NASA TECHNICAL MEMORANDUM 104077
AVSCOM TECHNICAL REPORT 91-B-012

RESIDUAL THERMAL AND MOISTURE INFLUENCES ON THE STRAIN ENERGY RELEASE RATE ANALYSIS OF LOCAL DELAMINATIONS FROM MATRIX CRACKS

T. K. O'Brien

JULY 1991

NASA

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

US ARMY
AVIATION
SYSTEMS COMMAND
AVIATION R&T ACTIVITY

SUMMARY

An analysis utilizing laminated plate theory is developed to calculate the strain energy release rate associated with local delaminations originating at off-axis, angle ply, matrix cracks in laminates subjected to uniaxial loads. The analysis includes the contribution of residual thermal and moisture stresses to the strain energy released. Examples are calculated for the strain energy release rate associated with local delaminations originating at 90 degree and angle ply (non-90 degree) matrix ply cracks in glass epoxy and graphite epoxy laminates. The solution developed may be used to assess the relative contribution of mechanical, residual thermal, and moisture stresses on the strain energy release rate for local delamination for a variety of layups and materials.

KEYWORDS: residual thermal stress, hygroscopic stress, moisture, strain energy release rate, delamination, matrix crack

NOMENCLATURE

A	Delaminated area
a	Delamination length
C	Compliance
E	Modulus of elasticity
G	Strain energy release rate
h	Ply thickness
K	Number of plies in a region
l	Length
m	Number of local delaminations
N	Number of plies in uncracked laminate
n	Number of cracked plies
$\{N\}$	Vector of force resultants
P	Applied axial mechanical load
$[\overline{Q}]$	Transformed reduced stiffness matrix
t	Thickness of a region
U	Strain energy
u	Strain energy density
V	Volume
W	Work
w	Width
α_1	Lamina coefficient of thermal expansion in the fiber direction
α_2	Lamina coefficient of thermal expansion transverse to the fiber direction
$\{\alpha\}_k$	Lamina coefficients of thermal expansion in the laminate coordinate system
$\{\overline{\alpha}\}$	Laminate coefficients of thermal expansion
β_1	Lamina coefficient of moisture expansion in the fiber direction
β_2	Lamina coefficient of moisture expansion transverse to the fiber direction
$\{\beta\}_k$	Lamina coefficients of moisture expansion in the laminate coordinate system
$\{\overline{\beta}\}$	Laminate coefficients of moisture expansion
ΔH	Percentage moisture weight gain
ΔT	Temperature differential from cure temperature to test temperature

2

ε_{ij}	Strain tensor
$\{\varepsilon\}$	Vector of in-plane strain components
$\{\varepsilon^0\}$	Vector of mid-plane strain components
$\{\kappa\}$	Vector of curvatures
σ_{ij}	Stress tensor
$\{\sigma\}$	Vector of in-plane stress components
$\eta_{xy,x}$	Laminate coefficient of mutual influence of the second kind
ν_{xy}	Laminate Poisson's ratio

Subscripts and superscripts

k	Ply number
LD	Locally delaminated region
LAM	Laminated region
M	Mechanical Loading
T	Thermal Loading
H	Hygroscopic loading
x,y,z	Laminate coordinates

INTRODUCTION

One common source of delamination in composite laminates are matrix cracks that form in the off-axis plies oriented at a non-zero degree angle to the applied load. If many similar plies are grouped together in the laminate, matrix cracks may form under a monotonically increasing load and initiate local delaminations [1]. More commonly, matrix cracks cause delaminations to form under cyclic loads, even if the laminate stacking sequence contains angle plies with only single ply thicknesses [2,3]. Hence, in order to predict the damage tolerance and fatigue behavior of these laminates, solutions are needed for the critical parameters that govern the formation of delamination from matrix cracks.

Previous studies have contrasted the difference between local delaminations induced by matrix cracks and the more commonly studied edge delaminations resulting from mismatch in lamina and laminate properties [4-8]. In several studies, the strain energy release rate associated with these localized delaminations has been shown to be a useful parameter in predicting the onset and accumulation of local delaminations through the thickness in flat,

3

tapered, and curved laminates [3,9,10]. In references 3&9, predictions were accomplished using a simple closed form equation for G that was derived using laminated plate theory. The purpose of this investigation was to develop a G analysis based on laminated plate theory that includes the contribution of the residual thermal and moisture stresses to the strain energy release rate associated with local delaminations from matrix cracks in laminates subjected to uniaxial loading.

STRAIN ENERGY RELEASE RATE ANALYSIS

Background

Figure 1 shows a simplified model of a composite laminate containing delaminations growing from a matrix crack. The composite gage length, l, is divided into a locally delaminated region (LD) whose length is the delamination length, a, and a laminated region (LAM) of length l-a. In reference 4, an equation for the strain energy release rate associated with local delamination growth was derived using the classical definition for G as

$$G = \frac{P^2}{2} \frac{dC}{dA} \tag{1}$$

where C is the laminate compliance and A is the surface area created by the local delamination. A series solution was developed assuming the composite displacements are the sum of the displacements in the laminated and locally delaminated regions, and the load carried by the two regions is equal to the applied load, P. This series solution resulted in the following expression

$$G = \frac{P^2}{2mw^2} \left(\frac{1}{t_{LD}E_{LD}} - \frac{1}{t_{LAM}E_{LAM}} \right) \tag{2}$$

where P is the applied axial mechanical load on the laminate, w is the laminate width, and m is the number of delaminations growing from the matrix crack. This parameter, m, would have a value of 2 if the cracked off-axis ply is in the interior of the laminate, and would have a value of 1 if the cracked off-axis ply is a surface ply. Furthermore, t_{LAM} is the laminate thickness, E_{LAM} is the laminate modulus (as calculated from laminated plate theory [11]), t_{LD} is the thickness of the locally delaminated region that carries the load

4

(i.e., the laminate thickness minus the thickness of the cracked off-axis plies), and E_{LD} is the modulus of the locally delaminated region (as calculated from laminated plate theory). Although figure 1 depicts a local delamination growing from a 90 degree matrix crack, eq (2) has also been shown to be valid for local delaminations extending from angle ply matrix cracks where the cracked ply is oriented at an arbitrary, non-90 degree, angle to the load axis [8]. In equation 2, G depends on the layup and the location of the delaminated interfaces, which is determined by the through-thickness location of the cracked angle ply in the laminate, but does not depend on delamination size.

In reference 6, a 3D finite element analysis was utilized to determine the strain energy release rate for local delamination in laminates containing 90 degree matrix cracks. The strain energy release rate for local delamination increased initially with the delamination length from the matrix crack but eventually approached a constant level asymptotically at a delamination length of 3 to 4 ply thickness from the matrix crack. For a given delamination length, the 3D analysis yielded plots of G across the laminate width. The asymptotic values of total G for local delamination calculated near the free edge agreed well with eq (2). The agreement was found near the free edge instead of in the interior of the laminated width because the modulus terms in eq (2) were calculated using laminated plate theory and not beam theory. Hence, as was shown in ref [6], eq (2) is sensitive to the mismatch in Poisson contraction of the original laminate and the locally delaminated region.

Analysis Derivation

Although eq (2) has been verified using the results of a 3D finite element analysis [6], it includes only the contribution of the mechanical load to the strain energy released as local delaminations grow. Residual thermal stresses and the stresses due to absorbed moisture may also contribute to the strain energy that is released as the delamination grows. Therefore, a new analysis was developed to account for these thermal and moisture influences.

Beginning with the definition of strain energy release rate,

$$G = \frac{dW}{dA} - \frac{dU}{dA} \qquad (3)$$

where W is the work performed by the applied mechanical load and U is the strain energy released as the crack extends, an expression

5

may be derived for G associated with local delaminations from matrix cracks that includes the influence of residual thermal and moisture stresses. The work performed on the laminate by the applied mechanical load is equal to the summation of the work performed on both the laminated and locally delaminated regions. Hence,

$$W(a) = W_{LAM}(a) + W_{LD}(a) \tag{4}$$

For any region, the work performed may be defined in terms of the strain and stress tensors resulting from an applied mechanical load on that region as

$$W = \int_V \varepsilon_{ij}^M \sigma_{ij}^M \, dV \tag{5}$$

Using the assumptions and matrix notation of classical laminated plate theory [11], eq (5) becomes

$$W = wl \sum_{k=1}^{K} \int_{z_{k-1}}^{z_k} (\varepsilon')_k^M (\sigma)_k^M \, dz \tag{6}$$

where the prime in $(\varepsilon')_k^M$ represents the transpose of the strain vector, and K is the number of plies in the region. For a laminate that does not undergo bending, and has equal ply thicknesses throughout the laminate, eq (6) becomes

$$W = \frac{wlt}{K} \sum_{k=1}^{K} (\varepsilon')_k^M (\sigma)_k^M \tag{7}$$

If the laminate in the region is asymmetric, however, bending and coupling contributions to the mechanical strain, and hence, to the work done on the region, may be included using the expressions given in the appendix. The strains in equations (6) and (7) are the strains associated with the applied mechanical load, P, for the region and are given by

6

$$\{\varepsilon\}_k^M = \left\{ \begin{array}{c} 1 \\ -v_{xy} \\ \eta_{xy,x} \end{array} \right\} \varepsilon_x^M \qquad (8)$$

where ε_x^M is the axial strain in the region due to the applied mechanical load, v_{xy} is the laminate Poisson's ratio for the region, and $\eta_{xy,x}$ is the laminate coefficient of mutual influence of the second kind for the region.

For the laminated region, $w_{LAM} = w$, $l_{LAM} = l-a$, $K = N$, and $t_{LAM} = Nh$, where N is the number of plies in the laminate and h is the ply thickness. Therefore, for an applied mechanical load, P,

$$\left(\varepsilon_x^M\right)_{LAM} = \frac{P}{whNE_{LAM}} \qquad (9)$$

For the locally delaminated region, $w_{LD} = w$, $l_{LD} = a$, $K = N-n$, where n is the number of cracked plies, and $t_{LD} = (N-n)h$. Therefore

$$\left(\varepsilon_x^M\right)_{LD} = \frac{P}{wh(N-n)E_{LD}} \qquad (10)$$

The Poisson's ratio, v_{xy}, and $\eta_{xy,x}$ for each region may be calculated from laminated plate theory [11]. Because $\eta_{xy,x}$ accounts for the coupling between the applied axial strain and in-plane shear, it will be zero for the laminated region if the original laminate is balanced and symmetric, but it may be non-zero for the locally delaminated region.

The stress vector, $\{\sigma\}_k^M$, in equations (6) and (7) contains the ply stresses associated with the applied axial mechanical load, P, for either region and is given by

$$\{\sigma\}_k^M = \left[\overline{Q}\right]_k \{\varepsilon\}_k^M \qquad (11)$$

where $\left[\overline{Q}\right]_k$, is the transformed reduced stiffness matrix for the kth ply as defined in ref [11]. Substituting eq (11) into eq (7) yields

$$W = \frac{2w l t}{K} \sum_{k=1}^{K} u_k^M \qquad (12)$$

where
$$u_k^M = \frac{1}{2} \{\varepsilon\}_k^M [\overline{Q}]_k \{\varepsilon\}_k^M \qquad (13)$$

For the model shown in fig.1, $w_{LAM} = w_{LD} = w$, $l_{LAM} = l$-a, $l_{LD} = a$, $K_{LAM} = N$, $K_{LD} = N$-n, $t_{LAM} = Nh$, and $t_{LD} = (N$-n$)h$. Combining these expressions with eqs (4) and (12) and noting that each region has equal ply thickness, $h = t/K$, yields

$$W(a) = 2wlh \, u_{LAM}^M + 2wah \left[u_{LD}^M - u_{LAM}^M \right] \qquad (14)$$

where
$$u_{LAM}^M = \sum_{k=1}^{N} u_k^M \qquad (15)$$

and
$$u_{LD}^M = \sum_{k=1}^{N-n} u_k^M \qquad (16)$$

Recalling that dA = mwda for a through-width local delamination growing uniformly from a matrix ply crack [4,8], and differentiating eq (14) with respect to delamination length, a, yields

$$\frac{dW}{dA} = \left(\frac{2h}{m} \right) (u_{LD}^M - u_{LAM}^M) \qquad (17)$$

The strain energy released in the laminate as the delamination grows is equal to the summation of the strain energy released in the laminated and locally delaminated regions. Hence,

$$U(a) = U_{LAM}(a) + U_{LD}(a) \qquad (18)$$

For any region, the strain energy released may be defined in terms of the strain and stress tensors for that region as

8

$$U = \frac{1}{2} \int_v \varepsilon_{ij} \, \sigma_{ij} \, dV \qquad (19)$$

Using the assumptions and matrix notation of classical laminated plate theory [11], eq (19) becomes

$$U = \frac{wl}{2} \sum_{k=1}^{K} \int_{z_{k-1}}^{z_k} \{\varepsilon'\}_k \, \{\sigma\}_k \, dz \qquad (20)$$

For a laminate that does not undergo bending, and has equal ply thicknesses throughout the laminate, eq (20) becomes

$$U = \frac{wlt}{2K} \sum_{k=1}^{K} \{\varepsilon'\}_k \, \{\sigma\}_k \qquad (21)$$

If the laminate in the locally delaminated region is asymmetric, however, bending and coupling contributions to the strain, and hence, to the strain energy in the region, may be included using the expressions given in the appendix.

The strains in equations (20) and (21) are the total strains that contribute to the release of strain energy as the delamination grows. This total strain may consist of strains resulting from the applied mechanical load (M), the residual thermal stresses in the region (T), and the hygroscopic stresses that develop as a function of absorbed moisture (H). The total strain may be written as

$$\{\varepsilon\}_k = \{\varepsilon\}_k^M + \{\varepsilon\}_k^T + \{\varepsilon\}_k^H \qquad (22)$$

where the mechanical strain is given by eq (8), with ε_x^M given by eqs (9) or (10) for the appropriate region. The thermal strains for any region are given by [12]

$$\{\varepsilon\}_k^T = \left\{ \begin{array}{c} \overline{\alpha}_x - \alpha_x^k \\ \overline{\alpha}_y - \alpha_y^k \\ \overline{\alpha}_{xy} - \alpha_{xy}^k \end{array} \right\} \Delta T \qquad (23)$$

where $\quad\overline{\alpha}$ = the coefficient of thermal expansion of the region

α^k = the coefficient of thermal expansion of the kth ply in the region

ΔT = the difference between the stress free temperature and the temperature of the ambient environment

The hygroscopic strains for any region are given by [12]

$$\{\epsilon\}_k^H = \left\{ \begin{array}{c} \overline{\beta}_x - \beta_x^k \\ \overline{\beta}_y - \beta_y^k \\ \overline{\beta}_{xy} - \beta_{xy}^k \end{array} \right\} \Delta H \tag{24}$$

where $\quad\overline{\beta}$ = the coefficient of moisture expansion of the region

β^k = the coefficient of moisture expansion of the kth ply in the region

ΔH = the percent weight difference between the ambient (wet) condition and the dry condition

The stress vector, $\{\sigma\}_k$, in equations (20) and (21) contains the ply stresses associated with the total strain for any region and is given by [11]

$$\{\sigma\}_k = [\overline{Q}]_k \{\epsilon\}_k \tag{25}$$

Substituting eq (25) into eq (21) and noting that each region has equal ply thickness, h=t/K, yields

$$U = wlh \sum_{k=1}^{K} u_k \tag{26}$$

where

$$u_k = \frac{1}{2} \{\epsilon\}_k [\overline{Q}]_k \{\epsilon\}_k \tag{27}$$

10

For the model shown in fig.1, $w_{LAM} = w_{LD} = w$, $l_{LAM} = l\text{-}a$, $l_{LD} = a$, $K_{LAM} = N$, $K_{LD} = N\text{-}n$, $t_{LAM} = Nh$, and $t_{LD} = (N\text{-}n)h$. Combining these expressions with eqs (18) and (26) yields

$$U(a) = wlh\ u_{LAM} + wah\ [u_{LD} - u_{LAM}] \tag{28}$$

where
$$u_{LAM} = \sum_{k=1}^{N} u_k \tag{29}$$

and
$$u_{LD} = \sum_{k=1}^{N-n} u_k \tag{30}$$

Recalling that dA = mwda for a through-width local delamination growing uniformly from a matrix ply crack [4,8], and differentiating eq (28) with respect to delamination length, a, yields

$$\frac{dU}{dA} = \left(\frac{h}{m}\right)(u_{LD} - u_{LAM}) \tag{31}$$

Substituting eqs (17) and (31) into eq (3) yields the strain energy release rate

$$G = \left(\frac{h}{m}\right)\left\{2(u_{LD}^{M} - u_{LAM}^{M}) - (u_{LD} - u_{LAM})\right\} \tag{32}$$

If only the mechanical strain due to the applied load is assumed to contribute to the strain energy released in the laminated and delaminated regions, then $u_{LD} = u_{LD}^{M}$, $u_{LAM} = u_{LAM}^{M}$, and eq (32) becomes

$$G = \left(\frac{h}{m}\right)\left\{u_{LD}^{M} - u_{LAM}^{M}\right\} \tag{33}$$

Eq (33) will yield the same strain energy release rate as the closed form eq (2) derived based on the change in compliance with delamination growth. If the laminate is completely dry, only the mechanical strain due to the applied load and the strains due to residual thermal stresses contribute to the strain energy released in

the laminated and delaminated regions. For this mechanical plus thermal case, the strain energy release rate, G^{M+T}, is given by eq (32), where ΔH, and hence $(\varepsilon)_k^H$ in eqs (22) and (24), are zero. If the laminate is not completely dry, which is typical of polymer matrix composites that absorbed moisture from the air in the ambient environment [8,12], all of the strains in eq (22) contribute to the strain energy released in the laminated and locally delaminated regions. For this mechanical plus thermal plus hygroscopic case, the strain energy release rate, G^{M+T+H}, is given by eq (32), where ΔH, and hence $(\varepsilon)_k^H$ in eqs (22) and (24), are not zero.

ANALYTICAL RESULTS

In this section, four laminates will be analyzed to illustrate the influence of residual thermal and moisture stresses on G for local delamination from matrix cracks. These results simply illustrate how eq (32) may be used to quantify G for a combined mechanical, thermal, and moisture loading condition. These results are not intended as a sensitivity study, nor should they be compared to each other, since all three load components are coupled, and only their relative magnitudes at the onset of delamination under several loadings and environmental conditions can reveal their relative significance.

In the derivation of G^{M+T+H} for edge delamination in ref [12], it was noted that the sublaminates in the delaminated region would be constrained to the laminate's free thermal and moisture expansions in the applied load direction by the test machine grips. Therefore, $\bar{\alpha}_x$ and $\bar{\beta}_x$ for the sublaminates were set equal to $\bar{\alpha}_x$ and $\bar{\beta}_x$ for the original laminate in eqs (23) & (24). The local delaminations modeled in this study do not extend to the grip. However, the sublaminates in the locally delaminated region may be constrained by the sublaminate in the laminated region. Therefore, results for particular laminates calculated using eq (32) will be plotted with and without this constraint to illustrate the influence of constraining the sublaminates' free thermal expansion on the calculated strain energy release rate.

Figure 2 shows the influence of residual thermal and moisture stresses on the strain energy release rate associated with a local delamination growing from a 90 degree matrix crack in the 0/90 interfaces of a $(0_2/90_4)_s$ glass epoxy laminate. Lamina properties

from ref [6] were used in the analysis along with the following thermal coefficients assumed to be typical for glass epoxy

$$\alpha_1 = 3.80 \ \mu\varepsilon/°C \ (2.11 \ \mu\varepsilon/°F)$$
$$\alpha_2 = 16.7 \ \mu\varepsilon/°C \ (9.28 \ \mu\varepsilon/°F)$$

and the following moisture expansion coefficients that were determined for graphite epoxy [12], but were also assumed to be reasonable for glass epoxy

$$\beta_1 = 0 \ \mu\varepsilon/weight\%$$
$$\beta_2 = 5560 \ \mu\varepsilon/weight\%$$

A mechanical axial load per unit width, N_x, of 648 kN/m (3703 lbs/in.), corresponding to an axial strain of 0.01, and a ΔT of -100°C (-180°F) were applied. This ΔT corresponds to the difference between the room temperature condition of 21°C (70°F) and a 121°C (250°F) cure temperature for glass epoxy. The glass epoxy ply thickness was assumed to be 0.203 mm (0.008 in.). Strain energy release rates are plotted as a function of the percentage moisture weight gain assuming either a constrained or unconstrained sublaminate as discussed previously. The strain energy release rate caused by mechanical loading only, G^M, calculated from eq (33) agreed with the value calculated from eq (2) and is shown on the ordinate. The strain energy release rate corresponding to a mechanical and thermal loading only, G^{M+T}, is also shown on the ordinate.

For the constrained sublaminate case, G^{M+T}, as calculated by eq (32), is higher than G^M for the same applied load. However, if the laminate absorbs moisture, the residual thermal stresses are relaxed, and the strain energy release rate, G^{M+T+H}, decreases depending on the percentage of moisture weight gain, ΔH.

For the constrained sublaminate case shown in fig. 2, a moisture weight gain of approximately 0.2% completely relaxes the residual thermal stresses, resulting in a strain energy release rate, G^{M+T+H}, that is equal to G^M due to the mechanical load alone. For the unconstrained sublaminate case, however, G^{M+T} calculated by eq (32) is only slightly higher than G^M for the same applied load. Furthermore, as the laminate absorbs moisture, the strain energy release rate, G^{M+T+H}, decreases to G^M at $\Delta H = 0.2\%$, but then increases monotonically with increasing ΔH.

13

Fig. 3 shows a similar relationship for the strain energy release rate associated with a local delamination growing from a 90 degree matrix crack in the -45/90 interfaces of a $(45/-45/90_4)_s$ glass epoxy laminate subjected to an N_x of 414 kN/m (2363 lbs/in.), corresponding to an axial strain of 0.01, and a ΔT of -100°C (-180°F). For the constrained sublaminate case, the strain energy release rate caused by mechanical loading only, G^M, increases slightly when the residual thermal stresses are included, G^{M+T}, and decreases when the moisture contribution is included, G^{M+T+H}. For the unconstrained sublaminate case, however, G^{M+T} is nearly identical to G^M for the same applied load. Furthermore, as the laminate absorbs moisture, the strain energy release rate, G^{M+T+H}, increases very slightly with increasing ΔH.

In ref. [8], a strain energy release rate analysis was developed for local delaminations forming in the $\theta/-\theta$ interfaces of a $(0/\theta/-\theta)_s$ laminate from a $-\theta$ degree matrix crack, where θ is greater than zero but less than ninety degrees. For these cases, the sublaminates that comprise the locally delaminated (LD) region consist of a pair of asymmetric $(0/\theta)$ sublaminates. Under the applied tension load, these sublaminates may undergo both shear and bending deformations depending on the degree of shear-extension and bending-extension coupling present. To account for these coupling effects, the modulus of the locally delaminated region, E_{LD}, in eq (2) was determined by prescribing an applied mechanical strain, ε_x, setting the applied N_y and M_y equal to 0, and setting γ_{xy}, κ_x and κ_{xy} equal to 0. The N_x that results from this analysis was used to determine E_{LD} as

$$E_{LD} = N_x/t_{LD}\varepsilon_x \qquad (34)$$

which may be used in eq (2) to calculate G^M and in eq (10) to calculate $(\varepsilon_x^M)_{LD}$. Furthermore, This loading may be prescribed to determine the ply stresses and strains used to calculate the strain energy and work terms in eq (32) where u_{LAM}, u_{LD}, u_{LAM}^M, and u_{LD}^M are described in the appendix.

Figure 4 shows the influence of residual thermal and moisture stresses on the strain energy release rate associated with a local delamination growing from a -15 degree matrix crack in the 15/-15 interfaces of a $(0/15/-15)_s$ graphite epoxy laminate. Lamina properties from ref [8] were used in the analysis along with the

following thermal and moisture coefficients for graphite epoxy from ref [12]

$$\alpha_1 = -0.41 \ \mu\varepsilon/°C \ (-0.23 \ \mu\varepsilon/°F)$$
$$\alpha_2 = 26.8 \ \mu\varepsilon/°C \ (14.9 \ \mu\varepsilon/°F)$$

$$\beta_1 = 0 \ \mu\varepsilon/weight\%$$
$$\beta_2 = 5560 \ \mu\varepsilon/weight\%$$

A mechanical axial load per unit width, N_x, of 1400 kN/m (8000 lbs/in.) and a ΔT of -156°C (-280°F) were applied. This ΔT corresponded to the difference between the room temperature condition of 21°C (70°F) and a 177°C (350°F) cure temperature for graphite epoxy. The N_x chosen is close to the average N_x at the onset of damage in $(0_2/15_2/-15_2)_s$ AS4/3501-6 graphite epoxy laminates [7,8]. In the analysis, the graphite epoxy ply thickness was assumed to be 0.254 mm (0.01 in.) which is twice the measured ply thickness. This allowed the analysis of a 6-ply $(0/15/-15)_s$ laminate instead of a 12-ply $(0_2/15_2/-15_2)_s$ laminate. Strain energy release rates are plotted as a function of the percentage moisture weight gain assuming either an unconstrained sublaminate or a sublaminate whose free thermal and moisture expansions in the load (X) direction were constrained, as discussed previously. Furthermore, because these sublaminates experience shear-extension coupling, G was also calculated for sublaminates whose free thermal and moisture shear expansions were constrained. Hence, $\overline{\alpha}_{xy}$ and β_{xy} for the sublaminates were set equal to $\overline{\alpha}_{xy}$ and β_{xy} for the original laminate. G was calculated with either a shear constraint alone (XY) or a shear constraint in addition to the constraint in the load direction (X&XY).

In all cases, the strain energy release rate caused by mechanical loading only, G^M, calculated from eq (33) agreed with the value calculated from eq (2) and is shown on the ordinate. The strain energy release rate corresponding to a mechanical and thermal loading only, G^{M+T}, is also shown on the ordinate. For the unconstrained and XY constrained cases, G^{M+T} calculated from eq (32), is higher than G^M for the same applied load. However, for the X constrained and X&XY constrained cases, G^{M+T} calculated from eq (32) is nearly identical to G^M for the same applied load. For the unconstrained and XY constrained cases, as the laminate absorbs moisture, the residual thermal stresses are relaxed, and the strain energy release rate, G^{M+T+H} calculated from eq (32) decreases

with increasing moisture weight gain, ΔH. However, for the X constrained and X&XY constrained cases, G^{M+T+H} calculated from eq (32) deviates only slightly from G^M with increasing moisture weight gain, ΔH.

Fig. 5 shows a similar relationship for the strain energy release rate associated with a local delamination growing from a -45 degree matrix crack in the 45/-45 interfaces of a $(0/45/-45)_s$ graphite epoxy laminate. An N_x of 1400 kN/m (8000 lbs/in.) and a ΔT of -156°C (-280°F) were applied. In all cases, the strain energy release rate caused by mechanical loading only, G^M, calculated from eq (33) agreed with the value calculated from eq (2) and is shown on the ordinate. The strain energy release rate corresponding to a mechanical and thermal loading only, G^{M+T}, is also shown on the ordinate. For the unconstrained and XY constrained cases, G^{M+T} calculated from eq (32) is significantly higher than G^M for the same applied load. However, for the X constrained and X&XY constrained cases, G^{M+T} calculated from eq (32) is only slightly higher than G^M for the same applied load. For the unconstrained and XY constrained cases, as the laminate absorbs moisture, the residual thermal stresses are relaxed, and the strain energy release rate, G^{M+T+H} calculated from eq (32) decreases with increasing moisture weight gain, ΔH. However, for the X constrained and X&XY constrained cases, G^{M+T+H} calculated from eq (32) deviates only slightly from G^M with increasing moisture weight gain, ΔH.

DISCUSSION

For the local delaminations that originate from 90 degree matrix ply cracks (figures 2,3), the constrained values for G^{M+T+H} are probably more physically realistic, because these delaminations are localized along the laminate's length. Hence, it is possible to visualize how the undamaged sublaminate may constrain the free thermal expansion of the locally damaged region in the load direction. These constrained results for the local delamination from 90 degree matrix cracks are also consistent with the constrained results for edge delamination [12] where G^{M+T} is greater than G^M, but G^{M+T+H} decreases with increasing moisture weight gain.

For the local delaminations that originate from angle ply (non-90 degree) matrix ply cracks (figures 4,5), the XY constraint has very little effect on G^{M+T+H} when the free thermal and moisture expansions in the X direction are either constrained or unconstrained. However, the X constrained values of G^{M+T+H} are

16

significantly different than the unconstrained values. Hence, the X or X&XY constrained values for G^{M+T+H} are probably more physically realistic because these delaminations are also localized along the laminate's length and extend only slightly into the laminate's width [7,8]. However, these constrained results indicate that residual thermal and moisture contributions to G for local delaminations that originate from $-\theta$ degree ply cracks in $(0/\theta/-\theta)_s$ graphite epoxy laminates are very small, unlike the more significant effects observed for edge delamination in quasi-isotropic graphite epoxy laminates [12].

Tests on laminates that are dry could conceivably be compared to tests that have absorbed a known moisture content to quantify the significance of these thermal and moisture contributions. This experimental verification has been attempted for edge delamination with some success [12]. However, as noted in ref [7], under monotonic loading of $(0/15/-15)_s$ graphite epoxy laminates, the first indication of damage that can be documented shows both matrix cracking and local delaminations present. Hence, the observed dependence of moisture content on the load or strain at the onset of damage may reflect the influence of thermal and moisture stresses on matrix cracking more than their influence on local delamination. Similar tests were conducted under cyclic loading [8], where it was documented that matrix cracking preceded the onset of local delamination. Although the moisture content before testing was determined in ref.8, no attempt was made to test dry laminates in fatigue because of their tendency to absorb moisture from the air during the course of the fatigue test.

Clearly, a precise experimental verification of eq (32) is difficult to achieve. However, the accuracy of eq (32) could be verified by comparing to 3D finite element analysis results that also incorporate thermal and moisture effects. Hopefully, these comparisons should help confirm the correct constraint assumptions for the cases studied.

CONCLUSIONS

A laminated plate theory analysis was developed to calculate the strain energy release rate associated with local delaminations originating at matrix ply cracks in laminates subjected to axial loads. The analysis included the contribution of residual thermal and moisture stresses to the strain energy released. Examples were calculated for the strain energy release rate associated with local delaminations originating at 90 degree and angle ply (non-90 degree)

matrix ply cracks in glass epoxy and graphite epoxy laminates, respectively. The solution developed may be used to assess the relative contribution of mechanical, residual thermal, and moisture stresses on the strain energy release rate for local delamination for a variety of layups and loading combinations.

REFERENCES

1. Crossman, F.W., and Wang, A.S.D., "The Dependence of Transverse Cracking and Delamination on Ply Thickness in Graphite/Epoxy Laminates," Damage in Composite Materials, ASTM STP 775, June,1982, p.118-139.

2. Reifsnider, K.L., and Talug, A., "Analysis of Fatigue Damage in Composite Laminates," Int. J. of Fatigue, Vol.3,No.1, Jan.1980, pp.3-11.

3. O'Brien, T.K., Rigamonti, M., and Zanotti, C., "Tension Fatigue Analysis and Life Prediction for Composite Laminates," Int. J. of Fatigue, Vol.11, No.6, Nov.,1989, pp.379-394.

4. O'Brien, T.K., "Analysis of Local Delaminations and Their Influence on Composite Laminate Behavior," Delamination and Debonding of Materials, ASTM STP 876, Oct.,1985, p.282-297.

5. O'Brien, T.K., Johnston, N.J., Raju, I.S., Morris, D.H., and Simonds, R.A., "Comparisons of Various Configurations of the Edge Delamination Test for Interlaminar Fracture Toughness," in Toughened Composites, ASTM STP 937, May, 1987, pp.199-221.

6. Salpekar, S.A., and O'Brien, T.K., "Combined Effect of Matrix Cracking and Free Edge on Delamination," Composite Materials: Fatigue and Fracture, 3rd Volume, ASTM STP 1110, 1991 (Also in NASA TM 102591, March, 1990).

7. O'Brien, T.K., and Hooper, S.J., "Local Delamination in Laminates with Angle Ply Matrix Cracks: Part I, Tension Tests and Stress Analysis," NASA TM 104055, June, 1991 (presented at the 4th ASTM Symposium on Composite Materials: Fatigue and Fracture, Indianapolis, Indiana, May, 1991).

8. O'Brien, T.K., "Local Delamination in Laminates with Angle Ply Matrix Cracks: Part II, Delamination Fracture Analysis and Fatigue Characterization," NASA TM 104076, June, 1991 (presented at the 4th ASTM Symposium on Composite Materials: Fatigue and Fracture, Indianapolis, Indiana, May, 1991).

9. Murri, G.B., O'Brien, T.K., and Salpekar, S.A., "Tension Fatigue of Glass/epoxy and Graphite/epoxy Tapered Laminates," NASA TM 102628, April, 1990 (presented at the 46th annual AHS Forum, Washington, DC, May, 1990).

10. Martin, R.H., and Jackson, W.C. "Damage Prediction in Cross-plied Curved Composite Laminates," NASA TM 104089, July, 1991 (presented at the 4th ASTM Symposium on Composite Materials: Fatigue and Fracture, Indianapolis, Indiana, May, 1991).

11. Jones, R.M., Mechanics of Composite Materials, McGraw Hill, Washington, D.C., 1975.

12. O'Brien, T.K., Raju, I.S., and Garber, D.P., "Residual Thermal and Moisture Influences on the Strain Energy Release Rate Analysis of Edge Delamination," J. Composites Technology and Research, Vol.8, No.2, Summer, 1986, pp.37-47.

APPENDIX - COUPLING EFFECTS

In appendix A of ref.12, expressions were derived for the strain energy associated with an asymmetric sublaminate that experienced bending-extension coupling. For these asymmetric sublaminates, the total strain at any point in the sublaminate is given by

$$\{\varepsilon\} = \{\varepsilon^0\} + z\{\kappa\} \tag{A1}$$

where $\{\varepsilon^0\}$ are the midplane strains, $\{\kappa\}$ are the midplane curvatures, and z is the distance from the midplane to the point of interest. The strain energy for these sublaminates was derived in ref.12 as

$$U = wl \sum_{k=1}^{K} l_k \, u_k \tag{A2}$$

where

$$u_k = u_{ek} + u_{ck} + u_{bk} \tag{A3}$$

The individual terms in eq (A3) correspond to the strain energy in the asymmetric sublaminate corresponding to extension, u_{ek}, bending, u_{bk}, and bending-extension coupling, u_{ck}, as defined in eqs (A10-12) in ref.12. Unfortunately, there were several typographical errors in these equations. On page 45 of appendix A in ref.12, in the introduction to eq (A8), the "mechanical" strain is defined as the "total" strain less the free thermal and moisture strains of the ply. This "mechanical" strain is

$$\{\varepsilon^0\} - \{\alpha\}_k \Delta T - \{\beta\}_k \Delta H \tag{A4}$$

where

$$\{\varepsilon^0\} = \{\varepsilon^0\}^M + \{\bar{\alpha}\}\Delta T + \{\bar{\beta}\}\Delta H \tag{A5}$$

Hence this "mechanical" strain was not the mechanical term $\{\varepsilon^0\}^M$ alone, but was actually the total strain described in eqs (22-24) of this paper, and eqs (12-14) of ref.12, which includes the mechanical strain and a thermal and moisture contribution corresponding to the difference in the free thermal and moisture expansions or

20

contractions of the laminated regions minus the thermal and moisture expansions or contractions of the individual plies in the regions. These are the strains that contribute to residual thermal and moisture stresses, and hence, contribute to the strain energy that is stored in each laminated region. In eq (A10) of appendix A in ref.12, This strain should have appeared on both sides, not just on the right hand side, of $[\overline{Q}]_k$. Hence, eq (A10) in ref.12 should have been

$$u_{ek} = \frac{1}{2}\left(\{\varepsilon^0\} - \{\alpha\}_k\Delta T - \{\beta\}_k\Delta H\right)'[\overline{Q}]_k\left(\{\varepsilon^0\} - \{\alpha\}_k\Delta T - \{\beta\}_k\Delta H\right) \tag{A6}$$

Equation (A11) was shown correctly in ref.12 as

$$u_{bk} = \frac{1}{6}\left(z_k^2 + z_k z_{k-1} + z_{k-1}^2\right)\{\kappa\}'[\overline{Q}]_k\{\kappa\} \tag{A7}$$

However, eq (A12) in ref.12 should have been

$$u_{ck} = \frac{1}{2}(z_k + z_{k-1})\{\kappa\}'[\overline{Q}]_k\left(\{\varepsilon^0\} - \{\alpha\}_k\Delta T - \{\beta\}_k\Delta H\right) \tag{A8}$$

The calculations and results reported in ref.12 were performed correctly using eqs (A6-A8) above, even though the errors noted were made in printing the text.

For the asymmetric locally delaminated regions modeled in this paper, u_k^M in eqs (15 &16) and u_k in eqs (29 & 30) were calculated using equation A3 (where u_{ek}, u_{bk} and u_{ck} are given by equations A6,A7, &A8) instead of using equations 13 and 27 alone. These u_k^M and u_k values were used in eqs (15,16,29,&30) to determine u_{LAM}^M, u_{LD}^M, u_{LAM}, and u_{LD}, respectively.

LIST OF FIGURES

Figure 1. Model of local delamination.

Fig.2 G^{M+T+H} FOR LOCAL DELAMINATION FROM 90 DEGREE MATRIX CRACK

$(0_2/90_4)_s$ Glass Epoxy

$N_x = 648$ kN/m
$\Delta T = -100°C$

G^{M+T}

G^M, eq(2)

Unconstrained

Constrained

G^{M+T+H}, kJ/m^2

ΔH, %

24

Fig.3 G^{M+T+H} FOR LOCAL DELAMINATION FROM 90 DEGREE MATRIX CRACK

$(45/-45/90_4)_s$ Glass Epoxy

$N_x = 414$ kN/m
$\Delta T = -100°C$

Unconstrained

Constrained

G^{M+T}

G^M, eq(2)

G^{M+T+H}, kJ/m^2

ΔH, %
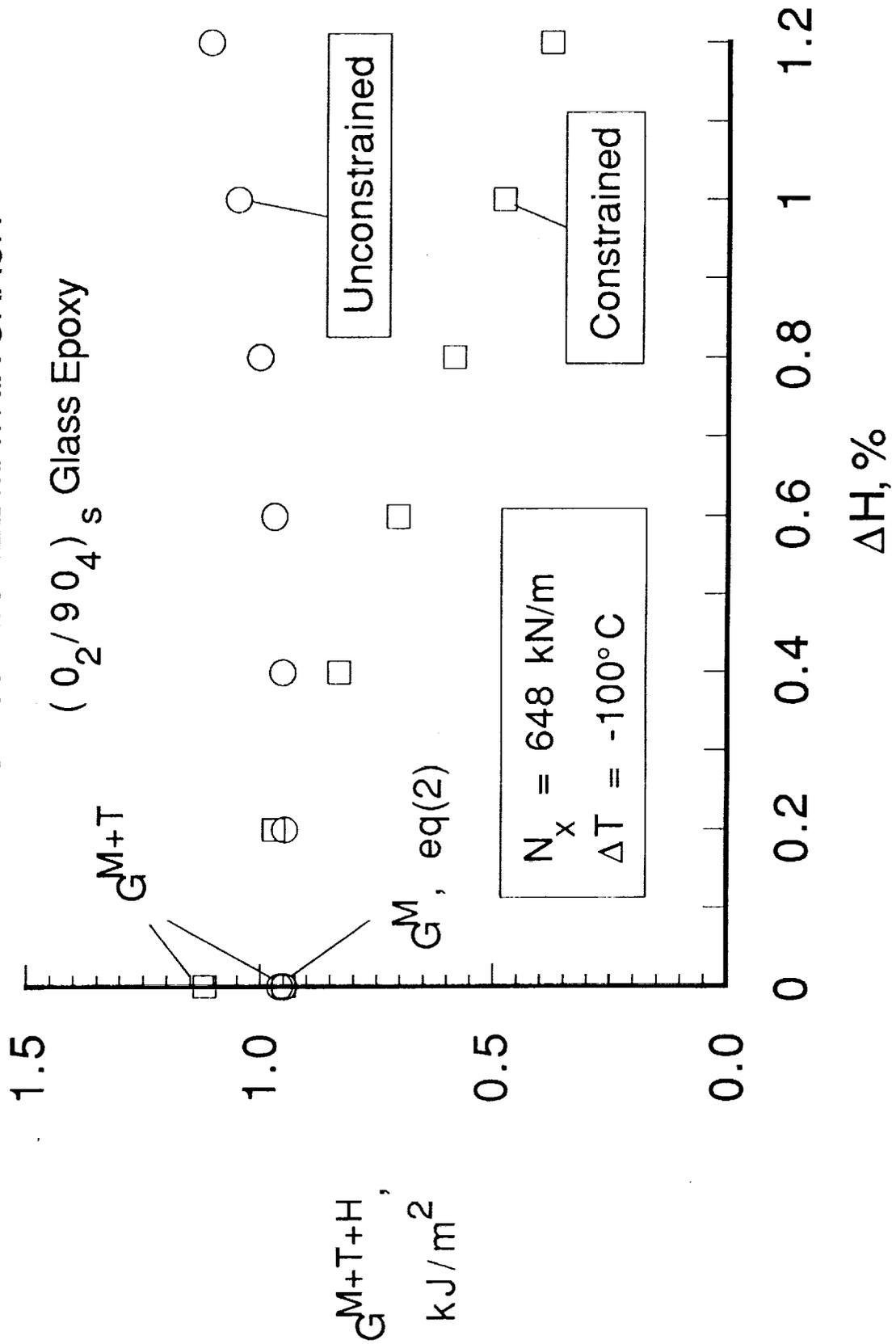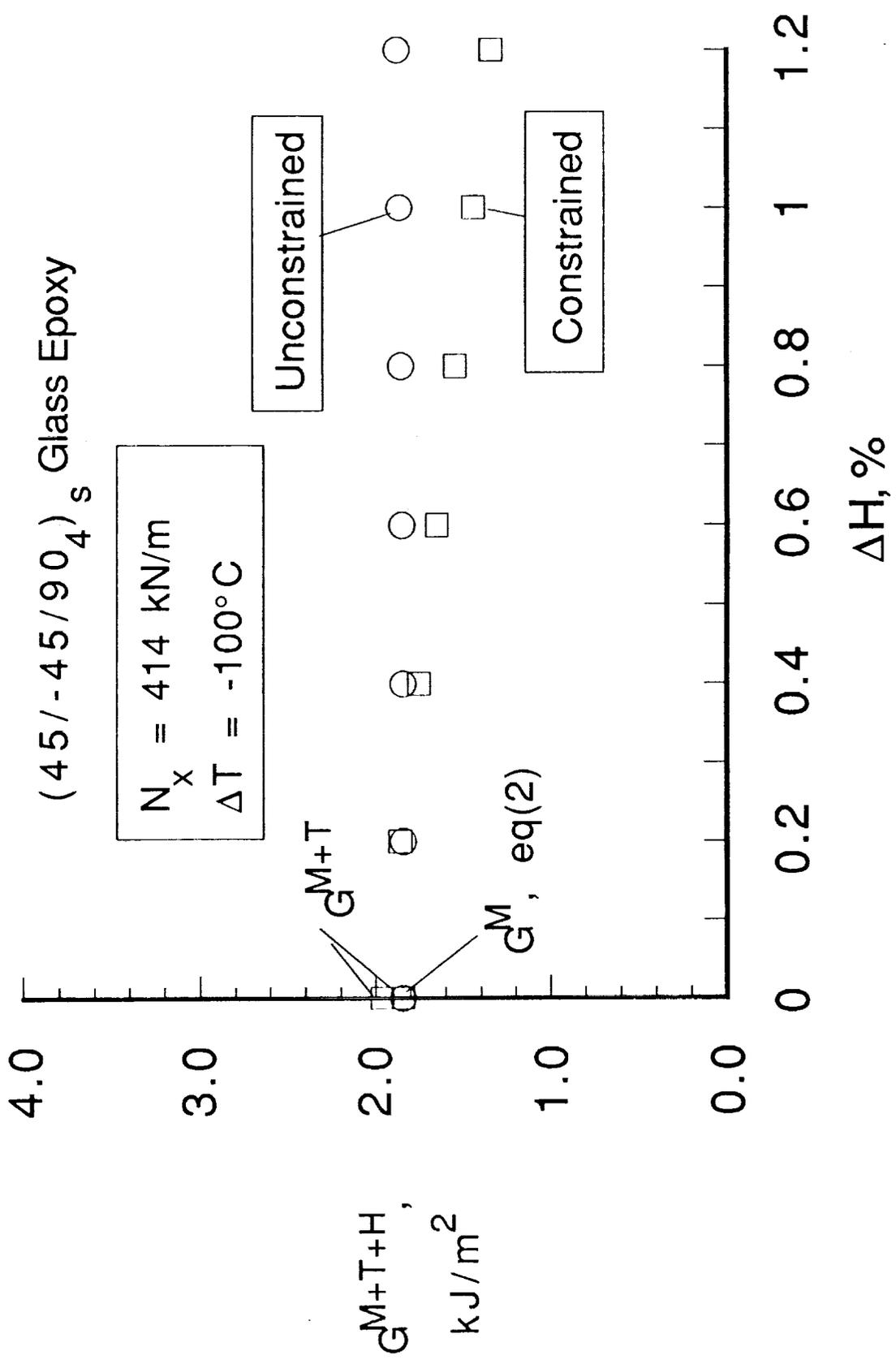
Fig.4 G^{M+T+H} FOR LOCAL DELAMINATION FROM -15 DEGREE MATRIX CRACK

$(0/15/-15)_s$ Graphite Epoxy

$N_x = 1400$ kN/m
$\Delta T = -156°C$

XY Constrained

Unconstrained

X & XY Constrained

X Constrained

G^{M+T}

G^M, eq(2)

G^{M+T+H}, kJ/m²

ΔH, %
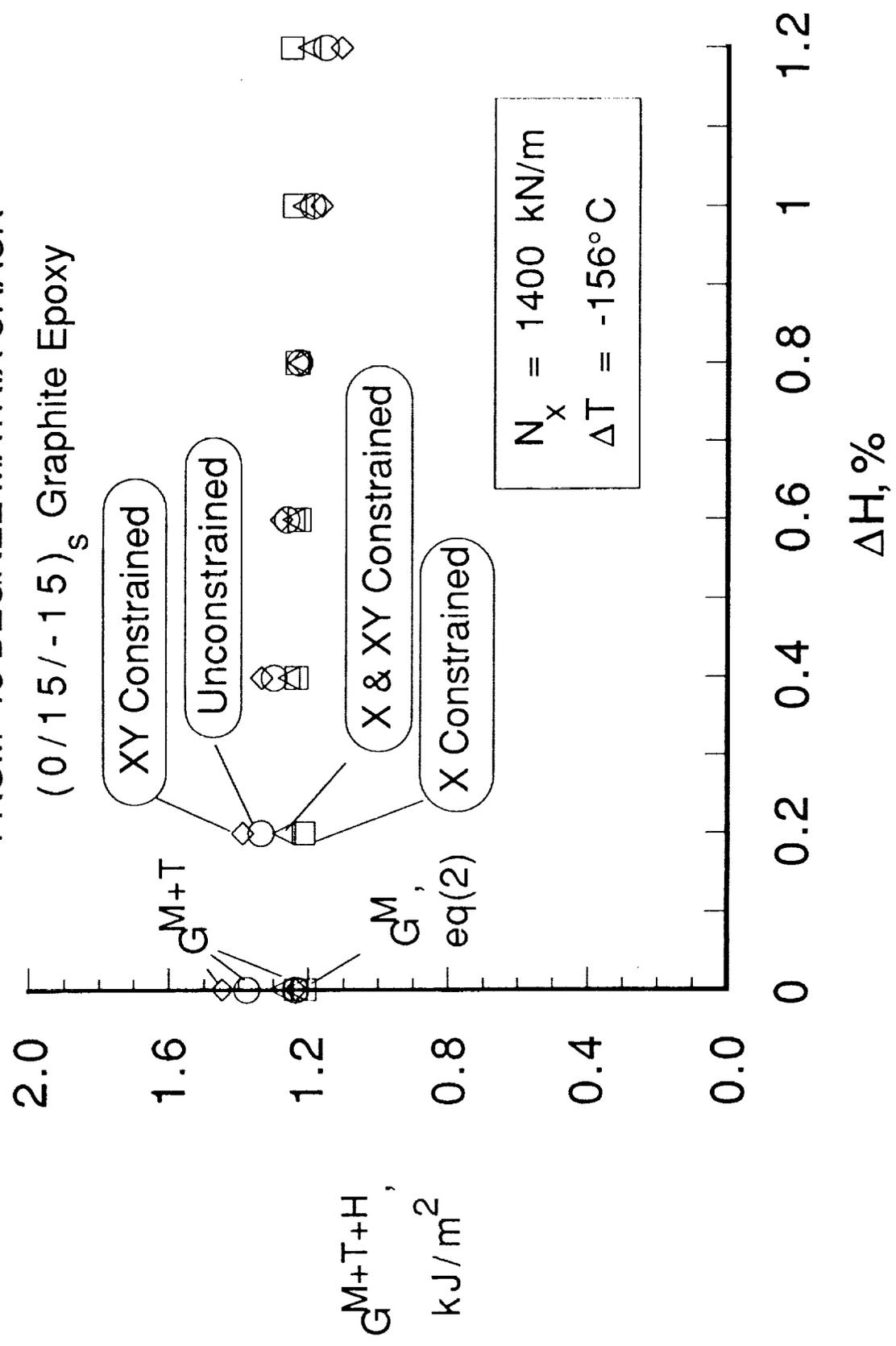
Fig.5 G^{M+T+H} FOR LOCAL DELAMINATION FROM -45 DEGREE MATRIX CRACK

$(0/45/-45)_s$ Graphite Epoxy

N_x = 1400 kN/m
ΔT = -156°C

XY Constrained

Unconstrained

X Constrained

X & XY Constrained

G^{M+T}

G^M, eq(2)

G^{M+T+H}, kJ/m²

ΔH,%

NASA

NASA
National Aeronautics and
Space Administration
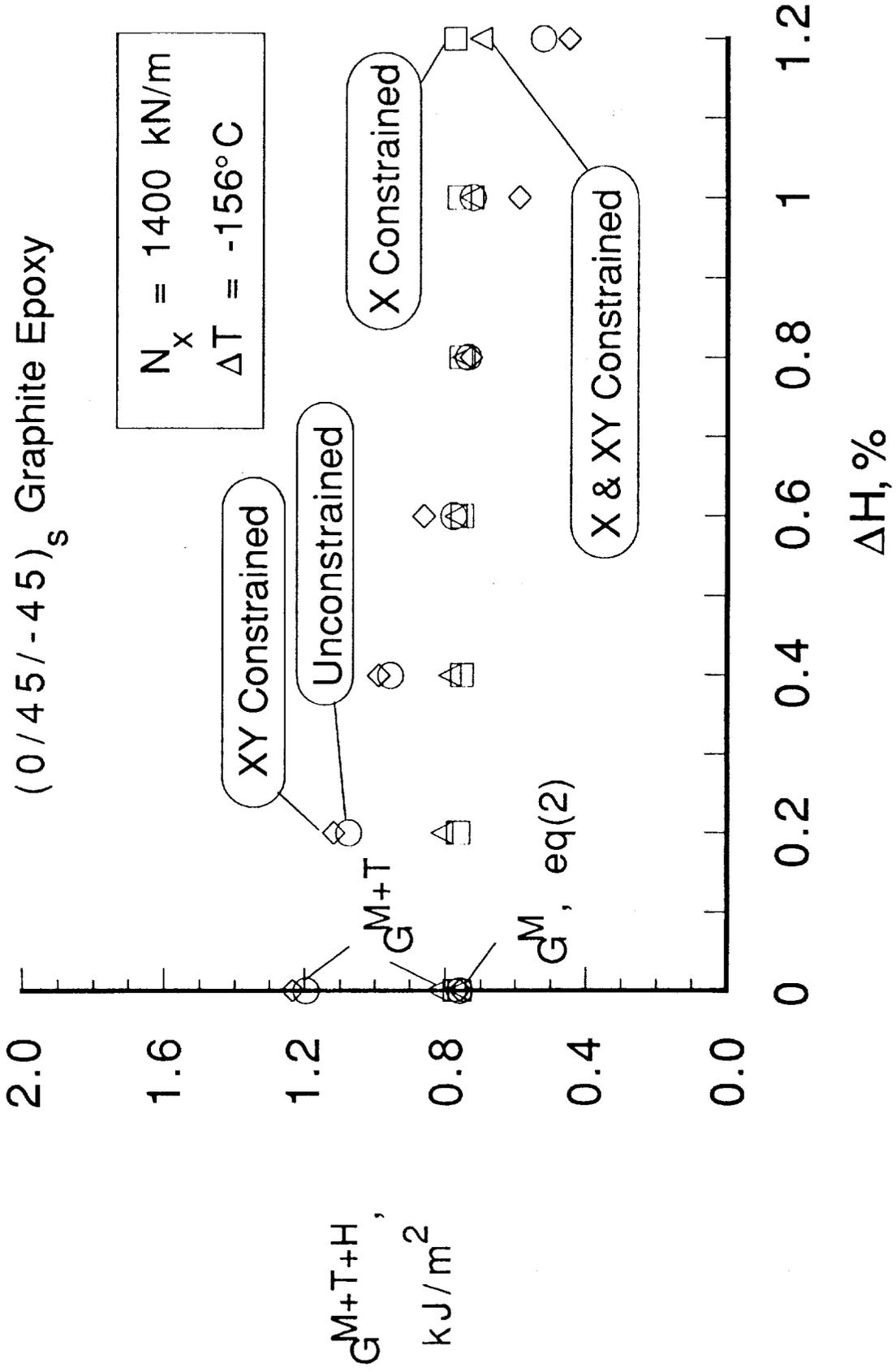
Report Documentation Page

1. Report No. NASA TM-104077 AVSCOM TR-91-B-012	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Residual Thermal and Moisture Influences on the Strain Energy Release Rate Analysis of Local Delaminations from Matrix Cracks		5. Report Date July 1991
		6. Performing Organization Code
7. Author(s) T. K. O'Brien		8. Performing Organization Report No.
		10. Work Unit No. 505-63-50-04
9. Performing Organization Name and Address NASA Langley Research Center, Hampton, VA 23665-5225 U.S. Army Aviation Research and Technology Activity (AVSCOM) Aerostructures Directorate Hampton, VA 23665-5225		11. Contract or Grant No.
		13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 U.S. Army Aviation Systems Command St. Louis, MO 63166		Technical Memorandum
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

An analysis utilizing laminated plate theory is developed to calculate the strain energy release rate associated with local delaminations originating at off-axis, angle ply, matrix cracks in laminates subjected to uniaxial loads. The analysis includes the contribution of residual thermal and moisture stresses to the strain energy released. Examples are calculated for the strain energy release rate associated with local delaminations originating at 90 degrees and angle ply (non-90 degrees) matrix ply cracks in glass epoxy and graphite epoxy laminates. The solution developed may be used to assess the relative contribution of mechanical, residual thermal, and moisture stresses on the strain energy release rate for local delamination for a variety of layups and materials.

17. Key Words (Suggested by Author(s)) Residual thermal stress Matrix crack Hygroscopic stress Moisture Strain energy release rate Delamination	18. Distribution Statement Unclassified - Unlimited Subject Category - 24

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 28	22. Price A03