pts, k = 1.500

k	ReF	ImF
0.000	0.0104	0.0000
1.500	1.0210	0.0903
3.000	−0.0041	0.0036
4.500	−0.0032	−0.0017
6.000	−0.0015	−0.0012

Implicit, 2nd order dt
dt = 0.03272   k = 1.500
Newton = 1
0.0100 < dz < 0.5190
0.0631 < CN < 3.2725
Npc = 64
Zmax = 20.   Nz = 61
Tmax = 63.   Nmax = 1920

Figure 14.− Harmonic response with improved mesh for $k = 1.5$.

21

Although the maximum frequency $k = 1.5$ for which calculations were shown is perhaps much higher than would be of interest for flutter, it has been assumed that the Mach number is approximately 0.7 throughout the presentation of these results. An increase in the Mach number for the linear problem or significant transonic effects would reduce the frequency range for which the conclusions are valid. Said differently, a finer mesh would be needed to produce comparable results as the Mach number was increased.

Conclusions

Recommendations for choosing some of the input parameters used in the CAP-TSD computer code have been made. These parameters include those associated with the mesh design and time step. These guidelines are based primarily on experience with a one-dimensional model problem used to study wave propagation in the vertical direction. Numerical simulations presented herein support the recommendations given.

References

1. Batina, John T., Seidel, David A., Bland, Samuel R., and Bennett, Robert M., "Unsteady Transonic Flow Calculations for Realistic Aircraft Configurations," *Journal of Aircraft*, Vol. 26, Jan. 1989, pp. 21–28.

2. Bland, Samuel R., "Interactive Grid Generation Program for CAP-TSD," NASA TM-102705, Oct. 1990.

3. Bland, Samuel R., "Personal Computer Study of Finite-Difference Methods for the Transonic Small Disturbance Equation," NASA TM-102582, Dec. 1989.

Report Documentation Page

NASA National Aeronautics and Space Administration

1. Report No. NASA TM 104083	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Suggestions for CAP-TSD Mesh and Time Step Input Parameters		5. Report Date June 1991
		6. Performing Organization Code
7. Author(s) Samuel R. Bland		8. Performing Organization Report No.
		10. Work Unit No. 509-10-02-03
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Virginia 23665-5225		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This paper gives suggestions for some of the input parameters used in the CAP-TSD computer code. These parameters include those associated with the mesh design and time step. The guidelines are based principally on experience with a one-dimensional model problem used to study wave propagation in the vertical direction.

17. Key Words (Suggested by Author(s)) Transonic Small Disturbance Theory Unsteady flow Finite-difference wave propagation CAP-TSD code	18. Distribution Statement Unclassified - Unlimited Subject Category - 02		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 23	22. Price A03

NASA FORM 1626 OCT 86