ASTROP Users Manual: A Program for Aeroelastic Stability Analysis of Propfans

G. V. Naray...   R. V. K...

NASA Technical Memorandum 4304

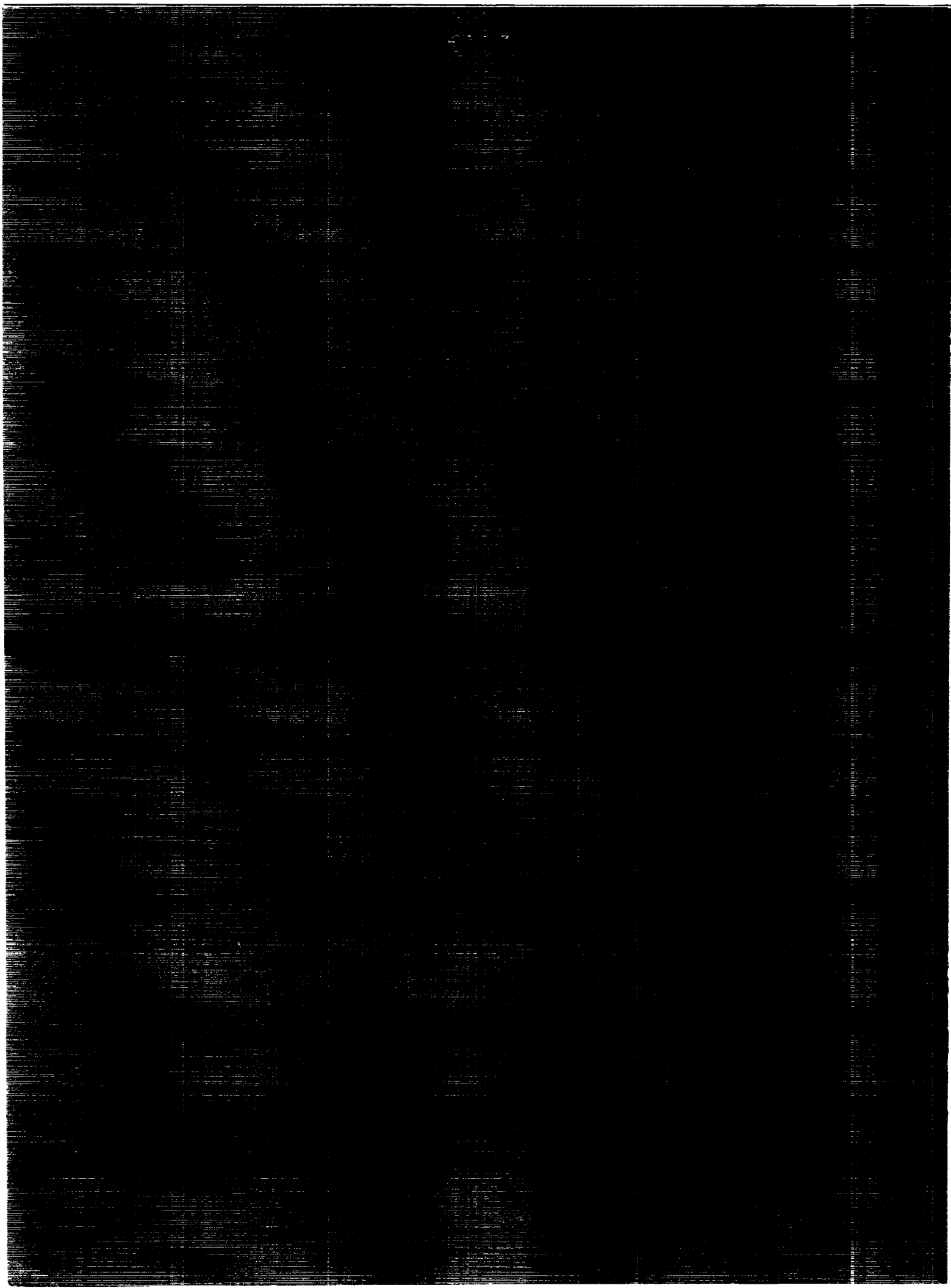# ASTROP2 Users Manual: A Program for Aeroelastic Stability Analysis of Propfans

G. V. Narayanan
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio

K. R. V. Kaza
Lewis Research Center
Cleveland, Ohio

Contents

Summary

This is a users manual for the Aeroelastic STability and Response Of Propulsion Systems computer program called ASTROP2. The ASTROP2 code performs aeroelastic stability analysis of rotating propfan blades. This analysis uses a two-dimensional, unsteady cascade aerodynamics model and a three-dimensional, normal-mode structural model. Analytical stability results from this code are compared with published experimental results of a rotating composite advanced turboprop model and of a nonrotating metallic wing model. This manual will help the user of ASTROP2 to set up the input deck for running the code, and it will provide the serious user with enough information to understand the mathematical development used in the code. In addition, this manual will enable the programmer to install and maintain the code in a computer environment.

The code is written in the FORTRAN language. ASTROP2 is currently operational on the Cray X/MP computer at the NASA Lewis Research Center.

Introduction

The thrust for improving fuel efficiency in aircraft propulsion has generated interest in advanced propellers, known as propfans. Assuring the structural integrity of propfans requires accurate flutter analysis. Consequently, an effort was undertaken at the NASA Lewis Research Center to develop refined analytical methods and associated computer codes for predicting propfan flutter. This effort was started with work described in reference 1. In this reference the aerodynamics model is based on two-dimensional cascade theory with a correction for blade sweep, and the structural model is an idealized swept beam for each blade. Continuation of this analytical work has resulted in the development of more refined analytical methods. The codes based on these methods are termed "ASTROP." ASTROP is an acronym that stands for aeroelastic stability and response of propulsion systems. ASTROP is a modal flutter analysis program that uses a finite element representation of the blade structure. It exists in two versions. One version, ASTROP2, the subject of this report, uses two-dimensional unsteady aerodynamics. The other version, ASTROP3, uses three-dimensional unsteady aerodynamics. Details of the ASTROP codes are presented in references 2 and 3.

This report is a users manual for the ASTROP2 code. This manual is written to help the user of ASTROP2 to set up the input deck for running the code and to provide the user with enough information to understand the mathematical development used in the code. In addition, this manual includes information on how to install and maintain the code in a computer environment. It is divided into two parts: Part 1 provides the user with information to execute ASTROP2 on the computer; part 2 discusses the theoretical development used in ASTROP2. Those users who are only interested in running ASTROP2 can skip part 2. Appendix A explains the major routines of ASTROP2. The other appendixes give details to clarify the descriptions in parts 1 and 2.

The flutter analysis is performed in three steps:

(1) A geometric nonlinear structural analysis of the rotating blade is performed using the finite element method. This analysis provides the steady-state deformed configuration and total differential stiffness of the propfan blade.

(2) The natural frequencies and mode shapes of the blade in its deformed state are calculated by using the differential stiffness matrix generated in step (1).

(3) The unsteady aerodynamic loads and the stability parameters of the propfan blade are calculated as described in part 2.

The first two steps can be performed by any finite element program. Currently, the ASTROP2 code is set up to use the NASA STructural ANalysis program (NASTRAN). Two forms of NASTRAN are available at Lewis; one is COSMIC/NASTRAN (ref. 4), and the other is MSC/NASTRAN (ref. 5), developed by the MacNeal Schwendler Corporation (MSC).

The steady-state deflected configuration of the rotating propfan blade is obtained from a geometric nonlinear analysis. For instance, this analysis is done in MSC/NASTRAN by using the available solution sequence number 64 (ref. 6). The normal-mode vibration analysis of the blade is done in MSC/NASTRAN by using the solution sequence number 63. The respective solution sequence numbers in COSMIC/NASTRAN are 4 and 9.

Symbols

$A_1, A_2 ... A_{NM}$ torsion contributions of NM modes about reference axis

$[A]$ aerodynamic matrix due to motion

a elastic axis location, nondimensional

b semichord

$\mathbf{c}$ chordline vector

c chord length

d distance of aeropoint from origin

$[E]$ matrix defined in equation (8c)

h plunging displacement

h_r plunging displacement of rth blade

$[K_r]$ generalized stiffness matrix for r^{th} blade

$[K_s]$ generalized stiffness matrix for system

k reduced frequency, $\omega b/V_e$

k_{rj} generalized stiffness of jth degree of freedom

$[L]$ aerodynamic coefficient matrix, equation (D4a)

$[LP]$ aerodynamic coefficient matrix, equation (D4b)

$[LPP]$	aerodynamic coefficient matrix, equation (D4c)
L_r	lift due to motion of rth blade per unit span, positive up
ℓ_{hh}	nondimensional lift coefficient due to plunging
$\ell_{h\alpha}$	nondimensional lift coefficient due to pitching
$\ell_{\alpha h}$	nondimensional moment coefficient due to plunging
$\ell_{\alpha\alpha}$	nondimensional moment coefficient due to pitching
M	axial Mach number
M_e	effective Mach number at aeropoint on leading edge
M_r	moment about elastic axis due to motion of rth blade per unit span, positive nose up
$[M_r]$	generalized mass matrix for r^{th} blade
$[M_s]$	generalized mass matrix for system
m_{rj}	generalized mass of jth degree of freedom on r^{th} blade
N	number of blades on rotor
NM	number of modes for each blade
$\mathbf{n}$	vector normal to a blade chord
$[P_s]$	matrix defined in equation (14a)
$\{Q(t)\}$	generalized force vector, equation (2d)
$\{Q_r(t)\}$	generalized force vector of rth blade
$[Q_s]$	matrix defined in equation (14b)
$\{q(t)\}$	generalized coordinate vector, equation (2c)
$\{q_r(t)\}$	generalized coordinate vector of rth blade
$\{q_0\}$	amplitude of $\{q(t)\}$
$\{q_0\}$	generalized amplitude vector for system
$\{\bar{q}_{r0}\}$	amplitude of $\{q_r(t)\}$
$\{q_{r0}\}$	generalized amplitude vector of rth blade
R	tip radius of blade
r	integer specifying blade number, $r = 1,2,...,n$

s	distance along leading edge (aeropoint)
s/c	ratio of blade-to-blade spacing to chord length
t	time
$\mathbf{t}$	tangent vector
V	axial velocity relative to blade
V_e	effective or relative velocity at point on leading edge
$[W]$	modal transformation matrix, equation (E1d)
$[WP]$	modal transformation matrix, equation (E1e)
$[WPP]$	modal transformation matrix, equation (E1f)
$W_1, W_2, ..., W_{NM}$	bending contributions of NM normal modes of reference axis
X, Y, Z	global coordinate axes (fig. 1)
α	pitching displacement
α_r	pitching displacement of rth blade, positive clockwise
β_n	interblade phase angle
$\beta_{0.75R}$	setting angle of blades at $0.75R$
γ	complex aeroelastic eigenvalue
ε	tolerance value
ζ_{rj}	structural damping ratio of jth mode of rth blade
Λ	structural or geometric blade sweep angle
μ	real part of
ν	imaginary part of
ρ	air density
Ω	rotor speed, rpm
ω	frequency at flutter
ω_0	reference frequency
ω_{rj}	natural frequency of rth blade in j^{th} mode

Part 1: Users Manual

Introduction

This part of the manual contains the documentation on the use and execution of ASTROP2. The ASTROP2 program uses the global blade coordinate system shown in figure 1. The X axis is the axis of rotation. It is assumed to be along the direction of axial flow and positive in the sense of flow. The blade pitch axis is the Y axis. It is taken normal to the X axis with positive values in the direction of increasing radius. The Z axis is then defined to form a right-hand coordinate system. Also, the plane of rotation is the plane formed by the Y and Z axes. The ASTROP2 input structure is explained here and later the input is described in detail.
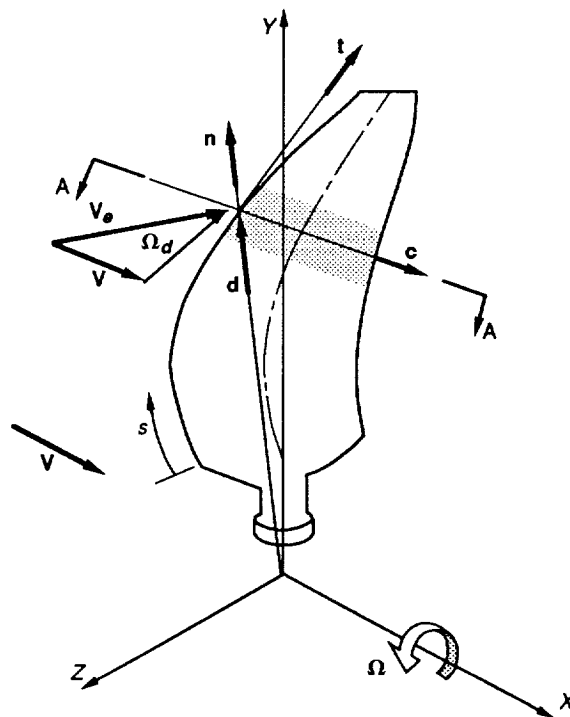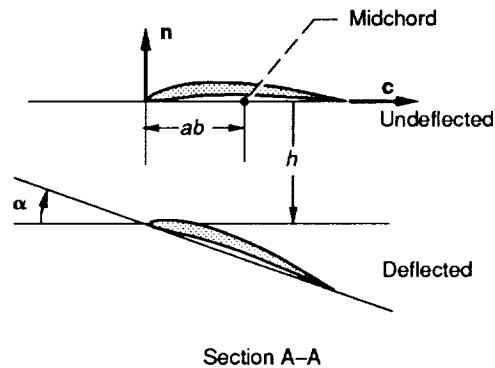

Figure 1. — ASTROP2 coordinate system for a propfan blade.

The required input to ASTROP2 can be classified as having two major data blocks in the following order of loading:

(1) Blade data and operating conditions: This group of data relates to the blade and its operating conditions. They are the problem title, the rotor speed Ω in revolutions per minute, the axial Mach number M, the reference frequency ω_0, the air density ρ, the number of blades on the rotor N, the number of points selected on the reference axis (NAEROP), and their locations on the blade in the global coordinate system (fig. 1). The code can accept these user inputs in the "keyword = value" format.

(2) Blade undeformed geometry, the number of modes (NMODE), and the blade's natural vibration frequencies and mode shapes: ASTROP2 needs the reference surface geometry of the propfan blade and its vibration frequencies, generalized masses, and mode shapes. If the finite element method with plate elements is used to model the blade, the reference surface is obtained from the midsurface grid points in space. At present the needed information is generated on the Cray X/MP computer at Lewis by using either MSC/NASTRAN or COSMIC/NASTRAN and is automatically read by ASTROP2. The routines in ASTROP2 that read the NASTRAN output are RDMSC and RDCOS. The input required for performing modal analysis using the COSMIC and MSC NASTRAN's is discussed in references 4 and 5, respectively. Sample job streams to execute these NASTRAN's at Lewis are given in figures 2(a) and (b) respectively. If NASTRAN is not used, the routines RDMSC and RDCOS must be replaced with similar routines suitable for reading the needed data.

Users who modify the code to provide these data must link the geometry and modal file to a FORTRAN logical unit. Presently a default value of 4 is used for this unit number, but the user can change the value by using the FORT keyword.

The output from ASTROP2 can be classified into five sections in the following order of printout:

(1) An echo listing of ASTROP2 input.
(2) Listing of specific blade data pertaining to
    (a) Reference line location, including the list of Y planes perpendicular to the blade axis and the list of the leading- and trailing-edge NASTRAN grid coordinates
    (b) X and Z coordinates of the user reference points (AEROPOINTS) in terms of Y coordinates
    (c) Tangent, chordline, and normal vectors at each reference point
    (d) The modal displacements and their derivatives at each reference point for all user-specified natural modes
    (e) The relative velocity at each reference point
    (f) Summary of these data in a report format

```
//jobname JOB (,dist), 'username', REGION=2560K,CLASS=A,
// MSGCLASS=A, MSGLEVEL=msglevel, TIME=(0,45), NOTIFY=userid,
// USER=userid, PASSWORD=XXXXXXXX
/*JOBPARM CARDS=90000
/*ROUTE PRINT VMLERC1.userid
/*ROUTE PUNCH VMLERC1.userid (not needed for DEST=VAX)
// EXEC NASTRAN, USERID=userid,JNAME=anyname,LINKLST=SLIST,DEST=VM (or VAX),
// TIME.NS=(0,30) (,VAXPW=XXXXXXXX for DEST=VAX)
//NSCOPY.INPUT DD *
***** INSERT COSMIC/NASTRAN INPUT for Modal Analysis *****
/*
//
```

(a) COSMIC/NASTRAN.

Figure 2. — Job streams for executing COSMIC/NASTRAN on Amdahl/MVS and MSC/NASTRAN on Cray/UNICOS at Lewis.

```
# USER=userid     PW=userpw
# QSUB -r     runmsc
# QSUB -eo
# QSUB -lm     1.0mw
# QSUB -lt     60
set -xvk
cd
rm  core
cat > nas_input << "EOF"
  .
    .
      .
         MSC/NASTRAN Input for Modal Analysis
    .
      .
         .
EOF
mscnast   in=nas_input   out=modal_output
rid=db.saved
```

(b) MSC/NASTRAN.

Figure 2. — Concluded.

(3) Listing of the lift submatrices ([L], [LP], and [LPP]) at all reference points for each interblade phase angle. Each submatrix is of order (2 x NMODE), where NMODE refers to the number of natural frequency modes included in the flutter analysis.

(4) Listing of the structural generalized mass, damping, and stiffness matrices and the aerodynamic submatrices for each interblade phase angle in the mistuned analysis.

(5) The aeroelastic frequencies and damping summary corresponding to the input operating condition. There are (N x NMODE) frequencies and damping values printed out, where N is the number of interblade phase angles (equal to the number of blades in the rotor) used in a mistuned analysis.

ASTROP2 Code Access

For the convenience of Lewis personnel the ASTROP2 code is readily available for execution on the Cray. Execute access has been provided for these users so that they need not take the trouble of obtaining the source code and then compiling and validating the code. The details for executing ASTROP2 on the Cray computer at Lewis are given here.

Use of ASTROP2 on Cray for Flutter Calculations

It is desirable for the user to review the material on obtaining the modal analysis output of NASTRAN (see refs. 5 and 6) before using ASTROP2 on the Cray. The procedure explained here guides the user on how to run ASTROP2 on the Cray.

(1) Execute NASTRAN and have the NASTRAN modal output on the Cray computer directed to the users directory. To execute NASTRAN at Lewis, use a job stream, given in figure 2, corresponding to COSMIC/NASTRAN or MSC/NASTRAN.

(2) Review the procedure on obtaining the numerical flutter frequency. This procedure is outlined in appendix B of this manual.

```
# USER=userid  PW=userpw
# QSUB -r runaap2
# QSUB -eo
# QSUB -lm  2.0mw
# QSUB -lt  60
set -xvk
cd
rm core
fetch  modal_out  -mUX  -t' fn=modal, ft=output' # from VM
#fetch modal_out  -mVX  -t' modal.output' # from VAX
chmod a+rx modal_out
astrop2=/aerospace2/smastrp/astrop2.exe
cat > aap2.inp <<"EOF"
   .

   .

   .

       ASTROP2 needed input

   .

   .

   .


EOF
ln  modal_out  fort.4
ln  aap2.inp  fort.5
$astrop2 < aap2.inp > astrop2.out
dispose  astrop2.out    -mUX  -t' fn=astrop2,ft=output' # to VM
#dispose astrop2.out  -mVX  -t' astrop2.output' # to VAX
```

Figure 3. — Job stream for executing ASTROP2 on Cray/UNICOS at Lewis.

(3) Set up a Cray job stream on a file as outlined in figure 3, and submit the job to the Cray for batch processing.

(4) If the flutter reference frequency input does not match with the calculated value, change the input to the calculated value and run it again. Repeat this step until the calculated and input frequency values agree.

(5) Repeat steps (3) and (4) for different operating conditions of the propfan or turbofan rotor system.

Sample Job Streams

A sample Cray job stream to run ASTROP2 at Lewis is given in figure 3. In this figure "modal_out" is the file name of the NASTRAN modal output obtained on executing the respective NASTRAN's. The input to ASTROP2 is contained in the file named "aap2.inp." The ASTROP2 output is written onto the file named "astrop2.out."

If user modifications are to be made to the ASTROP2 source code, figure 4 gives the Cray job stream for compiling and linking the ASTROP2 source code under UNICOS. In this figure the ASTROP2 source is contained in a file named "caap2.f," and "cft77" and "segldr" are the UNICOS commands to compile and link the code, respectively. The resulting ASTROP2 executable is stored in the file named "astrop2." A corresponding Cray job stream to run the modified ASTROP2 program under UNICOS is given in figure 5.

8

```
# USER=userid  PW=userpw
# QSUB -r compile
# QSUB -eo
# QSUB -lm  2.0mw
# QSUB -lt 60
set        -xvk
cd
rm   core
cft77   -l   list   -f    caap2.f
cat>segldr.inp<<"EOF"
Lib=/tpsw/imsl/imslib.a
dupentry=ignore
EOF
segldr   -o astrop2.exe  caap2.o segldr.inp
chmod a+rx astrop2.exe
```

Figure 4. — Job stream for compiling and linking modified ASTROP2 source code on Cray/UNICOS at Lewis.

```
# USER=userid   PW=userpw
# QSUB -r runaap2
# QSUB -eo
# QSUB -lm  2.0mw
# QSUB -lt 60
set -xvk
cd
rm core
fetch modal_out      - mUX      -t'fn=modal, ft=output' # from VM
chmod    a+rx       modal_out
astrop2=/aerospace2/smastrp/astrop2.exe
cat > aap2.inp <<"EOF"
 .
 .
 .
ASTROP2 needed input
 .
 .
 .
EOF
ln   modal_out     fort.4
ln       aap2.inp     fort.5
$astrop2 < aap2.inp > astrop2.out
dispose    astrop2.out   -mUX   -t'fn=astrop2,ft=output' # to VM
```

Figure 5. — Job stream for executing modified ASTROP2 on Cray/UNICOS at Lewis.

9

Description of User Input

The ASTROP2 code was developed with flexibility of input. Also, the program has user-friendly input descriptions. Each input is entered with a carriage return (ENTER key on some keyboards) as the last entry on an input line. Each ASTROP2 input line, thus entered, forms a logical record of input in the computer. The length of the input record is restricted to 80 printable characters.

Every ASTROP2 input line can be entered in a free format of the form

keyword = value

where **keywords** are names defined for the various needed input. An equal sign must follow the keyword on the line. The **value** on the right-hand side of the equal sign pertains to the self-explanatory keyword. There can be any number of blank spaces between the keyword and the equal sign or between the equal sign and the value (or values). Also, the keyword can begin at any place on the logical record; it need not necessarily start on the first column.

Some input to this code requires several values for the same keyword. In such cases several values may be entered with a space between the values. Continuation of several values on multiple lines is handled with a plus sign symbol (+) as the last character on the current line. The line immediately following the line with the + symbol must contain values for the keyword that needed multiple lines.

The ASTROP2 user can enter comments either on any input line or between the lines by prefixing the information with a dollar sign symbol ($). ASTROP2 skips any information on the input line past the $ symbol.

The following pages provide detailed descriptions of available keywords and valid values for the ASTROP2 code.

Input Data Keyword

AEROPOINTS COORDINATES

Description

This input set consists of NAEROP values for this keyword. Each value represents the global Y coordinate of the corresponding aeropoint. User must enter values in a numerical ascending order.

Format

AEROPOINTS COORDINATES = coord_value1 value2...

Default value

None. User must supply these values.

Units

Inches

Example

AEROPOINTS COORDINATES = 3.25 3.5922 +
    4.4717 5.6361 6.9739 8.3736 +
    9.7114 10.8758 11.7553 12.25

Remarks

1. It is assumed that NAEROP = 10 in the above example.
2. The program will compute the X and Z coordinates of the aeropoint corresponding to its Y values by interpolating values from the undeformed surface coordinates of the blade.

Input Data Keyword

AFDIR — axial flow direction

Description

This input defines the axial flow direction with respect to the NASTRAN coordinate system. Valid values are either X, Y, Z, $-X$, $-Y$, or $-Z$. However, this value cannot be the same as the value for the BSAXIS keyword.

Format

AFDIR = X, Y, Z, $-X$, $-Y$, or $-Z$

Default value

AFDIR = X

Example

AFDIR = Z

Remarks

1. This keyword along with the BSAXIS keyword will cause the transformation of the NASTRAN coordinate system into the ASTROP2 coordinate system.
2. The positive direction of the axial flow is defined as the projection on the X axis of a line joining the leading and trailing edges of the blade.

Input Data Keyword

AIR DENSITY

Description

This input defines the density of air that the blade is operating in at the analysis condition.

Format

AIR DENSITY = density_value

Default value

No default. User must supply this value.

Units

Pound second squared per inch to the fourth power

Example

RHOA = 1.1E-07

Remarks

The user can write RHOA as the keyword instead of AIR DENSITY.

Input Data Keyword

AMAT — aeromatrix computation

Description

This input defines an indicator for ASTROP2 either to compute the aeromatrix for use in the flutter analysis or to skip it.

Format

AMAT = ind_value

Default value

AMAT = yes

Example

AMAT = no

Remarks

1. The option of AMAT = no is equivalent to the modal vibration analysis of the blade, since aerodynamic forces are suppressed in the generalized equations of blade motion.
2. This option is useful for checking if the computed natural frequencies of the blade are the same as those in the NASTRAN model.

Input Data Keyword

    BSAXIS — blade span axis

Description

    This input defines the blade span axis of the NASTRAN model. Valid values are either X, Y, or Z. However, this value cannot be the same as the value for the AFDIR keyword.

Format

    BSAXIS = X, Y, or Z

Default value

    BSAXIS = Z

Example

    BSAXIS = Y

Remarks

    This keyword, along with the AFDIR keyword, will allow the easy transformation of the NASTRAN model into the ASTROP2 coordinate system.

Input Data Keyword

CAL—calculation method for interpolated modal translations and rotations at the aeropoints

Description

This input tells the ASTROP2 program the method to be used for interpolating translations and calculating the average rotational values at the aeropoints. Three valid values to this input are the numerals 1, 2, and 3. Values 1 and 2 refer to the calculation methods for rotational values at the grids used for interpolation. Value 3 to this input will skip these methods, so that the user can read in his or her own modal translation and rotational values and their derivatives at the aeropoints. In both methods 1 and 2 the NASTRAN displacements at the grids are used for interpolation. In method 1 the rotational values at the grids on the leading edge are calculated and averaged from varying translation values at the grids on the corresponding chordline. In method 2 the rotational values at the grids on the leading edge are averaged from the NASTRAN rotational values at grids on the corresponding chordline. From the translations and rotations at the grids on the leading edge, the values at the aeropoints along the leading edge of the blade are interpolated.

Format

CAL = value

Default value

CAL = 2

Example

CAL = 1

Remarks

None

Input Data Keyword

CASCADE — cascaded aerodynamic analysis indicator

Description

This input determines if unsteady, cascaded, or isolated aerodynamic lifts and moments are to be computed for the aeroelastic analysis. Valid values are "yes" or "no." If "yes," cascade effects are included in the unsteady aerodynamic lift and moments, under compressible airflow, for use in the flutter analysis.

Format

CASCADE = yes or no

Default value

CASCADE = yes

Remarks

1. This keyword with a value of "no" is identical in effect to the ISOLATED keyword.
2. Either this or the ISOLATED keyword is needed, but not both of them.

Input Data Keyword

FRF — flutter reference frequency ω_0

Description

This input defines the assumed flutter frequency of the turboprop.

Format

FRF = flutter_reference_freq_value

Default value

None. The user must supply this value.

Units

Radians per second

Examples

1. FRF = 1600
2. FLUTTER REFERENCE FREQUENCY = 800

Remarks

An initial entry may equal one of the natural frequencies of the blade.

Input Data Keyword

FSF — frequency scaling factor

Description

This input defines frequency scaling factors for a flutter analysis. The ratios are used by ASTROP2 as multiplying factors of the corresponding natural vibration frequencies obtained from the NASTRAN modal output. The number of ratio values in this input must equal the number of modes used in the flutter analysis.

Format

FSF = fsf_values

Default value

FSF = 1.0 1.0 1.0 1.0

Example

FSF = 0.9 0.9 0.95 1.0

Remarks

The number of values provided in the example assumes that four modes are used in the flutter analysis.

Input Data Keyword

FORT — FORTRAN unit number

Description

This input defines the FORTRAN logical unit number associated with a data set that contains NASTRAN modal analysis output. The defined value must be positive and greater than zero.

Format

FORT = unit_number

Default value

FORT = 4

Examples

1. FORT = 8
2. FORTRAN UNIT NUMBER = 8

Remarks

1. The value of the FORT or FORTRAN UNIT NUMBER keyword can be any number other than 5 or 6.
2. This unit must be linked with the modal output prior to executing ASTROP2.
3. The FORTRAN compiler restricts the maximum unit number to 99.

Input Data Keyword

GDAMP — generalized damping

Description

This input defines the generalized damping ratio for the normal modes used in the flutter analysis. The input set consists of NMODE values for this keyword. The default value of 0 is used for GDAMP if this keyword is not present in the input.

Format

GDAMP = gdamp_value

Default value

GDAMP = 0.0 0.0

Examples

1. GDAMP = 0.02 0.02
2. GENERALIZED DAMPING = 0.02 0.02

Remarks

1. The user can skip this keyword if the default values are to be used for GDAMP.
2. The number of default GDAMP values must equal the user-specified value for the NMODE keyword.

Input Data Keyword

GMASS — generalized masses

Description

This input defines the generalized mass values for the normal modes used in the flutter analysis. The input set consists of NMODE values for this keyword.

Format

GMASS = gmass_values

Default value

None

Examples

1. GMASS = 1.0 1.0 1.0 1.0
2. GENERALIZED MASSES = 1.0 1.0 1.0 1.0

Remarks

1. The user must provide the generalized mass values needed in the analysis.
2. The number of generalized mass values must equal the user-specified value for the NMODE keyword.
3. The generalized mass values can be computed and printed in NASTRAN.
4. The generalized masses must be real, positive values.

Input Data Keyword

    IDEBUG — print debug parameter

Description

    This input defines an optional debug print parameter value. The debug print parameter value can be either 0 or 1. A value of 1 will activate the listing of all intermediate computational values for debugging purposes. A value of 0 will suppress such an intermediate computational values listing.

Format

    IDEBUG = debug_param_value

Default value

    IDEBUG = 0

Example

    IDEBUG = 1

Remarks

    For a debug value of 1 the output listing will be large. In most production runs it is best to use the default value of 0.

Input Data Keyword

IPA — interblade phase angle (or angles)

Description

This input defines the values of a set of interblade phase angles for use in the analysis. The number of values in the set is equal to the value specified by the NIPA keyword. If a NIPA keyword does not exist, the set of values will be computed by the program.

Format

IPA = ipa_value

Default value

None

Units

Degrees

Examples

1. IPA = 225
2. IPA = 0 90 180 270

Remarks

Examples 1 and 2 correspond to NIPA values for the corresponding examples under the NIPA keyword.

Input Data Keyword

    ISOLATED — isolated blade indicator

Description

    This input determines if the unsteady, cascaded, or isolated aerodynamic lifts and moments are to be computed for the aeroelastic analysis. Valid values are "yes" or "no." If the given blade is isolated, no cascaded effects are present, and hence the unsteady, noncascaded aerodynamic lifts and moments, under compressible airflow, should be computed and used in the flutter analysis.

Format

    ISOLATED = yes or no

Default value

    ISOLATED = no

Remarks

    1. This keyword is identical in effect to the CASCADE keyword with a value of "no."
    2. Either this or the CASCADE keyword is needed, but not both of them.

Input Data Keyword

LECOY — lowest Y coordinate on leading edge

Description

This input defines the lowest Y coordinate value on the leading edge of the blade. This value helps ASTROP2 to determine one end (the beginning) of the blade leading edge. The other end is defined by the input blade geometry.

Format

LECOY = lecoy_value

Default value

None. The user must provide this keyword and a value as ASTROP2 input.

Units

Inches

Examples

1. LECOY = 3.25
2. LECOY = 0.0

Remarks ·

None

Input Data Keyword

LENODES — leading-edge nodes

Description

This input defines the user-defined nodes of the NASTRAN input geometry that lie on the blade leading edge. The set of nodes defined by this keyword will define the reference line (blade leading edge) for use in the flutter analysis. This keyword is an option for the user in cases where the program cannot identify the leading-edge nodes from the input blade geometry. The two such cases are (1) when chordwise NASTRAN nodes are not parallel to the axial flow direction and (2) when the change in the Y coordinate of consecutive chordwise nodes (excluding a leading-edge node) exceeds a value of 0.05 in.

Format

LENODES = lenode_values

Default value

The leading-edge nodes of the NASTRAN model with ASTROP2 Y coordinates greater than LECOY.

Example

LENODES = 51 59 67 76 85 94 103 112 121 130 139 148 +
157 166 175 184 193 202 211 220

Remarks

1. The ASTROP2 program assumes that the chordwise NASTRAN nodes of the input geometry are on lines parallel to the axial flow direction in order to identify the unique leading-edge nodes automatically. If this is not the case, the LENODES keyword must be used to define the reference line.
2. This keyword is not required but is recommended because the automatic leading-edge search algorithm is not robust enough to work for all blade finite element models.

Input Data Keyword

MACH — axial Mach number

Description

This input relates to the free-stream velocity, which is assumed to be parallel to the rotor axis. The input is in terms of Mach number, which is the ratio of the free-stream velocity to the speed of sound.

Format

MACH = Mach_number_value

Default value

None. The user must provide this keyword and a value as ASTROP2 input.

Examples

1. MACH = 0.4
2. MACH = 0.8

Remarks

1. The program will convert the Mach number value into the free-stream velocity when needed.
2. The user can write MACH NUMBER as the keyword with a blank space after MACH.

Input Data Keyword

        MISTUNED — mistuned analysis indicator

Description

        This input defines the nature of parameter changes between the various blades of the rotor. Mistuned analysis assumes that all blades are not identical. ASTROP2 handles input of randomly mistuned frequencies and masses but assumes identical mode shapes for the rotor blades. Keywords RMF and RMM are used to define the randomly mistuned frequency and mass values.

Format

        MISTUNED = yes or no

Default value

        MISTUNED = no

Remarks

        Please refer to the RMF and RMM keywords for additional input to be given along with this keyword.

Input Data Keyword

MODAL FREQUENCIES — modal frequencies of structure

Description

This input defines the modal (natural) frequencies of the structure. It is not normally required. The number of values for this input must match the number input on the NMODE keyword.

Format

MODAL FREQUENCIES = freq_values

Default value

In the absence of this keyword the modal frequency values are read from the NASTRAN modal output.

Examples

1. MODAL FREQUENCIES = 200 320 600 800
2. MFR = 200 320 600 800

Remarks

1. The example assumes that NMODE = 4.
2. MFR can be used as the keyword instead of MODAL FREQUENCIES.

Input Data Keyword

NAEROP — number of aeropoints

Description

This input defines the number of points on the reference line that will identify the midpoints of selected aerodynamic strips on the blade. The reference line used in ASTROP2 is the blade leading edge (LE). An explicit POINTS can also be used as the keyword.

Format

NAEROP = np_value

Default value

NAEROP = 7

Examples

1. NAEROP = 6
2. NAEROP = 8
3. POINTS = 10

Remarks

The default number of aeropoints is set to 7. The user can override this value with any valid number up to 20. The recommended value for good flutter results is any number greater than 6.

Input Data Keyword

NASTRAN TYPE

Description

This input defines the type of NASTRAN used in performing the modal analysis. The valid values are either MSC or COS (for COSMIC).

Format

NASTRAN TYPE = MSC or COS

Default value

None. The user must provide this keyword and specify a type as ASTROP2 input.

Example

NASTRAN TYPE = MSC

Remarks

This input lets ASTROP2 decide on the read format of NASTRAN modal output.

Input Data Keyword

NBLADES — number of blades

Description

This value defines the number of blades mounted on the rotor. It is used in the interblade phase angle calculations, and in turn, in the unsteady aerodynamic cascaded anlaysis.

Format

NBLADES = nb_value

Default value

None. The user must provide this keyword and specify a value as ASTROP2 input.

Examples

1. NBLADES = 10
2. NBLADES = 4
3. NBLADES = 1

Remarks

The value of NBLADES = 1 is for an isolated blade analysis.

Input Data Keyword

NIPA — number of interblade phase angles

Description

This input defines the number of interblade phase angles to be used in performing a tuned flutter analysis.

Format

NIPA = nipa_value

Default value

NIPA = number_of_blades value

Examples

1. NIPA = 1
2. NIPA = 4

Remarks

Do not use the NIPA keyword in a mistuned analysis.

Input Data Keyword

NMODE — number of normal modes

Description

This value defines the number of normal modes to be used in the flutter analysis. The information regarding the blade mode shapes will be read from the provided NASTRAN modal output.

Format

NMODE = nmode_value

Default value

NMODE = 4

Examples

1. NMODE = 2
2. NORMALMODES = 2

Remarks

1. The nmode_value can be any value from 1 to 6.
2. If the NASTRAN modal output contains less than six normal modes, this input must be less than or equal to the number of normal modes in the output.

Input Data Keyword

PRP — print parameter for NASTRAN geometry

Description

This input defines a print parameter for the option either to print the NASTRAN geometry model or to skip it. The model is printed in the ASTROP2 coordinate system. The valid value can be either 1 or 0. A value of 1 will trigger the geometry model printout in the ASTROP2 output.

Format

PRP = prp_value

Default value

PRP = 0

Examples

1. PRP = 1
2. PARAMETER FOR PRINT GEOMETRY = 1

Remarks

This paremter helps to check the geometry as read from the NASTRAN modal output.

Input Data Keyword

RMF — randomly mistuned frequencies

Description

This input defines the frequency values in the analysis of a randomly mistuned rotor. Each RMF card provides mistuned frequency values for a specific rotor blade. The number of frequency values in each RMF input must equal the number of modes used in this flutter analysis. The user must specify as many RMF cards as the number of blades in the rotor. The order of RMF cards is not important, but the blade number must increase sequentially starting with a value of 1.

Format

RMF = blade_no   rmf_values

Default value

The NASTRAN natural frequency values for a blade are assumed for all blades. The user can provide this keyword in a mistuned flutter analysis (when the TUNED = no keyword is present) and specify values as ASTROP2 input.

Units

Cycles per second

Example

RMF = 1   210.0   530.0
RMF = 3   210.0   530.0
RMF = 2   208.0   525.0
RMF = 4   208.0   525.0

Remarks

The example assumes that two modes are used in the flutter analysis of a four-blade rotor.

Input Data Keyword

RMM — randomly mistuned masses

Description

This input defines scale factors that multiply the generalized masses specified on the GMASS card. These are used to analyze a randomly mass mistuned rotor. Each RMM card provides mistuned mass values for a specific rotor blade. The number of mass values in each RMM input must equal the number of modes used in this flutter analysis. The user must specify as many RMM cards as the number of blades in the rotor. The order of RMM cards is not important, but the blade number must increase sequentially starting with a value of 1.

Format

RMM = blade_no   rmm_values

Default value

The NASTRAN generalized mass values for a blade are assumed for all blades. The user can provide this keyword in a mistuned flutter analysis (when the TUNED = no keyword is present) and specify values as ASTROP2 input.

Example

RMM = 1   0.998   1.004
RMM = 2   0.99   0.97
RMM = 4   1.003   0.992
RMM = 3   1.005   1.002

Remarks

The example assumes that two modes are used in the flutter analysis of a four-blade rotor.

Input Data Keyword

RPM — rotational speed

Description

This input defines the rotational speed of the rotor. The positive rotation vector of the rotor is along the positive axial flow direction.

Format

RPM = rpm_value

Default value

None. The user must provide this keyword and specify a value as ASTROP2 input.

Units

Revolutions per minute

Example

RPM = 7500

Remarks

1. The program will convert this value into revolutions per second, as needed.
2. The RPM value can be less than or equal to 0 (RPM $\leq$ 0).

Input Data Keyword

SPS — speed of sound

Description

This input defines the speed of sound through air.

Format

SPS = sps_value

Default value

SPS = 1130

Units

Feet per second

Example

SPS = 1140

Remarks

The program will convert this value into inches per second, as needed.

Input Data Keyword

STRUCTURAL SWEEP ANGLES

Description

    This input defines the structural or geometric sweep angles at aeropoints along the blade leading edge. This angle is the angle between the leading-edge tangent vector and the Y axis. If this keyword does not exist in ASTROP2 input, these values are computed by the program for use in the flutter analysis. If this keyword is used with nonzero values, the number of values input to this keyword must equal the number of aeropoints defined in the input.

Format

    STRUCTURAL SWEEP ANGLES = set_values

Default value

    As computed by ASTROP2

Example

    1. STRUCTURAL SWEEP ANGLES = 15 15 15 15 15 15 15 15
    2. STS = 45 45 45 45 45 45 45 45

Remarks

    1. Positive values of this keyword denote the backward sweep of the blade, and negative values denote forward sweep. In most cases, the values will be positive.
    2. An increase in sweep values will decrease the effective free-stream velocity in the analysis.

Input Data Keyword

    S/C — gap-to-chord ratio

Description

    This input defines the gap-to-chord ratios at each aeropoint on the blade reference axis. A value of 0 can be entered to use the computed s/c values within the program. The number of values input to this keyword must equal the number of aeropoints defined.

Format

    S/C = gap_to_chord values

Default value

    As computed by ASTROP2

Examples

    1. S/C = 50.0 50.0 50.0 50.0 50.0 50.0 50.0
    2. GAP TO CHORD RATIO = 0.0 (same as default)

Remarks

    1. These values are used in the unsteady cascaded aerodynamic calculations and have a strong effect on the flutter speed.
    2. If the user-input values are greater than 0.001, ASTROP2 will not compute s/c values.
    3. Computed s/c values depend on the location of the aeropoints, the number of blades, and the chord length at the corresponding aeropoint.

42

Input Data Keyword

TECOY — lowest Y coordinate on trailing edge

Description

This input defines the lowest Y coordinate value on the trailing edge of the blade. It helps ASTROP2 to determine one end (the beginning) of the blade trailing edge. The other end is defined by the input blade geometry.

Format

TECOY = tecoy_value

Default value

None. The user must provide this keyword and a value as ASTROP2 input.

Units

Inches

Examples

1. TECOY = 4.0
2. TECOY = 0.0

Remarks

None

Input Data Keyword

TENODES — trailing-edge nodes

Description

This input defines the user-defined nodes of the NASTRAN input geometry that lie on the blade trailing edge. The set of nodes defined by this keyword will define the trailing edge for use in the flutter analysis. This keyword is an option for the user in cases when the program cannot identify the trailing-edge nodes automatically from the input blade geometry. See the LENODES keyword for these cases.

Format

TENODES = tenode_values

Default value

The trailing-edge nodes of the NASTRAN model with ASTROP2 Y coordinates greater than TECOY.

Example

TENODES = 75 84 93 102 111 120 129 +
138 147 156 165 174 183 192 +
201 210 219 228

Remarks

1. The ASTROP2 program assumes that the chordwise NASTRAN nodes of the input geometry are on lines parallel to the axial flow direction. This helps in automatically identifying the unique nodes on the trailing edge. If this is not the case, the keyword TENODES must be used to define the trailing edge of the blade.
2. The number of nodes in TENODES need not be the same number as in the keyword LENODES.

Input Data Keyword

TITLE

Description

This input defines the title for a particular problem run.

Format

TITLE = title_for_the_problem

Default value

None

Example

TITLE = Test case of SR3C-X2 blade using Cray COSMIC/NASTRAN output

Remarks

Since the length of a line is limited to 80 characters, the maximum length of the problem title is 74 characters.

Input Data Keyword

TUNED — tuned analysis indicator

Description

This input defines a tuned analysis and assumes that all blades are identical.

Format

TUNED = yes or no

Default value

TUNED = yes

Example

TUNED = no

Remarks

1. If the flutter analysis is for a tuned cascade, one-blade information is enough for ASTROP2.
2. For mistuned flutter analysis, randomly mistuned frequency and mass values are the additional input that may be given. Please refer to the MISTUNED card for mistuned analyses.
3. This keyword with a value of "no" is identical to the MISTUNED keyword with a value of "yes."

Examples of Input

Two examples are considered for demonstrating ASTROP2. The ASTROP2 results are compared with the experimental data that are available for these examples. The aerodynamic models used in these examples are quite different. The first model pertains to the classical wing and uses unsteady isolated-airfoil theory (ref. 7). The second model is a single-rotor propfan with eight composite blades rotating at a given speed about the rotor axis and uses unsteady, cascade aerodynamic theory (ref. 8).

Although the ASTROP2 code was developed for propfans, the classical wing example is provided as an easy-to-run example for the user's convenience. The second example, with a multibladed rotor, is more appropriate for analysis by ASTROP2. These examples are discussed here in detail, including the input to ASTROP2, the output from ASTROP2, and the comparative results of analyses and experiments.

Example 1

This example, a flat-plate wing with 15° of feedback, is being provided to give users an analysis case that can easily be duplicated, in order to verify the working of ASTROP2 on their systems. However, it is an unusual type of structure to be analyzed with ASTROP2, which is specifically for rotating multiblade rotors. For this example, an isolated-wing aerodynamic module was added to the code. However, for multiblade rotors a cascade aerodynamic module is used. This wing has been tested in a wind tunnel for flutter at subsonic and supersonic speeds, and the results have been reported in reference 9. Flutter analyses for this wing have also been performed in reference 10. Comparison will be made in this example between the analysis from ASTROP2 and references 9 and 10 at a Mach number of 0.45. The 15° sweptback model had a constant chord of 2.07055 in. and a semispan of 5.52510 in. The wing geometry is shown in figure 6.

MSC/NASTRAN has been used in reference 10 to compute the analytical flutter, and the wing model is represented as both a beam and a plate. For convenience the NASTRAN input data for this example are given in appendix C. The flutter velocity and frequency for the beam and plate models of the wing are included in this report, as shown in table I.

Since isolated-wing aerodynamics is required for this analysis, the keyword CASCADE must be set to "no." This causes ASTROP2 to compute the appropriate aerodynamic noncascade coefficients, including the compressible effects of air.

Input to ASTROP2. — The input to ASTROP2 at flutter conditions is given in figure 7. The input data in this figure pertain to the 15° sweptback wing. The first four natural frequencies are used for the flutter calculations.

Output from ASTROP2. — An overview of output from ASTROP2 is given in the introduction to part 1. The output listing from ASTROP2 is not provided in this report. A summary of the tables similar to those given under example 2 is not provided for this example and is easily understandable from the ASTROP2 listing.
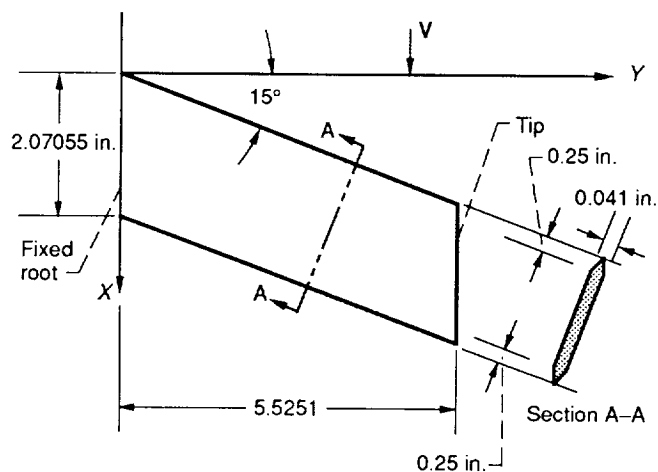

Figure 6. — 15° Sweptback wing model.

TABLE I. — COMPARISON OF MEASURED AND CALCULATED

FLUTTER CONDITIONS OF 15° SWEPTBACK WING

[Operating conditions: density, 1.1092×10^{-7} lbm sec²/in.⁴; speed of sound, 1135 ft/sec.]

	Flutter frequency, Hz	Flutter velocity, ft/sec	Reduced flutter frequency
MSC/NASTRAN (noncascade theory):			
Beam model	134	509	0.2854
Plate model	113	483	.2536
ASTROP2 (noncascade theory): Plate model	92.3	477	0.2098
Experimental result	120	495	0.2628

TITLE = Subsonic Flutter of 15 Degree Swept Blade - HA75E
IDEBUG = 0 $An Optional Debug Print
RPM = 0.0 $Rotational Speed in RPM
MACH NUMBER = 0.42 $Axial Mach Number Value
FLF= 580.0 $Estimated Flutter Frequency
AIR DENSITY = 1.0726E-07 $Density of Air
POINTS = 8.0 $Number of Aeropoints
AMAT = Yes $ If AMAT = Yes then Construct a New Aero Matrix
NBLADES = 1.0
AEROPOINTS COORDINATES = 0.0 0.7893 1.5786 2.3679 3.1572 +
                         3.9465 4.7358 5.5251
NMODE = 2.0 $Number of EIGENMODES Used
BSAXIS = Y $ Blade Axis
GMASS=0.00002414 0.00000868 0.00000850 0.00000762 $Generalized Mass
STRUCTURAL SWEEP ANGLES AT AEROPOINTS = 15 15 15 15 15 15 15 15
TUNED = Yes $Tuned Analysis
ISOLATED = Yes
NASTRAN TYPE = MSC $Could be COS or MSC
FORTRAN UNIT NUMBER = 4 $ NASTRAN Modal Output Unit
PRP = 1.0 $Parameter for Print Geometry
LECOY = 0.0 $Lowest Blade Axis Coordinate Value on Leading Edge
TECOY = 0.00 $Lowest Blade Axis Coordinate Value on Trailing Edge
NIPA = 1 $Number of Interblade Phase Angle
IPA = 0 $Interblade Phase Angle

Figure 7. — Input to ASTROP2 for example 1.

Results and discussion. — Table I compares the flutter frequency and velocity given by ASTROP2 and MSC/NASTRAN with experimental values. It can be seen that ASTROP2 gives a conservative flutter velocity. However, the MSC/NASTRAN flutter velocities bracket the experimental value. Further explanation of the results is outside the scope of this guide.

Example 2

Figure 8 shows a single rotor with eight composite propfan blades (SR3C-X2). This model was tested at Lewis; and subsonic wind tunnel flutter data are presented in reference 11. This model is a typical example of the type of rotor that can be analyzed by using ASTROP2.

A flutter condition was computed for the SR3C-X2 model and compared with the experimental results. In this rotor model, all blades are assumed to be identical. Tuned cascade analysis is used to compute the flutter condition. The finite element model of one SR3C-X2 blade is shown in figure 9. The

48
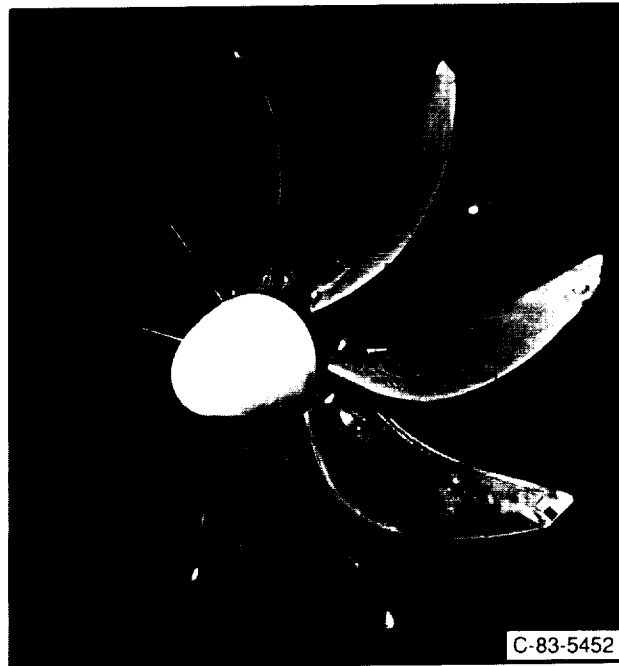
Figure 8. — SR3C-X2 propfan model in Lewis 8- by 6-Foot Supersonic/Transonic Wind Tunnel.
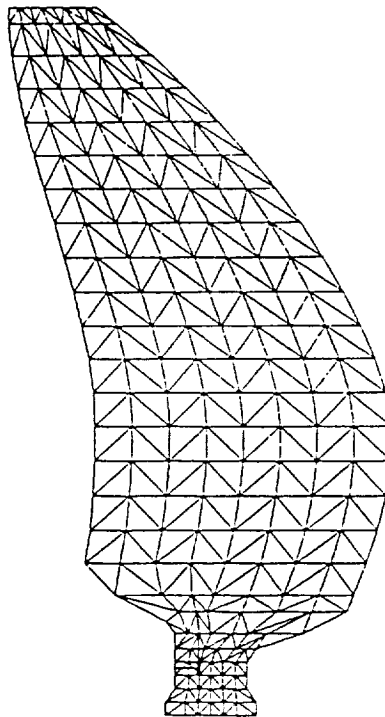
Figure 9. — Finite element model of SR3C-X2 propfan blade.

blade hub is assumed to be rigid. The blade vibration frequencies are calculated by COSMIC/NASTRAN. The access to NASTRAN input data for this example is given in appendix C. Four modes of blade vibration were used in this analysis. It has been found for this rotor that the first three coupled vibration modes are enough to predict the flutter speed (ref. 12). Table II gives the computed natural frequencies of the blade.

49

TABLE II. — GENERALIZED MASS, DAMPING,

AND STIFFNESS AND NATURAL FREQUENCIES

OF SR3C-X2 BLADE FROM NASTRAN

[Generalized mass, 1.0; generalized damping, 0.]

Mode	Generalized stiffness	Natural frequency for 6100 rpm and $\beta_{0.75R} = 61.2°$, Hz
1	1.93×10^6	221.08
2	6.38×10^6	402.13
3	1.92×10^7	698.20
4	2.63×10^7	816.94

TABLE III. — SELECTED AEROPOINTS ON

LEADING EDGE OF SR3C-X2 BLADE

Aeropoint	Global coordinate values		
	X	Y	Z
1	-2.1073	3.2500	0.3928
2	-2.2573	3.5922	.5111
3	-2.5878	4.4717	.7933
4	-2.8255	5.6361	1.0825
5	-2.5771	6.9739	1.1761
6	-1.8329	8.8736	.9662
7	-.8097	9.7114	.4315
8	.3191	10.8758	-.3642
9	1.2933	11.7553	-1.1815
10	1.9048	12.2500	-1.7386

The ASTROP2 code uses the blade leading edge as the reference axis for the aerodynamic analysis. Ten aerodynamic strips are selected on the blade in computing the generalized aerodynamic forces. The midlines (chords) of each strip intersect on the reference line at a point hereinafter referred to as the "aeropoint." Table III gives the location of the selected aeropoints on the undeformed SR3C-X2 blade in the global coordinate system (fig. 1). A sample blade cross section through the chord at any point can also be seen in figure 1.

Input to ASTROP2. — The input to ASTROP2 at the flutter condition is provided in figure 10. The input data in this figure pertain to the SR3C-X2 propfan blade.

Output from ASTROP2. — An overview of output from ASTROP2 is given in the introduction to part 1. The complete listing of the ASTROP2 output is not provided in this report for space considerations. However, the important pages of the output listing are summarized here. Tables II to V summarize the various input and other ASTROP2 computed quantities at each user-specified aeropoint on the blade. Table VI lists the directions of the tangent, chordline, and normal vectors as computed by ASTROP2. Table VII lists the plunging and pitching displacements and their first two derivatives for the first two mode shapes. Similar plunging and pitching displacements and their derivative values are printed out for every mode shape used.

Results and discussion. — Table VIII compares ASTROP2 flutter results with the experimental values. The ASTROP2 results in this table pertain to the tuned cascade of SR3C-X2 blades for a rotational speed of 6080 rpm at flutter. The flutter damping is the real part of the eigenvalue obtained from the solution for γ in equation (13) of part 2. A plot of the real part of the eigenvalue versus axial Mach number at a blade setting angle of 61.2° is shown in figure 11. Table VIII and figure 11 show that the analytically computed flutter velocity is conservative relative to the experimental value.

TITLE = Test Case of SR3C-X2 Blade Using Cray COSMIC/NASTRAN Output (alpha from ROT)
IDEBUG = 0 $An Optional Debug Print
RPM = 6080. $Rotational Speed in RPM
SPS = 1130.0 $Speed of Sound in ft/sec
MACH NUMBER = 0.60 $Axial Mach Number Value
FLF = 1640 $Estimated Flutter Frequency
NAEROP = 10.0 $Number of Aeropoints used
AIR DENSITY = 0.9976E.07 $Density of Air
AMAT = YES $ If AMAT = Yes Then Construct a New Aero Matrix
BSAXIS = Z $Blade Span Axis
NBLADES = 8.0 $Number of blades
$
AEROPOINTS COORDINATES = 3.25   3.5922 4.4717 5.6361 6.9739 +
                         8.3736   9.7114   10.8758   11.7553 12.25
$
EIGENMODES = 4.0 $Number of EIGENMODES Used
GENERALIZED MASS = 1.0 1.0 1.0 1.0 $Generalized Mass
CRITICAL DAMPING VALUES = 0.0
STRUCTURAL SWEEP ANGLES AT AEROPOINTS = 0
GAP TO CHOR RATIO = 0.0
TUNED = YES $Tuned Analysis
NASTRAN TYPE = COS
FORTRAN UNIT NUMBER = 4
PARAMETER FOR PRINT GEOMETRY = 0.0 $Parameter for Print Geometry
LECOY = 3.25 $Lowest Blade Axis Coordinate Value on Leading Edge
TECOY = 4.0 $Lowest Blade Axis Coordinate Value on Trailing Edge
NIPA = 1 $Number of Interblade Phase Angle
IPA = 225 $Interblade Phase Angle

Figure 10. — Input to ASTROP2 for example 2.

TABLE IV. — GEOMETRICAL PROPERTIES OF SR3C-X2 BLADE AT SELECTED AEROPOINTS

Aeropoint	Global Y coordinate	Geometrical sweep angle, deg	Semichord, in.	Stagger angle, deg	Gap-to-chord ratio, s/c
1	3.2500	-30.25	2.26	9.63	0.749
2	3.5922	-28.31	2.31	11.23	.782
3	4.4717	-24.05	2.46	15.04	.857
4	5.6361	-10.51	2.45	18.34	.942
5	6.9739	22.46	2.50	17.64	.948
6	8.3736	35.28	2.38	16.57	1.156
7	9.7114	45.74	2.04	14.58	1.591
8	10.8758	53.44	1.66	11.45	2.267
9	11.7553	57.26	1.29	9.16	3.257
10	12.2500	61.61	1.05	6.86	4.225

TABLE V. — AERODYNAMIC PROPERTIES OF SR3C-X2

BLADE AT SELECTED AEROPOINTS

Aeropoint	Global Y coordinate	Reduced frequency, k	Effective Mach number, M_e	Effective velocity, in./sec
1	3.2500	0.5110	0.5351	7256
2	3.5922	.5097	.5491	7446
3	4.4717	.5107	.5815	7885
4	5.6361	.4585	.6467	8769
5	6.9739	.4781	.6338	8594
6	8.8736	.4900	.5868	7957
7	9.7114	.4686	.5262	7135
8	10.8758	.4268	.4694	6366
9	11.7553	.3527	.4420	5993
10	12.2500	.3211	.3971	5384

Aeropoint	Global Y coordinate	Tangent vector component			Chordline vector component			Normal vector component		
		X	Y	Z	X	Y	Z	X	Y	Z
1	3.2500	-0.3991	0.8638	0.3072	0.8667	0.4700	-0.1672	-0.2888	0.2003	-0.9363
2	3.5922	-.3693	.8805	.2975	.8770	.4392	-.1947	-.3020	.1889	-.9344
3	4.4717	-.2996	.9130	.2762	.8953	.3621	-.2595	-.3370	.1694	-.9261
4	5.6361	-.0485	.9831	.1758	.9440	.0997	-.3146	-.3268	.1507	-.9330
5	6.9739	.3801	.9243	-.0378	.8786	-.3690	-.3030	-.2941	.0817	-.9523
6	8.8736	.5341	.8163	-.2197	.7645	-.5781	-.2852	-.3599	-.0156	-.9328
7	9.7114	.6093	.6981	-.3763	.6567	-.7013	-.2517	-.4432	-.0936	-.8915
8	10.8758	.6324	.5957	-.4952	.5767	-.7925	-.1985	-.5107	-.1601	-.8447
9	11.7553	.6278	.5409	-.5598	.5401	-.8264	-.1593	-.5487	-.2022	-.8109
10	12.2500	.6447	.4754	-.5986	.4970	-.8594	-.1195	-.5713	-.2207	-.7904

TABLE VII. — PLUNGING AND PITCHING MODAL DISPLACEMENTS AND THEIR DERIVATIVES AT AEROPOINTS OF SR3C-X2 BLADE

(a) First mode

Aeropoint	Plunging displacement, h	Pitching displacement, α	h'	α'	h''	α''
1	0.2976	-1.2908	-0.6111	1.3156	-2.1248	-8.6301
2	-.0985	-1.4774	-1.2954	-1.9407	-1.6192	-8.0724
3	-2.2262	-8.9819	-3.1200	-4.8253	-1.5747	3.1407
4	-6.8319	-7.1165	-4.3998	.0323	-1.0338	2.1422
5	-14.4850	-5.9894	-7.1812	.8441	-2.7125	-.2391
6	-27.8790	-5.7420	-8.8049	-.6052	-.2942	-.3571
7	-44.6850	-8.4228	-10.2870	-3.2436	-.7395	-.6940
8	-66.6160	-15.1850	-16.1250	-5.3830	-7.1867	-3.4197
9	-99.8500	-25.6760	-26.4060	-7.8140	-3.9630	.1497
10	-127.3800	-31.2990	-31.1740	-2.7429	-5.8780	12.070

(b) Second mode

Aeropoint	Plunging displacement, h	Pitching displacement, α	h'	α'	h''	α''
1	3.3984	1.3948	4.7260	5.3771	4.4850	-71.717
2	5.5797	.2623	6.1597	-10.5250	3.4293	-13.305
3	13.4020	-13.6900	9.9268	-11.8690	3.3602	-7.0570
4	25.8860	-13.2210	11.8440	-7.2005	.7699	-3.2508
5	43.6460	-26.7060	12.3580	-10.5780	.3639	-.8713
6	64.2280	-43.0360	12.7930	-11.3820	-.3017	-.5385
7	85.2930	-66.2800	10.1940	-16.2580	-2.5391	-.4045
8	95.2060	-96.2020	-2.1202	-19.8350	-12.6010	-6.7837
9	72.9140	-134.3100	-27.7310	-29.4650	-15.1220	.7409
10	38.2930	-159.0300	-46.5770	-20.7110	-18.1250	15.340

TABLE VIII. — COMPARISON OF MEASURED AND CALCULATED FLUTTER CONDITIONS OF SR3C-X2 EIGHT-BLADE ROTOR (EXAMPLE 2)

[Operating conditions: rotational speed, 6080 rpm; density, 9.976×10^{-8} lbm sec²/in.⁴; speed of sound, 1130 ft/sec; blade setting angle, 61.2°; interblade phase angle, 225°.]

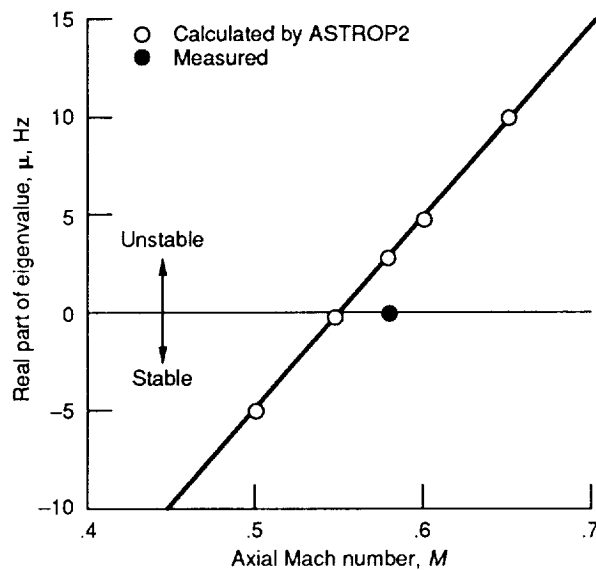|	Axial Mach number	Flutter frequency, $\omega/2\pi$, Hz	Flutter damping
Experimental result	0.58	255.0	-----
ASTROP2	0.55 .60	261.0 261.0	-0.26 4.80
ASTROP2 (interpolated)	0.56	261.0	0.00

Figure 11. — Comparison of measured and calculated SR3C–X2 flutter velocity.

Part 2: Theoretical Development

Introduction

The aeroelastic analysis presented herein is an extension of the work described in reference 13. The modal method of reference 13 uses the nonrotating beam modes in conjunction with a two-dimensional cascade theory in a stripwise manner. In the present work two refinements have been made. First, the normal modes and frequencies of the propfan blade, which is represented as a twisted plate, are used. Second, the two-dimensional cascade theory is corrected for blade sweep. Some aspects of the theory, which is briefly described in reference 2, are repeated here for completeness with emphasis on the refinements used in ASTROP2.

Structural Model

Experience has shown that a beam model for the propfan blade does not represent its true dynamic characteristics. Instead, a structural plate model for the blade should be used for this purpose. For instance, the finite element (FE) plate model can be used to obtain the vibration frequencies of the rotating propfan blade. Figure 9 shows the FE model of a typical propfan blade. The details of the FE method are well documented in the literature (ref. 14).

Aerodynamic Model

Two-dimensional aerodynamic strip theory is used in ASTROP2 to calculate the aerodynamic loads. The unsteady, two-dimensional, noncascade and cascade aerodynamic loads in subsonic flow are obtained by using the theories in references 7 and 8, respectively. In these theories the airfoil thickness, camber, and steady-state angle of attack are neglected and the flow is assumed to be isentropic and irrotational. At any radial station the relative Mach number is a function of the inflow conditions and the rotor speed. Most current propeller designs have supersonic flow at the tip and subsonic flow at the root. As a result, some

53

regions of the blade span encounter transonic flow. The Rao and Jones theory with a correction (ref. 2) for blade sweep at each radial station is used for a relative and effective Mach number less than 1.0. Since this theory is not valid for transonic and supersonic flow fields, provisions exist in ASTROP2 to add new routines when available. The motion-dependent aerodynamic lift and moment coefficients are expressed in reference 13 in nondimensional form. The details regarding the lift and moment expressions used in ASTROP2 are given in appendix D. These expressions account for the structural sweep of the propfan blade.

Formulation of Aeroelastic Equations

The following formulation pertains to a propfan or turbofan with N flexible, rotating, pretwisted, nonuniform, identical blades. The governing generalized equations of motion of the system, considering NM normal modes for each mode, are

$$[M_s]\{\ddot{q}(t)\} + [K_s]\{q(t)\} = \{Q(t)\}$$

(1)

where the subscript s stands for system, and

$$[M_s] = \begin{bmatrix} [M_1] & & & \\ & [M_r] & & 0 \\ 0 & & \ddots & \\ & & & \ddots \\ & & & & [M_N] \end{bmatrix}$$

(2a)

$$[K_s] = \begin{bmatrix} [K_1] & & & \\ & [K_r] & & 0 \\ 0 & & \ddots & \\ & & & \ddots \\ & & & & [K_N] \end{bmatrix}$$

(2b)

$$\{q(t)\} = \begin{Bmatrix} \{q_1(t)\} \\ \vdots \\ \{q_r(t)\} \\ \vdots \\ \{q_N(t)\} \end{Bmatrix}$$

(2c)

54

$$\{Q(t)\} = \begin{Bmatrix} \{Q_1(t)\} \\ \cdot \\ \cdot \\ \cdot \\ \{Q_r(t)\} \\ \cdot \\ \cdot \\ \cdot \\ \{Q_N(t)\} \end{Bmatrix} \qquad (2d)$$

Each vibration mode of every blade is taken as the generalized displacement state. In such a case $[M_r],[K_r]$ are diagonal matrices with m_{rj}, k_{rj} as diagonal elements for the jth mode of the rth blade.

The k_{rj} values are expressed in terms of m_{rj} as

$$k_{rj} = m_{rj}\omega_{rj}^2 + 2i\zeta_{rj}m_{rj}\omega_{rj}^2 \qquad (3)$$

where ω_{rj} and ζ_{rj} are the natural frequency and the structural damping ratio respectively, of the rth blade in the jth mode.

Assuming that the motion is simple harmonic at flutter, the generalized coordinate vector of the blade can be written as

$$\{q_r(t)\} = \{\bar{q}_{r0}\}e^{i\omega t} \qquad (4)$$

where $\{q_{r0}\}$ is the amplitude of $\{q_r(t)\}$.

The flutter mode can be near any one of the vibration modes or combination of these modes of the system. The phase angles between N identical propfan blades mounted on a rotor will be the same at flutter. The cascade of such a system is termed a "tuned cascade." According to Lane's (ref. 15) assumption, the interblade phase angles are restricted to N discrete values. The number N is equal to the number of blades in the system. Using Lane's assumption, the interblade phase angle β_n is given by

$$\beta_n = \frac{2\pi n}{N}, \quad n = 0,1,2,\dots,N-1 \qquad (5)$$

All blades in the tuned system have identical amplitudes in each of these phase angle modes. All the interblade phase angles are uncoupled for a tuned rotor. The flutter of an N-blade system can occur in any one of these interblade phase angle modes. It is adequate to analyze the motion of a tuned cascade by studying the motion of each interblade phase angle of the rth blade separately. Thus, equation (4) can be written for a tuned cascade as

$$\{q_r(t)\} = \{\bar{q}_{r0}\}e^{i(\omega t + \beta_n r)} \qquad (6)$$

In a system with N nonidentical blades, each blade can have different amplitudes and the phase angle between adjacent blades can vary. Such a system is termed a "mistuned system." The cascade of such a system is known as a "mistuned cascade." In an arbitrarily mistuned cascade the motion of the rth blade can be expressed as a linear combination of the motion in all possible interblade phase angle modes of the corresponding tuned cascade. Thus, equation (4) for the rth blade takes the form

$$\{q_r(t)\} = \sum_{n=0}^{N-1} \{\bar{q}_{(n+1)0}\}e^{i(\omega t + \beta_n r)} \qquad (7)$$

Now, the generalized coordinate vector $\{q(t)\}$ can be written as

55

$$\{q(t)\} = [E]\{q_0\}e^{i\omega t} \tag{8a}$$

where $\{q_0\}$ is the generalized amplitude vector for the system and $[E]$ is the matrix whose elements are functions of all possible interblade phase angles of the cascade (ref. 13). The vector $\{q_0\}$ and the matrix $[E]$ are given by

$$\{q_0\} = \begin{Bmatrix} \{q_{10}\} \\ \{q_{20}\} \\ \cdot \\ \cdot \\ \cdot \\ \{q_{N0}\} \end{Bmatrix} \tag{8b}$$

$$[E] = \begin{bmatrix} E_{1,0} & E_{1,1} & \cdots & E_{1,N-1} \\ E_{2,0} & E_{2,1} & \cdots & E_{2,N-1} \\ \vdots & \vdots & \cdots & \vdots \\ E_{N,0} & E_{N,1} & \cdots & E_{N,N-1} \end{bmatrix} \tag{8c}$$

where $E_{r,n} = \exp\left[i\dfrac{2\pi(r-1)n}{N}\right][I]$.

Each vector $\{Q_r(t)\}$ is the generalized aerodynamic force on the rth blade. The derivation for the generalized aerodynamic force vector $\{Q(t)\}$ is presented in appendix E. Rewriting $\{Q(t)\}$ in matrix form gives

$$\{Q(t)\} = \omega^2[E][A]\{q_0\}e^{i\omega t} \tag{9}$$

where ω is the frequency at flutter and $[A]$ is the system aerodynamic matrix defined in appendix E.

Substituting for $\{q(t)\}$ from equation (8a) and $\{Q(t)\}$ from equation (9) into equation (1) gives the flutter equation of an arbitrarily mistuned cascade as

$$-\omega^2[M_s]\{\bar{q}_0\} + [K_s]\{\bar{q}_0\} = \omega^2[E][A]\{q_0\} \tag{10}$$

where $\{\bar{q}_0\}$ is related to $\{q_0\}$ through the matrix $[E]$ as

$$\{\bar{q}_0\} = [E]\{q_0\} \tag{11}$$

Expressing equation (10) in terms of $\{q_0\}$ and rearranging terms gives

$$[K_s][E]\{q_0\} = \omega^2[[M_s][E] + [E][A]]\{q_0\} \tag{12}$$

Multiplying equation (12) throughout by $(1/\omega_0^2)[E]^{-1}$ yields the final form of the flutter equation formulated as an eigenvalue problem:

$$[P_s]\{q_0\} = \gamma[Q_s]\{q_0\} \tag{13}$$

56

$$[P_s] = \frac{1}{\omega_0^2}[E]^{-1}[K_s][E] \tag{14a}$$

$$[Q_s] = \left[[E]^{-1}[M_s][E] + [A]\right] \tag{14b}$$

$$\gamma = \left(\frac{\omega}{\omega_0}\right)^2 \tag{14c}$$

and ω_0 is the assumed reference frequency at flutter.

Note that the matrix $[A]$ in ASTROP2 is different than the one given in reference 13, since the eigenvalue problem is formulated in $\{q_0\}$ in the present report instead of $\{\overline{q}_0\}$ as in reference 13. Also, the matrix $[A]$ in reference 2 is related to $[A]$ in this report through ω^2 by

$$[A] \text{ in ref. 2 } = \omega^2[A] \text{ in present report} \tag{15}$$

Solution of Aeroelastic Equations

The aeroelastic stability boundaries are obtained by solving the standard complex eigenvalue problem represented by equation (13). The relation between the flutter frequency ω and the eigenvalue obtained from the solution for γ in equation (13) is given by

$$i\left(\frac{\omega}{\omega_0}\right) = i\sqrt{\gamma} = \mu + iv \tag{16}$$

Flutter occurs when μ is greater than zero.

The ASTROP2 code assembles and solves the generalized eigenvalue problem given in equation (13). The in-vacuum natural frequencies of the rotating blades can be retrieved by setting the aerodynamic matrix $[A]$ to zero and can be used to check the structural input to ASTROP2 before flutter computations. The procedure outlined in appendix B helps to determine the flutter frequencies.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, January 8, 1991

Appendix A

Major Routines in ASTROP2

The ASTROP2 code is written in FORTRAN language. It is designed to work on the Cray X/MP computer system but can be translated easily onto any other computer system. The program is written in a modular fashion. The existing routines and their purposes in ASTROP2 are given in table IX. They are listed there in alphabetical order.

TABLE IX. — EXISTING ROUTINES IN ASTROP2

Routine	Purpose
ALOGO	Prints ASTROP2 logo
AMAP	Computes the aerodynamic forces and hence P and Q matrices to formulate the eigenvalue problem
AVEMVR1	Calculates average α values at aeropoints from NASTRAN translational values
AVEMVR2	Calculates average α values at aeropoints from NASTRAN printed rotational values
CM1DW	Prints complex one-dimensional matrix with header
CM2DW	Prints complex two-dimensional matrix with header
CORTRS	Transforms MSC/NASTRAN or COSMIC/NASTRAN model to the ASTROP2 coordinate system
CTVP	Writes tabular values of complex matrix in report format
DATAP	Accepts user input for ASTROP2
DEINVE	Performs matrix multiplication $[E^{-1}E]$ for matrix $[A]$
DTIME	Prints date and time of day
ECHO	Prints an echo of ASTROP2 input
EXTEIG	Extracts eigenvalues and eigenvectors from equation (13). This routine uses IMSL routine EIGZC to obtain these values
FDDRP	Computes second derivative of function at points on blade
FUNCD	Computes first derivative of function at points on blade
GRIDPR	Prints grid points and their coordinates in a report format
HACAL	Interpolates h, and α, on leading edge along blade axis at aeropoints
HAINTS	Interpolates h, and α, along leading-edge curved line at aeropoints
HAPCAL	Interpolates h, and α, and their derivatives on leading-edge along blade axis at aeropoints
HAPCAS	Interpolates h, and α, and their derivatives along leading-edge curved line at aeropoints
HEADER	Skips a header and a problem title on a new page
LETEDG	Determines grids on blade leading and trailing edges
LIN2DN	Computes two-dimensional, aerodynamic, noncascade lift and moment coefficients on a blade strip
LOCGRI	Is a driver routine to determine X and Z coordinates of user-specified aeropoints
LOCXZ	Determines X and Z coordinates of aeropoints on leading edge

58

TABLE IX. — Concluded.

Routine	Purpose
LOCXZS	Determines X and Z coordinates of aeropoints on leading edge
MAIN	Is the main driver routine for ASTROP2
MDPRNT	Prints modal displacements in report form
NORMAL	Determines normal vector at aeropoint on leading edge
OUTT	Prints computed values of variables and arrays in a tabular form
RDCARD	Reads one logical record on ASTROP2 input file
RDCOS	Reads COSMIC/NASTRAN output to obtain blade model data including grid numbers, their coordinates, modal values, and corresponding mode shapes
RDMSC	Reads Cray MSC/NASTRAN output to obtain blade model data including grid numbers, their coordinates, modal values, and corresponding mode shapes
RDNASO	Is a driver routine to read NASTRAN output and to compute parameters for aeroanalysis
RELVEL	Computes the effective (relative) velocity at aeropoints
RLENC	Computes the arc length values of leading-edge grids for use as an independent coordinate in numerical interpolation
RMIDW	Writes out one-dimensional matrix containing real values
SDATA	Converts keyword input value in character format to real or integer data
SDOTP	Computes the dot product of two vectors, $\mathbf{A}$ and $\mathbf{B}$
SEQNOD	Determines the IGRID array index number for a specified grid number
SMAP	Assembles the generalized stiffness matrix $[GS]$
SORTX	Sorts numerical values in a one-dimensional array in an ascending order
SPLINT	Performs one-dimensional interpolation using bicubic spline routine
SUBSAP	Computes two-dimensional, aerodynamic, cascade lift and moment coefficients on a blade strip
TSNCAL	Computes direction vectors of tangent, chordline, and normal at user-specified aeropoints
TSNFAL	Computes direction vectors of tangent, streamline, and normal at all grid points on leading edge
TVP	Writes tabular values of real matrix in a report form
VCROSP	Performs vector product of two vectors, $\mathbf{A}$ and $\mathbf{B}$.

A more detailed explanation of the major routines is given here.

MAIN

MAIN is the main program for ASTROP2. It calls all important routines in a specific order. All the global variables in ASTROP2 are grouped and labeled in FORTRAN common statements.

DATAP

DATAP accepts the user input to ASTROP2. The data required in ASTROP2 are explained in part 1 of this report.

RDNASO

If the user has NASTRAN modal analysis output, RDNASO can read in the blade model, its vibration frequencies, and mode shapes. By this, the user can avoid explicit input of the blade model to ASTROP2.

After reading the blade model data, this routine calls the LETEDG routine for determining the leading and trailing edges. It is assumed in this routine that the reference line used for aeroelastic analysis in ASTROP2 is the leading edge. The user provides the aeropoints for use in this analysis. These points must lie on the reference line. At these points the plunging and pitching modal displacements and their derivatives are computed. They are represented in this manual as the $[W]$, $[WP]$, and $[WPP]$ matrices. The preprocessing work necessary to compute these matrices is done through this routine. A detailed explanation of how to compute these matrices is given in appendix E.

RELVEL

RELVEL computes the effective Mach number and hence the effective velocity at the specified aeropoints.

SMAP

SMAP formulates matrices $[M_s]$ and $[K_s]$ and passes this information through a common block to the AMAP routine.

AMAP

AMAP determines the aerodynamic load for each interblade phase angle mode and a given frequency. The SUBSAP routine is used to compute the lift coefficients (appendix D). AMAP calls this routine for each interblade phase angle and each blade vibration mode. It also computes its derivatives before using these values in the aerodynamic load calculations. These loads in conjunction with the mass, damping, and stiffness matrices are used to formulate the eigenvalue problem given in equation (13) in part 2 of this guide.

EXTEIG

EXTEIG solves equation (13) given in part 2 of this guide for the eigenvalues and the eigenvectors. The flutter frequency ratio and damping are related to each eigenfrequency through equation (16).

Appendix B

Procedure for Obtaining a Flutter Condition

The procedure for obtaining a flutter condition is outlined here for convenience. The ASTROP2 flow chart is given in figure 12. The iterative steps to be followed in order to determine the flutter boundary are as follows:

(1) Select a rotational speed Ω and assume an axial Mach number M.

(2) Input the blade geometry, the generalized masses, the generalized vibration frequencies, and the mode shapes as outlined in part 1 of this manual.

(3) Assume a reference frequency ω_0.

(4) Select either tuned or frequency-mistuned cascade analysis.

(5) For a tuned cascade, specify the interblade phase angles of interest.

(6) Run ASTROP2 and obtain outputs v and μ for all modes.

(7) For the least stable mode of interest, check whether v in equation (16) is within an acceptable tolerance of unity. If it is not, modify the reference frequency and repeat the process from step (3). As a hint, if v is greater than unity, increase the assumed reference frequency; if it is less than unity, decrease the assumed reference frequency.

(8) Check for the least stable mode whether μ in equation (16) is positive or negative. In either case, repeat the process by changing M in step (1) until $\mu = 0$ and $v = 1$ within an acceptable tolerance. As a hint, if μ is negative when v is close to unity, increase M; and if μ is positive when v is close to unity, decrease M. If μ is positive, an unstable condition exists.
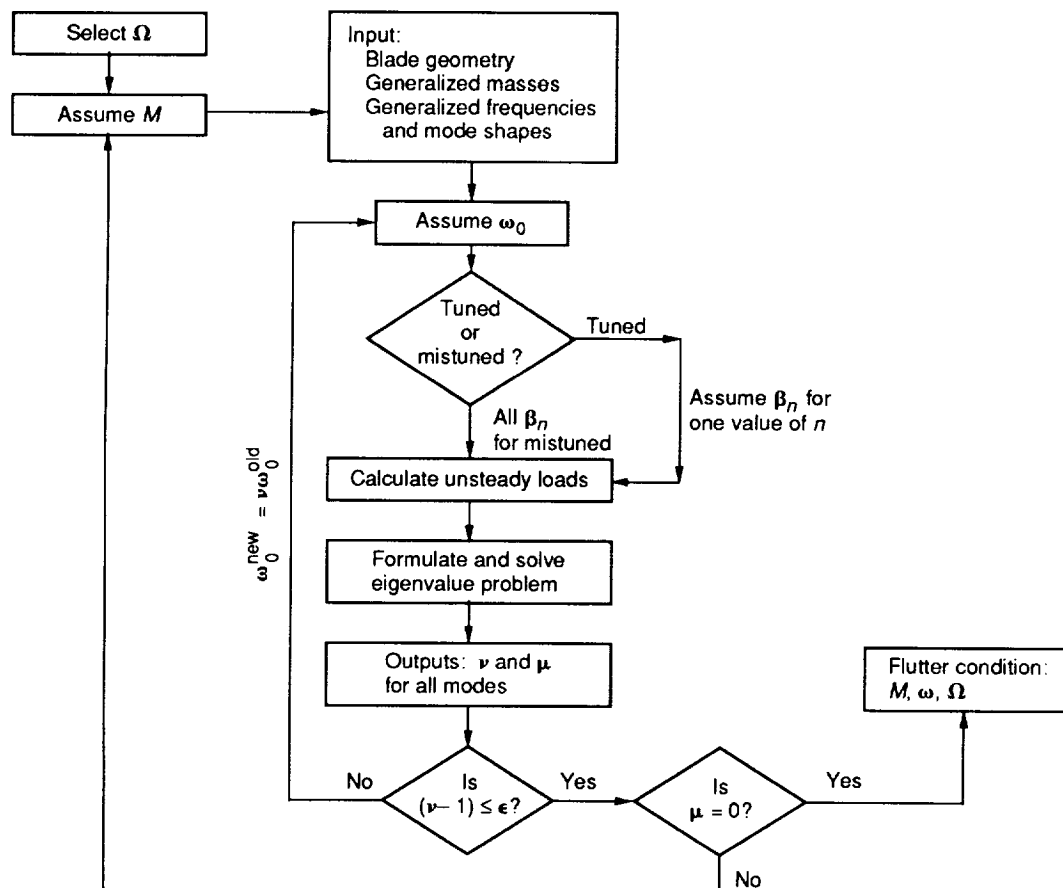

Figure 12. — ASTROP2 flowchart.

Appendix C

NASTRAN Input Data for Examples

The NASTRAN input data for example 1 are listed in figure 13, but the NASTRAN input data for example 2 are too long to be included in this manual. However, the access to the data for example 2 is described here.

```
ID MSC, HA75E
TIME 5
DIAG 14
SOL 75 $ NEW FLUTTER SOLUTION SEQUENCE
ALTER 997   $ V65
PURGE BHH/ALWAYS
CEND
TITLE = EXAMPLE HA75E: HALF SPAN 15-DEGREE SWEPT UNTAPERED WING
SUBT = KE - METHOD FLUTTER ANALYSIS, DOUBLET-LATTICE AERO
LABEL = 0.041 IN AL PLATE W/BEVELLED LEADING AND TRAILING EDGES
SEALL = ALL
SPC   = 1$ WING ROOT DEFLECTIONS AND PLATE IN-PLANE ROTATIONS FIXED
SDAMP = 2000
METHOD   = 10   $ MODIFIED GIVENS METHOD OF REAL EIGENVALUE EXTRACTION
FMETHOD = 30   $ KE-FLUTTER METHOD
SVEC    = ALL $ PRINT VIBRATION MODES
OUTPUT(PLOT)
CSCALE 2.0
PTITLE = STRUCTURAL ELEMENTS
 SET 1 = QUAD4
FIND SCALE, ORIGIN 1, SET 1
PLOT MODAL 0 ORIGIN 1, SET 1
OUTPUT (XYOUT)
CSCALE 2.0
CURVELINESYMBOL = 6
YTTITLE = DAMPING   G
YBTITLE = FREQUENCY   F (HZ)
XTITLE   = VELOCITY   V (FT/SEC)
XTGRID LINES = YES
XBGRID LINES = YES
YTGRID LINES = YES
YBGRID LINES = YES
UPPER TICS = -1
TRIGHT TICS = -1
BRIGHT TICS = -1
XYPLOT VG / 1(G,F) 2(G,F) 3(G,F)
BEGIN BULK
$***
***$
$***     15 DEG SWEPT WING GRID POINTS AND CONSTRAINTS
***$
$***
***$
GRID     1              .0       0.000   .0
GRID     2              .211491 0.7893   .0
GRID     3              .422983 1.5786   .0
GRID     4              .634474 2.3679   .0
GRID     5              .845966 3.1572   .0
GRID     6             1.05746 3.9465    .0
GRID     7             1.26895 4.7358    .0
GRID     8             1.48044 5.5251    .0
GRID     9              .258819 0.0      .0
GRID     10             .47031  .7893    .0
GRID     11             .681802 1.5786   .0
GRID     12             .893293 2.3679.  .0
GRID     13            1.10478 3.1572    .0
GRID     14            1.31628 3.9465    .0
GRID     15            1.52777 4.7358    .0
GRID     16            1.73926 5.5251    .0
GRID     17            1.03528 0.0       .0
GRID     18            1.24677 .7893     .0
GRID     19            1.45826 1.5786    .0
GRID     20            1.66975 2.3679    .0
GRID     21            1.88124 3.1572    .0
```

Figure 13. — MSC/NASTRAN input data for example 1.

62

```
GRID     22              2.09273 3.9465   .0
GRID     23              2.30422 4.7358   .0
GRID     24              2.51572 5.5251   .0
GRID     25              1.81173 0.0      .0
GRID     26              2.02322 .7893    .0
GRID     27              2.23471 1.5786   .0
GRID     28              2.44621 2.3679   .0
GRID     29              2.6577  3.1572   .0
GRID     30              2.86919 3.9465   .0
GRID     31              3.08068 4.7358   .0
GRID     32              3.29217 5.5251   .0
GRID     33              2.07055 0.0      .0
GRID     34              2.28204 .7893    .0
GRID     35              2.49353 1.5786   .0
GRID     36              2.70502 2.3679   .0
GRID     37              2.91652 3.1572   .0
GRID     38              3.12801 3.9465   .0
GRID     39              3.3395  4.7358   .0
GRID     40              3.55099 5.5251   .0
$***
***$
$***    15 DEG SWEPT WING STRUCTURAL ELEMENTS (CQUAD4)
***$
$***
***$
CQUAD4   1       1       1       2       10      9               +M000
+M000                    0.0     0.0      .041    .041
CQUAD4   2       1       2       3       11      10              +M001
+M001                    0.0     0.0      .041    .041
CQUAD4   3       1       3       4       12      11              +M002
+M002                    0.0     0.0      .041    .041
CQUAD4   4       1       4       5       13      12              +M003
+M003                    0.0     0.0      .041    .041
CQUAD4   5       1       5       6       14      13              +M004
+M004                    0.0     0.0      .041    .041
CQUAD4   6       1       6       7       15      14              +M005
+M005                    0.0     0.0      .041    .041
CQUAD4   7       1       7       8       16      15              +M006
+M006                    0.0     0.0      .041    .041
CQUAD4   8       1       9       10      18      17
CQUAD4   9       1       10      11      19      18
CQUAD4   10      1       11      12      20      19
CQUAD4   11      1       12      13      21      20
CQUAD4   12      1       13      14      22      21
CQUAD4   13      1       14      15      23      22
CQUAD4   14      1       15      16      24      23
CQUAD4   15      1       17      18      26      25
CQUAD4   16      1       18      19      27      26
CQUAD4   17      1       19      20      28      27
CQUAD4   18      1       20      21      29      28
CQUAD4   19      1       21      22      30      29
CQUAD4   20      1       22      23      31      30
CQUAD4   21      1       23      24      32      31
CQUAD4   22      1       25      26      34      33              +M007
+M007                    .041    .041     0.0     0.0
CQUAD4   23      1       26      27      35      34              +M008
+M008                    .041    .041     0.0     0.0
CQUAD4   24      1       27      28      36      35              +M009
+M009                    .041    .041     0.0     0.0
CQUAD4   25      1       28      29      37      36              +M010
+M010                    .041    .041     0.0     0.0
CQUAD4   26      1       29      30      38      37              +M011
+M011                    .041    .041     0.0     0.0
CQUAD4   27      1       30      31      39      38              +M012
+M012                    .041    .041     0.0     0.0
CQUAD4   28      1       31      32      40      39              +M013
+M013                    .041    .041     0.0     0.0
PSHELL   1       1       0.041   1                1
```

Figure 13. — Continued.

```
$***
***$
$***     15 DEG SWEPT WING MATERIAL PROPERTIES   ( ALUMINIUM )
***$
$***
***$
MAT1      1         8.5977+63.2554+6          0.097464
PARAM     WTMASS   .0025901
PARAM     COUPMASS1
PARAM     GRDPNT   17
PARAM     KDAMP   ·-1
PARAM     OPPHIPA 1
PARAM     LMODES   4                                                             +T2000
TABDMP1   2000
+T2000    0.0       0.01      1000.0    0.01      ENDT
SPC1      1         12345     9
SPC1      1         12345     25
SPC1      1         6         1         THRU      40
$ASET1    3         1         THRU      8
$ASET1    3         10        THRU      16
$ASET1    3         18        THRU      24
$ASET1    3         26        THRU      40
$***
***$
$***     15 DEG SWEPT WING AERODYNAMIC ELEMENT DESCRIPTION
***$
$***
***$
AERO      0                   2.0706    1.1092-7 1
CAERO1    101       1         0         6         4                   1         +CA101
+CA101    .0        .0        .0        2.07055   1.48044  5.52510 0.0          2.07055   +MK
MKAERO1   .45
+MK       .001      0.1       0.12      0.14      0.16      0.20                +MK
PAERO1    1
SET1      100       2         4         6         8         9         11        13        +S1
+S1       15        18        20        22        24        25        27        29        +S2
+S2       31        34        36        38        40
SPLINE1   100       101       101       124       100       .0
$***
***$
$***     15 DEG SWEPT WING EIGENVALUE AND FLUTTER CONTROL DATA
***$
$***
***$
EIGR      10        MGIV                                    4                   +ER
+ER       MAX                                                                   DENS
FLFACT    1         0.967                                                       MACH
FLFACT    2         .45
FLFACT    3         .20000    .16667    .14286    .12500    .11111    .1
FLUTTER   30        KE        1         2         3         L
PARAM     VREF      12.0
ENDDATA
EOF
```

Figure 13. — Concluded.

Example 1 pertains to an isolated, nonrotating-wing flutter, whereas example 2 pertains to the flutter of an SR3C–X2 propfan with eight blades rotating at a high rotational speed. The ASTROP2 analysis requires the natural frequencies and mode shapes of the blades. The detailed procedures for obtaining the modal output by using NASTRAN are outside the scope of this manual. These procedures are given in references 4 and 5 for COSMIC/NASTRAN and MSC/NASTRAN, respectively.

In example 1 the modal analysis of the wing is performed by using solution sequence 63 or 9 depending, respectively, on whether the MSC/NASTRAN or COSMIC/NASTRAN program is used. The inverse-power method of real eigenvalue extraction is used in example 1 with COSMIC/NASTRAN. The modified Givens method of real eigenvalue extraction is used in example 2 with MSC/NASTRAN.

Because of blade rotation the steady-state configuration must be determined in example 2 before the modal analysis is performed. Hence, two NASTRAN runs are required in example 2 and only one in example 1. Solution sequence 64 or 4 is used for determining the steady-state configuration of the rotating blades depending, respectively, on whether the MSC/NASTRAN or COSMIC/NASTRAN program is used.

NASTRAN Input Data for Example 1

Figure 13 gives the NASTRAN input data for example 1 for the MSC/NASTRAN run. The data file contains information for performing the flutter analysis by using MSC/NASTRAN, as done in reference 10.

Access to NASTRAN Input Data for Example 2

Two NASTRAN input data files are required in example 2: one to obtain the steady-state configuration, and the other to obtain the modal analysis results. Figure 14 gives the Cray job stream for accessing and obtaining a copy of these data files. The file named "ex2_r1_data" contains the COSMIC/NASTRAN input data for the SR3C–X2 turboprop that is used to perform an analysis for obtaining the differential stiffness (see ref. 4 for more details). The second file named "ex2_r2_data," contains the COSMIC/NASTRAN input data for the SR3C–X2 turboprop that is used to perform the modal analysis by using the differential stiffness of the first run. The resulting NASTRAN output from the modal analysis is used by ASTROP2 in the flutter analysis.

```
# USER=userid PW=userpw
# QSUB    - r     ex2_nas_data
# QSUB    -eo
# QSUB    -lm    1.0mw
# QSUB    -lt    60
set    -xvk
cd
rm  core
astd = /aerospace2/smastrp/astrop2
cp  $astd/ex2_r1_data    ex2_r1_data # This is first file
cp  $astd/ex2_r2_data    ex2_r2_data # This is second file
```

Figure 14. — Cray job stream for accessing NASTRAN input for example 2.

Appendix D

Expressions for Lift and Moment Due to Blade Motion

It is assumed that the lift and moment per unit span due to blade motion are linearly related to the displacements and their derivatives with respect to s. The expressions for lift and moment per unit span given in this appendix are for airfoil sections under subsonic compressible flow.

With the sign convention of the plunging displacement as positive downward and the pitching displacement as positive upward (fig. 1), the lift and moment per unit span at an aeropoint s on the rth blade reference axis for a specified frequency and blade sweep angle, in the notation of reference 9, are

$$L_r(s,t) = \pi\rho\omega^2 b^3 \left\{ \ell_{hh}\left(\frac{h_r}{b}\right) + \left(\frac{-i}{k}\right)(-1 - \ell_{hh})(\tan \Lambda)h_r' + \left(\frac{b}{k^2}\right)(\tan^2\Lambda)h_r'' - \ell_{h\alpha}\alpha_r \right.$$

$$\left. + b(\tan \Lambda)\left(\frac{1}{k^2} - \frac{i}{k}\left[\left(\frac{1}{2} + a\right) - \left(\frac{1}{2} - a\right)\ell_{hh}\right]\right)\alpha_r' - \frac{ab^2}{k^2}(\tan^2\Lambda)\alpha_r'' \right\}$$

(D1)

$$M_r(s,t) = \pi\rho\omega^2 b^4 \left\{ -\ell_{\alpha h}\left(\frac{h_r}{b}\right) - \left(\frac{-i}{k}\right)(\tan \Lambda)(a - \ell_{\alpha h})h_r' - \frac{ab}{k^2}(\tan^2\Lambda)h_r'' - \ell_{\alpha\alpha}\alpha_r \right.$$

(D2)

$$\left. - \frac{i}{k}b(\tan \Lambda)\left[-\frac{3}{8} - \left(\frac{1}{8} + a^2\right) + \frac{i}{k}\left(\frac{1}{2} - a\right) + \left(\frac{1}{4} - a^2\right)\ell_{hh}\right]\alpha_r' + \frac{b^2}{k^2}(\tan^2\Lambda)\left(\frac{1}{8} + a^2\right)\alpha_r'' \right\}$$

Here, the coefficients ℓ_{hh}, $\ell_{h\alpha}$, $\ell_{\alpha h}$, $\ell_{\alpha\alpha}$ are dimensionless functions of the reduced frequency k, the interblade phase angles β_n, the gap-to-chord ratio s/c, and the effective Mach number M_e. These are unsteady coefficients calculated for subsonic compressible flow and are obtained by using the theory in reference 8. The quantities h_r and α_r represent the plunging and pitching displacements of the rth blade reference axis at the sth aeropoint. The primes on h_r and α_r denote the differentiation with respect to the arc length measured on the reference line. Note that h_r and α_r are functions of time.

Expressions (D1) and (D2) can be rearranged in matrix form as

$$\begin{bmatrix} L_r(s,t) \\ M_r(s,t) \end{bmatrix} = \pi\rho\omega^2 [L]\begin{matrix} h_r \\ \alpha_r \end{matrix} + [LP]\begin{pmatrix} h_r' \\ \alpha_r' \end{pmatrix} + [LPP]\begin{pmatrix} h_r'' \\ \alpha_r'' \end{pmatrix}$$

(D3)

where

$$[L] = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix}$$

(D4a)

$$[LP] = \begin{bmatrix} \ell_{13} & \ell_{14} \\ \ell_{23} & \ell_{24} \end{bmatrix}$$

(D4b)

$$[LPP] = \begin{bmatrix} \ell_{15} & \ell_{16} \\ \ell_{25} & \ell_{26} \end{bmatrix}$$ (D4c)

and the coefficients ℓ_{11}, ℓ_{12},...,ℓ_{16}, and ℓ_{21},...,ℓ_{26} are defined below in terms of ℓ_{hh}, $\ell_{h\alpha}$, $\ell_{\alpha h}$, $\ell_{\alpha\alpha}$

$$\ell_{11} = -b^2\ell_{hh}$$ (D5a)

$$\ell_{12} = -b^3\ell_{h\alpha}$$ (D5b)

$$\ell_{13} = b^3\frac{i}{k}(1 + \ell_{hh})\tan \Lambda$$ (D5c)

$$\ell_{14} = b^4(\tan \Lambda)\left\{\frac{1}{k^2} - \frac{i}{k}\left[\left(\frac{1}{2} + a\right) - \left(\frac{1}{2} - a\right)\ell_{hh}\right]\right\}$$ (D5d)

$$\ell_{15} = \frac{b^4}{k^2}\tan^2\Lambda$$ (D5e)

$$\ell_{16} = -\frac{b^5}{k^2} a \tan^2\Lambda$$ (D5f)

$$\ell_{21} = -b^3\ell_{\alpha h}$$ (D5g)

$$\ell_{22} = -b^4\ell_{\alpha\alpha}$$ (D5h)

$$\ell_{23} = -b^4\frac{i}{k}(a - \ell_{\alpha h})\tan \Lambda$$ (D5i)

$$\ell_{24} = -b^5\frac{i}{k}(\tan \Lambda)\left[-\frac{3}{8} - \left(\frac{1}{8} + a^2\right) + \frac{i}{k}\left(\frac{1}{2} - a\right) + \left(\frac{1}{4} - a^2\right)\ell_{hh}\right]$$ (D5j)

$$\ell_{25} = -\frac{b^5}{k^2} a \tan^2\Lambda$$ (D5k)

$$\ell_{26} = \frac{b^6}{k^2}\left(\frac{1}{8} + a^2\right)\tan^2\Lambda$$ (D5l)

67

Appendix E

Computation of Generalized Aerodynamic Forces

A brief discussion of the computational steps involved in computing the generalized aerodynamic forces is presented here. The plunging h_r and pitching α_r displacements of a blade at an aeropoint (see fig. 1) and their first two derivatives, h_r', h_r'' and α_r', α_r'', with respect to s, are

$$\begin{bmatrix} h_r(t) \\ \alpha_r(t) \end{bmatrix} = [W]\{q_r(t)\} \tag{E1a}$$

$$\begin{bmatrix} h_r'(t) \\ \alpha_r'(t) \end{bmatrix} = [WP]\{q_r(t)\} \tag{E1b}$$

$$\begin{bmatrix} h_r''(t) \\ \alpha_r''(t) \end{bmatrix} = [WPP]\{q_r(t)\} \tag{E1c}$$

where

$$[W] = \begin{bmatrix} W_1 & W_2 & \cdots & W_{NM} \\ A_1 & A_2 & \cdots & A_{NM} \end{bmatrix} \tag{E1d}$$

$$[WP] = \begin{pmatrix} W_1' & W_2' & \cdots & W_{NM}' \\ A_1' & A_2' & \cdots & A_{NM}' \end{pmatrix} \tag{E1e}$$

$$[WPP] = \begin{pmatrix} W_1'' & W_2'' & \cdots & W_{NM}'' \\ A_1'' & A_2'' & \cdots & A_{NM}'' \end{pmatrix} \tag{E1f}$$

Here W_j and A_j, $j = 1,...,NM$, are the modal displacement amplitudes and rotations, respectively, and the prime denotes the differentiation with respect to the arc length measured on the reference line.

The numerical steps for computing the elements of the $[W]$, $[WP]$, and $[WPP]$ matrices are as follows:

(1) The tangent vector t is computed at the aeropoints by using the leading edge as the reference curve.

(2) The chordline vectors at these aeropoints are computed next. The chord is taken to be perpendicular to the reference curve at the aeropoint. For this purpose a plane is drawn perpendicular to the tangent vector at the aeropoint. The point of intersection of this plane with the blade trailing edge lies on the chordline vector c at the aeropoint. A chordline vector c is drawn by joining this point from the corresponding aeropoint. The chordline vectors are thus determined at all selected aeropoints.

(3) The normal vectors at aeropoints on the blade are computed. The normal vector n at an aeropoint is the vector cross product of the tangent and the chordline vectors at that aeropoint.

(4) The plunging displacements of the jth mode along the normal vector at an aeropoint, represented by W_j, are the dot product of the normal vector and the corresponding j^{th} modal displacement vector. These W_j values for all modes at an aeropoint are thus calculated. and they are represented by the first-row elements of the $[W]$ matrix as in equation (E1d).

(5) The pitching displacements of the jth mode about the tangent vector at an aeropoint, represented by A_j, are the dot product of the tangent vector and the corresponding j^{th} modal rotation vector. These A_j values for all modes at an aeropoint are thus calculated, and they are represented by the second-row elements of the $[W]$ matrix as in equation (E1d).

(6) In addition, W_j', A_j', W_j'', and A_j'' values required to calculate the generalized aerodynamic forces of the swept blade are computed by the finite difference method for all modes at an aeropoint. It can be seen in equations (E1e) and (E1f) that these values represent the first- and second-row elements of the $[WP]$ and $[WPP]$ matrices, respectively.

These plunging and pitching displacements and their derivatives are used in the equations for lift and moment due to blade motion given in appendix D. After substituting for h_r, h_r', and h_r'' and α_r, α_r', and α_r'' equation (E1) in expression (D3), $L_r(s,t)$ and $M_r(s,t)$ can be rewritten as

$$\begin{bmatrix} L_r(s,t) \\ M_r(s,t) \end{bmatrix} = \pi\rho\omega^2 [LW]\{q_r(t)\} \qquad (E2)$$

where

$$[LW] = [L][W] + [LP][WP] + [LPP][WPP] \qquad (E3)$$

and $\{q_r(t)\}$ is the generalized coordinate vector of the rth blade.

Once $L_r(s,t)$ and $M_r(s,t)$ on the rth blade are known. the generalized aerodynamic force vector $\{Q_r(t)\}$ on this blade for NM normal modes can be written as

$$\{Q_r(t)\} = \pi\rho\omega^2 \int_0^\ell [W]^T \begin{bmatrix} L_r(s,t) \\ M_r(s,t) \end{bmatrix} ds \qquad (E4)$$

where ℓ represents the length of the reference axis.

Substituting for $L_r(s,t)$ and $M_r(s,t)$ from equation (E2) and $\{q_r(t)\}$ from equation (7) in this expression and rearranging terms gives the following expression for the vector $\{Q_r(t)\}$ for a mistuned cascade:

$$\{Q_r(t)\} = \omega^2 \sum_{N=0}^{N-1} [A]_{n,r} \{q_{(n+1)0}\} e^{i(\omega_t + \beta_n)} \qquad (E5)$$

where the matrix $[A]_{n,r}$ is

$$[A]_{n,r} = \pi\rho \int_0^\ell [W]^T [LW] ds \qquad (E6)$$

69

The trapezoidal rule of numerical integration is used to calculate the submatrix $[A]_{s,r}$ and hence $\{Q_r(t)\}$. Therefore, collecting all vectors ($\{Q_r\}$, $r = 1,2,...,N$) and assembling them allows the generalized aerodynamic force vector $\{Q(t)\}$ for the system to be written in matrix form as

$$\{Q(t)\} = \omega^2 [E][A]\{q_0\}e^{i\omega t} \qquad (E7)$$

where $[E]$ and $\{q_0\}$ are as defined in equation (8) and $[A]$ is the system aerodynamic matrix for the N blades given by

$$[A] = \begin{bmatrix} [A]_{0,0} & [A]_{0,1} & \cdots & [A]_{0,N-1} \\ [A]_{1,0} & [A]_{1,1} & & [A]_{1,N-1} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ [A]_{N-1,0} & [A]_{N-1,1} & & [A]_{N-1,N-1} \end{bmatrix} \qquad (E8)$$

References

1. Mehmed, O., Kaza, K.R.V., Kielb, R.E., and Lubomski, J.F.: Bending-Torsion Flutter of a Highly Swept Advanced Turboprop. NASA TM–82975, 1981.

2. Kaza, K.R.V., Mehmed, O., Narayanan, G.V., and Murthy. D.V.: Analytical Flutter Investigation of a Composite Propfan Model. J. Aircraft, vol. 26, no. 8, Aug. 1989, pp. 772–780.

3. Kaza, K.R.V., Mehmed, O., Williams, M., and Moss, L.A.: Analytical and Experimental Investigation of Mistuning in Propfan Flutter. 28th Structures, Structural Dynamics and Materials Conference, Part 2A, AIAA, 1987, pp. 98–110. (Also, NASA TM–88959.)

4. NASTRAN User's Manual. Vols. 1 and 2, NASA SP–222. 1986.

5. MSC/NASTRAN User's Manual. Version 66, Vols. 1 and 2, MacNeal-Schwendler Corp., Nov. 1988.

6. Lawrence, C., Aiello, R.A., Ernst, M.A., McGee, O.G.: A NASTRAN Primer for the Analysis of Rotating Flexible Blades. NASA TM–89861, 1987.

7. Bland, S.R.: Development of Low-Frequency Kernel Function Aerodynamics for Comparison With Time-Dependent Finite Difference Methods. NASA TM–83283, 1982.

8. Rao, B.M.; and Jones, W.P.: Unsteady Airloads on a Cascade of Staggered Blades in Subsonic Flow. Unsteady Phenomena in Turbomachinery, AGARD–CP–177, 1975, pp. 32–1 to 32–10.

9. Barmby, J.G., Cunningham, H.J.; and Garrick, I.E.: Study of Effects of Sweep on the Flutter of Cantilever Wings. NACA TN–2121, 1950.

10. MSC/NASTRAN Handbook for Aeroelastic Analysis. Version 65, Vols. 1 and 2, Nov. 1987.

11. Mehmed, O.; and Kaza, K.R.V.: Experimental Classical Flutter Results of a Composite Advanced Turboprop Model. NASA TM–88792, 1986.

12. Woolston, D.S.; and Runyan, H.L.: On the Use of Coupled Modal Functions in Flutter Analysis. NACA TN–2375, 1951.

13. Kaza, K.R.V.; and Kielb, R.E.: Flutter of Turboprop Rotors With Mistuned Blades. AIAA J., vol. 22, no. 11, Nov. 1981, pp. 1618–1625.

14. Zienkiewicz, O.C.: The Finite Element Method. 3rd ed., McGraw-Hill, 1977.

15. Lane, F.: System Mode Shapes in the Flutter of Compressible Blade Rows. J. Aeronaut. Sci., vol. 23, no. 1, Jan. 1956, pp. 54–66.

NASA
National Aeronautics and
Space Administration

Report Documentation Page

1. Report No. NASA TM-4304	2. Government Accession No.	3. Recipient's Catalog No.

4. Title and Subtitle ASTROP2 Users Manual: A Program for Aeroelastic Stability Analysis of Propfans		5. Report Date **August 1991**
		6. Performing Organization Code

7. Author(s) G.V. Narayanan and K.R.V. Kaza	8. Performing Organization Report No. E-5768
	10. Work Unit No. 535-03-01 505-63-1B

9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum

12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001	14. Sponsoring Agency Code

16. Abstract

This is a user's manual for the aeroelastic stability and response of propulsion systems computer program called ASTROP2. The ASTROP2 code performs aeroelastic stability analysis of rotating propfan blades. This analysis uses a two-dimensional, unsteady cascade aerodynamics model and a three-dimensional, normal-mode structural model. Analytical stability results from this code are compared with published experimental results of a rotating composite advanced turboprop model and of a nonrotating metallic wing model. This manual will help the user of ASTROP2 set up the input deck for running the code, and it will provide the serious user with enough information to understand the mathematical development used in the code. In addition, this manual will enable the programmer to install and maintain the code in a computer environment. The code is written in the FORTRAN language. ASTROP2 is currently operational on the Cray X/MP computer at the NASA Lewis Research Center.

17. Key Words (Suggested by Author(s)) Aeroelasticity; Flutter; Propeller; Propfan; Vibration	18. Distribution Statement Unclassified – Unlimited Subject Category 39

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 76	22. Price* A05