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Title: ACTIVE CONTROL OF PANEL VIBRATIONS INDUCED BY BOUNDARY-LAYER FLOW

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1. Introduction

In recent years, active control of sound and vibration in aeroelastic panel has gained a great deal of attention due to many possible applications to aerospace and related industries. In the absence of a flow field, such problems have been studied by several authors. For an overview of this subject, one is referred to the paper [1] – [4]. Most recent advances in active control of sound and vibration can be found in the conference proceeding [5].

In the current research project, we seek to investigate some problems in active control of panel vibration excited by a boundary-layer flow over a flat plate. In the first phase of this investigation, we have studied the optimal control problem of vibrating elastic panel induced by a fluid dynamical loading. For a simply-supported rectangular plate, the vibration control problem can be analyzed by a modal analysis. The control objective is to minimize the total cost functional, which is the sum of a vibrational energy and the control cost. By means of the modal (eigenfunction) expansion, the dynamical equation for the plate and the cost functional are reduced to a system of ordinary differential equations and the cost functions for the modes. For the linear elastic plate, the modes become uncoupled. The control of each modal amplitude reduces to the so-called linear regulator problem in control theory. Such problem can then be solved by the method of adjoint state. This method was used successfully in our previous study on the control of thermal fluctuation in a shear flow [6]. The optimality system of equations was solved numerically by a shooting method. The results of this investigation will be summarized in this report. We have also begun to look into the control problem for a nonlinear panel vibration. Analytically we have obtained some preliminary results. Numerical solution of the optimal control of a nonlinear panel (or beam) in one space dimension is in progress. Further development of the vibration control problem will be detailed in the next report.

2. Results on Active Control of Panel Vibrations

Let us consider a viscous flow past over the elastic panel. The flow is governed by the well know Navier-Stokes equation:

$$\frac{\partial \bar{u}}{\partial t} + (\bar{u} \cdot \nabla)\bar{u} = -\frac{1}{\rho}\nabla p + \nu \Delta \bar{u}, \tag{1}$$

where the notations are standard. For a slightly compressible flow, the continuity equation reads

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{u}) = 0. \tag{2}$$

The panel is regarded as an elastic plate with thickness h, Young's modulus E and Poisson's ratio γ. Under a uniform tension with $N > 0$ (or compression with $N < 0$) and the fluctuating wall pressure, the vertical displacement ζ of the plate satisfies the following equation:

$$\rho_w \frac{\partial^2 \zeta}{\partial t^2} = N\Delta\zeta - D\Delta^2\zeta + p_w + q(\bar{x}, t) \tag{3}$$

where ρ_w is the plate density, p_w the wall pressure fluctuation, and q is the applied force as the active control. The constant D is the stiffness of the plate defined by

$$D = \frac{Eh^3}{12(1-\gamma^2)}. \tag{4}$$

According to the boundary-layer theory, given an upstream velocity field $\bar{U}$, the flow near the plate can be determined by the Prandtl's approximation. In particular, if the panel is located on the $x-y$ plane, the pressure gradient $\dfrac{\partial p}{\partial z}$ across the boundary-layer is nearly constant, where $\bar{x} = (x, y, z)$. Suppose that the mean flow outside the boundary-layer is parallel to the plate so that $\bar{U} = (U_\infty, 0, 0) + \bar{U}_1(\bar{x}, t)$, where $\bar{U}_1$ is a small perturbation. To derive the equations for acoustic quantities $\bar{u}, p_1$ and ρ_1, we let

$$\bar{u} = \bar{u}_0 + \bar{u}_1, p = p_0 + p_1 \quad \text{and} \quad \rho = \rho_0 + \rho_1 \tag{5}$$

where $\bar{u}_0, p_0$ and ρ_0 are flow variables associated with the mean flow. As in the stability analysis, we introduce a parallel flow approximation. Then, in view of (5), one obtains the acoustic equations from (1) and (2) by linearization:

$$\frac{\partial \bar{u}_1}{\partial t} + (\bar{u}_0 \cdot \nabla)\bar{u}_1 = -\frac{1}{\rho_0}\nabla p_1, \tag{6}$$

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \bar{u}_1 + \rho_1 \bar{u}_0) = 0. \tag{7}$$

For an isentropic flow, ρ_1 and p_1 are related by

$$\rho_1 = p_1/c^2, \tag{8}$$

where c is the speed of sound for the unperturbed flow. Aside from a static displacement, the vibration of the panel is described by the perturbation w of equation (3) as follows:

$$\rho_w \frac{\partial^2 w}{\partial t^2} = N\Delta w - D\Delta^2 w + f + q(\bar{x}, t), \tag{9}$$

where $f = \tilde{p}_w$ is the fluctuating part of the wall pressure excited by the unsteady boundary-layer flow. The coupling of the acoustic equations (6) and (7), and the plate equation is through the boundary conditions. For the plate equation (9), since it is simply supported by a periodic structure, we need only to analyze the problem over a fundamental domain $0 \le x \le a, 0 \le y \le b$ and impose the boundary conditons:

$$w(x,y,t) = 0 \ \text{ at } \ x = 0, a; \ y = 0, b. \tag{10}$$

Since the pressure gradient $\dfrac{\partial p}{\partial z} = 0$ across the boundary, the wall pressure $\tilde{p}_w$ can be determined from the perturbed potential flow field $\bar{U}_1$ through an approximate Euler's equation, that is,

$$\tilde{p}_w = F(\bar{U}_1). \tag{11}$$

To counter this excitation, a control force $q(x,y,t)$ was introduced in (9). The objective of the active control is to minimize the average vibrational energy and the control cost:

$$J(q) = \frac{1}{2T} \int_0^T \int_D \{\alpha(\frac{\partial w}{\partial t})^2 + \beta(\Delta w)^2 + \gamma|\nabla w|^2 + kq^2\} dt dx dy, \tag{12}$$

where the time T may be infinite, D is the basic domain $\{0 \le x \le a, 0 \le y \le b\}$; α, β, γ and k are positive constants. In the language of the optimal control of a distributed parameter system, the equation (9) is known as the equation of state and $J(q)$ defined by (12), the objective or cost functional. Here the physical problem of vibrational control reduces to an optimization problem: Given the wall pressure excitation $\tilde{p}_w$, find an optimal control $q^*(x,y,t)$ from a certain admissible class Q of functions which minimizes the objective functional $J(q)$, that is,

$$J(q^\star) = \min\{J(q),\ q \text{ in } Q\}. \tag{13}$$

To obtain an analytical solution, we consider the case of simply supported boundary conditions:

$$w(x,y,t) = 0 \text{ at } x = 0, a;\ y = 0, b,$$

$$\frac{\partial^2 w}{\partial x^2}(x,y,t) = 0 \text{ at } x = 0, a \text{ and } \frac{\partial^2 w}{\partial y^2}(x,y,t) = 0 \text{ at } y = 0, b. \tag{14}$$

The initial conditions are given by

$$w(x,y,0) = w_0(x,y), \qquad \frac{\partial w}{\partial t}(x,y,0) = w_1(x,y). \tag{15}$$

It is well known that the set of functions

$$\varphi_{mn}(x,y) = 2\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y, \quad m,n = 1,2,\cdots \tag{16}$$

are orthogonal eigenfunctions associated with the plate equation (9) and the corresponding eigenvalues are

$$\lambda_{mn} = N[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] + D[(\frac{n\pi}{a})^2 + (\frac{m\pi}{b})^2]^2. \tag{17}$$

In terms of the above eigenfunctions, we can expand the displacement w, the wall pressure f and the control as follows:

$$w(x,y,t) = \sum_{m,n=1}^{\infty} w_{m,n}(t)\varphi_{mn}(x,y), \tag{18}$$

$$f(x,y,t) = \sum_{m,n=1}^{\infty} f_{m,n}(t)\varphi_{mn}(x,y) \tag{19}$$

and

$$q(x,y,t) = \sum_{m,n=1}^{\infty} q_{m,n}(t)\varphi_{mn}(x,y), \tag{20}$$

where the coefficients w_{mn} etc. are computed by

$$w_{mn} = \langle w, \varphi_{mn} \rangle = \int_0^a \int_0^b w(x,y,t)\varphi_{mn}(x,y)dxdy,$$

and so on. A substitution of the expansions (18)-(20) into the equations (9), (15) and (12) yields the following uncoupled system of equations:

$$\begin{cases} \rho_w \ddot{w}_{mn} + \lambda_{mn} w_{mn} = f_{mn}(t) + q_{mn}(t) \ , \\ w_{mn}(0) = w_{0,mn}, \ \dot{w}_{mn}(0) = w_{1,mn} \end{cases} \tag{21}$$

and

$$J(q) = \sum_{m,n=1}^{\infty} J_{mn}(q) \ , \tag{22}$$

where

$$J_{mn}(q) = \frac{1}{2T} \int_0^T \{\alpha \dot{w}_{mn}^2(t) + \mu_{mn} w_{mn}^2(t) + k q_{mn}^2(t)\} dt \tag{23}$$

for $m, n = 1, 2, \cdots$. Since the modes are uncoupled, if the cost J_{mn} for each mode is minimized, so does the total cost J.

For a given (m,n)-mode, dropping all the subscripts, we are led to consider the so-called "linear regulator" problem in optimal control: Find the control q in the equation

$$\begin{cases} \rho \ddot{w} + \lambda w = f(t) + q(t) \ , \\ w(0) = w_0, \ \dot{w}(0) = w_1 \ , \end{cases} \tag{24}$$

which minimizes

$$J(q) = \frac{1}{2T}\int_0^T \{\alpha\dot{w}^2 + \mu w^2 + kq^2\}dt\ , \tag{25}$$

where μ_{mn} is given as in λ_{mn} with D and N replaced by β and γ, respectively. By the method of adjoint state,[7], for the cost to be minimal, the state w and its adjoint v must satisfy the optimality system:

$$\begin{cases} \rho\ddot{w} + \lambda w = f(t) + \frac{1}{k}(\alpha w - v), \\ w(0) = w(0),\ \ \dot{w}(0) = w_1, \end{cases} \tag{26}$$

and

$$\begin{cases} \rho\ddot{v} + \lambda v = (\alpha\lambda + \mu)w, \\ v(T) = \dot{v}(T) = 0. \end{cases} \tag{27}$$

The optimal control $q^\star$ is given by $q = \frac{1}{k}(\alpha w - v)$. One notes that, due to the coupling between w and v, the above system (26)–(27) is a two-point boundary-value problem. Numerically it can be solved by the shooting method. Some numerical results for the original modal equations (21) and (23) have been obtained.

For example, we choose $a = 4\pi, b = \pi, w_0 = w_1 \equiv 0$ and $T = 4$, and set

$$f_{mn}(t) = \frac{1}{(m^2+n^2)}\cos(m^2+n^2)^{1/2}t,\ \ m,n = 1,2,\ldots.$$

All the parameters are taken to be one except for β, which is zero. The maximal amplitude of vibration under a optimal control has been computed and some results, corresponding to 4 modes ($m+n=4$), are shown in Fig. 1 to Fig. 3. In the above figures, the solid curves represent the controlled amplitudes, which are in contrast with the uncontrolled ones. It is seen that the control is very effective in reducing the vibration amplitudes. For an independent interest, the controlled mode shape at $t = 4$ is plotted as shown in Fig. 4.

3. Active Control of Panel Fluttering

The problem of panel flutter is a classical problem in aeroelasticity. The problem is of great concern to those who are involved in the design and operation of airplanes. The fluttering is a phenomenon of nonlinear self-excited panel vibrations. To include the nonlinear effect, we shall confine our analysis to the case for which the panel is long in the y-direction so that the problem involves only one space-dimension x. Here in contrast with Eq. (9), the equation of motion for the panel reads:[8]

$$\begin{aligned} \rho\frac{\partial^2 w}{\partial t^2} &= (N + N_x)\frac{\partial^2 w}{\partial x^2} - D\frac{\partial^4 w}{\partial x^4} + f(x,t) + q(x,t), \\ w &= \frac{\partial^2 w}{\partial x^2} = 0 \text{ at } x = 0, a, \\ w(x,0) &= w_0, \ \frac{\partial w}{\partial t}(x,0) = w_1, \end{aligned} \tag{28}$$

where

$$N_x = \frac{Eh}{2a}\int_0^a \left(\frac{\partial w}{\partial x}\right)^2 dx$$

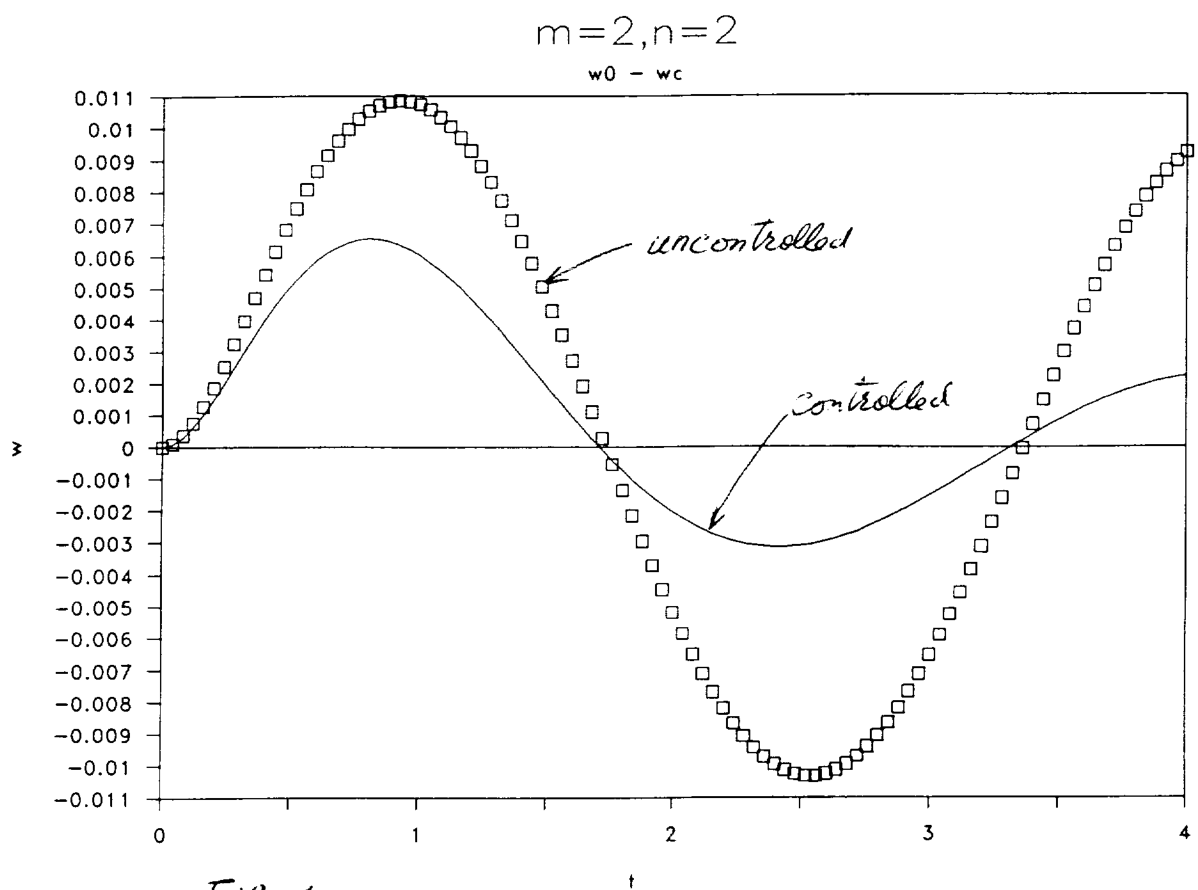
is the nonlinear effect due to streching, E and h are the Young's modulus and the panel thickness, respectively. Without the active control q, the nonlinear panel vibration due to the aerodynamic forcing f has been studied extensively. However, a comprehensive study of the active control problem is still lacking. Subject to the optimality criterion $J(q)$ given by Eq. (12), we have studied this problem analytically by a modal expansion. To this end, we first derived the optimality system for the state equation (28), in which the optimal control must satisfy the following terminal-value problem:

$$\begin{aligned}
\rho\frac{\partial^2 q}{\partial x^2} &= (N+N_x)\frac{\partial^2 q}{\partial x^2} - D\frac{\partial^4 q}{\partial x^4} - 2\delta\langle\frac{\partial^2 w}{\partial x^2}, q\rangle\frac{\partial^2 w}{\partial x^2} \\
&+ \frac{1}{k}(\alpha\frac{\partial^2 w}{\partial t^2} + \beta\frac{\partial^4 w}{\partial x^4} + \gamma\frac{\partial^2 w}{\partial x^2}), \\
q &= \frac{\partial^2 q}{\partial x^2} = 0 \text{ at } x = 0, a, \\
q &= 0 \text{ and } \frac{\partial q}{\partial x} + \frac{\alpha}{k\rho}\frac{\partial w}{\partial t} = 0 \text{ at } t = T,
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
\delta &= (\frac{Eh}{2a}) \text{ and} \\
\langle w, q\rangle &= \int_0^a wq\,dx.
\end{aligned}$$

Since the optimality system (28) and (29) is now nonlinear, the modal expansion (18) – (20) is x alone yields an infinite system of coupled equations. We approximate the system by a modal truncation. The truncated system is sovled numerically. The numerical study is still in progress. The numerical solution will be presented in the next report.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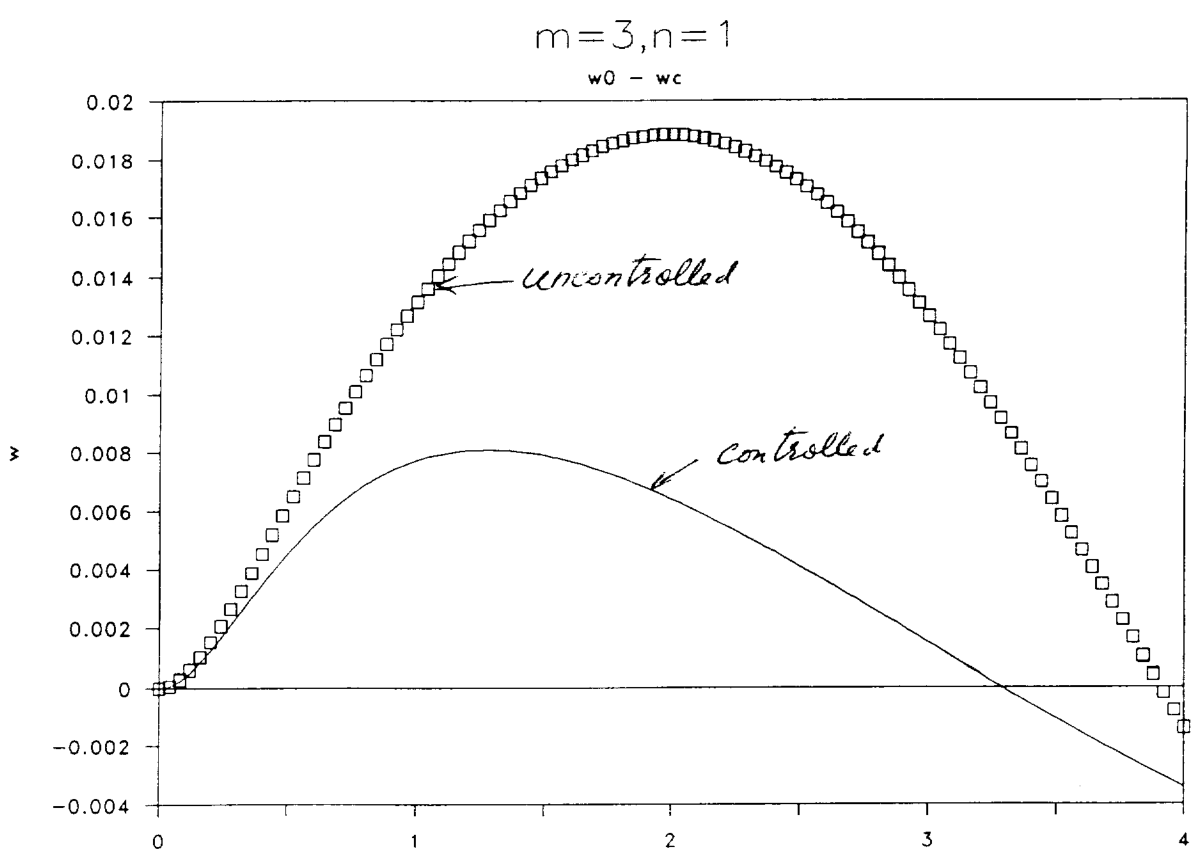
w0 - wc
uncontrolled
controlled
w
t

Fig. 1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w0 - wc
uncontrolled
controlled
w

Fig. 2.

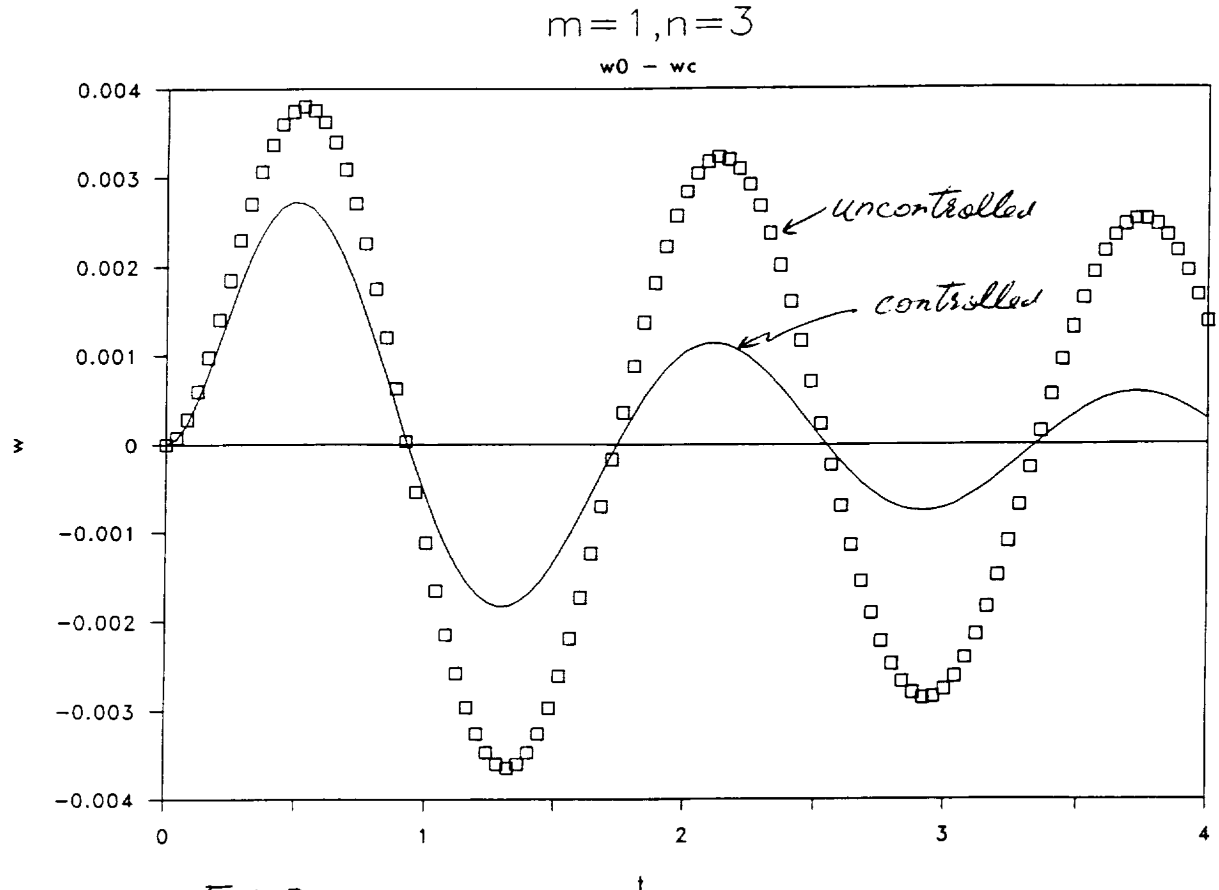

Fig. 3

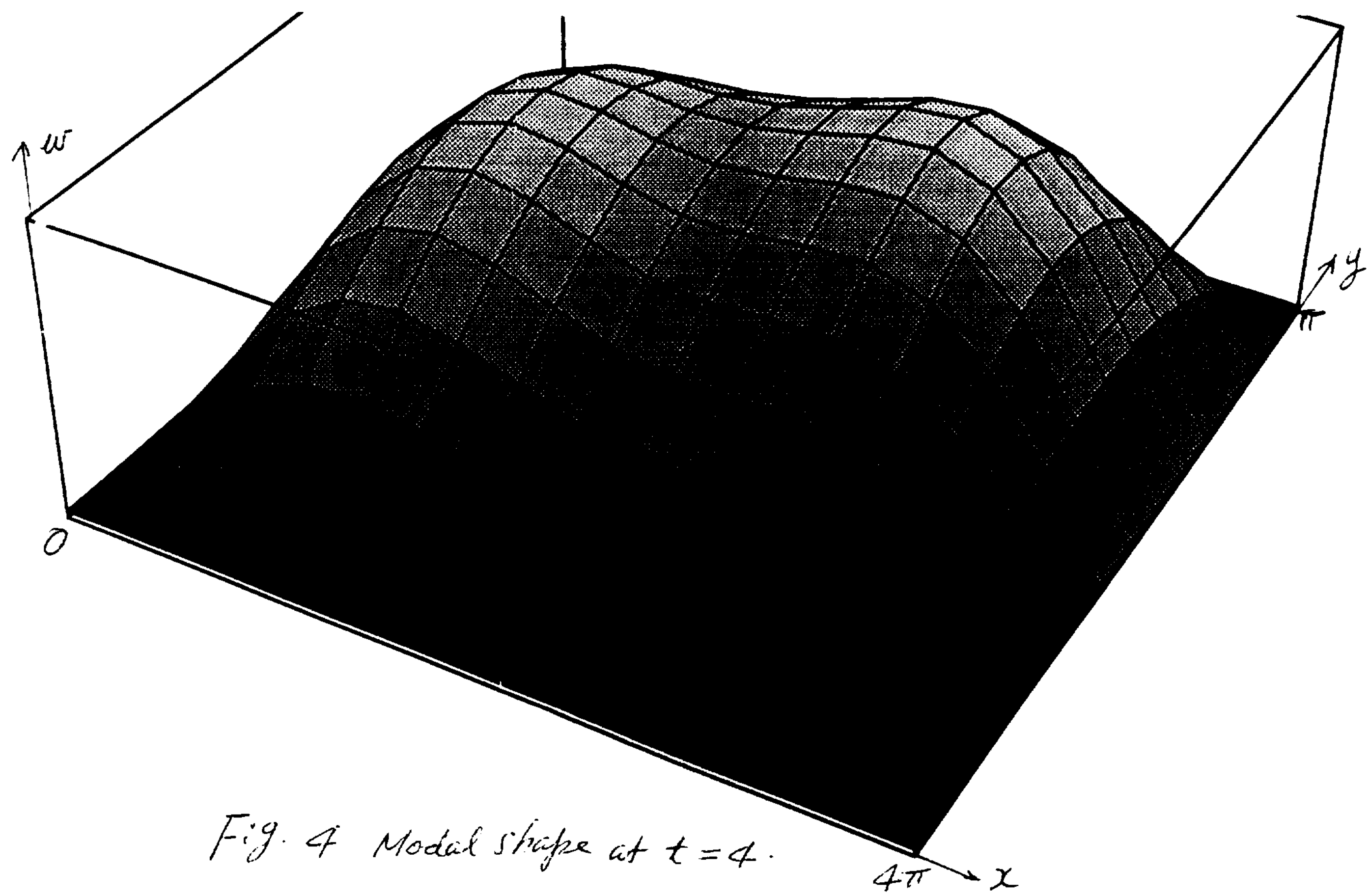

Fig. 4 Modal shape at $t=4$.

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