NEUTRINO DEGENERACY AND COSMOLOGICAL NUCLEOSYNTHESIS, REVISITED *

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ABSTRACT

A reexamination of the effects of non-zero degeneracies on Big Bang Nucleosynthesis is made. As previously noted, non-trivial alterations of the standard model conclusions can be induced only if excess lepton numbers L_i, comparable to photon number densities η_τ, are assumed (where $\eta_\tau \sim 3 \times 10^9 \eta_b$). Furthermore, the required lepton number densities $(L_i \eta_\tau)$ must be different for ν_e than for ν_μ and ν_τ. It is shown that this loophole in the standard model of nucleosynthesis is robust and will not vanish as abundance and reaction rate determinations improve. However, it is also argued that theoretically $\mid L_e \mid \sim \mid L_\mu \mid \sim \mid L_\tau \mid \sim \eta_b \ll \eta_\tau$ which would preclude this loophole in standard unified models.

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The baryon number of the Universe is a quantity of fundamental interest. Baryon number violation is a process of considerable importance in both cosmology and particle physics. In many grand unification models, as well as in recent studies of the electroweak model at high temperatures, violation of B is accompanied by violation of lepton number L, generally with conservation of some linear combination of the two ($B - L$ for instance, in minimal SU$_5$). In cosmology, the baryon to photon ratio, though small ($\eta \equiv (n_B - n_{\bar{B}})/n_\gamma = O(10^{-10})$), has a major influence on the primordial abundances of the light elements; may determine when the Universe became matter-dominated; and is an important parameter in theories of galaxy formation and dark matter determinations. Clearly it is a quantity whose value we would like to know. Fortunately, standard big bang nucleosynthesis can provide us with relatively tight constraints, $\eta_{10} \equiv \eta/10^{-10} = 2.8\text{--}4.0$ [1]. If lepton number violation is of the same order as baryon number violation then it has a negligible effect on nucleosynthesis. However, the question remains, can nucleosynthesis give any constraints on the violation of lepton number? To answer this requires a relatively straightforward adjustment to the usual calculation of the primordial abundances of the light elements. Although large lepton asymmetries can cause difficulties with models for galaxy formation [2], they do not preclude the possibility of galaxy formation when additional physics (e.g. late time phase transitions [3]) is taken into account.

Since we know from redshift-luminosity measurements that the total density of the Universe is not much more than critical we can infer a limit on baryon number from

$$\Omega_B h_0^2 \theta^{-3} = 3.53 \times 10^7 \eta \tag{1}$$

where the Hubble parameter $H_0 = h_0 100$ km s^{-1} Mpc^{-1} and θ is the microwave background temperature in units of 2.75 K. If we assume that $\Omega = 1$ (Ω_{total}, not Ω_B) then age of the Universe arguments constrain $\Omega h_0^2 \lesssim 0.25$ [4] and thus $\eta_{10} \lesssim 70$. More generally, we

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can safely assume that $\Omega_B h_0^2 < 1$ which implies $\eta_{10} \lesssim 280$. In addition, the upper limit on the fractional excess of charge in the universe $(n_e - n_p)/n_p$ is of order 10^{-18} [5] so we can assume that the lepton number due to electrons and positrons is small. Any additional asymmetry in the electron family must therefore be in the form of neutrino degeneracy. The approach we take then is to allow the three known neutrinos to have chemical potentials $(\xi_i \equiv \mu_{\nu_i}/T_\nu)$ up to the value 54, at which point the energy density of that species reaches the critical value for closure, $\rho_c = 3H_0^2/8\pi G$ (see equation 3 below).

The exploration of neutrino chemical potentials for big bang nucleosynthesis has been performed on several occasions in the last few years [6–10], however a number of the relevant reaction rates in the calculation have been updated recently as a result of improved measurements. In particular, since the recent calculation of Terasawa and Sato [8] an improved measurement of the neutron half-life [11] dramatically narrows the uncertainty in the n$\leftrightarrow$p rates [1,12]. In addition the rates D(d,n)^{3}He, D(d,p)T, ^{4}He(t,γ)^{7}Li, ^{7}Li(p,α)^{4}He and ^{7}Be(n,p)^{7}Li have changed [13]. The purpose of this paper is to investigate the effect of these improved input parameters on the baryon and lepton asymmetries of the Universe to accurately assess the current situation.

In addition to improvements in the numerical computation of primordial abundances, the comparison between the predicted abundances and the observational determinations has become more stringent. For example the older bounds on the ^{4}He abundance $Y_p < 0.25$ or 0.254 have now been improved to $Y_p = 0.23 \pm 0.01$, and the bound on the lithium abundance which has ranged from ^{7}Li/H $< (3-10) \times 10^{-10}$ is now down to ^{7}Li/H$\lesssim 1.4 \times 10^{-10}$ [1]. In what follows we will examine the effect of these changes on the neutrino degeneracy loophole. We will conclude that the situation is not qualitatively different from the previous conclusions, despite more stringent input data. Furthermore, we will show that loopholes in the standard big bang nucleosynthesis conclusions about baryon density

will always persist if one allows the introduction of different but specific combinations of values for ξ_e and ξ_μ and/or ξ_τ. While the values required will seem unphysically large, as we will discuss, this loophole will nonetheless remain.

Our calculation is based on the code of Wagoner et al [14], with the n$\leftrightarrow$p rates calculated as described in [1]. We also use the most recent reaction rates of Caughlan and Fowler [13] and the recent data on the neutron mean-life. The observational limits that we use on the light elements are those found in ref. [1]:

$$0.22 \leq Y_p \leq 0.24 \tag{2a}$$

$$1.8 \times 10^{-5} \leq X_d \tag{2b}$$

$$X_d + X_{^3He} \leq 10^{-4} \tag{2c}$$

$$1.0 \times 10^{-10} \leq X_{^7Li} + X_{^7Be} \leq 1.4 \times 10^{-10} \tag{2d}$$

where Y_p is the mass fraction of ^{4}He and X refers to number density relative to hydrogen. The most recent calculations [1] combined with the above observational bounds indicate that the **simplest** version, ie. the standard model of nucleosynthesis, is consistent with these observations. The consistency occurs when $2.8 \leq \eta_{10} \leq 3.3(4.0)$. (The higher value up to $\eta_{10} = 4.0$ is allowed when the uncertainties in key ^{7}Li rates are taken into account, see eg. ref. [1,15].) Through the years, there have been numerous attempts at altering the conclusions of the standard model, by (usually) complicating the input. For example early claims of allowing $\Omega_B = 1$ because of inhomogeneities or decaying particles have largely been dispelled. The nagging alternative of including neutrino degeneracy is the one we discuss here.

Introducing neutrino degeneracy has two effects: (1) weak reaction rates are altered by the change in the electron neutrino distribution function, thus changing the equilibrium

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ratio of neutron to proton densities, and (2) the energy density of neutrinos increases, speeding up the expansion of the universe. Thus ξ_e has two effects, whereas ξ_μ and ξ_τ only affect the expansion rate. Furthermore, this means that ξ_μ and ξ_τ are interchangeable as far as their effects are concerned.

The energy density of neutrinos (and antineutrinos) can be evaluated analytically:

$$\rho_\nu = \sum_{e,\mu,\tau} \frac{\pi^2}{15}(kT_\nu)^4 \left[\frac{7}{8} + \frac{15}{4\pi^2}(\xi_i^2 + \xi_i^4/2\pi^2)\right] ; \qquad \xi_i \equiv \mu_{\nu_i}/T_\nu \tag{3}$$

where we assume the tau neutrino is effectively massless for $T \sim 1$ MeV. For a single neutrino species, closure density is achieved for $\xi = 53.8$ (for a Hubble parameter of 100 km s^{-1}Mpc^{-1}). The lepton number of the universe is then

$$L_i \equiv \frac{n(\nu_i) - n(\bar{\nu}_i)}{n(\gamma)} = \frac{1}{12\zeta(3)}\left(\frac{T_\nu}{T_\gamma}\right)^3 (\pi^2\xi_i + \xi_i^3) \tag{4}$$

(where ζ is the Riemann zeta function and $\zeta(3) \approx 1.202$). If lepton number is conserved then ξ_i is a constant, except during e$^+$–e$^-$ annihilation.

Since the chemical potentials for μ and τ neutrinos appear only in the expression for the energy density, introducing ξ_μ, ξ_τ is clearly equivalent to introducing additional neutrino flavors. We can write

$$N_{\nu\text{eff}} = 3 + \sum_{\mu,\tau} \frac{30}{7\pi^2}(\xi_i^2 + \xi_i^4/2\pi^2). \tag{5}$$

where $N_{\nu\text{eff}}$ parameterizes the neutrinos in terms of massless neutrino species. Following the Z^0 results from LEP we will naturally assume that there are three neutrino species and that the tau neutrino is light. Note that although $\xi_i \sim 50$ corresponds to $N_{\nu\text{eff}} \sim 10^5$, one could equivalently add 10^5 fermion degrees of freedom which do not couple to the Z^0 and hence avoid the LEP limit.

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The effect of neutrino degeneracy on the helium abundance can be readily understood: Increasing ξ_μ or ξ_τ (with either sign) raises the expansion rate before nucleosynthesis. This leads to a higher freeze-out temperature for weak interactions and hence an increased yield of ^{4}He. (This of course, is the same argument as for an increased number of neutrino flavors.) The dramatic increase in the expansion rate (due to the equivalent of $\sim 10^5$ neutrinos) is allowable when one notes that although raising ξ_e also affects the expansion rate (increasing it), in addition it changes the weak reaction rates. The neutron to proton density ratio (in equilibrium) is given by

$$n/p = \exp(-\Delta m/T - \xi_e) \qquad (6)$$

where $\Delta m \equiv m_n - m_p = 1.29$ MeV, so increasing ξ_e leads to a smaller value of n/p when the weak rates freeze out and hence a smaller yield of ^{4}He. Hence, for fixed η the two quantities ξ_e and ξ_μ/ξ_τ can be played off against each other without grossly affecting the ^{4}He abundance. Changing the sign of ξ_e has the opposite effect on the weak reaction rates, but gives the same contribution to the energy density. As η increases so does ^{4}He production as the nuclear reactions producing ^{4}He become more efficient relative to the expansion rate. Increasing the expansion rate (by increasing $\xi_{e,\mu,\tau}$) and decreasing $(n/p)_F$ (by increasing ξ_e) can readily compensate for an increase in η.

D+^{3}He and ^{7}Li production are far less sensitive to n/p at freeze-out as their abundance is primarily determined by competition between nuclear reaction rates and the expansion rate. The longer the nuclear rates are in equilibrium, the more D and ^{3}He are destroyed. For $\eta_{10} \gtrsim 3$ ($\lesssim 3$) the production of mass 7 nuclei (^{7}Be, ^{7}Li) increases (decreases) with increasing η. Thus an increase in $\xi_{e,\mu,\tau}$ can always compensate for an increase in η. At fixed η relatively large increases in ξ_e (driving $n/p \to 0$ exponentially) are necessary to compensate increases in $\xi_{\mu,\tau}$. In the case of D+^{3}He, the increase in ξ_e shuts down D

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and ^{3}He production. For $\eta_{10} \gtrsim 3$ the increase in ξ_e shuts off ^{7}Be(n,p), the destruction channel for mass 7, and results in more ^{7}Li. For $\eta_{10} \lesssim 3$ an increase in ξ_e results in less ^{7}Li production (via ^{4}He(t,γ)) and greater destruction (via ^{7}Li(p,α))—the net result being a decrease in ^{7}Li.

In figures 1a–d we show the regions of the ξ_e–ξ_μ plane (taking $\xi_\tau = \xi_\mu$) allowed by the observational limits for $\eta_{10} = 2.8, 3.3, 10, 280$ and for $\tau_n = 889.6$ sec. The curves in the figures are iso-abundance curves at the observational bounds given in (2a–2d). The allowed region is shown in bold. For $\eta_{10} < 2.7$ the region in which the ^{4}He limits are satisfied has $X_d + X_{^3\text{He}} > 10^{-4}$.

At $\eta_{10} = 2.7$ we are able to meet the constraints for $\xi_\mu = \xi_\tau = 0$ and $\xi_e = 0.1$. As we increase η the yields of deuterium and ^{3}He begin to drop and at $\eta_{10} = 2.8$ we are able to have $\xi_e = \xi_\mu = \xi_\tau = 0$ (fig. 1a). (If we increase ξ_μ sufficiently (to ~ 25) the increased expansion rate reduces the time available for nucleosynthesis and brings Y_p back down to ~ 0.24 [16]. However the D and ^{3}He abundances continue to rise, so this region is still ruled out. This corresponds to ~ 18000 neutrino flavors [17].) $\eta_{10} = 2.8$ corresponds to the lower limit of ref. [1]. Here the limits on the chemical potentials are $|\xi_\mu| < 0.6$ (if $\xi_\mu = \xi_\tau$) and $-0.02 < \xi_e < 0.1$. The lepton numbers are then constrained by $|L_\mu| < 0.15$ and $-0.0050 < L_e < 0.025$. As η rises further the deuterium and ^{3}He abundances continue to drop, while Y_p begins to increase. (Raising η causes nucleosynthesis to begin earlier, giving a larger value of n/p at the onset, and hence increasing Y_p.) The limits on $X_{^7\text{Li}} + X_{^7\text{Be}}$ also begin to move to higher values of the chemical potentials, leaving the allowed region around $0 \lesssim \xi_e \lesssim 0.1, 0 \lesssim \xi_\mu \lesssim 2$ for $\eta_{10} = 3.3$ (fig. 1b). This is the highest value of η for which the origin falls within the allowed region. As we continue to raise η the observationally allowed region moves to higher values of ξ_e and ξ_μ. When we reach $\eta_{10} = 280$ (where $\Omega_B = 1$ for $H_0 = 100$ km s^{-1}Mpc^{-1}) we find we require $\xi_e \approx 1.6, \xi_\mu \approx 40$ in order to satisfy the

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observational constraints.

In fig. 2 we show the same contours as a function of η_{10} and ξ_e for $\xi_\mu = \xi_\tau = 0$. Here it is clear that for $\xi_e = 0$ the limits on the baryon to photon ratio are $2.8 < \eta_{10} < 3.3$ in agreement with ref. [1]. However a value of ξ_e between -0.01 and $+0.09$ is still permissible.

We have seen therefore that the standard nucleosynthesis bound $\eta_{10} \leq 3.3(4.0)$ can be bypassed by introducing two new parameters and in fact allow for $\Omega_B = 1$. It is amusing to note that in addition to the limit on η_{10}, the parameters ξ_e and ξ_μ/ξ_τ are sufficient for obliterating the cosmological bound on the number of neutrino flavors. Indeed by choosing $\xi_\mu = \xi_\tau = 0$ and ξ_e at its limiting value of ~ 1.6, the cosmological bound becomes $N_\nu \lesssim 1.8 \times 10^5$. A further twist arises if for some reason one were compelled to take $\xi_{\mu,\tau} \sim O(\xi_e)$. In that event, an extra source of energy density comparable to that given by the bound on N_ν, $N_\nu \sim O(10^5)$, is needed to account for the necessary expansion.

Despite the baroque nature of the preceeding discussion, the nucleosynthesis loophole based on non-vanishing neutrino chemical potentials does not lead to any direct inconsistencies, per se. Furthermore it is one that we do not forsee as disappearing, either because of improved cross-section measurements or abundance determinations. This can be seen by an examination of figs. 1a–d. The closest observational bound that could close the loophole is the upper bound on $(D+{}^3He)/H$. For $(D+{}^3He)/H \lesssim 8$ the window in the $\eta_{10} = 280$ plot closes, but then the standard model is also ruled out for any value of η_{10}.

We do not however wish to leave the reader with the impression that we are advocating this route for achieving a large value for Ω_B, or that one should ignore the cosmological bound on N_ν. In principle it may be possible to have L_i (ξ_i) be non-zero, but any realistic theory to date would only predict $L_i \sim O(\eta)$. It would also be peculiar to have L_μ or L_τ much different from L_e as would be required to have nucleosynthesis go through the loophole [18]. Furthermore, unless the baryon asymmetry is generated very late, electroweak

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baryon number violation which conserves $B - L$ would require that the total lepton asymmetry $L = L_e + L_\mu + L_\tau = \eta$ ($L = n_L/n_\gamma = n_B/n_\gamma$) [19]. Hence, degenerate nucleosynthesis which requires $L_e, L_{\mu,\tau} \gg \eta$ and $|L_{\mu,\tau}| > L_e$ would necessitate a very special cancellation to achieve $L = \eta$. Thus while we cannot categorically eliminate large neutrino degeneracies as a loophole for cosmological nucleosynthesis it is clear that they are exceedingly unlikely on general theoretical grounds.

REFERENCES

1. T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive and H-S. Kang, in press Ap. J. (1991);

   K.A. Olive, D.N. Schramm, G.Steigman and T.Walker, Phys. Lett. **B236** (1990) 454

2. G. Steigman, in Nucleosynthesis and its Implications on Nuclear and Particle Physics, eds. J. Audouze and N. Mathieu, NATO ASI Series (1985) **163** 45

3. C. Hill, D.N. Schramm and J. Fry, Comm. Nucl. Part. Phys. **19** (1989) 25

4. K. Freese and D.N. Schramm, Nucl. Phys. **B233** (1989) 167

5. R.A. Lyttleton and H. Bondi, Proc. R. Soc. London **A252** (1959) 313

6. Y. David and H. Reeves, Phil. Trans. R. Soc. Lond. **A 296** (1980) 415;

   R.J. Scherrer, Mon. No. R. Astr. Soc. **205** (1983) 683;

   G. Beaudet and A. Yahil. Ap. J. **218** (1977) 253

7. N. Terasawa and K. Sato, Ap. J. **294** (1985) 9

8. N. Terasawa and K. Sato, Progr. Theor. Phys. **80** (1988) 468

9. A. Bianconi, H.W. Lee and R. Ruffini, University of Rome preprint (1990)

10. H-S. Kang and G. Steigman, in preparation (1991)

11. W. Mampe, P. Ageron, C. Bates, J. Pendlebury and A. Steyerl, Phys. Rev. Lett. **63** (1989) 593

12. D. Dicus, E. Kolb, A. Gleeson, E. Sundarshan, V. Teplitz and M. Turner, Phys. Rev. **D26** (1982) 2694

13. G.R. Caughlan and W.A. Fowler, Atom. Dat. Nucl. Dat. Tab. **40** (1988) 283

14. R.V. Wagoner, W.A. Fowler and F. Hoyle, Ap. J. **148** (1967) 3;

    R.V. Wagoner, Ap. J. Suppl. Ser. **162** (1969) 247

15. L. Krauss and P. Romanelli, Ap. J. **358** (1990) 47

16. H-S. Kang, in preparation

17. J. Ellis and K.A. Olive, Nucl. Phys. **B223** (1983) 252

18. K. Enqvist and D.V. Nanopoulos, Prog. Part. Nucl. Phys. **16** (1985) 1;

    E.W. Kolb and M.S. Turner, Ann. Rev. Nucl. Part. Sci. **33** (1983) 645

19. E.W. Kolb and M.S. Turner, Mod. Phys. Lett. **A2** (1987) 285

FIGURE CAPTIONS

1a. Limiting contours in the ξ_μ–ξ_e plane for $\eta_{10} = 2.8$, $\xi_\tau = \xi_\mu$ and $\tau_n = 889.6$ sec. The limits shown are those of equations 2. The region of the plane allowed by all limits is shown in bold.

1b. As fig. 1a but with $\eta_{10} = 3.3$.

1c. As fig. 1a but with $\eta_{10} = 10$.

1d. As fig. 1a but with $\eta_{10} = 280$ and on a larger scale.

2. The same limits in the ξ_e–η_{10} plane with $\xi_\tau = \xi_\mu = 0$.

ξ_e

$\frac{D+{}^3He}{H}=10^{-4}$

$\frac{{}^7Li}{H}=(1.0-1.4)\times10^{-10}$

$Y_p=.24$

$Y_p=.22$

1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0

$\eta_{10}=2.8$

0 1 2 3 4 5 6 7 8 9 10

$\xi_{\mu,\tau}$