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EQUATIONS OF MOTION FOR A FLEXIBLE SPACECRAFT -- LUMPED PARAMETER IDEALIZATION

by

Joel Storch
Stephen Gates

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16. Abstract

The equations of motion for a flexible vehicle capable of arbitrary translational and rotational motions in inertial space accompanied by small elastic deformations are derived in an unabridged form. The vehicle is idealized as consisting of a single rigid body with an ensemble of mass particles interconnected by massless elastic structure. The internal elastic restoring forces are quantified in terms of a stiffness matrix. A transformation and truncation of elastic degrees of freedom is made in the interest of numerical integration efficiency. Deformation dependent terms are partitioned into a hierarchy of significance. The final set of motion equations are brought to a fully assembled first order form suitable for direct digital implementation. A FORTRAN program implementing the equations is given and its salient features described.

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The final chapter of this report pertains to a FORTRAN computer program which implements and numerically integrates the complete set of equations. The salient features of the program, its subroutines, and the input and output data are described. An annotated flowchart along with a full listing of the code is provided.

It is noteworthy that while the idealization and methodology applied in this report are essentially those of Likins,[5] the equations formulated herein are unique from those developed in Reference 5, and indeed the distinction is fundamental. It was the express desire to avoid the kinematic restrictions required there to effect a coordinate transformation on the elastic deflections which motivated this approach.

From an applications standpoint, the basic discretization of the vehicle of interest is performed in the manner of lumped mass structural dynamics modeling. The required stiffness matrix which quantifies the internal elastic restoring forces can in general be obtained from preprocessed linear structural finite element analysis programs (e.g., NASTRAN). Because of the mass particle idealization of the elastic domain, only translational displacements are defined at those points, hence any finite element model used to provide stiffness matrix information must be purged of any rotational degrees of freedom that may exist. This requirement is easily satisfied through the application of the static condensation procedure. Thus the analyst is afforded these familiar and versatile structural modeling techniques augmented by the arbitrary motion capability.

The motion equations formulated here are complete and unabridged for a single unconstrained flexible vehicle. However, they could, in a straightforward fashion, be coupled to the dynamics equations for other independent bodies to form an articulated system. This can be done through the identification and elimination of interbody constraint forces/torques and redundant kinematic variables. Indeed, it is for just such an application that these equations are intended. Specifically they are to represent a generic flexible payload to be terminally attached to the

Space Shuttle Orbiter remote manipulator system, which is an articulated chain of rigid and flexible bodies. For this case the model's rigid-body is taken to be the payload grapple fixture with all outboard structure represented by the particle assemblage.

CHAPTER 2

PRELIMINARIES

2.1 Vehicle Idealization

The system being analyzed (see Figure 1) consists of a single rigid body and an attached flexible appendage. The appendage is idealized as a system of particles connected by massless elastic structure. There is no articulation between the appendage and rigid base, i.e., the appendage is "cantilevered" to the rigid body. At an arbitrary point, 0_g, of the rigid body we locate the origin of the body fixed frame which rotates as the body rotates in inertial space. The vector $\vec{R}$ serves to determine the position of 0_g relative to the inertially fixed point 0. The particle masses m_i $(i = 1,2,\ldots,n)$ are located via the position vectors $\vec{r}_i$ relative to 0_g in the undeformed state. The elastic displacement of m_i is $\vec{q}_i$, measured in the body frame.

Many space vehicles or parts of spacecraft can be approximated in this manner. A specific example is the Shuttle Remote Manipulation System in which the "appendage" corresponds to a flexible payload and the "rigid base" to the grapple fixture (this component being attached to the orbiter through the links of the manipulator arm).

2.2 External and Internal Forces

With the ultimate goal in mind of applying the present analysis to more complicated situations, we wish to accommodate all forces and torques which will arise when the system in Figure 1 is attached to other spacecraft components. Hence, at the point 0_g, let there be a force-torque pair: $\vec{f}^{o}(t)$, $\vec{\tau}^{o}(t)$. In the domain of the elastic appendage we have an external force $\vec{f}^{i}(t)$ acting upon the point mass m_i.

As an example, in the case of the Shuttle Remote Manipulator Arm, $\vec{f}^{o}$ and $\vec{\tau}^{o}$ would represent the force and torque exerted by the end-effector on the grapple fixture.

We assume that we are given a stiffness matrix reflecting the mutual elastic forces between the mass points of the appendage. Assemble the elastic displacements as

$$\underset{\sim}{q} = (q_x^1\ q_y^1\ q_z^1\ q_x^2\ q_y^2\ q_z^2,\ \ldots,\ q_x^n\ q_y^n\ q_z^n\ )^T$$

(x,y,z) refer to Cartesian components along the axes of the body fixed frame.

If [K] is the stiffness matrix, then $-[K]\underset{\sim}{q}$ is the vector of elastic forces exerted on the point masses. Assume that [K] is partitioned such that the vector of elastic forces are ordered exactly as the elements in $\underset{\sim}{q}$. Note that in generating [K] the appendage is cantilevered to the rigid base.

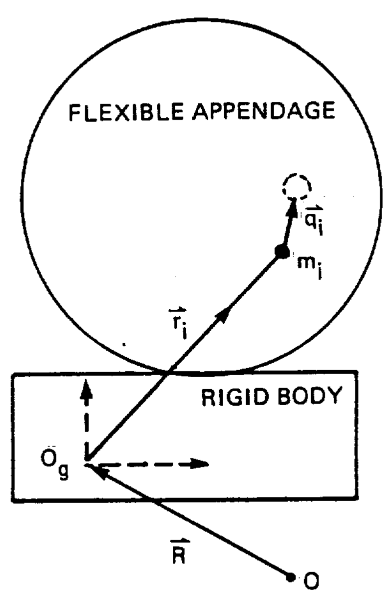


Figure 1. Idealized vehicle.

CHAPTER 3

EQUATIONS OF MOTION

3.1 Particle Translational Equations

$\vec{v}^i$, the inertial velocity of the i^{th} particle, is given by

$$\vec{v}^i = \frac{d}{dt}(\vec{R} + \vec{r}^i + \vec{q}^i)$$

Let $\begin{pmatrix} u \\ v \\ w \end{pmatrix}$ and $\underline{\omega}$ represent the (absolute) velocity and angular velocity of the body frame resolved in body axes, we then have

$$\underline{v}^i = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \underline{\omega} \times (\underline{r}^i + \underline{q}^i) + \dot{\underline{q}}^i$$

Differentiating this expression we arrive at the particle acceleration

$$\frac{d}{dt}\underline{v}^i = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - (\underline{r}^i + \underline{q}^i) \times \dot{\underline{\omega}} + \ddot{\underline{q}}^i$$

$$+ \underline{\omega} \times \left[\begin{pmatrix} u \\ v \\ w \end{pmatrix} + 2\dot{\underline{q}}^i\right] + \underline{\omega} \times [\underline{\omega} \times (\underline{r}^i + \underline{q}^i)]$$

Expanding the cross product in the last term and using the matrix-vector form for the cross product $\underline{a} \times \underline{b} \equiv [a]^{\sim}\underline{b}$ where $[\underline{a}]^{\sim} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$

the particle acceleration can be written as

$$\frac{d}{dt}\underline{v}^i = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - ([\underline{r}^i]^{\sim} + [\underline{q}^i]^{\sim})\dot{\underline{\omega}} + \ddot{\underline{q}}^i + [\underline{\omega}]^{\sim}\begin{pmatrix} u \\ v \\ w \end{pmatrix} + 2[\underline{\omega}]^{\sim}\dot{\underline{q}}^i$$

$$+ (\underline{\omega}^T\underline{r}^i)\underline{\omega} - ||\underline{\omega}||^2\,\underline{r}^i + (\underline{\omega}^T\underline{q}^i)\underline{\omega} - ||\underline{\omega}||^2\underline{q}^i \qquad (3\text{-}1)$$

If $\underline{f}^i$ is the external force on the i^{th} particle and $\underline{f}_e^i$ the elastic force exerted on the i^{th} particle by the rest of the assemblage (both resolved along body axes), the translational equation is

$$\underline{f}^i + \underline{f}_e^i = m_i \frac{d}{dt}\underline{v}^i$$

$$(i = 1,2,\ldots,n)$$

Partitioning the stiffness matrix into (3×3) arrays

$$[K] = \begin{pmatrix} [K_{11}] & [K_{12}]\ldots[K_{1n}] \\ [K_{21}] & [K_{22}]\ldots[K_{2n}] \\ \vdots & \vdots \quad\quad \vdots \\ [K_{n1}] & [K_{n2}]\ldots[K_{nn}] \end{pmatrix}$$

$$\underline{f}_e^i = -\sum_{j=1}^{n} [K_{ij}]\,\underline{q}^j$$

Employing Eq. (3-1) for the particle acceleration, the translation equations for the appendage particles may be assembled as

$$\begin{bmatrix} m^1 \\ \hline m^2 \\ \hline \cdot \\ \cdot \\ \cdot \\ m^n \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - \begin{bmatrix} m_1(\tilde{\underline{r}}^1 + \tilde{\underline{q}}^1) \\ \hline m_2(\tilde{\underline{r}}^2 + \tilde{\underline{q}}^2) \\ \hline \cdot \\ \cdot \\ \cdot \\ m_n(\tilde{\underline{r}}^n + \tilde{\underline{q}}^n) \end{bmatrix} \dot{\underline{\omega}} + \begin{bmatrix} m^1 & \underline{0} & \underline{0} & \cdots & \underline{0} \\ \underline{0} & m^2 & \underline{0} & \cdots & \underline{0} \\ \underline{0} & \underline{0} & m^3 & \cdots & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot \\ \underline{0} & \underline{0} & \cdot & \cdot & m^n \end{bmatrix} \ddot{\underline{q}}$$

$$+ [K]\underline{q} = \begin{bmatrix} \underline{f}^1 \\ \underline{f}^2 \\ \vdots \\ \underline{f}^n \end{bmatrix} + \underline{u}_v \qquad (3\text{-}2)$$

where we have introduced the symbol $m^i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix} (i = 1,2,\ldots,n)$

and the nonlinear kinematic term $\underline{u}_v$ is given by

$$\underline{u}_v = - \begin{bmatrix} m_1\tilde{\underline{\omega}} \\ \hline m_2\tilde{\underline{\omega}} \\ \hline \vdots \\ \hline m_n\tilde{\underline{\omega}} \end{bmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} - 2 \begin{bmatrix} m_1\tilde{\underline{\omega}}\,\dot{\underline{q}}^1 \\ \hline m_2\tilde{\underline{\omega}}\,\dot{\underline{q}}^2 \\ \hline \vdots \\ \hline m_n\tilde{\underline{\omega}}\,\dot{\underline{q}}^n \end{bmatrix} - \begin{bmatrix} m_1\underline{\omega} \cdot (\underline{r}^1 + \underline{q}^1)\underline{\omega} \\ \hline m_2\underline{\omega} \cdot (\underline{r}^2 + \underline{q}^2)\underline{\omega} \\ \hline \vdots \\ \hline m_n\underline{\omega} \cdot (\underline{r}^n + \underline{q}^n)\underline{\omega} \end{bmatrix}$$

$$+ ||\underline{\omega}||^2 \begin{bmatrix} m_1(\underline{r}^1 + \underline{q}^1) \\ \hline m_2(\underline{r}^2 + \underline{q}^2) \\ \hline \vdots \\ \hline m_n(\underline{r}^n + \underline{q}^n) \end{bmatrix} \qquad (3\text{-}3)$$

Equation (3-2) constitutes a set of 3n scalar differential equations.

3.2 Vehicle Translational Equations

For the composite system (rigid body and appendage) the sum of the external forces equals the total mass times the acceleration of the mass center.

If m_b is the mass of the rigid body and $\underline{s}$ is the vector from O_g to the mass center of the rigid body (expressed in the body frame)

$$m\underline{c} = \sum_{i=1}^{n} m_i(\underline{r}^i + \underline{q}^i) + m_b\underline{s} \qquad (3\text{-}4)$$

$m = m_b + \sum_{i=1}^{n} m_i$ is the total mass.

$\underline{c}(t)$ is the vector position of the instantaneous mass center relative to O_g.

The acceleration of the mass center is: $\frac{d^2}{dt^2}(\vec{R} + \vec{c})$

$$\frac{d^2\vec{R}}{dt^2} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + \underline{\omega} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad \text{(in body frame)}$$

$$m\frac{d^2\underline{c}}{dt^2} = \sum_{i=1}^{n} m_i\ddot{\underline{q}}^i - m\underline{c} \times \dot{\underline{\omega}} + 2\underline{\omega} \times \sum_{i=1}^{n} m_i\dot{\underline{q}}^i + \underline{\omega} \times (\underline{\omega} \times m\underline{c})$$

Expressing this last term as: $(\underline{\omega} \cdot m\underline{c})\underline{\omega} - ||\underline{\omega}||^2 m\underline{c}$ the vehicle translational equation assumes the form

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - [m\underline{c}]^{\sim}\dot{\underline{\omega}} + [m^1 \quad m^2 \quad \ldots \quad m^n]\ddot{\underline{q}} = \sum_{i=0}^{n} \underline{f}^i + \underline{u}_t \qquad (3\text{-}5)$$

The nonlinear term $\underline{u}_t$ is given by

$$\underline{u}_t = -m\underline{\omega} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} - 2\underline{\omega} \times \sum_{i=1}^{n} m_i \dot{\underline{q}}^i - (\underline{\omega} \cdot m\underline{c})\underline{\omega} + ||\underline{\omega}||^2 m\underline{c} \tag{3-6}$$

3.3 Vehicle Rotational Equations

For the composite system (rigid body and appendage) the sum of the external torques taken about the mass center equals the time rate of change of the angular momentum taken about the mass center.

Let $[I_b]$ be the inertia matrix of the rigid body with respect to a coordinate system located at the mass center of the rigid body and parallel to the body fixed axes system at O_g.

$$[I_b] = \iiint [\underline{\lambda} \cdot \underline{\lambda} E - \underline{\lambda}\underline{\lambda}^T]\ dm$$

$\underline{\lambda}$ is the position vector of a mass element dm in the rigid body relative to the rigid body mass center and the integration is performed over the region occupied by the rigid base. [E] denotes the unit matrix.

The system angular momentum can be split into two parts:

$\underline{H}_b$ — angular momentum of rigid base

$\sum_{i=1}^{n} \underline{H}_i$ — angular momentum of appendage particles

Let $\underline{\ell}^i$ and $\underline{\ell}$ denote the position vectors from the system mass center to m_i and a generic mass element in the rigid body respectively.

$$\underline{H}_b = \iiint \underline{\ell} \times \frac{d}{dt} \underline{\ell}\ dm$$

$$\underline{H}_i = \underline{\ell}^i \times m_i \frac{d}{dt} \underline{\ell}^i$$

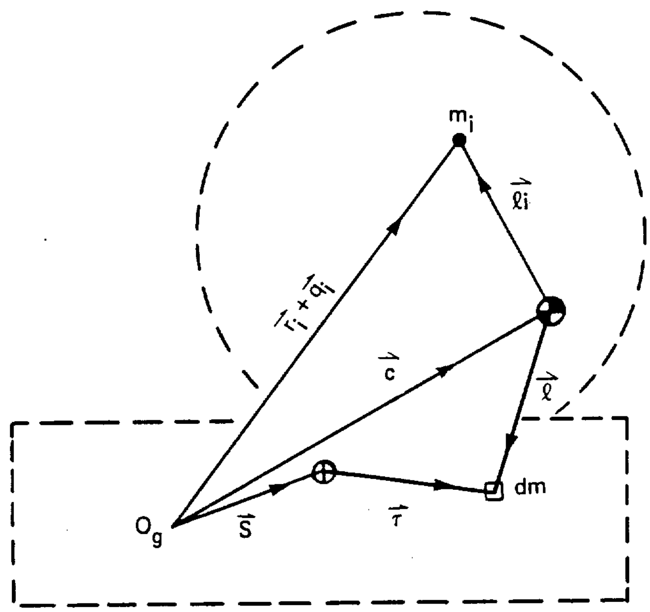


Figure 2. Vector geometry.

From Figure 2, $\underline{\ell}^i = \underline{r}^i + \underline{q}^i - \underline{c}$, $\underline{\ell} = \underline{s} + \underline{\lambda} - \underline{c}$.

Inserting this expression for $\underline{\ell}$ into the integral definition of $\underline{H}_b$ and recalling the definition of $[I_b]$ and the fact that $\iiint \underline{\lambda}\ dm = \underline{0}$ we arrive at the following expression for $\underline{H}_b$

$$\underline{H}_b = [I_b]\underline{\omega} + m_b(\underline{s} - \underline{c}) \times \frac{d}{dt}(\underline{s} - \underline{c})$$

Thus

$$\frac{d}{dt}\underline{H}_b = [I_b]\dot{\underline{\omega}} + \underline{\omega} \times [I_b]\underline{\omega} + m_b(\underline{s} - \underline{c}) \times [\dot{\underline{\omega}} \times \underline{s} + \underline{\omega} \times (\underline{\omega} \times \underline{s})]$$
$$- m_b(\underline{s} - \underline{c}) \times \frac{d^2}{dt^2}\underline{c}$$

Turning to the angular momentum of the i^{th} particle

$$\frac{d}{dt}\underline{H}^i = \underline{\ell}^i \times m_i \frac{d^2}{dt^2}\underline{\ell}^i$$

Now

$$\frac{d^2}{dt^2}\underline{\ell}^i = -(\underline{r}^i + \underline{q}^i) \times \dot{\underline{\omega}} + \ddot{\underline{q}}^i + 2\underline{\omega} \times \dot{\underline{q}}^i + [\underline{\omega} \cdot (\underline{r}^i + \underline{q}^i)]\underline{\omega}$$

$$- ||\underline{\omega}||^2(\underline{r}^i + \underline{q}^i) - \frac{d^2}{dt^2}\underline{c}$$

Combining the above expressions for $\frac{d}{dt}\underline{H}_b$ and $\frac{d}{dt}\underline{H}^i$ (with the substitution for $\frac{d^2}{dt^2}\underline{\ell}^i$) the terms involving $\frac{d^2}{dt^2}\underline{c}$ conveniently cancel leaving the following result for the time derivative of the angular momentum

$$\frac{d}{dt}\underline{H} = ([I_b] - m_b(\underline{s} - \underline{c})^{\sim}\tilde{\underline{s}})\dot{\underline{\omega}} + \underline{\omega} \times [I_b]\underline{\omega} + \sum_{i=1}^{n} m_i[\underline{\ell}^i]^{\sim}\ddot{\underline{q}}^i$$

$$+ m_b(\underline{s} - \underline{c}) \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})] - \sum_{i=1}^{n} m_i[\underline{\ell}^i]^{\sim}(\underline{r}^i + \underline{q}^i)^{\sim}\dot{\underline{\omega}}$$

$$+ 2\sum_{i=1}^{n} m_i[\underline{\ell}^i]^{\sim}\tilde{\underline{\omega}}\,\dot{\underline{q}}^i + \sum_{i=1}^{n} [\underline{\omega} \cdot (\underline{r}^i + \underline{q}^i)]m_i\,\underline{\ell}^i \times \underline{\omega}$$

$$- ||\underline{\omega}||^2 \sum_{i=1}^{n} m_i\,\underline{\ell}^i \times (\underline{r}^i + \underline{q}^i) \qquad (3\text{-}7)$$

The system rotational motion equation is

$$\frac{d}{dt}\underline{H} = \underline{\tau}^{o} - \underline{c} \times \underline{f}^{o} + \sum_{i=1}^{n} (\underline{r}^{i} + \underline{q}^{i} - \underline{c}) \times \underline{f}^{i}$$

$$= \underline{\tau}^{o} - \underline{c} \times \sum_{j=0}^{n} \underline{f}^{j} + \sum_{i=1}^{n} (\underline{r}^{i} + \underline{q}^{i}) \times \underline{f}^{i}$$

If we were to insert Eq. (3-7) into this last equation we would have a valid equation for system rotation. Note, however, that this equation would not depend upon $(\dot{u},\dot{v},\dot{w})$ explicitly. Since the translational equations (3-5) depend upon $\dot{\underline{\omega}}$ explicitly, and we wish to have a final set of equations with a symmetric coefficient matrix of the generalized accelerations, we force the coupling between the rotational equations and the acceleration vector $(\dot{u}\ \dot{v}\ \dot{w})$ by the following device.

Take Eq. (3-5) and (3-6) and solve for $\sum_{j=0}^{n} \underline{f}^{j}$. The system rotational motion equation then becomes

$$\frac{d}{dt}\underline{H} = \underline{\tau}^{o} + \sum_{i=1}^{n} (\underline{r}^{i} + \underline{q}^{i}) \times \underline{f}^{i} - [m\underline{c}]^{\sim} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + [\underline{c}]^{\sim}[m\underline{c}]^{\sim}\, \dot{\underline{\omega}}$$

$$- \underline{c} \times \sum_{i=1}^{n} m_i\, \ddot{\underline{q}}^{i} - [m\underline{c}]^{\sim}\, \tilde{\underline{\omega}} \begin{pmatrix} u \\ v \\ w \end{pmatrix} - 2[\underline{c}]^{\sim}\, \tilde{\underline{\omega}} \sum_{i=1}^{n} m_i \dot{\underline{q}}^{i}$$

$$- (\underline{\omega} \cdot m\underline{c})\, \underline{c} \times \underline{\omega} \qquad (3\text{-}8)$$

Combining Eq. (3-7) and (3-8), we arrive at the final desired form for the vehicle rotational equation

$$[m\underline{c}]^{\sim}\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{pmatrix} + [I(t)]\ \dot{\underline{\omega}} + \left[m_1(\underline{r}^1 + \underline{q}^1)^{\sim} \mid m_2(\underline{r}^2 + \underline{q}^2)^{\sim} \mid \dots \mid m_n(\underline{r}^n + \underline{q}^n)^{\sim}\right]\ddot{\underline{q}}$$

$$= \underline{\tau}^o + \sum_{i=1}^{n} (\underline{r}^i + \underline{q}^i) \times \underline{f}^i + \underline{u}_r \qquad (3\text{-}9)$$

Here

$$[I(t)] = [I_b] - m_b(\underline{s} - \underline{c})^{\sim}\tilde{\underline{s}} - \sum_{i=1}^{n} m_i[\underline{\ell}^i]^{\sim}(\underline{r}^i + \underline{q}^i)^{\sim} - [\tilde{\underline{c}}][m\underline{c}]^{\sim}$$

which can be simplified to

$$[I(t)] = [I_b] - m_b[\tilde{\underline{s}}]^2 - \sum_{i=1}^{n} m_i(\tilde{\underline{r}}^i + \tilde{\underline{q}}^i)^2 \qquad (3\text{-}10)$$

In this form we recognize $[I(t)]$ as the inertia matrix of the vehicle about point O_g.

The nonlinear rotation term $\underline{u}_r$ is given by

$$\underline{u}_r = -[m\underline{c}]^{\sim}\tilde{\underline{\omega}}\begin{pmatrix}u\\ v\\ w\end{pmatrix} - 2[\underline{c}]^{\sim}[\underline{\omega}]^{\sim}\sum_{i=1}^{n} m_i\dot{\underline{q}}^i - (\underline{\omega}\cdot m\underline{c})[\underline{c}]^{\sim}\underline{\omega}$$

$$- [\underline{\omega}]^{\sim}[I_b]\underline{\omega} - m_b(\underline{s} - \underline{c}) \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})] - 2\sum_{i=1}^{n} m_i[\underline{\ell}^i]^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i$$

$$- \sum_{i=1}^{n} [\underline{\omega}\cdot(\underline{r}^i + \underline{q}^i)]m_i\ \underline{\ell}^i \times \underline{\omega} + ||\underline{\omega}||^2 \sum_{i=1}^{n} m_i\ \underline{\ell}^i \times (\underline{r}^i + \underline{q}^i)$$

(3-11)

CHAPTER 4

EXPANSION AND PARTITIONING OF TERMS

4.1 Deformation Dependent Coefficients

In the equations developed thus far, specifically Eq. (3-2), (3-5), and (3-9), we have isolated the accelerations on the left hand sides of the respective equations. The acceleration coefficients are time dependent through the elastic deformations. It is quite desirable from an applications viewpoint to rank the constituents in these coefficients in accordance with their relative magnitude. Thus, in a computer simulation, one can choose to omit certain terms and speed up execution with a minimal impact on computed results.

We will rank terms amongst three categories:

(1) Terms independent of $\underline{q}$.

(2) Terms first order in $\underline{q}$.

(3) Terms second order in $\underline{q}$.

The majority of coefficients are directly identifiable in this hierarchy. We have two coefficients which require additional attention: $m\underline{c}$ and $[I(t)]$.

From Eq. (3-4), $m\underline{c} = m_b\,\underline{s} + \sum_{i=1}^{n} m_i\underline{r}^i + \sum_{i=1}^{n} m_i\underline{q}^i(t)$. The first two terms are of category 1 and the sum will be denoted by $m\underline{c}_0$. The time-dependent term will be denoted by $m\underline{c}_1(t)$.

The matrix $[I(t)]$ is given by Eq. (3-10) and can be written as

$$[I(t)] \;=\; [I_1] + [I_2(t)] + [I_3(t)]$$

The three matrices $[I_1]$, $[I_2(t)]$, and $[I_3(t)]$ are of category 1, 2, and 3 respectively and are given by

$$[I_1] = [I_b] - m_b[\tilde{\underline{s}}]^2 - \sum_{i=1}^{n} m_i[\tilde{\underline{r}}^i]^2$$

$$[I_2(t)] = -\sum_{i=1}^{n} m_i([\tilde{\underline{r}}^i][\tilde{\underline{q}}^i] + [\tilde{\underline{q}}^i][\tilde{\underline{r}}^i])$$

$$[I_3(t)] = -\sum_{i=1}^{n} m_i[\tilde{\underline{q}}^i]^2$$

4.2 Nonlinear Kinematic Terms

In this section, we concentrate upon the three nonlinear terms: $\underline{u}_t$, $\underline{u}_v$, and $\underline{u}_r$ appearing on the right hand sides of the motion equations. Following a procedure similar to that of the previous section, the nonlinear terms are partitioned amongst three categories:

(1) Nonlinear terms independent of $\underline{q}$, $\dot{\underline{q}}$

(2) Nonlinear terms first order in $\underline{q}$, $\dot{\underline{q}}$

(3) Nonlinear terms second order in $\underline{q}$, $\dot{\underline{q}}$

Accordingly, from Eq. (3-6), $\underline{u}_t = \underline{u}_t^{(1)} + \underline{u}_t^{(2)}$ with

$$\underline{u}_t^{(1)} = -m[\underline{\omega}]^{\sim} \begin{pmatrix} u \\ v \\ w \end{pmatrix} - (\underline{\omega} \cdot m\underline{c}_0)\underline{\omega} + ||\underline{\omega}||^2 m\underline{c}_0 \qquad (4\text{-}1)$$

$$\underline{u}_t^{(2)} = -2[\underline{\omega}]^{\sim} \sum_{i=1}^{n} m_i \dot{\underline{q}}^i - (\underline{\omega} \cdot m\underline{c}_1)\underline{\omega} + ||\underline{\omega}||^2 m\underline{c}_1 \qquad (4\text{-}2)$$

In a similar manner from Eq. (3-3)

$$\underline{u}_v = \underline{u}_v^{(1)} + \underline{u}_v^{(2)}$$

$$\underline{u}_v^{(1)} = -\begin{bmatrix} m_1\tilde{\underline{\omega}} \\ m_2\tilde{\underline{\omega}} \\ \vdots \\ m_n\tilde{\underline{\omega}} \end{bmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{bmatrix} m_1(\underline{\omega} \cdot \underline{r}^1)\underline{\omega} \\ m_2(\underline{\omega} \cdot \underline{r}^2)\underline{\omega} \\ \vdots \\ m_n(\underline{\omega} \cdot \underline{r}^n)\underline{\omega} \end{bmatrix} + ||\underline{\omega}||^2 \begin{bmatrix} m_1\underline{r}^1 \\ m_2\underline{r}^2 \\ \vdots \\ m_n\underline{r}^n \end{bmatrix} \quad (4\text{-}3)$$

$$\underline{u}_v^{(2)} = -2\begin{bmatrix} m_1\tilde{\underline{\omega}}\,\dot{\underline{q}}^1 \\ m_2\tilde{\underline{\omega}}\,\dot{\underline{q}}^2 \\ \vdots \\ m_n\tilde{\underline{\omega}}\,\dot{\underline{q}}^n \end{bmatrix} - \begin{bmatrix} m_1(\underline{\omega} \cdot \underline{q}^1)\underline{\omega} \\ m_2(\underline{\omega} \cdot \underline{q}^2)\underline{\omega} \\ \vdots \\ m_n(\underline{\omega} \cdot \underline{q}^n)\underline{\omega} \end{bmatrix} + ||\underline{\omega}||^2 \begin{bmatrix} m_1\underline{q}^1 \\ m_2\underline{q}^2 \\ \vdots \\ m_n\underline{q}^n \end{bmatrix} \quad (4\text{-}4)$$

<u>Expansion of $\underline{u}_r$</u>

The two terms in $\underline{u}_r$ (Eq. (3-11)) depending upon $\dot{\underline{q}}$ can be combined as

$$-2\sum_{i=1}^{n} m_i([\underline{c}]^{\sim} + [\underline{\ell}^i]^{\sim})[\underline{\omega}]^{\sim}\dot{\underline{q}}^i = -2\sum_{i=1}^{n} m_i(\underline{r}^i + \underline{q}^i)^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i$$

$$= -2\sum_{i=1}^{n} m_i[\underline{r}^i]^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i - 2\sum_{i=1}^{n} m_i[\underline{q}^i]^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i$$

The third term in $\underline{u}_r$ can be expressed as

$$(\underline{\omega} \cdot m\underline{c})[\underline{c}]^{\sim}\underline{\omega} = (\underline{\omega} \cdot m\underline{c}_0)[\underline{c}_0]^{\sim}\underline{\omega} + (\underline{\omega} \cdot m\underline{c}_0)[\underline{c}_1]^{\sim}\underline{\omega} + (\underline{\omega} \cdot m\underline{c}_1)[\underline{c}_0]^{\sim}\underline{\omega}$$
$$+ (\underline{\omega} \cdot m\underline{c}_1)[\underline{c}_1]^{\sim}\underline{\omega}$$

For the last two terms in $\underline{u}_r$ the following expansions are useful

$$\begin{aligned}[\underline{\omega} \cdot (\underline{r}^i + \underline{q}^i)]m_i\underline{\ell}^i \times \underline{\omega} &= (\underline{\omega} \cdot \underline{r}^i)m_i(\underline{r}^i - \underline{c}_0) \times \underline{\omega} \\ &+ (\underline{\omega} \cdot \underline{q}^i)m_i(\underline{r}^i - \underline{c}_0) \times \underline{\omega} \\ &+ (\underline{\omega} \cdot \underline{r}^i)m_i(\underline{q}^i - \underline{c}_1) \times \underline{\omega} \\ &+ (\underline{\omega} \cdot \underline{q}^i)m_i(\underline{q}^i - \underline{c}_1) \times \underline{\omega}\end{aligned}$$

$$\underline{\ell}^i \times (\underline{r}^i + \underline{q}^i) = \underline{r}^i \times \underline{c}_0 + \underline{r}^i \times \underline{c}_1 + \underline{q}^i \times \underline{c}_0 + \underline{q}^i \times \underline{c}_1$$

Collecting terms in $\underline{u}_r$ independent of deformation

$$\begin{aligned}\underline{u}_r^{(1)} = &-[m\underline{c}_0]^{\sim}[\underline{\omega}]^{\sim}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - (\underline{\omega} \cdot m\underline{c}_0)[\underline{c}_0]^{\sim}\underline{\omega} - \underline{\omega} \times [I_b]\underline{\omega} \\ &- m_b(\underline{s} - \underline{c}_0) \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})] - \sum_{i=1}^{n} m_i(\underline{\omega} \cdot \underline{r}^i)(\underline{r}^i - \underline{c}_0) \times \underline{\omega} \\ &+ ||\underline{\omega}||^2 \sum_{i=1}^{n} m_i\underline{r}^i \times \underline{c}_0\end{aligned}$$

The expression for $\underline{u}_r^{(1)}$ can be further simplified by use of the following identities

$$\sum_{i=1}^{n} m_i \, \underline{r}^i \times \underline{c}_0 \;=\; -m_b \, \underline{s} \times \underline{c}_0$$

$$\sum_{i=1}^{n} m_i (\underline{\omega} \cdot \underline{r}^i)(\underline{r}^i - \underline{c}_0) \times \underline{\omega} \;=\; -[\underline{\omega}]^{\sim} \sum_{i=1}^{n} m_i \, \underline{r}^i \, \underline{r}^{iT} \, \underline{\omega}$$

$$+ \; [\underline{\omega} \cdot (m_b \underline{s} - m\underline{c}_0)] \underline{c}_0 \times \underline{\omega}$$

$$-m_b(\underline{s} - \underline{c}_0) \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})] \;=\; -(\underline{\omega} \cdot \underline{s}) m_b \, \underline{s} \times \underline{\omega} + (\underline{\omega} \cdot \underline{s}) m_b \, \underline{c}_0 \times \underline{\omega}$$

$$- \; ||\underline{\omega}||^2 \, m_b \, \underline{c}_0 \times \underline{s}$$

Incorporating these results we arrive at the final expression

$$\underline{u}_r^{(1)} \;=\; -[m\underline{c}_0]^{\sim}[\underline{\omega}]^{\sim} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + [\underline{\omega}]^{\sim} \sum_{i=1}^{n} m_i \, \underline{r}^i \, \underline{r}^{iT} \, \underline{\omega} - \underline{\omega} \times [I_b]\underline{\omega}$$

$$- \; m_b (\underline{s} \cdot \underline{\omega}) \underline{s} \times \underline{\omega} \tag{4-5}$$

Collecting first order deformation dependent terms in $\underline{u}_r$

$$\underline{u}_r^{(2)} \;=\; -[m\underline{c}_1]^{\sim}[\underline{\omega}]^{\sim} \begin{pmatrix} u \\ v \\ w \end{pmatrix} - 2\sum_{i=1}^{n} m_i [\underline{r}^i]^{\sim}[\underline{\omega}]^{\sim} \dot{\underline{q}}^i - (\underline{\omega} \cdot m\underline{c}_0)[\underline{c}_1]^{\sim}\underline{\omega}$$

$$- \; (\underline{\omega} \cdot m\underline{c}_1)[\underline{c}_0]^{\sim}\underline{\omega} + m_b \underline{c}_1 \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})]$$

$$- \sum_{i=1}^{n} [(\underline{\omega} \cdot \underline{q}^i) m_i (\underline{r}^i - \underline{c}_0) \times \underline{\omega} + (\underline{\omega} \cdot \underline{r}^i) m_i (\underline{q}^i - \underline{c}_1) \times \underline{\omega}]$$

$$+ \; ||\underline{\omega}||^2 \sum_{i=1}^{n} m_i (\underline{r}^i \times \underline{c}_1 + \underline{q}^i \times \underline{c}_0)$$

The expression for $\underline{u}_r^{(2)}$ can be further simplified by use of the following identities

$$\sum_{i=1}^{n} m_i(\underline{r}^i \times \underline{c}_1 + \underline{q}^i \times \underline{c}_0) = m_b\, \underline{c}_1 \times \underline{s}$$

$$-\sum_{i=1}^{n} [(\underline{\omega} \cdot \underline{q}^i)m_i(\underline{r}^i - \underline{c}_0) \times \underline{\omega} + (\underline{\omega} \cdot \underline{r}^i)m_i(\underline{q}^i - \underline{c}_1) \times \underline{\omega}] =$$

$$[\underline{\omega}]^{\sim}\sum_{i=1}^{n} m_i(\underline{r}^i\underline{q}^{iT} + \underline{q}^i\underline{r}^{iT})\underline{\omega} + (\underline{\omega} \cdot m\underline{c}_1)\underline{c}_0 \times \underline{\omega} + [\underline{\omega} \cdot (m\underline{c}_0 - m_b\underline{s})]\underline{c}_1 \times \underline{\omega}$$

$$m_b\, \underline{c}_1 \times [\underline{\omega} \times (\underline{\omega} \times \underline{s})] = m_b(\underline{\omega} \cdot \underline{s})\underline{c}_1 \times \underline{\omega} - m_b||\underline{\omega}||^2\, \underline{c}_1 \times \underline{s}$$

$$\underline{u}_r^{(2)} = -[m\underline{c}_1]^{\sim}[\underline{\omega}]^{\sim}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - 2\sum_{i=1}^{n} m_i[\underline{r}^i]^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i$$

$$+ [\underline{\omega}]^{\sim}\sum_{i=1}^{n} m_i(\underline{r}^i\underline{q}^{iT} + \underline{q}^i\underline{r}^{iT})\underline{\omega} \qquad (4\text{-}6)$$

Collecting second order deformation dependent terms in $\underline{u}_r$ and simplifying

$$\underline{u}_r^{(3)} = -2\sum_{i=1}^{n} m_i[\underline{q}_i]^{\sim}[\underline{\omega}]^{\sim}\dot{\underline{q}}^i + [\underline{\omega}]^{\sim}\sum_{i=1}^{n} m_i\underline{q}^i\underline{q}^{iT}\underline{\omega} \qquad (4\text{-}7)$$

$\underline{u}_r = \underline{u}_r^{(1)} + \underline{u}_r^{(2)} + \underline{u}_r^{(3)}$ where the terms on the right hand side are given by Eq. (4-5) - (4-7).

CHAPTER 5

MODAL COORDINATE TRANSFORMATION

When the number of particles in the appendage idealization becomes large, high frequencies obtain which make numerical integration difficult. We will describe a truncated coordinate transformation to circumvent this difficulty. Note that the treatment to follow is somewhat heuristic and hence requires good engineering judgement and caution in its implementation.

Since the high frequencies arise from the appendage vibration, it is natural to start with its governing equation (3-2, 3-3). Consider the case where no external forces act, $\underline{\omega} = \underline{0}$ and $(\dot{u},\dot{v},\dot{w}) = \underline{0}$. The "constrained" appendage equation then assumes the familiar form

$$[M]\ddot{\underline{q}} + [K]\underline{q} = \underline{0} \qquad \text{where } [M] = \text{diag}(m^1, m^2, \ldots, m^n)$$

The natural frequencies, ω_i, and corresponding mode shapes, $\underline{v}^i$, are determined from

$$([K] - \omega^2[M])\underline{V} = \underline{0} \qquad (5\text{-}1)$$

For the vehicle we are treating here, the appendage is rigidly attached to the base body hence no rigid body modes are present in the above eigenvalue problem. Equivalently, [K] is positive definite. Since [K] and [M] are symmetric and positive definite there exists 3n independent eigenvectors $\underline{v}^i$ corresponding to positive eigenvalues ω_i^2, even if there are multiple eigenvalues.

We assume that the eigenvectors are normalized such that $(\underline{v}^i, [M]\underline{v}^i) = 1$. It follows that $(\underline{v}^i, [K]\underline{v}^i) = \omega_i^2$. We can always create a mutually orthogonal set such that

$$(\underline{v}^i, [M]\underline{v}^j) = 0 = (\underline{v}^i, [K]\underline{v}^j) \qquad (i \neq j)$$

In actual computation we can deal with a simpler eigenvalue problem than that presented by Eq. (5-1). Specifically we will transform Eq. (5-1) to an ordinary symmetric eigenvalue problem. Introduce the change of variables: $\underline{W} = [M]^{1/2}\underline{V}$. Since [M] is diagonal, $[M]^{1/2}$ is a diagonal matrix whose elements are the square roots of the corresponding elements in [M]. The eigenvalue problem transforms into

$$[K][M]^{-1/2}\underline{W} = \omega^2[M]^{1/2}\underline{W} \quad \text{or} \quad [\mathcal{G}]\underline{W} = \omega^2\underline{W} \qquad (5\text{-}2)$$

where $[\mathcal{G}]$ is the symmetric matrix: $[\mathcal{G}] = [M]^{-1/2}[K][M]^{-1/2}$

It is easily verified that if the eigenvectors $\underline{W}^i$ $(i = 1,2,\ldots,3n)$ of Eq. (5-2) are orthogonal (which can always be done) then the corresponding eigenvectors of (5-1) satisfy all orthogonality and normality conditions specified above.

Order the eigenvalues such that $\omega_1^2 \leq \omega_2^2 \leq \ldots \leq \omega_{3n}^2$ and let $[\Phi]$ be the $(3n \times t)$ matrix whose columns are the eigenvectors $\underline{v}^1, \underline{v}^2, \ldots, \underline{v}^t$ $(t \leq 3n)$. We now make the transformation

$$\underline{q} = [\Phi]\underline{\eta} \qquad (5\text{-}3)$$

This is not a coordinate transformation in the strict sense, since $[\Phi]$ does not have an inverse when $t < 3n$. The appendage deformation is now characterized by t "modal coordinates" instead of the original 3n deformation coordinates. We formally make the substitution, Eq. (5-3), into the full set of motion equations.

Substituting into the appendage deformation equation (3-2), premultiplying by $[\Phi]^T$ and recalling the orthogonality and normality conditions we arrive at

$$[\Phi]^T \begin{bmatrix} m^1 \\ m^2 \\ \vdots \\ m^n \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - [\Phi]^T \begin{bmatrix} m_1(\underline{r}^1 + \underline{q}^1)^{\sim} \\ m_2(\underline{r}^2 + \underline{q}^2)^{\sim} \\ \vdots \\ m_n(\underline{r}^n + \underline{q}^n)^{\sim} \end{bmatrix} \dot{\underline{\omega}} + \ddot{\underline{\eta}} + \begin{bmatrix} \omega_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & & & 0 \\ 0 & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \omega_t^2 \end{bmatrix} \underline{\eta} =$$

$$[\Phi]^T \begin{bmatrix} \underline{f}^1 \\ \underline{f}^2 \\ \vdots \\ \underline{f}^n \end{bmatrix} + [\Phi]^T \underline{u}_v \qquad (5\text{-}4)$$

The vehicle translational equations (3-5) become

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} - [m\underline{c}]^{\sim}\dot{\underline{\omega}} + [m^1 \; m^2 \; \ldots \; m^n][\Phi]\ddot{\underline{\eta}} = \sum_{i=0}^{n} \underline{f}^i + \underline{u}_t$$

The vehicle rotational equations (3-9) become

$$[m\underline{c}]^{\sim} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + [I(t)]\dot{\underline{\omega}} + \left[m_1(\underline{r}^1 + \underline{q}^1)^{\sim} \,\middle|\, m_2(\underline{r}^2 + \underline{q}^2)^{\sim} \,\middle|\, \ldots \,\middle|\, m_n(\underline{r}^n + \underline{q}^n)^{\sim}\right][\Phi]\ddot{\underline{\eta}} =$$

$$\underline{\tau}^0 + \sum_{i=1}^{n} (\underline{r}^i + \underline{q}^i) \times \underline{f}^i + \underline{u}_r$$

The assembled equations of motion in matrix form are presented in Figure 3.

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} & -[m\underline{c}]^{\sim} & [m^1 m^2 \ldots m^n]\,[\Phi] \\
\hline
[m\underline{c}]^{\sim} & [I(t)] & [m_1(\underline{r}^1+\underline{g}^1)^{\sim} \cdots m_n(\underline{r}^n+\underline{g}^n)^{\sim}]\,[\Phi] \\
\hline
[\Phi]^T \begin{bmatrix} m^1 \\ m^2 \\ \vdots \\ m^n \end{bmatrix} & -[\Phi]^T \begin{bmatrix} m_1(\underline{r}^1+\underline{g}^1)^{\sim} \\ m_2(\underline{r}^2+\underline{g}^2)^{\sim} \\ \vdots \\ m_n(\underline{r}^n+\underline{g}^n)^{\sim} \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & & \\ 0 & 0 & 0 & \cdots & & 1 \end{bmatrix}
\end{array}\right]
\left[\begin{array}{c} \dot{u} \\ \dot{v} \\ \dot{w} \\ \hline \dot{\underline{\omega}} \\ \hline \ddot{\underline{\eta}} \end{array}\right]
$$

$$
= \left[\begin{array}{c}
\sum_{i=0}^{n} \underline{f}^i \\
\hline
\underline{\tau}^0 + \sum_{i=1}^{n} (\underline{r}^i + \underline{g}^i) \times \underline{f}^i \\
\hline
[\Phi]^T \begin{pmatrix} \underline{f}^1 \\ \underline{f}^2 \\ \vdots \\ \underline{f}^n \end{pmatrix}
\end{array}\right]
- \left[\begin{array}{c}
\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
\hline
\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
\hline
\begin{bmatrix} \omega_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdot & & \\ 0 & & & \cdot & \\ \vdots & & & & \cdot \\ 0 & 0 & \cdots & \cdots & \omega_t^2 \end{bmatrix} \underline{\eta}
\end{array}\right]
+ \left[\begin{array}{c}
\underline{u}_t \\
\hline
\underline{u}_r \\
\hline
[\Phi]^T\, \underline{u}_v
\end{array}\right]
$$

Figure 3. Assembled equations of motion.

CHAPTER 6

SYSTEM KINETIC ENERGY

The kinetic energy of the vehicle is the sum of the translational and rotational kinetic energy of the rigid body and the kinetic energy of the particles comprising the appendage

$$T = \frac{1}{2} m_b v_b^2 + \frac{1}{2} \underline{\omega} \cdot [I_b]\underline{\omega} + \frac{1}{2} \sum_{i=1}^{n} m_i v_i^2$$

$\underline{v}_b = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \underline{\omega} \times \underline{s}$ is the velocity of the mass center of the base

$\underline{v}^i = \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \underline{\omega} \times (\underline{r}^i + \underline{q}^i) + \dot{\underline{q}}^i$ is the velocity of i^{th} particle

Forming the inner products $(\underline{v}_b, \underline{v}_b);(\underline{v}^i, \underline{v}^i)$ and recalling Eq. (3-10) for $[I(t)]$ the kinetic energy can be written as

$$\begin{aligned} T = {} & \frac{1}{2} m(u^2 + v^2 + w^2) + \frac{1}{2} \underline{\omega}^T [I(t)]\underline{\omega} + \frac{1}{2} \sum_{i=1}^{n} \{\dot{\underline{q}}^i\}^T m_i \dot{\underline{q}}^i \\ & - \frac{1}{2} (uvw)[m\underline{c}]^{\sim}\underline{\omega} + \frac{1}{2} \underline{\omega}^T [m\underline{c}]^{\sim} \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \frac{1}{2} (uvw) \sum_{i=1}^{n} m_i \dot{\underline{q}}^i \\ & + \frac{1}{2} \sum_{i=1}^{n} m_i \{\dot{\underline{q}}^i\}^T \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \frac{1}{2} \underline{\omega}^T \sum_{i=1}^{n} m_i (\underline{r}^i + \underline{q}^i)^{\sim} \dot{\underline{q}}^i - \frac{1}{2} \sum_{i=1}^{n} m_i \{\dot{\underline{q}}^i\}^T (\underline{r}^i + \underline{q}^i)^{\sim} \underline{\omega} \end{aligned}$$

We now rewrite those terms in T which depend upon $\dot{\underline{q}}^i$ in terms of $\dot{\underline{\eta}}$

$$\sum_{i=1}^{n} \{\dot{\underline{q}}^i\}^T m_i \dot{\underline{q}}^i = \dot{\underline{q}}^T [M] \dot{\underline{q}} = \dot{\underline{\eta}}^T [\Phi]^T [M][\Phi] \dot{\underline{\eta}} = \dot{\underline{\eta}}^T [E] \dot{\underline{\eta}}$$

[E] is the $(t \times t)$ identity matrix

$$(uvw) \sum_{i=1}^{n} m_i \dot{\underline{q}}^i = (uvw) \left[m^1 \mid m^2 \ldots \mid m^n \right] \begin{pmatrix} \dot{\underline{q}}^1 \\ \dot{\underline{q}}^2 \\ \vdots \\ \dot{\underline{q}}^n \end{pmatrix}$$

$$= (uvw)[m^1 \; m^2 \; \ldots \; m^n][\Phi]\dot{\underline{\eta}}$$

$$\underline{\omega}^T \sum_{i=1}^{n} m_i (\underline{r}^i + \underline{q}^i)^{\sim} \dot{\underline{q}}^i = \underline{\omega}^T [m_1 (\underline{r}^1 + \underline{q}^1)^{\sim} \; \ldots \; m_n (\underline{r}^n + \underline{q}^n)^{\sim}][\Phi]\dot{\underline{\eta}}$$

The kinetic energy can be written as the quadratic form $T = \frac{1}{2} \underline{U}^T [A] \underline{U}$ where [A] is the coefficient matrix (symmetric) of the generalized accelerations appearing in the equations of motion (see Figure 3) and $\underline{U}$ is the vector of non-holonomic velocities

$$\underline{U} = \left(uvw \mid \underline{\omega}^T \mid \dot{\underline{\eta}}^T \right)^T$$

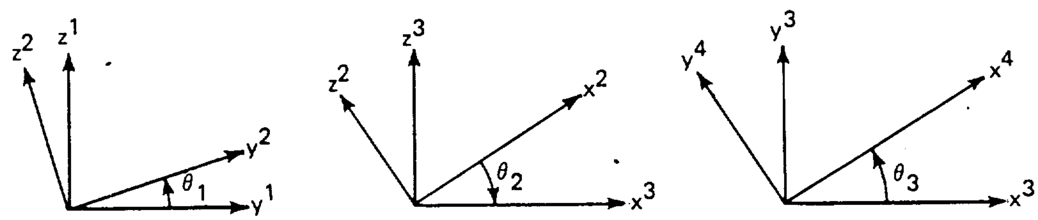
Since $[I_b]$ is positive definite, an inspection of the initial expression for T reveals that $T \geq 0$ for all $\underline{U}$. If $T = 0$ then $\vec{v}_b = \underline{\omega} = \underline{v}^i = \underline{0}$ $(i = 1,2,\ldots,n)$. But $\vec{v}_b = \underline{0} = \underline{\omega}$ implies $(uvw) = \underline{0}$ and $\underline{v}^i = \underline{0} = (uvw) = \underline{\omega}$ implies $\dot{\underline{q}} = \underline{0}$. Hence $[\Phi]\dot{\underline{\eta}} = \underline{0}$. Since the columns of $[\Phi]$ are linearly independent we must have $\dot{\underline{\eta}} = \underline{0}$ also. In other words, $T = 0$ if and only if $\underline{U} = \underline{0}$. This argument proves that [A] is positive definite and consequently nonsingular (see Chapter 8 where we require $[A]^{-1}$).

Note that if we replace the rigid body by a particle then $\underline{s} = \underline{0}$ and $[I_b] = [0]$. We still have $T \geq 0$ but if $T = 0$ we can only argue that $(uvw) = \underline{0}$. We can have $T = 0$ for nonzero $\underline{\omega}$ and $\dot{\underline{\eta}}$ as long as $\underline{\omega} \times (\underline{r}^i + \underline{q}^i) + \dot{\underline{q}}^i = 0$ $(i = 1,2,\ldots,n)$. Thus for this later case [A] is positive semi-definite. In particlar [A] will be singular. The situation here can be understood by simply enumerating the degrees of freedom involved. Originally we had a system consisting of a rigid body and n particles: (6 + 3n) degrees of freedom. The number of dynamic equations was also (6 + 3n). When degenerating the rigid body to a particle we have a system of (n + 1) particles: (3n + 3) degrees of freedom. However, when we retain the same equations of motion as in the original case (6 + 3n) there will clearly be a redundancy present. Indeed, this explains why [A] is singular for the degenerate case. Consequently, we cannot use the equations developed here for a system composed solely of particles; at least not without modification.

CHAPTER 7

KINEMATICAL RELATIONSHIPS

Let the transformation from the inertial frame $\{x^1, y^1, z^1\}$ to the body frame $\{x^4, y^4, z^4\}$ be arrived at by a sequence of three Euler angles $\theta_1, \theta_2, \theta_3$ as depicted below.



$$R^{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix} \qquad R^{23} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}$$

$$R^{34} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$[R^{ij}]$ is the transformation matrix from frame 'j' to frame 'i'

Concatenating transformations, $[R^{14}] = [R^{12}][R^{23}][R^{34}]$

$$[R^{14}] = \begin{pmatrix} \cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\ \cos\theta_1\sin\theta_3 + \sin\theta_1\sin\theta_2\cos\theta_3 & \cos\theta_1\cos\theta_3 - \sin\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2 \\ \sin\theta_1\sin\theta_3 - \cos\theta_1\sin\theta_2\cos\theta_3 & \sin\theta_1\cos\theta_3 + \cos\theta_1\sin\theta_2\sin\theta_3 & \cos\theta_1\cos\theta_2 \end{pmatrix} \tag{7-1}$$

We next derive the relationship between the body frame angular velocity $\underline{\omega}$ (expressed in body coordinates) and the Euler rates $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$. Let $\{\underline{i}_p, \underline{j}_p, \underline{k}_p\}$ be the set of unit vectors along the axes of frame 'p' (p = 1,2,3,4) .

$$\vec{\omega} = \dot{\theta}_1\vec{i}_1 + \dot{\theta}_2\vec{j}_2 + \dot{\theta}_3\vec{k}_3$$

To express $\vec{\omega}$ in the body frame, we must use the representation of the unit vectors in frame 4. With the aid of the transformations listed above we arrive at

$$\underline{\omega} = \begin{pmatrix} \dot{\theta}_1\cos\theta_2\cos\theta_3 + \dot{\theta}_2\sin\theta_3 \\ \dot{\theta}_2\cos\theta_3 - \dot{\theta}_1\cos\theta_2\sin\theta_3 \\ \dot{\theta}_3 + \dot{\theta}_1\sin\theta_2 \end{pmatrix} \quad \text{(in body frame)}$$

This system can be inverted to yield

$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = \begin{bmatrix} \dfrac{\cos\,\theta_3}{\cos\,\theta_2} & \dfrac{-\sin\,\theta_3}{\cos\,\theta_2} & 0 \\ \sin\,\theta_3 & \cos\,\theta_3 & 0 \\ -\cos\,\theta_3\tan\,\theta_2 & \sin\,\theta_3\tan\,\theta_2 & 1 \end{bmatrix} \underline{\omega} \qquad (\cos\,\theta_2 \neq 0) \tag{7-2}$$

CHAPTER 8

EQUATIONS OF MOTION — FIRST ORDER FORM

The assembled motion equations (Figure 3) can be written as

$$[A(t)]\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \hline \dot{\underline{\omega}} \\ \hline \ddot{\underline{\eta}} \end{bmatrix} = \underline{F} + \underline{U} - \begin{bmatrix} \underline{0} \\ \hline \underline{0} \\ \hline \omega_1^2\eta_1 \\ \omega_2^2\eta_2 \\ \vdots \\ \omega_t^2\eta_t \end{bmatrix} \tag{8-1}$$

where

$$\underline{F} = \begin{bmatrix} \sum_{i=0}^{n} \underline{f}^i \\ \hline \underline{T}^0 + \sum_{i=1}^{n} (\underline{r}^i + \underline{q}^i) \times \underline{f}^i \\ \hline [\Phi]^T \begin{pmatrix} \underline{f}^1 \\ \underline{f}^2 \\ \vdots \\ \underline{f}^n \end{pmatrix} \end{bmatrix} \tag{8-2}$$

and

$$
\underline{U} = \begin{bmatrix} \underline{u}_t \\ \hdashline \underline{u}_r \\ \hdashline [\Phi]^T \underline{u}_v \end{bmatrix} \tag{8-3}
$$

Let R_x, R_y, R_z be the components of the inertial position vector of O_g (origin of body frame) resolved along inertial axes and $[\Gamma]$ denote the matrix in Eq. (7-2). The kinematic relationships can now be written

$$
\begin{pmatrix} \dot{R}_x \\ \dot{R}_y \\ \dot{R}_z \end{pmatrix} = [R^{14}] \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \qquad \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = [\Gamma]\underline{\omega}
$$

Define $[\Omega^2] = \mathrm{diag}(\omega_1^2, \omega_2^2, \ldots, \omega_t^2)$. The state vector $\underline{Y}$ is defined to be

$$
\underline{Y} = (R_x R_y R_z \theta_1 \theta_2 \theta_3 \underline{\eta}^T \; uvw \; \underline{\omega}^T \; \underline{\dot{\eta}}^T)^T \tag{8-4}
$$

The equations of motion written in first order form are

$$
\frac{d}{dt}\underline{Y} = \begin{bmatrix} [R^{14}] \begin{pmatrix} u \\ v \\ w \end{pmatrix} \\ [\Gamma]\underline{\omega} \\ \underline{\dot{\eta}} \\ A^{-1}\left[\underline{F} + \underline{U} - \begin{pmatrix} \underline{0} \\ [\Omega^2]\underline{\eta} \end{pmatrix}\right] \end{bmatrix} \tag{8-5}
$$

This system of (2t + 12) first order equations can be integrated numerically with appropriate initial conditions.

CHAPTER 9

DIGITAL SIMULATION

This chapter is concerned with the FORTRAN computer program which implements and numerically integrates the complete set of first order ordinary differential equations presented in Chapter 8, Eq. (8-5). A description of the main program, its subroutines, and the input data is given. An annotated flowchart of the program is given in Figure 4 and a complete listing of the program and its subroutines is provided in Appendix A. An example of the input data for a sample vehicle is provided in Appendix B. The code is liberally commented throughout and in most instances the FORTRAN variable names are mnemonically similar to the corresponding analytical quantities. Virtually all computations involving real number quantities are performed in (IBM) double precision. External subroutines from the double precision IMSL library[8] are used to perform certain standard computations. IMSL subroutine "EIGRS" is used for eigenvalue/eigenvector extraction and subroutine "LEQT1P" is used to solve simultaneous linear equations. In addition, IMSL subroutine "USPLT" is used to generate time history graphs of selected elements of the vector

$$\{R_x R_y R_z \;\theta_1 \theta_2 \theta_3 \;\underline{q}^{1^T} \cdots \underline{q}^{n^T} \;uvw\; \omega_1 \omega_2 \omega_3 \;\dot{\underline{q}}^{1^T} \cdots \dot{\underline{q}}^{n^T}\} \qquad (9\text{-}1)$$

via the line printer.

Throughout the program deformation dependent terms are arranged and computed hierarchically as quantities involving structural deflections to the first and second degree. Similarly the nonlinear kinematic terms

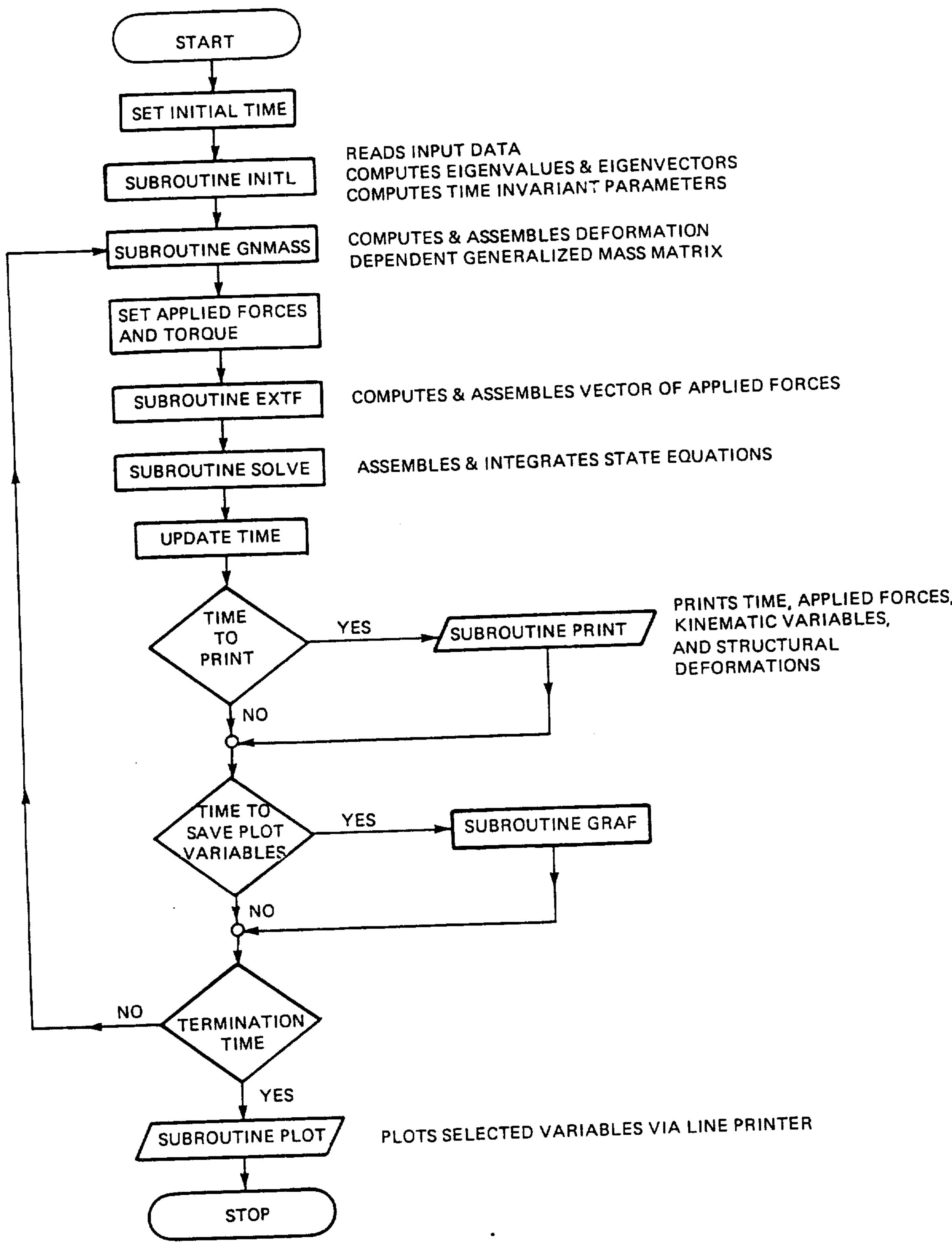


Figure 4. Program flowchart.

are organized into the three categories of Section 4.2 with the contributions of each group of terms being computed independently. This partitioned structure of the computations provides the capability to assess the influence of these higher order terms on the final solution and upon such analysis bypass those deemed negligible.

Main Program

The main program is simply an executive module which calls the appropriate subroutines in the proper order. The reader will note that if external forces are required, these must be explicitly coded either in the main program or as individual subroutines. If the external forces are time dependent, it is essential that they be recomputed prior to each call to subroutine "EXTF" (see comments in main program). For the system in Figure 1 the external excitation is accommodated via the three arrays: FØ,* TAUØ, FP.

FØ — sum of external forces on rigid body

TAUØ — sum of external moments on rigid body taken about body frame origin.

FØ and TAUØ are three-dimensional vectors whose elements refer to components along body frame axes.

FP(I,J) — is the I^{th} component of the external force acting upon particle J in the appendage (I = 1,2,3; J = 1,2,...,N).

The external forces for each of the "N" particles comprising the appendage are resolved along body axes.

Subroutines

Subroutine INITL reads in all program input data and performs consistency checks. Selected input data is echo printed. The eigenvalues

---

* "Ø" denotes the number zero.

and eigenvectors of the standard symmetric eigenvalue problem given by Eq. (5-2) are computed via a call to IMSL subroutine EIGRS. The eigenvectors are then transformed to those corresponding to Eq. (5-1). All time-invariant terms of the generalized mass matrix of Figure 3 are computed. Finally, the initial conditions on the particle displacements, modal coordinates, and the respective time derivatives are set.

Subroutine GNMASS computes and assembles the deformation dependent generalized mass matrix of Figure 3.

Subroutine EXTF computes and assembles the generalized force vector $\underline{F}$ of Eq. (8-2).

Subroutine NLKT computes and assembles the vector of nonlinear kinematic terms of Eq. (8-3).

Subroutine SOLVE computes the transformation matrices given by Eq. (7-1) and (7-2). The set of simultaneous equations given by Eq. (8-1) are solved via a call to IMSL subroutine LEQT1P. The state vector Eq. (8-4) is assembled and its time derivative, Eq. (8-5), evaluated. The value of the state vector is advanced one time step via a call to subroutine ODESLV.

Subroutine ODESLV integrates the state equation, Eq. (8-5), using the Adams method with third order differences.

Subroutine PRINT is executed only at print-time intervals specified in the input (see below). When called, the subroutine prints the time, force, and torque on the rigid body, applied forces on the particles and all the variables of the vector given in Eq. (9-1).

Entry point GRAF in subroutine PRINT stores selected variables for plotting at a specified time interval (see namelist items DTG and IPLOT below).

Program Input Data

Program input data is read in during execution of subroutine INITL. Input is achieved through four READ-NAMELIST combinations and a single unformatted READ of the stiffness matrix. It is worth noting that while the code given in Appendix A requires the stiffness matrix (described in Section 3.1) and from this and the appendage mass matrix (assembled internally) computes the constrained appendage eigenvalues and eigenvectors, it could be modified to read in the appropriate eigenvalues/eigenvectors directly. The four NAMELIST inputs are defined below, and their use illustrated in Appendix B.

(1) NAMELIST/INPUT/MØ, N, MASS, RM, IØ, S, NT; contains all mass and geometry data as well as the number of modes to be retained.

MØ = mass of rigid body (real)

N = number of particles (integer)

MASS = masses of particles 1 through N (real N × 1 array)

RM = position vectors of particles 1 through N prior to deformation, expressed in body frame (real 3 × N array)

IØ = inertia matrix of the rigid body with respect to a frame located at the rigid body mass center with axes parallel to body frame (real 3 × 3 array)

S = position vector from body frame origin to mass center of rigid body expressed in body frame (real 3 × 1 array)

NT = number of modes to be retained; modes 1 through NT are used (integer)

(2) NAMELIST/KIN/UVW, OMEGA, R, THETA: contains initial conditions for kinematic variables.

UVW = initial velocity vector of body frame origin, expressed in body frame coordinates (real 3 × 1 array)

OMEGA = initial angular velocity vector of body frame with respect to inertial frame, components expressed in body frame (real 3 × 1 array)

R = initial inertial position vector of body frame origin, components expressed in inertial frame (real 3 × 1 array)

THETA = initial 1-2-3 Euler angles of body frame with respect to inertial frame (real 3 × 1 array)

(3) NAMELIST/RUN/DT, TSTOP, DTP, DTG: contains numerical integration parameters and print and plot time intervals.

DT = integration time step in seconds (real)

TSTOP = integration termination time in seconds (real)

DTP = print output time interval in seconds; output printed every DTP seconds (real)

DTG = plot output time interval in seconds; selected variables plotted every DTG seconds (real)

(4) NAMELIST/PLOT/IPLOT: specifies which elements of vector in Eq. (9-1) are to be plotted via line printer.

IPLOT = integer array with the integers corresponding to those elements of the vector in Eq. (9-1) that are to be plotted versus time (see sample use in Appendix B).

APPENDIX A

FORTRAN PROGRAM LISTING

```
LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED              DATE 82.271/12.43.05          PAGE   1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT COSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

                C
                C  *******************************************************************
                C  *THIS PROGRAM SOLVES THE EQUATIONS OF MOTION OF A VEHICLE
                C  *CONSISTING OF A RIGID BASE WITH AN ATTACHED FLEXIBLE APPENDAGE.
                C  *THE APPENDAGE IS IDEALIZED AS A COLLECTION OF PARTICLES CONNECTED
                C  *BY MASSLESS ELASTIC STRUCTURE. IN ADDITION TO EXTERNAL FORCES ACTING
                C  *UPON EACH OF THE PARTICLES, A FORCE AND TORQUE ARE ACCOMMODATED AT
                C  *THE GRAPPLE FIXTURE CORRESPONDING TO THE ORIGIN OF BODY FRAME..
                C  *(WRITTEN BY JOEL STORCH & STEPHEN GATES  C.S.D.L.  BASED UPON
                C  * C.S.D.L.  REPORT #R1582  SEPT. 1982)
                C  *******************************************************************
                C
                C
                C   NOTE: ARRAYS ARE DIMENSIONED TO ACCOMODATE A MAXIMUM OF 50 PARTICLES
                C
   ISN 0002           IMPLICIT REAL*8(A-H,O-Z)
   ISN 0003           DIMENSION F0(3),TAU0(3),FP(3,50)
   ISN 0004           COMMON /TIME/ DT,TSTOP,DTP,DTG
   ISN 0005           T=0.0
                C   INPUT PROGRAM DATA AND CALCULATE ALL TIME INVARIANT PARAMETERS.
                C
   ISN 0006           CALL INITL
   ISN 0007           TP=DTP
   ISN 0008           TG=DTG
                C
                C   CALCULATE GENERALIZED MASS MATRIX
                C
   ISN 0009     10    CALL GNMASS
                C
                C   INPUT VALUES REQUIRED FOR 'EXTF'
                C   (ALL VECTORS EXPRESSED IN BODY FRAME)
                C
                C   F0 - EXTERNAL FORCE ON M0 (AT ORIGIN OF BODY FRAME)
                C   TAU0 - EXTERNAL TORQUE AT LOCATION OF BODY FRAME ORIGIN.
                C   FP - VECTOR OF EXTERNAL FORCES ON PARTICLES 1,2,...,N.
                C
   ISN 0010           CALL EXTF(F0,TAU0,FP)
                C
                C   CALCULATE NON-LINEAR KINEMATIC TERMS
                C
   ISN 0011           CALL NLKT
                C
                C   INTEGRATE EQUATIONS OF MOTION
                C
   ISN 0012           CALL SOLVE
   ISN 0013           T=T+DT
   ISN 0014           IF(T .LT. TP) GO TO 15
```

```
                                                                00005400
ISN 0026          CALL PRINT(T,F0,TAU0,FP)                      00005500
ISN 0027          TP=TP+DTP                                     00005600
ISN 0028    15    IF(T .LT. TG) GO TO 20                        00005700
ISN 0030          CALL GRAF(T)                                  00005800
ISN 0031          TG=TG+DTG                                     00005900
ISN 0032    20    IF(T .LT. TSTOP) GO TO 10                     00006000
ISN 0034          CALL PLOT                                     00006100
ISN 0035          STOP                                          00006200
ISN 0036          END
```

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

STATISTICS SOURCE STATEMENTS = 35, PROGRAM SIZE = 2002, SUBPROGRAM NAME = MAIN

STATISTICS NO DIAGNOSTICS GENERATED

****** END OF COMPILATION ****** 280K BYTES OF CORE NOT USED

```
LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED          DATE 82.246/12.21.45       PAGE   1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

  ISN 0002            SUBROUTINE SKEW(V,A)                                              00006300
               C                                                                        00006400
               C    THIS SUBROUTINE CREATES THE SKEW SYMMETRIC MATRIX CORRESPONDING     00006500
               C    TO THE VECTOR "V".                                                  00006600
               C                                                                        00006700
  ISN 0003            REAL*8 V,A                                                        00006800
  ISN 0004            DIMENSION V(3),A(3,3)                                             00006900
  ISN 0005            A(1,1)=0.0                                                        00007000
  ISN 0006            A(1,2)=-V(3)                                                      00007100
  ISN 0007            A(1,3)=V(2)                                                       00007200
  ISN 0008            A(2,1)=V(3)                                                       00007300
  ISN 0009            A(2,2)=0.0                                                        00007400
  ISN 0010            A(2,3)=-V(1)                                                      00007500
  ISN 0011            A(3,1)=-V(2)                                                      00007600
  ISN 0012            A(3,2)=V(1)                                                       00007700
  ISN 0013            A(3,3)=0.0                                                        00007800
  ISN 0014            RETURN                                                            00007900
  ISN 0015            END                                                               00008000

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      14, PROGRAM SIZE =      388, SUBPROGRAM NAME =  SKEW

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  280K BYTES OF CORE NOT USED
```

```
LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED              DATE 82.246/12.21.46           PAGE   1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)
                                                                                                  00008100
  ISN 0002             SUBROUTINE CROSS(A,B,C)                                                    00008200
                 C                                                                                00008300
                 C   THIS SUBROUTINE CALCULATES THE VECTOR CROSS PRODUCT                          00008400
                 C              A X B =C                                                          00008500
                 C                                                                                00008600
  ISN 0003             REAL*8 A(3),B(3),C(3)                                                      00008700
  ISN 0004             C(1)=A(2)*B(3)-A(3)*B(2)                                                   00008800
  ISN 0005             C(2)=A(3)*B(1)-A(1)*B(3)                                                   00008900
  ISN 0006             C(3)=A(1)*B(2)-A(2)*B(1)                                                   00009000
  ISN 0007             RETURN                                                                     00009100
  ISN 0008             END

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =        7, PROGRAM SIZE =       484, SUBPROGRAM NAME = CROSS

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  280K BYTES OF CORE NOT USED
```

```
REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

ISN 0002             SUBROUTINE INITL
               C
               C   THIS SUBROUTINE READS IN PROGRAM DATA AND CALCULATES ALL TIME
               C   INVARIANT PARAMETERS.
               C
ISN 0003             IMPLICIT REAL*8(A-H,O-Z)
ISN 0004             REAL*8 MASS,M0,K,MC0,INERT1,MRM,I0,M0S
ISN 0005             REAL*4 PLTDAT
ISN 0006             DIMENSION MASS(50),RM(3,50),K(150,150),MC0(3),INERT1(3,3),
                    1 MRM(3,50),CMAT(3,3),A12(3,3),A23(3,150),A(156,156),Q(3,50),
                    1 QDOT(3,50),UVW(3),OMEGA(3),R(3),THETA(3),I0(3,3),S(3),M0S(3),
                    2 AV(11325),FREQ(150),EV(150,150),WK(150),IPLOT(42),PLTDAT(100,20),
                    3 WKM(3,150),ETA(150),ETAD(150)
ISN 0007             COMMON /CONST/ TM,MASS,MC0,MRM,CMAT,N,N3,N3P6,NT,NTP6,N0
ISN 0008             COMMON /AMAT/ A,A12,INERT1,A23
ISN 0009             COMMON /STATE/ R,THETA,Q,UVW,OMEGA,QDOT
ISN 0010             COMMON /GEOM/ RM
ISN 0011             COMMON /TIME/ DT,TSTOP,DTP,DTG
ISN 0012             COMMON /RIGID/ M0S,I0,S
ISN 0013             COMMON /PLOTT/ PLTDAT,IPLOT,NP
ISN 0014             COMMON /MODCO/ ETA,ETAD
ISN 0015             COMMON /MODES/ EV,FREQ
ISN 0016             EQUIVALENCE(K(1,1),EV(1,1))
ISN 0017             NAMELIST /INPUT/ M0,N,MASS,RM,I0,S,NT
ISN 0018             NAMELIST /KIN/ UVW,OMEGA,R,THETA
ISN 0019             NAMELIST /RUN/ DT,TSTOP,DTP,DTG
ISN 0020             NAMELIST /PLOT/ IPLOT
               C
               C             DESCRIPTION OF NAMELIST VARIABLES
               C
               C   "M0" IS THE MASS OF THE RIGID BASE
               C
               C   "N" IS THE NUMBER OF PARTICLES THAT COMPRISE THE FLEXIBLE APPENDAGE
               C
               C   "MASS" CONTAINS THE MASSES OF PARTICLES 1 TO N
               C
               C   "RM" CONTAINS THE POSITION VECTORS OF PARTICLES 1 THRU N  IN THE
               C    UNDEFORMED STATE EXPRESSED IN THE BODY FRAME.
               C
               C   "I0" IS THE INERTIA MATRIX OF THE RIGID BASE WITH RESPECT TO A
               C        FRAME LOCATED AT THE MASS CENTER OF THE BASE AND PARALLEL
               C        TO THE BODY FIXED AXIS SYSTEM.
               C
               C   "S"  IS THE VECTOR FROM THE BODY FRAME ORIGIN TO THE MASS CENTER
               C        OF THE RIGID BODY.
               C
               C   "NT" NUMBER OF RETAINED MODES IN APPENDAGE VIBRATION
               C
               C   "UVW" IS THE INITIAL VELOCITY OF THE BODY FRAME ORIGIN EXPRESSED
               C         IN BODY COORDINATES.
               C
               C   "OMEGA" IS THE INITIAL ANGULAR VELOCITY OF THE BODY FRAME EXPRESSED
               C           IN BODY COORDINATES.
```

```
              C
              C
              C    "R" IS THE INITIAL INERTIAL POSITION VECTOR OF THE BODY FRAME
              C       ORIGIN
              C
              C    "THETA" IS THE INITIAL SET OF ATTITUDE ANGLES FOR THE BODY FRAME
              C
              C    "DT" IS THE INTEGRATION TIME STEP
              C
              C    "TSTOP" IS THE TERMINAL TIME FOR THE SIMULATION
              C
              C    "DTP" IS THE TIME INTERVAL BETWEEN PRINTOUTS
              C
              C    "DTG" IS THE TIME INTERVAL BETWEEN PLOTTED POINTS
              C
              C    "IPLOT" IS AN ARRAY INDICATING VARIABLES TO BE PLOTTED.
              C          NUMBERING CORRESPONDS TO LOCATION IN STATE VECTOR.
              C          POINTS ARE PLOTTED EVERY "DTG" SECONDS.
              C
ISN 0021            READ(5,INPUT)
ISN 0022            IF(NT .LE. 3*N) GO TO 8
ISN 0024            WRITE(6,112) NT,N
ISN 0025            STOP
ISN 0026      8     WRITE(6,100) M0
ISN 0027            DO 10 I=1,N
ISN 0028      10    WRITE(6,101) I,MASS(I),(RM(J,I),J=1,3)
ISN 0029      100   FORMAT(1H1,20X,'MASS OF RIGID BASE=',E12.4,' SLUGS',//,1H ,T10,
                   1 'PARTICLE',T21,'MASS(SLUGS)',T41,'POSITION(FT.)',/)
ISN 0030      101   FORMAT(1H ,T12,I3,T21,F7.2,T34,3(F7.2,2X))
ISN 0031            WRITE(6,106) ((I0(I,J),J=1,3),I=1,3)
ISN 0032            WRITE(6,107) S,NT
ISN 0033            READ(5,KIN)
ISN 0034            WRITE(6,104) R,THETA
ISN 0035      103   FORMAT(1H0,5X,'INITIAL VELOCITY=',3F7.2,3X,'FT/SEC',4X,
                   1 'INITIAL ANGULAR VELOCITY=',3F7.2,'  DEG/SEC')
ISN 0036            WRITE(6,103) UVW,OMEGA
ISN 0037      104   FORMAT(1H0,5X,'INITIAL POSITION=',3F7.2,3X,'FT',4X,
                   1 '  INITIAL ATTITUDE=',3F8.3,' DEG')
ISN 0038            READ(5,RUN)
ISN 0039            WRITE(6,105) DT,TSTOP,DTP,DTG
ISN 0040            READ(5,PLOT)
ISN 0041            II=0
ISN 0042            DO 108 I=1,42
ISN 0043            IF(IPLOT(I) .EQ. 0) GO TO 109
ISN 0045      108   II=II+1
ISN 0046      109   IF(II .EQ. 0) GO TO 111
ISN 0048            WRITE(6,110) (IPLOT(I),I=1,II)
ISN 0049      110   FORMAT(1H0,5X,'VARIABLES PLOTTED',4X,42(I2,1X))
ISN 0050      105   FORMAT(1H0,5X,'TIME STEP=',E12.4,' SEC',3X,'TERMINATION TIME=',
                   1 E12.4,' SEC',2X,'PRINT INTERVAL=',E12.4,' SEC','PLOT INTERVAL=',
                   2 E12.4,' SEC')
ISN 0051      112   FORMAT(1H0,3X,I2,' MODES REQUESTED',2X,I3,' PARTICLES IN MODEL')
ISN 0052      106   FORMAT(1H0,T15,'INERTIA MATRIX OF RIGID BODY(SLUG FT**2)',
                   1 //,(T15,3E13.5))
ISN 0053      107   FORMAT(1H0,15X,'S=',3E13.5,' FT',3X,I2,' CONSTRINED APPENDAGE MODE
                   1S RETAINED')
              C
              C    CHANGE ANGULAR VELOCITY & ATTITUDE TO RADIAN MEASURE
```

```
                C
ISN 0054        111   DTR=DATAN(1.000)/45.
ISN 0055              DO 15 I=1,3
ISN 0056              OMEGA(I)=DTR*OMEGA(I)
ISN 0057        15    THETA(I)=DTR*THETA(I)
                C
                C   TM - TOTAL BODY MASS
                C
ISN 0058              TM=M0
ISN 0059              DO 20 I=1,N
ISN 0060        20    TM=TM+MASS(I)
ISN 0061              DO 30 J=1,3
ISN 0062              MOS(J)=M0*S(J)
ISN 0063        30    MCO(J)=0.0
ISN 0064              DO 40 I=1,N
ISN 0065              DO 45 J=1,3
ISN 0066              MRM(J,I)=MASS(I)*RM(J,I)
ISN 0067        45    MCO(J)=MCO(J)+MRM(J,I)
ISN 0068        40    CONTINUE
ISN 0069              DO 42 I=1,3
ISN 0070        42    MCO(I)=MCO(I)+MOS(I)
ISN 0071              N3=3*N
ISN 0072              N3P6=N3+6
ISN 0073              NTP6=NT+6
ISN 0074              NO=2*NTP6
ISN 0075              NP=0
                C
                C   READ IN STIFFNESS MATRIX
                C
ISN 0076              READ(8) NDOF,((K(I,J),J=1,NDOF),I=1,NDOF)
ISN 0077              IF(NDOF .EQ. N3) GO TO 400
ISN 0079              WRITE(6,102) N,NDOF
ISN 0080              STOP
ISN 0081        400   CONTINUE
ISN 0082        102   FORMAT(1H0,10X,'INCONSISTENT DATA',2X,I3,' PARTICLES',2X,I3,
                     1 ' DEGREES OF FREEDOM IN STIFFNESS MATRIX')
                C
                C   GET CONSTRAINED FREQUENCIES AND MODE SHAPES OF APPENDAGE
                C
ISN 0083              L=1
ISN 0084              DO 300 J=1,N3
ISN 0085              LC=1+J/3
ISN 0086              IF( (J-3*(J/3)) .EQ. 0) LC=LC-1
ISN 0088              DO 300 I=1,J
ISN 0089              LR=1+I/3
ISN 0090              IF( (I-3*(I/3)) .EQ. 0) LR=LR-1
ISN 0092              AV(L)=K(I,J)/DSQRT(MASS(LR)*MASS(LC))
ISN 0093              L=L+1
ISN 0094        300   CONTINUE
ISN 0095              CALL EIGRS(AV,N3,1,FREQ,EV,150,WK,IER)
ISN 0096              IF(IER .EQ. 0) GO TO 310
ISN 0098              WRITE(6,301) IER
ISN 0099        301   FORMAT(1H0,10X,'ERROR FROM IMSL ROUTINE "EIGRS" ERROR CODE=',I4)
ISN 0100              STOP
                C
                C   TRANSFORM EIGENVECTORS
                C
ISN 0101        310   DO 311 L=1,N
```

```
ISN 0102           C1=DSQRT(MASS(L))
ISN 0103           DO 311 I=1,3
ISN 0104           IR=3*(L-1)+I
ISN 0105           DO 311 J=1,N3
ISN 0106      311  EV(IR,J)=EV(IR,J)/C1
ISN 0107           DO 320 I=1,N3
ISN 0108           FHZ=DSQRT(FREQ(I))/6.283185
ISN 0109           WRITE(6,321) I,FHZ,(EV(J,I),J=1,N3)
ISN 0110      321  FORMAT(1H0,3X,'MODE ',I2,' FREQUENCY=',E12.4,1X,'(HZ)',/,1H ,
                  1 'MODE SHAPE: ',(9(G12.4,1X)))
ISN 0111      320  CONTINUE
              C
              C    THE ROUTINE "EIGRS" RETURNS AN ORTHONORMAL SET OF EIGENVECTORS.
              C    THIS IS ESSENTIAL SINCE WE ASSUME IN THE DERIVATION THAT THE
              C    EIGENVECTORS(OF THE ORIGINAL GENERALIZED EIGENVALUE PROBLEM)
              C    ARE ORTHOGONAL WITH RESPECT TO MASS(STIFFNESS) AND NORMALIZED
              C    WITH RESPECT TO MASS.
              C
              C
              C    CALCULATE TIME INVARIANT PART OF INERTIA MATRIX "INERT1"
              C    AND "CMAT"
              C
ISN 0112           DO 50 I=1,3
ISN 0113           DO 50 J=1,3
ISN 0114           INERT1(I,J)=0.0
ISN 0115           CMAT(I,J)=0.0
ISN 0116       50  CONTINUE
ISN 0117           DO 60 I=1,N
ISN 0118           XS=RM(1,I)**2
ISN 0119           YS=RM(2,I)**2
ISN 0120           ZS=RM(3,I)**2
ISN 0121           INERT1(1,1)=INERT1(1,1)+MASS(I)*(YS+ZS)
ISN 0122           INERT1(1,2)=INERT1(1,2)-MASS(I)*RM(1,I)*RM(2,I)
ISN 0123           INERT1(1,3)=INERT1(1,3)-MASS(I)*RM(1,I)*RM(3,I)
ISN 0124           INERT1(2,2)=INERT1(2,2)+MASS(I)*(XS+ZS)
ISN 0125           INERT1(2,3)=INERT1(2,3)-MASS(I)*RM(2,I)*RM(3,I)
ISN 0126           INERT1(3,3)=INERT1(3,3)+MASS(I)*(XS+YS)
ISN 0127           CMAT(1,1)=CMAT(1,1)+MASS(I)*XS
ISN 0128           CMAT(2,2)=CMAT(2,2)+MASS(I)*YS
ISN 0129           CMAT(3,3)=CMAT(3,3)+MASS(I)*ZS
ISN 0130       60  CONTINUE
ISN 0131           CMAT(1,2)=-INERT1(1,2)
ISN 0132           CMAT(1,3)=-INERT1(1,3)
ISN 0133           CMAT(2,3)=-INERT1(2,3)
ISN 0134           INERT1(1,1)=INERT1(1,1)+I0(1,1)+M0*(S(2)**2+S(3)**2)
ISN 0135           INERT1(1,2)=INERT1(1,2)+I0(1,2)-M0*S(1)*S(2)
ISN 0136           INERT1(1,3)=INERT1(1,3)+I0(1,3)-M0*S(1)*S(3)
ISN 0137           INERT1(2,2)=INERT1(2,2)+I0(2,2)+M0*(S(1)**2+S(3)**2)
ISN 0138           INERT1(2,3)=INERT1(2,3)+I0(2,3)-M0*S(2)*S(3)
ISN 0139           INERT1(3,3)=INERT1(3,3)+I0(3,3)+M0*(S(1)**2+S(2)**2)
ISN 0140           DO 70 I=1,3
ISN 0141           DO 70 J=1,3
ISN 0142           IF(I .LE. J) GO TO 70
ISN 0144           INERT1(I,J)=INERT1(J,I)
ISN 0145           CMAT(I,J)=CMAT(J,I)
ISN 0146       70  CONTINUE
              C
              C    CREATE TIME INVARIANT PORTIONS OF GENERALIZED MASS MATRIX "A"
```

```
              C
              C   (1,2) PARTITION "A12"
              C
   ISN 0147          CALL SKEW(MC0,A12)
   ISN 0148          DO 80 I=1,3
   ISN 0149          DO 80 J=1,3
   ISN 0150          IF(I .EQ. J) GO TO 80
   ISN 0152          A12(I,J)=-A12(I,J)
   ISN 0153      80  CONTINUE
              C
              C   (2,3) PARTITION "A23"
              C
   ISN 0154          DO 90 I=1,N
   ISN 0155          L=3*I-2
   ISN 0156      90  CALL SKEW(MRM(1,I),WKM(1,L))
   ISN 0157          DO 92 I=1,3
   ISN 0158          DO 92 J=1,NT
   ISN 0159          A23(I,J)=0.0
   ISN 0160          DO 94 L=1,N3
   ISN 0161      94  A23(I,J)=A23(I,J)+WKM(I,L)*EV(L,J)
   ISN 0162      92  CONTINUE
              C
              C   STORE CONSTANT PARTITIONS OF "A"
              C
   ISN 0163          DO 200 I=1,NTP6
   ISN 0164          DO 200 J=1,NTP6
   ISN 0165          A(I,J)=0.0
   ISN 0166     200  CONTINUE
              C
              C   CREATE (1,1) PARTITION
              C
   ISN 0167          DO 210 I=1,3
   ISN 0168          DO 210 J=1,3
   ISN 0169          IF(I .EQ. J) A(I,J)=TM
   ISN 0171     210  CONTINUE
              C
              C   CREATE (1,3) PARTITION
              C
   ISN 0172          DO 215 I=1,3
   ISN 0173          DO 215 J=1,N3
   ISN 0174     215  WKM(I,J)=0.0
   ISN 0175          DO 220 L=1,N
   ISN 0176          JS=3*L-2
   ISN 0177          DO 221 I=1,3
   ISN 0178          WKM(I,JS)=MASS(L)
   ISN 0179          JS=JS+1
   ISN 0180     221  CONTINUE
   ISN 0181     220  CONTINUE
   ISN 0182          DO 283 I=1,3
   ISN 0183          DO 283 J=1,NT
   ISN 0184          JP6=J+6
   ISN 0185          A(I,JP6)=0.0
   ISN 0186          DO 281 L=1,N3
   ISN 0187     281  A(I,JP6)=A(I,JP6)+WKM(I,L)*EV(L,J)
   ISN 0188     283  CONTINUE
              C
              C   CREATE (3,3) PARTITION
              C
```

```
                                                                        00037700
  ISN 0189            DO 230 L=1,NT                                     00037800
  ISN 0190       230  A(L+6,L+6)=1.                                     00037900
                 C                                                      00038000
                 C    SET INITIAL DEFORMATION AND RATE TO ZERO          00038100
                 C                                                      00038200
  ISN 0191            DO 250 I=1,N                                      00038300
  ISN 0192            DO 250 J=1,3                                      00038400
  ISN 0193            Q(J,I)=0.0                                        00038500
  ISN 0194            QDOT(J,I)=0.0                                     00038600
  ISN 0195       250  CONTINUE                                          00038700
  ISN 0196            DO 293 I=1,NT                                     00038800
  ISN 0197            ETA(I)=0.0                                        00038900
  ISN 0198            ETAD(I)=0.0                                       00039000
  ISN 0199       293  CONTINUE                                          00039100
  ISN 0200            RETURN                                            00039200
  ISN 0201            END
*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      200, PROGRAM SIZE =    103026, SUBPROGRAM NAME = INITL

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          232K BYTES OF CORE NOT USED
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LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED          DATE 82.246/12.21.50        PAGE   1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

  ISN 0002            SUBROUTINE GNMASS
                C
                C   THIS SUBROUTINE CALCULATES THE GENERALIZED MASS MATRIX "A"
                C
  ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
  ISN 0004            REAL*8 MASS,MC0,MRM,INERT1,MQ,MC1,INERT2,INERT3
  ISN 0005            DIMENSION MASS(50),MC0(3),MRM(3,50),CMAT(3,3),A(156,156),
                     1 A12(3,3),INERT1(3,3),A23(3,150),Q(3,50),MQ(3,50),MC1(3),
                     2 WM(3,3),INERT2(3,3),A23Q(3,150),INERT3(3,3),QDOT(3,50),UVW(3),
                     3 OMEGA(3),R(3),THETA(3)
  ISN 0006            COMMON /CONST/ TM,MASS,MC0,MRM,CMAT,N,N3,N3P6,NT,NTP6,NO
  ISN 0007            COMMON /AMAT/ A,A12,INERT1,A23
  ISN 0008            COMMON /STATE/ R,THETA,Q,UVW,OMEGA,QDOT
  ISN 0009            COMMON /TDEPV/ MQ,MC1
  ISN 0010            COMMON /MODES/ EV(150,150),FREQ(150)
  ISN 0011            DO 10 I=1,3
  ISN 0012      10    MC1(I)=0.0
  ISN 0013            DO 40 I=1,N
  ISN 0014            DO 45 J=1,3
  ISN 0015            MQ(J,I)=MASS(I)*Q(J,I)
  ISN 0016      45    MC1(J)=MC1(J)+MQ(J,I)
  ISN 0017      40    CONTINUE
                C
                C   COMPUTE "A" NEGLECTING TIME DEPENDENT(DEFORMATION DEPENDENT) TERMS
                C
  ISN 0018            DO 50 J=1,3
  ISN 0019            JP3=J+3
  ISN 0020            DO 50 I=1,3
  ISN 0021      50    A(I,JP3)=A12(I,J)
  ISN 0022            DO 60 I=1,3
  ISN 0023            IP3=I+3
  ISN 0024            DO 60 J=1,3
  ISN 0025            JP3=J+3
  ISN 0026      60    A(IP3,JP3)=INERT1(I,J)
  ISN 0027            DO 70 J=1,NT
  ISN 0028            JP6=J+6
  ISN 0029            DO 70 I=1,3
  ISN 0030            IP3=I+3
  ISN 0031      70    A(IP3,JP6)=A23(I,J)
                C
                C   ADD IN FIRST ORDER DEFORMATION TERMS
                C
  ISN 0032            CALL SKEW(MC1,WM)
  ISN 0033            DO 80 J=1,3
  ISN 0034            JP3=J+3
  ISN 0035            DO 80 I=1,3
  ISN 0036      80    A(I,JP3)=A(I,JP3)-WM(I,J)
                C
                C   FIRST ORDER DEFORMATION TERMS IN INERTIA MATRIX - "INERT2"
                C
  ISN 0037            DO 85 I=1,3
  ISN 0038            DO 85 J=1,3
  ISN 0039      85    INERT2(I,J)=0.0
```

```
ISN 0040          DO 90 L=1,3
ISN 0041          SUM=0.0
ISN 0042          DO 100 I=1,N
ISN 0043    100   SUM=SUM+MRM(L,I)*Q(L,I)
ISN 0044          SUM=2.0*SUM
ISN 0045          LL=L+1
ISN 0046          DO 110 II=1,2
ISN 0047          IF(LL .GT. 3) LL=1
ISN 0049          INERT2(LL,LL)=INERT2(LL,LL)+SUM
ISN 0050          LL=LL+1
ISN 0051    110   CONTINUE
ISN 0052    90    CONTINUE
ISN 0053          SUM=0.0
ISN 0054          DO 120 I=1,N
ISN 0055    120   SUM=SUM-(MRM(1,I)*Q(2,I)+MRM(2,I)*Q(1,I))
ISN 0056          INERT2(1,2)=SUM
ISN 0057          SUM=0.0
ISN 0058          DO 130 I=1,N
ISN 0059    130   SUM=SUM-(MRM(1,I)*Q(3,I)+MRM(3,I)*Q(1,I))
ISN 0060          INERT2(1,3)=SUM
ISN 0061          SUM=0.0
ISN 0062          DO 140 I=1,N
ISN 0063    140   SUM=SUM-(MRM(2,I)*Q(3,I)+MRM(3,I)*Q(2,I))
ISN 0064          INERT2(2,3)=SUM
ISN 0065          DO 150 I=1,3
ISN 0066          DO 150 J=1,3
ISN 0067          IF(I .LE. J) GO TO 150
ISN 0069          INERT2(I,J)=INERT2(J,I)
ISN 0070    150   CONTINUE
ISN 0071          DO 160 I=1,3
ISN 0072          IP3=I+3
ISN 0073          DO 160 J=1,3
ISN 0074          JP3=J+3
ISN 0075    160   A(IP3,JP3)=A(IP3,JP3)+INERT2(I,J)
ISN 0076          DO 170 I=1,N
ISN 0077          L=3*I-2
ISN 0078    170   CALL SKEW(MQ(1,I),A23Q(1,L))
ISN 0079          DO 180 I=1,3
ISN 0080          IP3=I+3
ISN 0081          DO 180 J=1,NT
ISN 0082          JP6=J+6
ISN 0083          DO 185 L=1,N3
ISN 0084    185   A(IP3,JP6)=A(IP3,JP6)+A23Q(I,L)*EV(L,J)
ISN 0085    180   CONTINUE
            C
            C  ADD IN SECOND ORDER DEFORMATION TERMS - "INERT3"
            C
ISN 0086          DO 190 I=1,3
ISN 0087          DO 190 J=1,3
ISN 0088    190   INERT3(I,J)=0.0
ISN 0089          DO 200 L=1,3
ISN 0090          SUM=0.0
ISN 0091          DO 210 I=1,N
ISN 0092    210   SUM=SUM+MQ(L,I)*Q(L,I)
ISN 0093          LL=L+1
ISN 0094          DO 220 II=1,2
ISN 0095          IF(LL .GT. 3) LL=1
ISN 0097          INERT3(LL,LL)=INERT3(LL,LL)+SUM
```

```
  ISN 0098          LL=LL+1                                                    00050400
  ISN 0099    220   CONTINUE                                                   00050500
  ISN 0100    200   CONTINUE                                                   00050600
  ISN 0101          SUM=0.0                                                    00050700
  ISN 0102          DO 240 I=1,N                                               00050800
  ISN 0103    240   SUM=SUM-MQ(1,I)*Q(2,I)                                     00050900
  ISN 0104          INERT3(1,2)=SUM                                            00051000
  ISN 0105          SUM=0.0                                                    00051100
  ISN 0106          DO 250 I=1,N                                               00051200
  ISN 0107    250   SUM=SUM-MQ(1,I)*Q(3,I)                                     00051300
  ISN 0108          INERT3(1,3)=SUM                                            00051400
  ISN 0109          SUM=0.0                                                    00051500
  ISN 0110          DO 260 I=1,N                                               00051600
  ISN 0111    260   SUM=SUM-MQ(2,I)*Q(3,I)                                     00051700
  ISN 0112          INERT3(2,3)=SUM                                            00051800
  ISN 0113          DO 270 I=1,3                                               00051900
  ISN 0114          DO 270 J=1,3                                               00052000
  ISN 0115          IF(I .LE. J) GO TO 270                                     00052100
  ISN 0117          INERT3(I,J)=INERT3(J,I)                                    00052200
  ISN 0118    270   CONTINUE                                                   00052300
  ISN 0119          DO 280 I=1,3                                               00052400
  ISN 0120          IP3=I+3                                                    00052500
  ISN 0121          DO 280 J=1,3                                               00052600
  ISN 0122          JP3=J+3                                                    00052700
  ISN 0123    280   A(IP3,JP3)=A(IP3,JP3)+INERT3(I,J)                          00052800
              C                                                                00052900
  ISN 0124    302   FORMAT(1H0,//,1H ,10X,'A MATRIX')                          00053000
  ISN 0125    301   FORMAT(1H0,2X,15(F7.2,1X))                                 00053100
  ISN 0126          RETURN                                                     00053200
  ISN 0127          END                                                        00053300

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =     126, PROGRAM SIZE =     8156, SUBPROGRAM NAME =GNMASS

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          256K BYTES OF CORE NOT USED
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REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

ISN 0002          SUBROUTINE EXTF(F0,TAU0,FP)
            C
            C    THIS SUBROUTINE ASSEMBLES THE FORCE VECTOR  "F" IN THE
            C    MOTION EQUATIONS AND IS PARTITIONED AS: FORCRES FOR BODY
            C    TRANSLATION, FORCES FOR BODY ROTATION, AND FORCES FOR
            C    PARTICLE TRANSLATION.
            C
            C    INPUT TO SUBROUTINE
            C
            C    F0 - EXTERNAL FORCE ON M0 (AT ORIGIN OF BODY FRAME)
            C    TAU0 - EXTERNAL TORQUE AT LOCATION OF BODY FRAME ORIGIN.
            C    FP - VECTOR OF EXTERNAL FORCES ON PARTICLES 1,2,....,N.
            C    (ALL VECTORS EXPRESSED IN BODY FRAME)
            C
ISN 0003          IMPLICIT REAL*8(A-H,O-Z)
ISN 0004          REAL*8 MASS,MC0,MRM
ISN 0005          DIMENSION F0(3),TAU0(3),FP(3,50),Q(3,50),RM(3,50),MASS(50),MC0(3),
                 1 MRM(3,50),CMAT(3,3),F(156),SUM(3),WV(3),WV1(3),QDOT(3,50),UVW(3),
                 2 OMEGA(3),R(3),THETA(3),WV2(150),PHI(150,150),WS(150)
ISN 0006          COMMON /STATE/ R,THETA,Q,UVW,OMEGA,QDOT
ISN 0007          COMMON /GEOM/ RM
ISN 0008          COMMON /CONST/ TM,MASS,MC0,MRM,CMAT,N,N3,N3P6,NT,NTP6,N0
ISN 0009          COMMON /FORCE/ F
ISN 0010          COMMON /MODES/ PHI,WS
ISN 0011          DO 10 J=1,3
ISN 0012    10    F(J)=F0(J)
ISN 0013          DO 20 I=1,N
ISN 0014          DO 20 J=1,3
ISN 0015    20    F(J)=F(J)+FP(J,I)
ISN 0016          DO 30 J=1,3
ISN 0017    30    SUM(J)=0.0
ISN 0018          DO 40 I=1,N
ISN 0019          DO 50 J=1,3
ISN 0020    50    WV(J)=RM(J,I)+Q(J,I)
ISN 0021          CALL CROSS(WV,FP(1,I),WV1)
ISN 0022          DO 60 J=1,3
ISN 0023    60    SUM(J)=SUM(J)+WV1(J)
ISN 0024    40    CONTINUE
ISN 0025          DO 70 J=1,3
ISN 0026          JP3=J+3
ISN 0027    70    F(JP3)=TAU0(J)+SUM(J)
ISN 0028          L=1
ISN 0029          DO 80 I=1,N
ISN 0030          DO 85 J=1,3
ISN 0031          WV2(L)=FP(J,I)
ISN 0032          L=L+1
ISN 0033    85    CONTINUE
ISN 0034    80    CONTINUE
ISN 0035          DO 90 I=1,NT
ISN 0036          IP6=I+6
ISN 0037          F(IP6)=0.0
ISN 0038          DO 95 L=1,N3
ISN 0039    95    F(IP6)=F(IP6)+PHI(L,I)*WV2(L)
```

```
  ISN 0040     90     CONTINUE                                                    00058700
  ISN 0041            RETURN                                                      00058800
  ISN 0042            END                                                         00058900

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      41, PROGRAM SIZE =      2734, SUBPROGRAM NAME =  EXTF

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                               280K BYTES OF CORE NOT USED
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REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

 ISN 0002          SUBROUTINE NLKT
            C
            C   THIS SUBROUTINE CALCULATES THE NON-LINEAR KINEMATIC TERMS IN THE
            C   MOTION EQUATIONS. THESE TERMS ARE ASSEMBLED INTO THE VECTOR "U".
            C
 ISN 0003          IMPLICIT REAL*8(A-H,O-Z)
 ISN 0004          REAL*8 MASS,MCO,MRM,MQ,MC1,MOS,IO
 ISN 0005          DIMENSION MASS(50),MCO(3),MRM(3,50),CMAT(3,3),MQ(3,50),MC1(3),
                  1 QDOT(3,50),UVW(3),OMEGA(3),UT(3),UR(3),UV(150),U(156),WV1(3),
                  2 WV2(3),WV3(3),WV4(3),WM1(3,3),Q(3,50),R(3),THETA(3),MOS(3),
                  3 IO(3,3),S(3),WS(150),PHI(150,150)
 ISN 0006          COMMON /CONST/ TM,MASS,MCO,MRM,CMAT,N,N3,N3P6,NT,NTP6,NO
 ISN 0007          COMMON /TDEPV/ MQ,MC1
 ISN 0008          COMMON /STATE/ R,THETA,Q,UVW,OMEGA,QDOT
 ISN 0009          COMMON /FICFRC/ U
 ISN 0010          COMMON /RIGID/ MOS,IO,S
 ISN 0011          COMMON /MODES/ PHI,WS
 ISN 0012          EQUIVALENCE(U(1),UT(1)),(U(4),UR(1))
            C
            C   CALCULATE DEFORMATION INDEPENDENT TERMS
            C
 ISN 0013          CALL CROSS(UVW,OMEGA,WV1)
 ISN 0014          SUM=OMEGA(1)*MCO(1)+OMEGA(2)*MCO(2)+OMEGA(3)*MCO(3)
 ISN 0015          OMS=OMEGA(1)**2+OMEGA(2)**2+OMEGA(3)**2
 ISN 0016          DO 20 J=1,3
 ISN 0017      20  UT(J)=TM*WV1(J)-SUM*OMEGA(J)+OMS*MCO(J)
 ISN 0018          CALL CROSS(MCO,WV1,WV2)
 ISN 0019          DO 30 J=1,3
 ISN 0020      30  WV3(J)=CMAT(J,1)*OMEGA(1)+CMAT(J,2)*OMEGA(2)+CMAT(J,3)*OMEGA(3)
 ISN 0021          CALL CROSS(OMEGA,WV3,WV4)
 ISN 0022          DO 40 J=1,3
 ISN 0023      40  UR(J)=WV2(J)+WV4(J)
 ISN 0024          SUM=OMEGA(1)*S(1)+OMEGA(2)*S(2)+OMEGA(3)*S(3)
 ISN 0025          CALL CROSS(MOS,OMEGA,WV2)
 ISN 0026          DO 42 J=1,3
 ISN 0027      42  WV3(J)=IO(J,1)*OMEGA(1)+IO(J,2)*OMEGA(2)+IO(J,3)*OMEGA(3)
 ISN 0028          CALL CROSS(OMEGA,WV3,WV4)
 ISN 0029          DO 44 J=1,3
 ISN 0030      44  UR(J)=UR(J)-SUM*WV2(J)-WV4(J)
 ISN 0031          DO 50 I=1,N
 ISN 0032          L=3*I-2
 ISN 0033          DO 55 J=1,3
 ISN 0034      55  WV2(J)=MASS(I)*WV1(J)
 ISN 0035          SUM=OMEGA(1)*MRM(1,I)+OMEGA(2)*MRM(2,I)+OMEGA(3)*MRM(3,I)
 ISN 0036          DO 60 J=1,3
 ISN 0037          WV3(J)=SUM*OMEGA(J)
 ISN 0038      60  WV4(J)=OMS*MRM(J,I)
 ISN 0039          LL=L
 ISN 0040          DO 70 J=1,3
 ISN 0041          UV(LL)=WV2(J)-WV3(J)+WV4(J)
 ISN 0042      70  LL=LL+1
 ISN 0043      50  CONTINUE
 ISN 0044          DO 74 I=1,NT
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ISN 0045              IP6=I+6                                                     00064300
ISN 0046              U(IP6)=0.0                                                  00064400
ISN 0047              DO 74 L=1,N3                                                00064500
ISN 0048        74    U(IP6)=U(IP6)+PHI(L,I)*UV(L)                                00064600
                C                                                                 00064700
                C  CALCULATE FIRST ORDER DEFORMATION DEPENDENT TERMS              00064800
                C                                                                 00064900
ISN 0049              DO 80 J=1,3                                                 00065000
ISN 0050        80    WV2(J)=0.0                                                  00065100
ISN 0051              DO 90 I=1,N                                                 00065200
ISN 0052              DO 95 J=1,3                                                 00065300
ISN 0053        95    WV2(J)=WV2(J)+MASS(I)*QDOT(J,I)                             00065400
ISN 0054        90    CONTINUE                                                    00065500
ISN 0055              CALL CROSS(OMEGA,WV2,WV3)                                   00065600
ISN 0056              SUM=OMEGA(1)*MC1(1)+OMEGA(2)*MC1(2)+OMEGA(3)*MC1(3)         00065700
ISN 0057              DO 100 J=1,3                                                00065800
ISN 0058       100    UT(J)=UT(J)-2.0*WV3(J)-SUM*OMEGA(J)+OMS*MC1(J)              00065900
ISN 0059              CALL CROSS(MC1,WV1,WV2)                                     00066000
ISN 0060              DO 110 I=1,3                                                00066100
ISN 0061              DO 110 J=1,3                                                00066200
ISN 0062       110    WM1(I,J)=0.0                                                00066300
ISN 0063              DO 120 I=1,N                                                00066400
ISN 0064              DO 125 J=1,3                                                00066500
ISN 0065       125    WM1(J,J)=WM1(J,J)+2.0*MRM(J,I)*Q(J,I)                       00066600
ISN 0066              WM1(1,2)=WM1(1,2)+MRM(1,I)*Q(2,I)+MRM(2,I)*Q(1,I)           00066700
ISN 0067              WM1(1,3)=WM1(1,3)+MRM(1,I)*Q(3,I)+MRM(3,I)*Q(1,I)           00066800
ISN 0068              WM1(2,3)=WM1(2,3)+MRM(2,I)*Q(3,I)+MRM(3,I)*Q(2,I)           00066900
ISN 0069       120    CONTINUE                                                    00067000
ISN 0070              DO 130 I=1,3                                                00067100
ISN 0071              DO 130 J=1,3                                                00067200
ISN 0072              IF(I .LE. J) GO TO 130                                      00067300
ISN 0074              WM1(I,J)=WM1(J,I)                                           00067400
ISN 0075       130    CONTINUE                                                    00067500
ISN 0076              DO 140 J=1,3                                                00067600
ISN 0077       140    WV3(J)=WM1(J,1)*OMEGA(1)+WM1(J,2)*OMEGA(2)+WM1(J,3)*OMEGA(3)  00067700
ISN 0078              CALL CROSS(OMEGA,WV3,WV4)                                   00067800
ISN 0079              SUM=0.0                                                     00067900
ISN 0080              DO 150 I=1,N                                                00068000
ISN 0081       150    SUM=SUM+MRM(1,I)*QDOT(1,I)+MRM(2,I)*QDOT(2,I)+MRM(3,I)*QDOT(3,I)  00068100
ISN 0082              DO 160 J=1,3                                                00068200
ISN 0083       160    WV1(J)=SUM*OMEGA(J)                                         00068300
ISN 0084              DO 170 J=1,3                                                00068400
ISN 0085       170    WV3(J)=0.0                                                  00068500
ISN 0086              DO 180 I=1,N                                                00068600
ISN 0087              SUM=MRM(1,I)*OMEGA(1)+MRM(2,I)*OMEGA(2)+MRM(3,I)*OMEGA(3)   00068700
ISN 0088              DO 190 J=1,3                                                00068800
ISN 0089       190    WV3(J)=WV3(J)+SUM*QDOT(J,I)                                 00068900
ISN 0090       180    CONTINUE                                                    00069000
ISN 0091              DO 200 J=1,3                                                00069100
ISN 0092       200    UR(J)=UR(J)+WV2(J)+WV4(J)+2.*(WV3(J)-WV1(J))                00069200
ISN 0093              DO 300 I=1,N                                                00069300
ISN 0094              L=3*I-2                                                     00069400
ISN 0095              CALL CROSS(OMEGA,QDOT(1,I),WV1)                             00069500
ISN 0096              DO 310 J=1,3                                                00069600
ISN 0097       310    WV2(J)=MASS(I)*WV1(J)                                       00069700
ISN 0098              SUM=OMEGA(1)*MQ(1,I)+OMEGA(2)*MQ(2,I)+OMEGA(3)*MQ(3,I)      00069800
ISN 0099              DO 320 J=1,3                                                00069900
ISN 0100              WV3(J)=SUM*OMEGA(J)                                         00070000
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ISN 0101    320   WV4(J)=OMS*MQ(J,I)
ISN 0102          LL=L
ISN 0103          DO 330 J=1,3
ISN 0104          UV(LL)=UV(LL)-2.*WV2(J)-WV3(J)+WV4(J)
ISN 0105    330   LL=LL+1
ISN 0106    300   CONTINUE
ISN 0107          DO 307 I=1,NT
ISN 0108          IP6=I+6
ISN 0109          U(IP6)=0.0
ISN 0110          DO 307 L=1,N3
ISN 0111    307   U(IP6)=U(IP6)+PHI(L,I)*UV(L)
                C
                C  CALCULATE SECOND ORDER DEFORMATION DEPENDENT TERMS
                C
ISN 0112          DO 311 I=1,3
ISN 0113          DO 311 J=1,3
ISN 0114    311   WM1(I,J)=0.0
ISN 0115          DO 321 I=1,N
ISN 0116          DO 331 J=1,3
ISN 0117    331   WM1(J,J)=WM1(J,J)+MQ(J,I)*Q(J,I)
ISN 0118          WM1(1,2)=WM1(1,2)+MQ(1,I)*Q(2,I)
ISN 0119          WM1(1,3)=WM1(1,3)+MQ(1,I)*Q(3,I)
ISN 0120          WM1(2,3)=WM1(2,3)+MQ(2,I)*Q(3,I)
ISN 0121    321   CONTINUE
ISN 0122          DO 340 I=1,3
ISN 0123          DO 340 J=1,3
ISN 0124          IF(I .LE. J) GO TO 340
ISN 0126          WM1(I,J)=WM1(J,I)
ISN 0127    340   CONTINUE
ISN 0128          DO 350 J=1,3
ISN 0129    350   WV1(J)=WM1(J,1)*OMEGA(1)+WM1(J,2)*OMEGA(2)+WM1(J,3)*OMEGA(3)
ISN 0130          CALL CROSS(OMEGA,WV1,WV2)
ISN 0131          SUM=0.0
ISN 0132          DO 360 I=1,N
ISN 0133    360   SUM=SUM+MQ(1,I)*QDOT(1,I)+MQ(2,I)*QDOT(2,I)+MQ(3,I)*QDOT(3,I)
ISN 0134          DO 370 J=1,3
ISN 0135          WV3(J)=SUM*OMEGA(J)
ISN 0136    370   WV4(J)=0.0
ISN 0137          DO 380 I=1,N
ISN 0138          SUM=OMEGA(1)*MQ(1,I)+OMEGA(2)*MQ(2,I)+OMEGA(3)*MQ(3,I)
ISN 0139          DO 385 J=1,3
ISN 0140    385   WV4(J)=WV4(J)+SUM*QDOT(J,I)
ISN 0141    380   CONTINUE
ISN 0142          DO 390 J=1,3
ISN 0143    390   UR(J)=UR(J)+WV2(J)+2.*(WV4(J)-WV3(J))
ISN 0144          RETURN
ISN 0145          END
```

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*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =     144, PROGRAM SIZE =     8226, SUBPROGRAM NAME = NLKT

*STATISTICS*  NO  DIAGNOSTICS GENERATED
                                                              244K BYTES OF CORE NOT USED
****** END OF COMPILATION ******
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LEVEL 2.3.0 (JUNE 78)                    OS/360  FORTRAN H EXTENDED            DATE 82.246/12.21.56          PAGE   1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

  ISN 0002           SUBROUTINE SOLVE
              C
              C    THIS SUBROUTINE ASSEMBLES THE EQUATIONS OF MOTION IN FIRST ORDER
              C    FORM AND SOLVES THE SET OF SIMULTANEOUS DIFFERENTIAL EQUATIONS
              C                 (SIZE 6*N+12)
  ISN 0003           IMPLICIT REAL*8(A-H,O-Z)
  ISN 0004           REAL*8 MASS,MCO,MRM,INERT1
  ISN 0005           DIMENSION MASS(50),MCO(3),MRM(3,50),CMAT(3,3),A(156,156),A12(3,3),
                    1 INERT1(3,3),A23(3,150),Q(3,50),QDOT(3,50),UVW(3),OMEGA(3),
                    2 R(3),THETA(3),F(156),U(156),R14(3,3),GAMA(3,3),
                    3 B(156),YDOT(312),Y(312),WV1(3),WV2(3),WV3(156),
                    4 QV(150),QDOTV(150),AV(12246)
  ISN 0006           EQUIVALENCE (QV(1),Q(1,1)),(QDOTV(1),QDOT(1,1))
  ISN 0007           COMMON /CONST/ TM,MASS,MCO,MRM,CMAT,N,N3,N3P6,NT,NTP6,NO
  ISN 0008           COMMON /AMAT/ A,A12,INERT1,A23
  ISN 0009           COMMON /STATE/ R,THETA,Q,UVW,OMEGA,QDOT
  ISN 0010           COMMON /FORCE/ F
  ISN 0011           COMMON /FICFRC/ U
  ISN 0012           COMMON /TIME/ DT,TSTOP,DTP,DTG
  ISN 0013           COMMON /MODES/ PHI(150,150),WS(150)
  ISN 0014           COMMON /MODCO/ ETA(150),ETAD(150)
  ISN 0015           DATA GAMA/8*0.0,1.0/,IPASS/0/
              C
              C    CALCULATE R14 -TRANSFORMATION FROM BODY FRAME TO INERTIAL FRAME
              C
  ISN 0016           S1=DSIN(THETA(1))
  ISN 0017           C1=DCOS(THETA(1))
  ISN 0018           S2=DSIN(THETA(2))
  ISN 0019           C2=DCOS(THETA(2))
  ISN 0020           S3=DSIN(THETA(3))
  ISN 0021           C3=DCOS(THETA(3))
  ISN 0022           R14(1,1)=C2*C3
  ISN 0023           R14(1,2)=-C2*S3
  ISN 0024           R14(1,3)=S2
  ISN 0025           R14(2,1)=C1*S3+S1*S2*C3
  ISN 0026           R14(2,2)=C1*C3-S1*S2*S3
  ISN 0027           R14(2,3)=-S1*C2
  ISN 0028           R14(3,1)=S1*S3-C1*S2*C3
  ISN 0029           R14(3,2)=S1*C3+C1*S2*S3
  ISN 0030           R14(3,3)=C1*C2
              C
              C    CALCULATE "GAMA" - TRANSFORMS ANGULAR VELOCITY TO ATTITUDE RATES
              C
  ISN 0031           GAMA(1,1)=C3/C2
  ISN 0032           GAMA(1,2)=-S3/C2
  ISN 0033           GAMA(2,1)=S3
  ISN 0034           GAMA(2,2)=C3
  ISN 0035           T2=S2/C2
  ISN 0036           GAMA(3,1)=-C3*T2
  ISN 0037           GAMA(3,2)=S3*T2
  ISN 0038           DO 30 I=1,6
  ISN 0039      30   B(I)=F(I)+U(I)
  ISN 0040           DO 40 I=1,NT
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ISN 0041            IP6=I+6
ISN 0042      40    B(IP6)=F(IP6)+U(IP6)-WS(I)*ETA(I)
              C
              C   STORE UPPER TRIANGLE OF "A" IN "AV"
              C
ISN 0043            L=1
ISN 0044            DO 300 J=1,NTP6
ISN 0045            DO 300 I=1,J
ISN 0046            AV(L)=A(I,J)
ISN 0047            L=L+1
ISN 0048     300    CONTINUE
ISN 0049            CALL LEQT1P(AV,1,NTP6,B,156,0,D1,D2,IER)
ISN 0050            IF(IER .EQ. 0) GO TO 60
ISN 0052            WRITE(6,61) IER
ISN 0053            STOP
ISN 0054      61    FORMAT(1H0,5X,'ERROR DETECTED BY IMSL LIBRARY ROUTINE "LEQT1P"
                   1ERROR CODE=',I3)
ISN 0055      60    IF(IPASS .EQ. 1) GO TO 105
ISN 0057            IPASS=1
              C
              C   SET INITIAL VALUE OF "Y"
              C
ISN 0058            DO 70 I=1,3
ISN 0059            IP3=I+3
ISN 0060            Y(I)=R(I)
ISN 0061      70    Y(IP3)=THETA(I)
ISN 0062            DO 80 I=1,NT
ISN 0063      80    Y(I+6)=ETA(I)
ISN 0064            DO 90 I=1,3
ISN 0065            L=NTP6+I
ISN 0066            LL=L+3
ISN 0067            Y(L)=UVW(I)
ISN 0068      90    Y(LL)=OMEGA(I)
ISN 0069            DO 100 I=1,NT
ISN 0070            L=12+NT+I
ISN 0071     100    Y(L)=ETAD(I)
              C
              C   SET UP "YDOT"
              C
ISN 0072     105    DO 110 I=1,3
ISN 0073            WV1(I)=R14(I,1)*UVW(1)+R14(I,2)*UVW(2)+R14(I,3)*UVW(3)
ISN 0074     110    WV2(I)=GAMA(I,1)*OMEGA(1)+GAMA(I,2)*OMEGA(2)+GAMA(I,3)*OMEGA(3)
ISN 0075            DO 120 I=1,3
ISN 0076            YDOT(I)=WV1(I)
ISN 0077     120    YDOT(I+3)=WV2(I)
ISN 0078            DO 130 I=1,NT
ISN 0079     130    YDOT(6+I)=ETAD(I)
ISN 0080            DO 140 I=1,NTP6
ISN 0081     140    YDOT(NTP6+I)=B(I)
              C
              C   UPDATE VARIABLES IN STATE VECTOR
              C
ISN 0082            CALL ODESLV(NO,Y,YDOT,DT)
ISN 0083            DO 150 I=1,3
ISN 0084            R(I)=Y(I)
ISN 0085     150    THETA(I)=Y(I+3)
ISN 0086            DO 160 I=1,NT
ISN 0087     160    ETA(I)=Y(6+I)
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LEVEL 2.3.0 (JUNE 78)          SOLVE          OS/360  FORTRAN H EXTENDED                DATE 82.246/12.21.56        PAGE   3

  ISN 0088          DO 170 I=1,3                                                          00085900
  ISN 0089          L=NTP6+I                                                              00086000
  ISN 0090          LL=L+3                                                                00086100
  ISN 0091          UVW(I)=Y(L)                                                           00086200
  ISN 0092    170   OMEGA(I)=Y(LL)                                                        00086300
  ISN 0093          DO 180 I=1,NT                                                         00086400
  ISN 0094    180   ETAD(I)=Y(NT+12+I)                                                    00086500
              C                                                                           00086600
              C   COMPUTE NEW VALUES FOR "Q" AND "QDOT"                                   00086700
              C                                                                           00086800
  ISN 0095          DO 200 I=1,N3                                                         00086900
  ISN 0096          QV(I)=0.0                                                             00087000
  ISN 0097          QDOTV(I)=0.0                                                          00087100
  ISN 0098          DO 220 L=1,NT                                                         00087200
  ISN 0099          QV(I)=QV(I)+PHI(I,L)*ETA(L)                                           00087300
  ISN 0100    220   QDOTV(I)=QDOTV(I)+PHI(I,L)*ETAD(L)                                    00087400
  ISN 0101    200   CONTINUE                                                              00087500
  ISN 0102          RETURN                                                                00087600
  ISN 0103          END                                                                   00087700

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      102, PROGRAM SIZE =    107808, SUBPROGRAM NAME = SOLVE

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                    256K BYTES OF CORE NOT USED
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LEVEL 2.3.0 (JUNE 78)                OS/360  FORTRAN H EXTENDED          DATE 82.246/12.21.58          PAGE  1

REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

ISN 0002              SUBROUTINE ODESLV(N,Y,DERIV,H)
ISN 0003              IMPLICIT REAL*8(A-H,O-Z)
ISN 0004              DIMENSION DERIV(N),Y(N),DERIVO(312),BD1(312,2),BD2(312,2),BD3(312)
ISN 0005              DATA INTF/1/,C1/0.0/,C2/0.0/,C3/0./
           C
           C   THIS SUBROUTINE INTEGRATES THE FIRST ORDER SYSTEM OF ORDINARY
           C   DIFFERENTIAL EQUATIONS  "DY/DT=DERIV" BY THE ADAMS METHOD
           C   USING THIRD ORDER DIFFERENCES.
           C     N- SIZE OF SYSTEM
           C     Y- VECTOR OF INITIAL VALUES ON INPUT. "Y" IS OVERWRITTEN
           C        WITH THE NEW SOLUTION
           C     H- STEP SIZE
           C
ISN 0006              IF(N .LE. 312) GO TO 10
ISN 0008              WRITE(6,12) N
ISN 0009              STOP
ISN 0010   12         FORMAT(1H0,5X,'ERROR IN SUBROUTINE **ODESLV** CALLED WITH STATE
                     1SIZE =',I3,'  EXCEEDS DIMENSION SIZE OF ARRAYS')
ISN 0011   10         GO TO(1000,2000,3000,4000),INTF
           C
           C    FIRST CALL TO ROUTINE  - EULER INTEGRATION
           C
ISN 0012   1000  DO 20 I=1,N
ISN 0013   20         DERIVO(I)=DERIV(I)
ISN 0014              INTF=2
ISN 0015              GO TO 5000
           C
           C    SECOND CALL TO ROUTINE - FIRST ORDER DIFFERENCES
           C
ISN 0016   2000  DO 30 I=1,N
ISN 0017              BD1(I,1)=DERIV(I)-DERIVO(I)
ISN 0018              BD1(I,2)=BD1(I,1)
ISN 0019   30         DERIVO(I)=DERIV(I)
ISN 0020              C1=.5
ISN 0021              INTF=3
ISN 0022              GO TO 5000
           C
           C    THIRD CALL TO ROUTINE - SECOND ORDER DIFFERENCES
           C
ISN 0023   3000  DO 40 I=1,N
ISN 0024              BD1(I,2)=DERIV(I)-DERIVO(I)
ISN 0025              BD2(I,1)=BD1(I,2)-BD1(I,1)
ISN 0026              BD2(I,2)=BD2(I,1)
ISN 0027              DERIVO(I)=DERIV(I)
ISN 0028   40         BD1(I,1)=BD1(I,2)
ISN 0029              INTF=4
ISN 0030              C2=5.0/12.0
ISN 0031              GO TO 5000
           C
           C    ADAMS METHOD WITH 3RD ORDER DIFFERENCES.
           C
ISN 0032   4000  DO 50 I=1,N
ISN 0033              BD1(I,2)=DERIV(I)-DERIVO(I)
```

```
  ISN 0034           BD2(I,2)=BD1(I,2)-BD1(I,1)                                 00093100
  ISN 0035           BD3(I)=BD2(I,2)-BD2(I,1)                                   00093200
  ISN 0036           DERIVO(I)=DERIV(I)                                         00093300
  ISN 0037           BD1(I,1)=BD1(I,2)                                          00093400
  ISN 0038     50    BD2(I,1)=BD2(I,2)                                          00093500
  ISN 0039           C3=3.0/8.0                                                 00093600
  ISN 0040           GO TO 5000                                                 00093700
               C                                                                00093800
               C   UPDATE VECTOR 'Y'                                            00093900
               C                                                                00094000
  ISN 0041     5000  DO 60 I=1,N                                                00094100
  ISN 0042     60    Y(I)=Y(I)+H*(DERIV(I)+C1*BD1(I,2)+C2*BD2(I,2)+C3*BD3(I))   00094200
  ISN 0043           RETURN                                                     00094300
  ISN 0044           END                                                        00094400

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      43, PROGRAM SIZE =     16838, SUBPROGRAM NAME =ODESLV

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                              276K BYTES OF CORE NOT USED
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REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

  ISN 0002            SUBROUTINE PRINT(T,F0,TAU0,FP)
  ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
  ISN 0004            REAL*8 MASS,MC0,MRM
  ISN 0005            REAL*4 TD(3),OD(3),PLTDAT
  ISN 0006            DIMENSION R(3),THETA(3),Q(3,50),UVW(3),OMEGA(3),QDOT(3,50),
                     1 MC0(3),MRM(3,50),CMAT(3,3),MASS(50),F0(3),TAU0(3),FP(3,50),
                     2 IPLOT(42),PLTDAT(100,20)
  ISN 0007            COMMON /STATE/R,THETA,Q,UVW,OMEGA,QDOT
  ISN 0008            COMMON /CONST/ TM,MASS,MC0,MRM,CMAT,N,N3,N3P6,NT,NTP6,NO
  ISN 0009            COMMON /PLOTT/ PLTDAT,IPLOT,NP
  ISN 0010            WRITE(6,100) T
  ISN 0011            WRITE(6,200) F0,TAU0
  ISN 0012            DO 201 I=1,N
  ISN 0013      201   WRITE(6,202) I,(FP(J,I),J=1,3)
  ISN 0014            DO 10 I=1,3
  ISN 0015            TD(I)=57.29578*THETA(I)
  ISN 0016      10    OD(I)=57.29578*OMEGA(I)
  ISN 0017            WRITE(6,110) R,TD,UVW,OD
  ISN 0018            DO 20 I=1,N
  ISN 0019      20      WRITE(6,120) I,(Q(J,I),J=1,3)
  ISN 0020            DO 30 I=1,N
  ISN 0021      30      WRITE(6,130) I,(QDOT(J,I),J=1,3)
  ISN 0022      100   FORMAT(1H0,///,5X,'TIME=',F10.3,' SEC')
  ISN 0023      110   FORMAT(1H ,8X,'R=',3(2X,1PE11.4),' FT',/,4X,'THETA=',
                     1 3(2X,1PE11.4),' DEG',/,6X,'UVW=',3(2X,1PE11.4),' FT/SEC',/,
                     2 4X,'OMEGA=',3(2X,1PE11.4),' DEG/SEC',/)
  ISN 0024      120   FORMAT(1H ,5X,'Q',I3,'=',3(2X,1PE11.4),' FT')
  ISN 0025      130   FORMAT(1H ,2X,'QDOT',I3,'=',3(2X,1PE11.4),' FT/SEC')
  ISN 0026      200   FORMAT(1H0,8X,'F0=',3(2X,1PE11.4),' LB',/,
                     1 6X,'TAU0=',3(2X,1PE11.4),' FT LB')
  ISN 0027      202   FORMAT(1H ,4X,'F',I3,'=',3(2X,1PE11.4),' LB')
  ISN 0028            RETURN
  ISN 0029            ENTRY GRAF(T)
  ISN 0030            IF(IPLOT(1) .EQ. 0) GO TO 203
                C
                C   STORE VARIABLES FOR PLOTTING
                C
  ISN 0032            NP=NP+1
  ISN 0033            IF(NP .GT. 100) GO TO 203
  ISN 0035            PLTDAT(NP,1)=T
  ISN 0036            DO 300 I=1,42
  ISN 0037            NY=IPLOT(I)
  ISN 0038            IF(NY .EQ. 0) GO TO 203
  ISN 0040            IF(NY .GT. 3) GO TO 310
                C
                C  STORE INERTIAL POSITION
                C
  ISN 0042            PLTDAT(NP,I+1)=R(NY)
  ISN 0043            GO TO 300
  ISN 0044      310   IF(NY .GT. 6) GO TO 320
                C
                C   STORE ATTITUDE ANGLES IN DEGREES
                C
```

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LEVEL 2.3.0 (JUNE 78)          PRINT          OS/360  FORTRAN H EXTENDED          DATE 82.246/12.21.59          PAGE   2

  ISN 0046           PLTDAT(NP,I+1)=57.29578*THETA(NY-3)
  ISN 0047           GO TO 300
  ISN 0048     320   IF(NY .GT. N3P6) GO TO 330
               C
               C    STORE DEFORMATION COORDINATES
               C
  ISN 0050           L=NY-6
  ISN 0051           I1=1+L/3
  ISN 0052           I2=L-3*(I1-1)
  ISN 0053           IF(I2 .NE. 0) GO TO 321
  ISN 0055           I1=I1-1
  ISN 0056           I2=3
  ISN 0057     321   PLTDAT(NP,I+1)=Q(I2,I1)
  ISN 0058           GO TO 300
  ISN 0059     330   IF(NY .GT. (N3P6+3)) GO TO 340
               C
               C    STORE TRANSLATIONAL VELOCITY
               C
  ISN 0061           PLTDAT(NP,I+1)=UVW(NY-N3P6)
  ISN 0062           GO TO 300
  ISN 0063     340   IF(NY .GT. (N3P6+6)) GO TO 350
               C
               C    STORE ANGULAR VELOCITY IN DEG./SEC.
               C
  ISN 0065           PLTDAT(NP,I+1)=57.29578*OMEGA(NY-(N3+9))
  ISN 0066           GO TO 300
               C
               C    STORE DEFORMATION RATES
               C
  ISN 0067     350   L=NY-(N3+12)
  ISN 0068           I1=1+L/3
  ISN 0069           I2=L-3*(I1-1)
  ISN 0070           IF(I2 .NE. 0) GO TO 351
  ISN 0072           I1=I1-1
  ISN 0073           I2=3
  ISN 0074     351   PLTDAT(NP,I+1)=QDOT(I2,I1)
  ISN 0075     300   CONTINUE
  ISN 0076     203   RETURN
  ISN 0077           END

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      76, PROGRAM SIZE =      2324, SUBPROGRAM NAME = PRINT

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  268K BYTES OF CORE NOT USED
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```
REQUESTED OPTIONS: NOOBJ,TERM,,NOXREF,,NOMAP,,,NAME(MAIN),AD(NONE),OPT(0),,,FLAG(I),SIZE(384K),LC(60),

OPTIONS IN EFFECT: NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

                                                                                     00103700
  ISN 0002           SUBROUTINE PLOT                                                 00103800
  ISN 0003           IMPLICIT REAL*8(A-H,O-Z)                                        00103900
  ISN 0004           REAL*4 RAN,PLTDAT                                               00104000
  ISN 0005           DIMENSION IPLOT(42),PLTDAT(100,20),IT(144),RAN(4),IC(10),       00104100
                    1 IMAG4(5151)                                                    00104200
  ISN 0006           COMMON /PLOTT/ PLTDAT,IPLOT,NP                                  00104300
  ISN 0007           DATA IT(1)/0/,RAN/4*0./,IC(1)/1H*/                              00104400
               C                                                                     00104500
               C     CALCULATE NUMBER OF VARIABLES TO BE PLOTTED                     00104600
               C                                                                     00104700
  ISN 0008           NV=0                                                            00104800
  ISN 0009           DO 100 I=1,42                                                   00104900
  ISN 0010           IF(IPLOT(I) .EQ. 0) GO TO 110                                   00105000
  ISN 0012     100   NV=NV+1                                                         00105100
  ISN 0013     110   IF(NV .EQ. 0) RETURN                                            00105200
  ISN 0015           DO 120 IP=1,NV                                                  00105300
  ISN 0016           CALL USPLT(PLTDAT(1,1),PLTDAT(1,IP+1),100,NP,1,1,IT,RAN,IC,1,   00105400
                    1 IMAG4,IER)                                                     00105500
  ISN 0017     120   CONTINUE                                                        00105600
  ISN 0018           RETURN                                                          00105700
  ISN 0019           END

*OPTIONS IN EFFECT*NAME(MAIN) NOOPTIMIZE LINECOUNT(60) SIZE(0384K) AUTODBL(NONE)

*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK NOOBJECT NOMAP NOFORMAT GOSTMT NOXREF NOALC NOANSF TERM IBM FLAG(I)

*STATISTICS*     SOURCE STATEMENTS =      18, PROGRAM SIZE =     21736, SUBPROGRAM NAME =  PLOT

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                276K BYTES OF CORE NOT USED

*STATISTICS*  NO  DIAGNOSTICS THIS STEP
```

APPENDIX B

SAMPLE INPUT DATA

This appendix provides an illustrative example of the program NAMELIST input data corresponding to the vehicle in Figure 5. The vector geometry and inertia matrix for that vehicle are

$$\vec{S} = -\frac{b}{2}\,\underline{i}_4 + \frac{b}{2}\,\underline{j}_4 + \frac{h}{2}\,\underline{k}_4$$

$$\vec{r}^{\,1} = -\frac{b}{2}\,\underline{i}_4 + (b + L)\underline{j}_4 + \frac{h}{2}\,\underline{k}_4$$

$$\vec{r}^{\,2} = -\frac{b}{2}\,\underline{i}_4 + (b + 2L)\underline{j}_4 + \frac{h}{2}\,\underline{k}_4$$

$$\vec{r}^{\,3} = -\frac{b}{2}\,\underline{i}_4 - L\,\underline{j}_4 + \frac{h}{2}\,\underline{k}_4$$

$$\vec{r}^{\,4} = -\frac{b}{2}\,\underline{i}_4 - 2L\,\underline{j}_4 + \frac{h}{2}\,\underline{k}_4$$

$$[I_b] = \frac{m_b}{12}\begin{bmatrix} (b^2 + h^2) & 0 & 0 \\ 0 & (b^2 + h^2) & 0 \\ 0 & 0 & 2b^2 \end{bmatrix}$$

$$\vec{R} = R_x\,\underline{i}_1 + R_y\,\underline{j}_1 + R_z\,\underline{k}_1$$

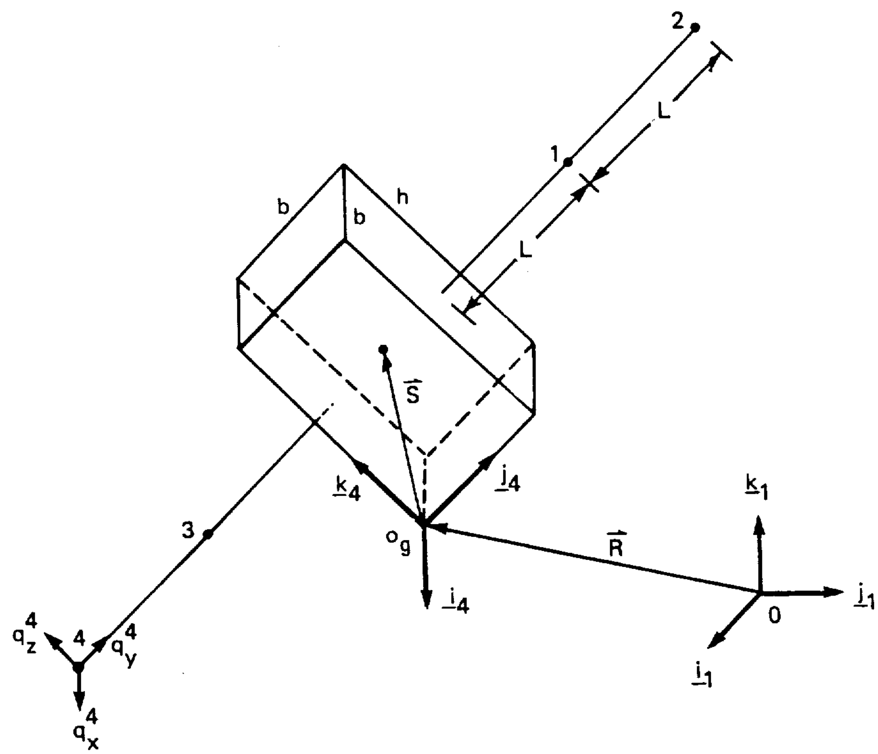

Figure 5. Example vehicle.

The namelist input items given below correspond to the following dimensions and masses

$$m_b = 5 \text{ slugs}$$

$$m_1 = m_3 = 1 \text{ slug}$$

$$m_2 = m_4 = 0.5 \text{ slug}$$

$$b = 1 \text{ ft}$$

$$h = 2 \text{ ft}$$

$$L = 10 \text{ ft}$$

Also two constrained modes are to be used in the simulation. The initial conditions on the kinematic variables are

@ t_0	$R_x = 1 \cdot 10^3$ ft	$\theta_1 = 5$ degrees
	$R_y = 2 \cdot 10^3$ ft	$\theta_2 = 20$ degrees
	$R_z = 3 \cdot 10^3$ ft	$\theta_3 = 0$ degrees
	$u = 0$ ft/s	$\omega_1 = 0$ deg/s
	$v = 5$ ft/s	$\omega_2 = 10$ deg/s
	$w = 0$ ft/s	$\omega_3 = 0$ deg/s

Note that the program in Appendix A sets the initial particle deflections, modal coordinates and the respective time derivatives to zero (see subroutine INITL).

The numerical integration is to proceed from time = 0 (set internally, see main program) to a final time of 60 seconds using an integration time step of 0.01 second. The print time step is to be 6 seconds and the plot time step 0.6 second.

The following variables are to be plotted versus time: R_y, θ_2, q_x^4, q_y^4, q_z^4, v, ω_2.

NAMELIST Input Data

```
&INPUT MØ = 5.0, N = 4, MASS = 1.0, 0.5, 1.0, 0.5,
RM = -0.5, 11.0, 1.0, -0.5, 21.0, 1.0, -0.5, -10.0, 1.0, -0.5,
     -20.0, 1.0,
IØ = 2.083, 0.0, 0.0, 0.0, 2.083, 0.0, 0.0, 0.0, 0.833,
S = -0.5, 0.5, 1.0, NT = 2 &END

&KIN R = 1.E3, 2.E3, 3.E3, THETA = 5.0, 20.0, 0.0,
UVW = 0.0, 5.0, 0.0, OMEGA = 0.0, 10.0, 0.0 &END

&RUN DT = 0.01, TSTOP = 60.0, DTP = 6.0, DTG = 0.6 &END

&PLT IPLOT = 2, 5, 16, 17, 18, 20, 23 &END
```

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2. Likins, P.W., "Analytical Dynamics and Nonrigid Spacecraft Simulation," Technical Report 32-1593, Jet Propulsion Laboratory, Pasadena, CA 1974.

3. Bodley, C.S., A.D. Devers, A.C. Park, and H.P. Frisch, "A Digital Computer Program for the Dynamic Interaction Simulation of Controls and Structures (DISCOS)," Volume I, NASA Technical Paper 1219, 1978.

4. Gates, S.S., "DISCOS Method for Incorporation of Finite-Element Model," Draper Intralab Memo DYN-82-1, January 1982.

5. Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," Technical Report 32-1329, Revision 1, Jet Propulsion Laboratory, Pasadena, CA, 1970.

6. Likins, P.W., "Finite Element Appendage Equations for Hybrid Coordinate Dynamic Analysis," <u>International Journal of Solids and Structures</u>, Volume 8, pp. 709-731, 1972.

7. Storch, J., "Dynamic Equations for Arbitrary Motion of a Flexible Body - Finite Element Idealization," Draper Intralab Memo DYN-82-4, January 1982.

8. IMSL Library Reference Manual, Edition 8, International Mathematical and Statistical Libraries, Inc., Houston, TX, 1980.