NASA Contractor Report 187607

ICASE Report No. 91-61

ICASE

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ABSTRACT

The two-equation K-ε model is used to analyze turbulent separated flow past a backward-facing step. It is shown that if the model constants are modified to be consistent with the accepted energy decay rate for isotropic turbulence, the dominant features of the flow field — namely — the size of the separation bubble and the streamwise component of the mean velocity, can be accurately predicted. In addition, except in the vicinity of the step, very good predictions for the turbulent shear stress, the wall pressure and the wall shear stress are obtained. The model is also shown to provide good predictions for the turbulence intensity in the region downstream of the reattachment point. Estimated long-time growth rates for the turbulent kinetic energy and dissipation rate of homogeneous shear flow are utilized to develop an optimal set of constants for the two equation K-ε model. The physical implications of the model performance are also discussed.

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1. INTRODUCTION

The accurate prediction of turbulent separated flows is crucial to the analysis of many physical systems. One of the most commonly used approaches is to Reynolds average the equations of motion and apply a one-point Reynolds stress closure (Launder & Spalding, 1974; Speziale, 1991). Among the various methods used for one-point closure, the two-equation turbulence models are probably the most popular since they offer an acceptable compromise between the more accurate but computationally expensive full Reynolds stress closure models and the less rigorous one-equation or algebraic models.

In its standard form the two-equation K-ε model consists of a representation for the eddy viscosity in terms of the turbulent kinetic energy and the turbulent dissipation rate which are themselves represented in terms of modeled transport equations (Launder & Spalding, 1974). The K-ε model involves the introduction of five constants which include the proportionality constant in the eddy viscosity, followed by the two constants in the equation for the turbulent dissipation, and two additional constants which represent the ratios of the eddy viscosity to the diffusivities of the turbulent kinetic energy and the turbulent dissipation. Typically these constants are calibrated based on the available experimental findings for some simplified flow configurations (Launder & Spalding, 1974; Rodi, 1980).

A widely used benchmark test for studying the performance of two-equation turbulence models involves the physical configuration of an abrupt expansion in a channel — the backward-facing step (cf., figure 1). The flow separates at the corner and is characterized by the presence of a large recirculation region which is straddled by a shear layer. The separated flow reattaches at a downstream location x_r and is followed by a flow recovery region. For fully-developed turbulent flow, an attached boundary layer exists in the region adjacent to the upper wall. It is the presence of such diverse features that had prompted many researchers in the past to use this flow configuration as a benchmark test case for analyzing the predictive capability of turbulence models. In particular, a variety of two-equation turbulence models have been tested and compared with the experimental data of Kim, Kline & Johnston (1980) and Eaton & Johnston (1981) and a description of these and other related results may be found in Kline, Cantwell & Johnston (1981).

It is also generally believed that the standard K-ε model (Launder & Spalding, 1974), with wall functions, underpredicts the reattachment point by a substantial amount of the order of 20-25%. To overcome this deficiency several alternative forms of the K-ε model have been developed over the years. Among these, Sindir (1984) made modifications to account for streamline curvature based on the algebraic stress model of Gibson (1985) and obtained a modest amount of improvement. Hanjalic, Launder, & Schiestel (1981) proposed a multiple scale K-ε model wherein the turbulent kinetic energy K and the turbulent dissipation rate ε were decomposed into low and high wavenumber parts and such a model was used by Chen (1986) to obtain significantly improved results for the separated flow past a backward-facing step. The multiple scale representation is also the crux of the models based on the renormalization group theory (Yakhot & Orszag, 1986). On the other hand Speziale & Ngo (1989) reported comparable improvements for the backward-facing step problem based on an anisotropic K-ε model implying that the main source of the errors could be due to the use of an isotropic eddy-viscosity in the standard K-ε model. However, Avva, Kline & Ferziger (1988) have suggested that the large underprediction of the reattachment point attributed to the standard K-ε model was mainly due to the under-resolution of the computational domain.

The present work is primarily aimed at the development of modifications for the standard K-ε model to improve its predictive capability. It is shown that if the standard K-ε model is modified to properly represent the decay rate of turbulent kinetic energy in isotropic turbulence its predictive capability is considerably enhanced. Computations are performed based on a finite-volume method and it is demonstrated that the model can accurately predict the dominant features of the flow field. These include the size of the separation bubble, the velocity profiles, the wall pressure and the wall shear stress — quantities which are of considerable use from the engineering point of view. In this context, the criteria for an optimal choice of model constants based on the growth rates of turbulent kinetic energy and dissipation is developed. The physical implications of these findings are also discussed.

2. FORMULATION OF THE PHYSICAL PROBLEM

The problem to be considered is the fully-developed turbulent flow of an incompressible viscous fluid past a backward-facing step. A schematic of the flow field is shown in figure 1. The incoming flow separates in the vicinity of the step corner and reattaches at a distance x_r. The inlet channel has a length L_i and a height h_1 while the channel downstream of the step has a length L_c and a height h_2. The governing equations consist of the Reynolds averaged Navier-Stokes and the continuity equations which may be expressed as:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0 \tag{1}$$

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} = -\frac{\partial \overline{p}}{\partial x} + \nu\left(\frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{u}}{\partial y^2}\right) - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} \tag{2}$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} = -\frac{\partial \overline{p}}{\partial y} + \nu\left(\frac{\partial^2 \overline{v}}{\partial x^2} + \frac{\partial^2 \overline{v}}{\partial y^2}\right) - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} \tag{3}$$

where, $\overline{u}$ and $\overline{v}$ are the mean velocity components in the x and y directions; $\overline{p}$ is the modified mean pressure; τ_{xx}, τ_{xy} and τ_{yy} are the components of the Reynolds stress tensor $\tau_{ij} \equiv \overline{u'_i u'_j}$; and ν is the kinematic viscosity. In the standard $K-\varepsilon$ model with isotropic eddy-viscosity the Reynolds stress tensor takes the form (see, Launder and Spalding, 1974; Rodi, 1980)

$$\tau_{ij} = \frac{2}{3}K\delta_{ij} - 2C_\mu\frac{K^2}{\varepsilon}\overline{S}_{ij} \tag{4}$$

where

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{5}$$

is the mean rate of strain tensor, $K \equiv \frac{1}{2}\tau_{ii}$ is the turbulent kinetic energy, ε is the turbulent dissipation rate, and $\overline{u}_i = (\overline{u}, \overline{v})$ is the mean velocity vector. The governing equations for K and ε may be modeled by the following transport equations (Launder & Spalding, 1974).

$$\frac{\partial K}{\partial t} + \bar{u}\frac{\partial K}{\partial x} + \bar{v}\frac{\partial K}{\partial y} = \mathcal{P} - \varepsilon + \frac{\partial}{\partial x}\left[\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_K}\right)\frac{\partial K}{\partial y}\right] \quad (6)$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}\frac{\partial \varepsilon}{\partial x} + \bar{v}\frac{\partial \varepsilon}{\partial y} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + \frac{\partial}{\partial x}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] \quad (7)$$

where,

$$\nu_T \equiv C_\mu \frac{K^2}{\varepsilon} \quad (8)$$

is the eddy viscosity,

$$\mathcal{P} = -\tau_{xx}\frac{\partial \bar{u}}{\partial x} - \tau_{xy}\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right) - \tau_{yy}\frac{\partial \bar{v}}{\partial y} \quad (9)$$

is the turbulence production, and C_μ, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$, σ_K and σ_ε are dimensionless constants. At this point a brief discussion of the method by which these constants are obtained is in order. The quantities σ_K and σ_ε are the ratios of the eddy viscosity to the diffusivity of the turbulent kinetic energy and turbulent dissipation, respectively. Both quantities are of order 1 and for the standard model the values of σ_K and σ_ε are estimated to be about 1 and 1.3, respectively, based on the results from two-dimensional shear flows. To evaluate $C_{\varepsilon 2}$ a simple model based on homogeneous grid generated turbulence (where production and diffusion are absent) is used. Based on the available experimental evidence and the results from direct simulations a value between 1.8 and 2.0 is recommended (with 1.92 being the recommended value for the standard $K-\varepsilon$ model). The quantity C_μ, is typically evaluated based on two-dimensional shear layers that are in local equilibrium. For this case, production and dissipation balance each other and from (6), (8) & (9) it can be shown that $\sqrt{C_\mu} = -\overline{uv}/K$. The measurements of Bradshaw, Ferriss & Atwell (1975) have shown that $\overline{uv}/K \approx -0.3$ for such a case and hence a value of $C_\mu \approx 0.09$ is typically used. Next, $C_{\varepsilon 1}$ is evaluated from (7) by assuming that in the near-wall regions production approximately equals dissipation and the convection of dissipation is negligible. Under these conditions with the added assumption of the logarithmic velocity profile it can be shown that

$$C_{\varepsilon 1} \approx C_{\varepsilon 2} - \frac{\kappa^2}{\sigma_\varepsilon \sqrt{C_\mu}} \qquad (10)$$

Herein the von Kármán constant, $\kappa = 0.41$, and the above expression is used to estimate $C_{\varepsilon 1} \approx 1.44$.

The constants for the standard K–ε model will now be reconsidered in an effort to improve its predictive capability. In this regard it should be noted that the first term in the transport equation for turbulent dissipation (7) is the production of dissipation while the second term represents the destruction of dissipation. The constant $C_{\varepsilon 2}$ which appears in the destruction term also plays a crucial role in the decay of turbulence. For the case of isotropic turbulence the decay of turbulent kinetic energy can be shown to be (Reynolds, 1987)

$$K(t) = K_o \left[1 + (C_{\varepsilon 2} - 1) \frac{\varepsilon_o}{K_o} t \right]^{-1/(C_{\varepsilon 2} - 1)} \qquad (11)$$

which is consistent with the experimentally observed decay rate of $K \sim t^{-1.2}$ (Comte-Bellot & Corrsin, 1971) when $C_{\varepsilon 2} = 11/6$. The standard K–ε model on the other hand, uses $C_{\varepsilon 2} = 1.92$ which corresponds to a power-law decay with $K \sim t^{-1.1}$. In addition, more recent studies on the modeling of pressure-strain correlations for the full Reynolds stress closure recommend the asymptotically correct decay rate of $K \sim t^{-1.2}$ (which again corresponds to $C_{\varepsilon 2} = 11/6$) for homogeneous shear flows (Speziale, Sarkar & Gatski, 1991). Thus, $C_{\varepsilon 2}$ is set to 11/6 in the modified K–ε model while all other constants are left unchanged.

For the above model, the Reynolds averaged equations (1)-(3), (6) and (7) are to be solved subject to the following boundary conditions:

(a) inlet profiles for $\bar{u}$, K and ε are specified five step heights upstream of the step corner. For this purpose, a separate Reynolds-averaged calculation for the developing turbulent channel flow is performed. The results from these computations at an appropriate location near the outlet of the channel (determined by matching with the experimental data of Eaton & Johnston, 1980) are used as the conditions at the inlet.

(b) Conservative extrapolated outflow conditions are applied thirty step heights downstream of the

step corner; these conditions involve the following: i) the $\bar{v}$-component of the velocity for the cells at the outflow boundary are obtained by extrapolation; ii) the $\bar{u}$-component of the velocity is then computed by the application of a mass balance; and iii) the scalar quantities such as pressure, turbulent kinetic energy and turbulent dissipation are all obtained by extrapolation. It was found that a downstream channel length of about thirty stepheights was needed to ensure that the local error for all the quantities was of the same order as the interior values.

(c) At the upper and the lower walls and along the step the law of the wall is applied in the standard two-layer form, wherein

$$\bar{u}^+ = \frac{1}{\kappa} \, ln \, y^+ + 5, \qquad \varepsilon = C_\mu^{3/4} \frac{K^{3/2}}{y}$$

and the normal derivative of K is taken to vanish at the wall. These conditions are applied at the first grid point y away from the wall if $y^+ \equiv y u_\tau / v \geq 11.6$ given that $\bar{u}^+ = \bar{u}/u_\tau$ (u_τ is the shear velocity and $\kappa = 0.41$ is the von Kármán constant); and if $y^+ < 11.6$, then $\bar{u}$, K and ε are interpolated to their wall values based on viscous sublayer constraints. It should be noted that the law of the wall does not formally apply to separated turbulent boundary layers. However, since the separation point is fixed at the corner of the backstep and since the flowfield is solved iteratively with the shear velocity u_τ updated until convergence, major errors do not appear to result from its use (Avva et al., 1988; Speziale & Ngo, 1989).

The governing equations (1)-(3), (6) and (7) with the boundary conditions are discretized based on a finite volume method and applied for the flow past a backward-facing step. The resulting system of algebraic equations are solved iteratively by a line relaxation method with the repeated application of the tridiagonal matrix solution algorithm (see, Lilley & Rhode, 1980). Computations are performed for the configuration wherein the expansion ratio, E is 1:3 (i.e., step height to outlet channel height, $h:h_2$) and the Reynolds number $Re = 132,000$ (based on the inlet centerline mean velocity and outlet channel height). This configuration was selected based on the fact that it has been used by a number of previous researchers to calibrate their turbulence models (Kline et al., 1981) and because of the available experimental data (Kim et al., 1980; Eaton & Johnston, 1981).

The issue of adequate numerical resolution for this particular configuration has been considered in the past by several researchers (Avva et al., 1988; Thangam & Hur, 1991). Based on their

findings a finite-volume scheme with a 200×100 nonuniform mesh (which is known to yield results that are within 0.3% of the grid independent solution) was used in the present study. The particular type of nonuniform computational mesh used for this study is shown in figure 2 (wherein, for clarity only the alternate grid boundaries are shown over a shortened region); and for this mesh, the magnitude of y^+ of the cells adjacent to the solid boundaries has a maximum value of about 6. The computed solution was assumed to have converged to its steady state when the magnitude of the relative average difference between successive iterations for all the velocity components, pressure, the mass residual, and the location of the reattachment point was less than 10^{-4}. Approximately 2000 iterations were needed for the convergence of the standard K-ε model which corresponds to about 18 minutes of CPU time in a partially vectorized mode on the CRAY-YMP supercomputer using 64-bit precision. In the following section the computational results are presented and discussed.

3. DISCUSSION OF THE RESULTS

The system of governing equations outlined in §2 are solved using the modified K-ε model for the case of flow past a backward-facing step of 1:3 expansion ratio (step height to downstream channel height ratio) at a flow Reynolds number of 132,000 (based on the inlet centerline velocity and the outlet channel height). The results of the computations are presented in figures 3-5 and compared with the experimental data of Kim *et al.* (1980) and Eaton & Johnston (1981).

In figure 3 (a), the contours of meanflow streamlines are shown for the modified K-ε model. As can be seen, the computed streamlines show a reattachment at $x_r/h \approx 7.0$, which is in excellent agreement with the experimental mean reattachment point (~ 7.0). This is in contrast to most of the earlier results based on the standard K-ε model which underpredict the reattachment point by as much as 20% (Kline *et al.*, 1981). In fact, even the substantially more elaborate nonlinear models have difficulty in predicting the reattachment point accurately (Speziale, 1991). In the present study, calculations were performed using the same computational mesh and flow conditions for the standard K-ε model (wherein, $C_{\varepsilon 2} = 1.92$) to yield a reattachment point $x_r/h \approx 6.0$. Thus, a change of about 5% in the value of the $C_{\varepsilon 2}$ (from 1.92 to 11/6) causes a 15% reduction in the reattachment length.

We now consider the profiles of the streamwise component of the mean velocity. In figure 3 (b), the predicted values of the streamwise mean velocity profiles are compared with the experimental data. As can be seen, there is very good agreement between the computations and the experimental findings over the entire flow field.

The profiles of the dimensionless turbulence shear stress are shown next in figure 4 (a) at selected locations in the streamwise direction. As can be seen the turbulence shear stresses are also well predicted and in good agreement with the experimental results. In figure 4 (b) the turbulence intensity profiles are shown and compared with the experimental results. While the model significantly underpredicts the normal stresses in the vicinity of the reattachment point, the overall agreement can still be considered good.

In figures 5 (a)-(b) the variation of the experimental and computed wall pressure coefficient,

C_p (defined as $C_p = 2(p - p_r)/\rho U_r^2$ where, p_r and U_r are the pressure and velocity, on the centerline at the inlet) are shown along the top and the bottom walls downstream of the step for the modified K-ε model. As can be seen, there is very good agreement between the experimental and computational findings.

Another parameter of importance is the wall shear stress which could be expressed in its dimensionless form as $c_f = 2\tau_w/\rho U_r^2$. Since, the experimental data for the particular configuration used is not available in the literature, the scaled data of Driver & Seegmiller (1985) which corresponds to an expansion ratio of 1:9 is used. It is known that except in the region very close to the step wall the variation of the dimensionless wall shear stress ratio, c_f / c_{fout} (wherein, c_{fout} corresponds to the fully-developed value) with the normalized distance, $(x-x_r)/x_r$ is essentially independent of the expansion ratio (Adams & Eaton, 1988). Based on this premise, the experimental results (with an expansion ratio of 1:9 and a reattachment point, $x_r/h \approx 6.26$) are scaled and compared with computational results (with an expansion ratio of 1:3 and a reattachment point, $x_r/h \approx 7.0$) and shown in figure 5 (c). As can be seen the computational results are in good agreement with the experimental findings.

We now consider some of the physical implications of the results presented so far. It should be noted at the outset that the two-equation K-ε model used here is based on the assumption that the eddy viscosity is isotropic (i.e., same for all components of the Reynolds stress). Thus the results obtained clearly show that for even for such diversified flows as the separated flow past the backward -facing step which is characterized by a large recirculating region (wherein the normal-stresses and shear-stresses are of the same order, and where both are considerably smaller than the inertial quantities) the isotropic turbulence model is adequate. In fact, most of the physical quantities of interest could be accurately computed based on the modified K-ε model, although the anisotropic K-ε models or the full Reynolds stress closure model should be preferred for analyzing the recirculating region, particularly in the vicinity of the step (Speziale, 1991).

Since the flow past the backward-facing step is characterized by a shear layer that straddles a large recirculation zone, the following interpretation for the size of the separation bubble is con-

sidered next. In a homogeneous shear flow, equations (6) and (7) can be combined to yield a long-time behavior for the turbulence kinetic energy K and dissipation ε of the form (Speziale & Mac Giolla Mhuiris, 1989)

$$K \sim exp\,(\lambda t^*)$$

$$\varepsilon \sim exp\,(\lambda t^*)$$

where t^* is the time (nondimensionalized by the shear rate) and λ is the growth rate given by

$$\lambda = \left[\frac{C_\mu (C_{\varepsilon 2} - C_{\varepsilon 1})^2}{(C_{\varepsilon 1} - 1)\,(C_{\varepsilon 2} - 1)}\right]^{1/2} \tag{12}$$

Therefore, the eddy viscosity also has the form

$$\nu_T \sim exp\,(\lambda t^*)$$

Hence the growth rate of the eddy viscosity is intimately tied to the three model constants, C_μ, $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ for shear flows. Purely for pedagogical reasons as well as for consistency (such as equation 10) the value of $C_{\varepsilon 1}$ is maintained between 1.40 and 1.45, although there are several two equation models wherein $C_{\varepsilon 1}$ can have a significantly different value (Yakhot & Orszag, 1986). For a specified value of $C_{\varepsilon 1}$ an increase in C_μ or $C_{\varepsilon 2}$ would cause an increase in the growth rate λ, and therefore in ν_T. An increase in ν_T would make the flow field more dissipative and would be expected to reduce the size of the separation bubble.

In this regard, it should be noted that for the standard K-ε model (with $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$), the growth rate $\lambda \approx 0.225$ and the reattachment point $x_r/h \approx 6.0$; whereas for the modified K-ε model (with $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 11/6$), the corresponding growth rate $\lambda \approx 0.195$ with the reattachment point $x_r/h \approx 7.0$. Calculations for other values of $C_{\varepsilon 2}$ indicate the same trend.; for example, when $C_{\varepsilon 2} = 1.70$ (corresponding to the growth rate $\lambda \approx 0.140$), $x_r/h \approx 9.65$. Thus, the separation bubble size is nearly in inverse proportion to the growth rate. In addition, several additional computations were performed for different values of C_μ, $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ such that for each set the growth rate $\lambda \approx 0.2$. For all these cases, the separation bubble size $x_r/h \approx 7.0$ and the mean velocity profiles were observed to be in agreement with the experimental values. How-

ever, the profiles of the turbulence stresses as well as intensity tend to be significantly affected when the model constants deviate too far (i.e., > 15%) from that of the standard K-ε model. In this context it should be pointed out that the magnitude of σ_K and σ_ε themselves do not significantly (i.e., of the order of 1% in terms of the mean flow quantities) affect the above findings so long as they are not appreciably different from unity. In the present study, this premise was verified by performing computations with several values of σ_K and σ_ε ranging from 0.7 to 1.4.

These findings however, should not be taken to imply as an unequivocal endorsement for the standard K-ε model. The two equation models are based on the assumption that the local characteristics of turbulence can be represented by a single velocity scale and that the individual components of the Reynolds stress are related to this scale through an eddy-viscosity relationship. Such an approach leads to a model that cannot account for the multiple scales present in more complex flows. Furthermore, there are certain flow situations where a particular aspect of the motion may be solely due to the anisotropy of the Reynolds stress components (for example, the turbulent secondary motion in straight, noncircular ducts), and for such flows it is important to model accurately the anisotropy of the Reynolds stress (Speziale, 1991). In addition, the model is purely dissipative and is unable to account for the relaxational effects of the Reynolds stress. Partly to overcome such shortcomings models based on anisotropic generalization of the eddy viscosity have been developed (for an overview, see, Speziale; 1991) and successfully applied for the prediction of turbulent separated flows (Speziale & Ngo, 1989). In general such models tend to be complex and in some instances have been known to be numerically dispersive to the extent of requiring special treatment (Speziale & Ngo, 1989).

In this regard, it should be noted that very close to the step wall another, much weaker, secondary recirculating layer is present (see, figure 3). The simple isotropic eddy-viscosity model outlined here cannot be expected to predict accurately the details, shape, or the size of such features of the flow field. Consequently, the pressure coefficients, the wall shear stress and the shape of the streamlines would be affected in the vicinity of the step. However, what this study has shown is that in spite of these shortcomings, the simple two-equation model based on isotropic eddy viscosity with optimally selected model constants can accurately predict the dominant features of recirculating flow past the backward-facing step.

4. CONCLUSIONS

An analysis of two-equation turbulence models of the K-ε type is presented to improve their predictive capability for separated flows. The constants of the K-ε model are optimally selected based on the accepted energy decay rate in isotropic turbulence and then applied to the prediction of turbulent separated flow past the backward-facing step.

The model is shown to accurately predict the dominant features of the flow field. The well-predicted features include the size of the separation bubble and the mean velocity. In addition, except in the vicinity of the step, the wall pressure and the wall shear stress are also well predicted. Furthermore, the model is shown to provide very good predictions for the turbulence shear stress, while its predictive capability for the turbulence intensity is good except in the region near the re-attachment point.

Since the turbulent separated flow past a backward-facing step is characterized by the presence of a large recirculation region that is straddled by a shear layer, an empirically derived constraint for the model constants is proposed based on the observed long-time growth rate of turbulent kinetic energy and dissipation rate in homogeneous shear layers.

In summary, the findings of this study indicate that with minor modifications the standard two-equation K-ε turbulence model based on isotropic eddy viscosity can be a viable option for the prediction of turbulent separated flows past the backward-facing step.

Acknowledgments

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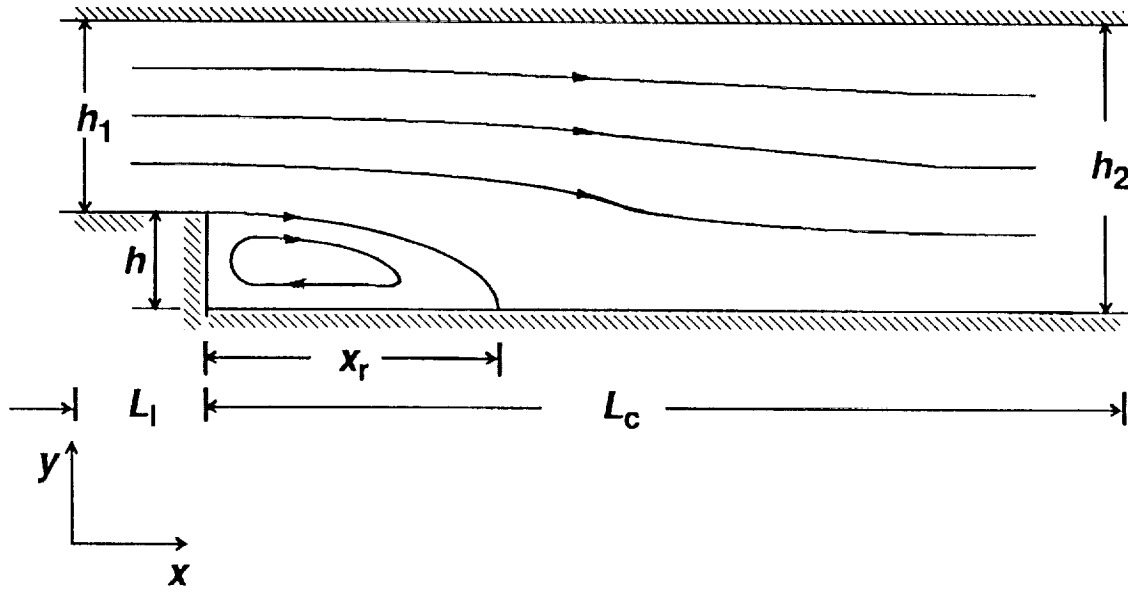
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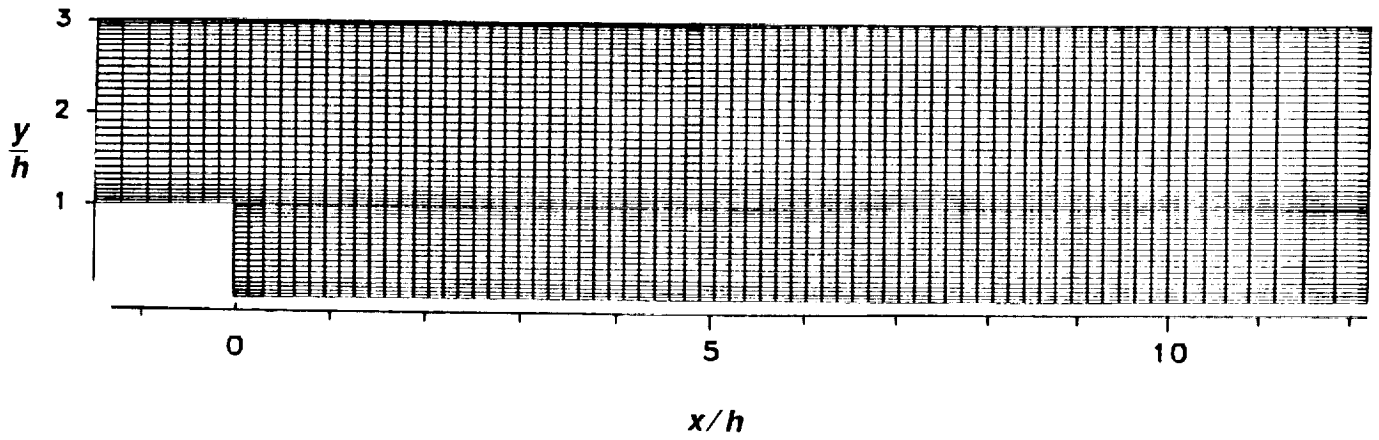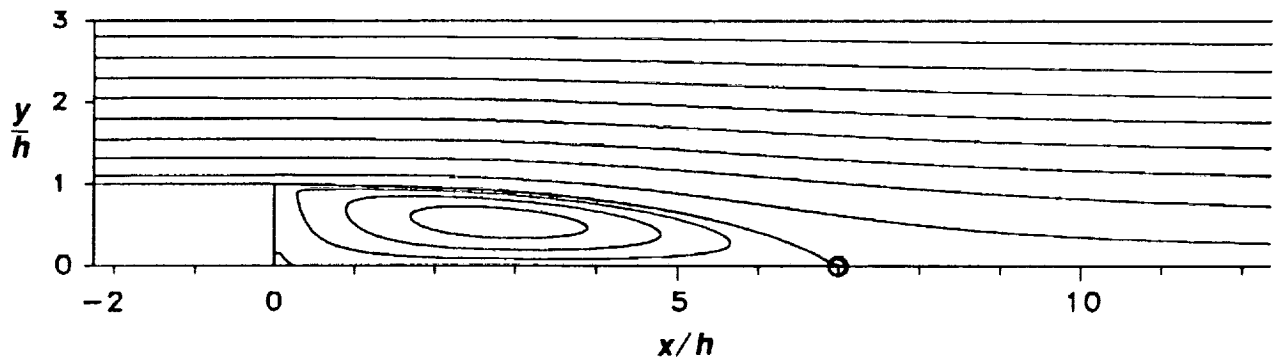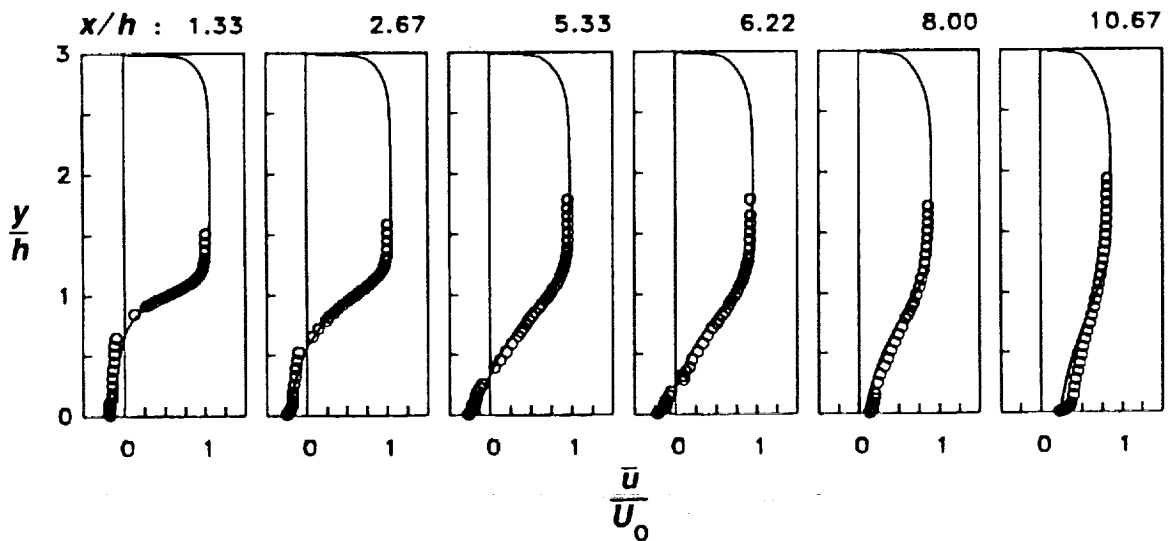Fig. 1 Physical Configuration and Coordinate System

Fig. 2 Computational grid (200x100 mesh; L_l = 5; L_c = 30; E = 1:3)
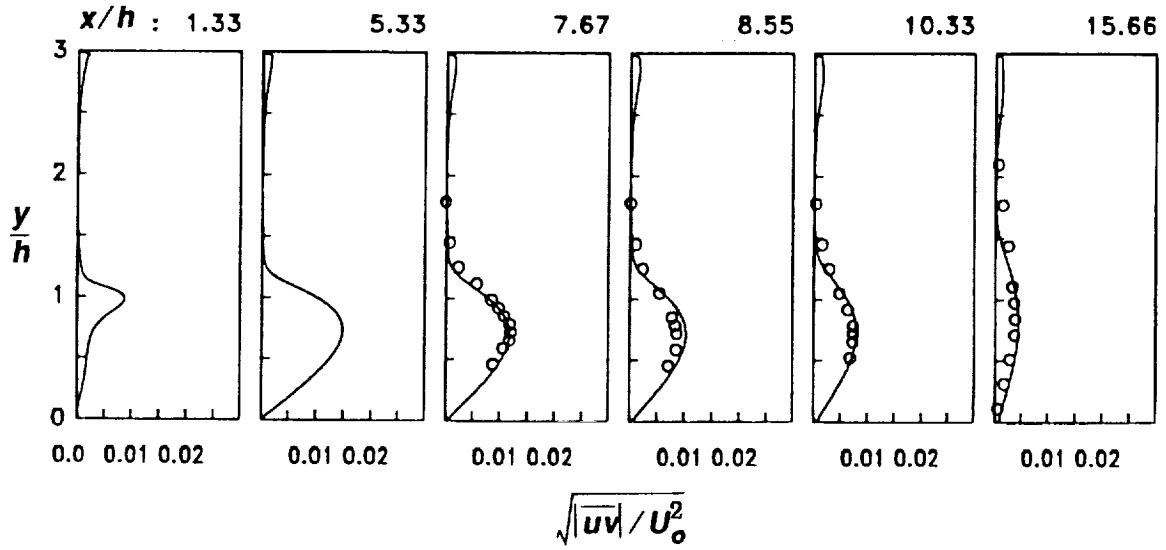
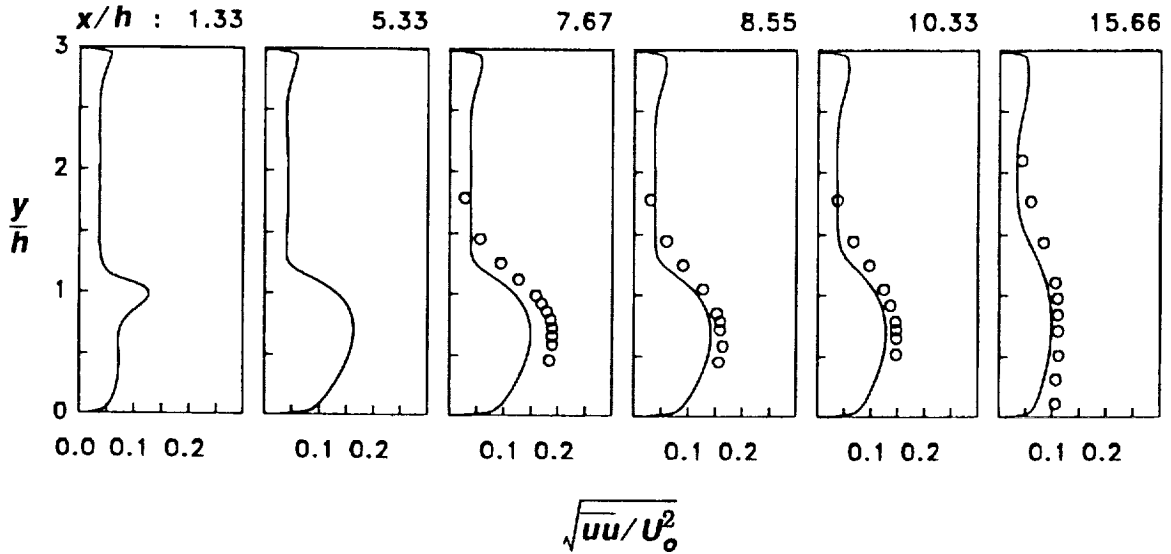**(a) Streamline contours (o experimental mean reattachment point)**

**(b) Streamwise component of the mean velocity profiles at selected locations
(—— computations; o experiments of Kim *et al*, 1980; Eaton & Johnston, 1981)**

Fig. 3 Computed mean flow field for the modified K-ε Model [$E = 1$:3; $Re = 132,000$; $C_{\varepsilon 1} = 1.44$; $C_{\varepsilon 2} = 11/6$; $C_\mu = 0.09$; $\sigma_K = 1.0$; $\sigma_\varepsilon = 1.3$; $K = 0.41$;]
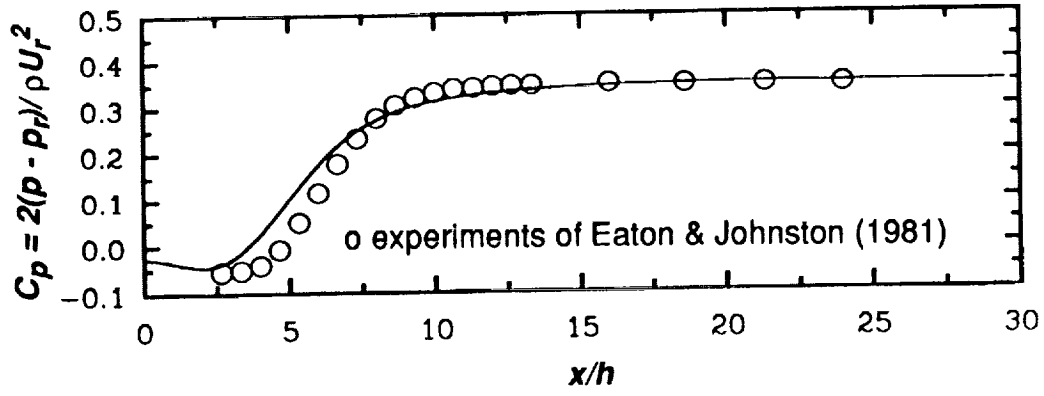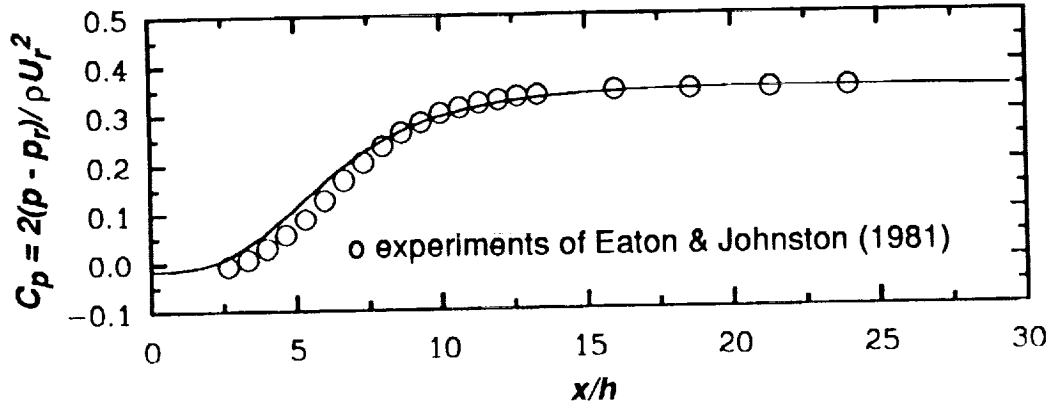
(a) Turbulence shear stress at selected locations

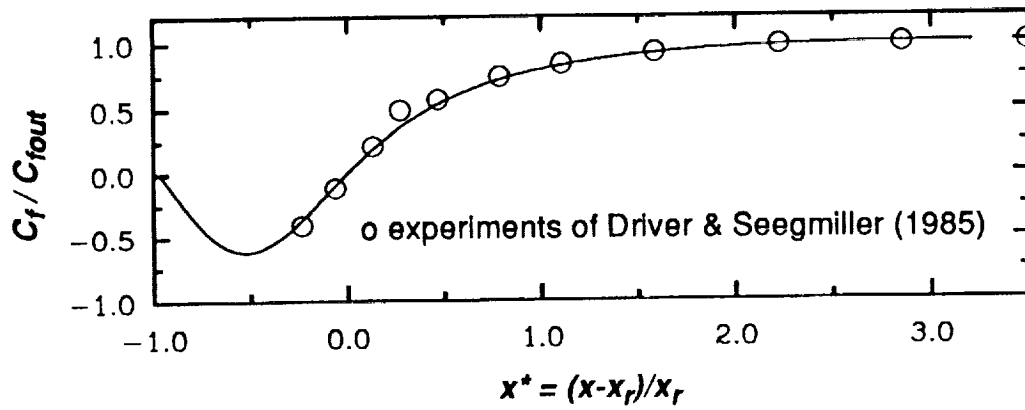(b) Turbulence intensity at selected locations

Fig. 4 Turbulence stress profiles for the modified K-ε Model [E=1:3; Re=132,000; $C_{\varepsilon 1}$ = 1.44; $C_{\varepsilon 2}$ = 11/6; C_μ = 0.09; σ_K = 1.0; σ_ε = 1.3; ——— computations; o experiments of Kim *et al*, 1980; Eaton & Johnston, 1981]

(a) Pressure coefficient along the bottom wall

Plot (a): y-axis $C_p = 2(p - p_r)/\rho U_r^2$ with values 0.5, 0.4, 0.3, 0.2, 0.1, 0.0, -0.1; x-axis x/h from 0 to 30. o experiments of Eaton & Johnston (1981)

(b) Pressure coefficient along the top wall

Plot (b): y-axis $C_p = 2(p - p_r)/\rho U_r^2$ with values 0.5, 0.4, 0.3, 0.2, 0.1, 0.0, -0.1; x-axis x/h from 0 to 30. o experiments of Eaton & Johnston (1981)

(c) Shear stress along the bottom wall

Plot (c): y-axis C_f / C_{fout} with values 1.0, 0.5, 0.0, -0.5, -1.0; x-axis $x^* = (x - x_r)/x_r$ from -1.0 to 3.0. o experiments of Driver & Seegmiller (1985)

Fig. 5 The wall distribution of the pressure coefficients and the shear stress [o experiments; —— computations for the modified K-ε model with $E = 1{:}3$; $Re = 132{,}000$; $C_{\varepsilon 1} = 1.44$; $C_{\varepsilon 2} = 11/6$; $C_\mu = 0.09$; $\sigma_K = 1.0$; $\sigma_\varepsilon = 1.3$; $\kappa = 0.41$]

NASA National Aeronautics and Space Administration	Report Documentation Page	

1. Report No. NASA CR-187607 ICASE Report No. 91-61	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ANALYSIS OF TWO-EQUATION TURBULENCE MODELS FOR RECIRCULATING FLOWS		5. Report Date July 1991
		6. Performing Organization Code
7. Author(s) S. Thangam		8. Performing Organization Report No. 91-61
		10. Work Unit No. 505-90-52-01
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No. NAS1-18605
		13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225		Contractor Report
		14. Sponsoring Agency Code

15. Supplementary Notes
Langley Technical Monitor: Michael F. Card Submitted to Journal of Fluid Engineering Final Report

16. Abstract
The two-equation K-ε model is used to analyze turbulent separated flow past a backward-facing step. It is shown that if the model constants are modified to be consistent with the accepted energy decay rate for isotropic turbulence, the dominant features of the flow field -- namely -- the size of the separation bubble and the streamwise component of the mean velocity, can be accurately predicted. In addition, except in the vicinity of the step, very good predictions for the turbulent shear stress, the wall pressure and the wall shear stress are obtained. The model is also shown to provide good predictions for the turbulence intensity in the region downstream of the reattachment point. Estimated long-time growth rates for the turbulent kinetic energy and dissipation rate of homogeneous shear flow are utilized to develop an optimal set of constants for the two equation K-ε model. The physical implications of the model performance are also discussed.

17. Key Words (Suggested by Author(s)) turbulent separated flow; two equation K-ε model; backward-facing step	18. Distribution Statement 34 - Fluid Mechanics and Heat Transfer Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 22	22. Price A03