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SOLAR NEUTRINOS AND THE MSW EFFECT FOR THREE-NEUTRINO MIXING*

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X. Shi

Department of Physics,
The University of Chicago
5640 S. Ellis Avenue, Chicago, IL 60637

David N. Schramm

Department of Physics,
Department of Astrophysics & Astronomy,
The University of Chicago,
5640 S. Ellis Avenue, Chicago, IL 60637
and
NASA/Fermilab Astrophysics Center,
Fermi National Accelerator Laboratory,
Box 500, Batavia, IL 60510-500

ABSTRACT

We consider three-neutrino MSW mixing, assuming $m_3 \gg m_2 > m_1$ as expected from theoretical consideration if neutrinos have mass, and calculate the corresponding mixing parameter space allowed by both the ^{37}Cl and Kamiokande II experiments. We also calculate the expected depletion for the ^{71}Ga experiment. Finally, we explore a range of theoretical uncertainty due to possible astrophysical effects by varying the ^{8}B neutrino flux and redoing the MSW mixing calculation.

I. Introduction:

The well known solar neutrino problem centers on the fact that the measured time averaged solar neutrino flux is lower than theoretical estimates by a factor of 2∼3 in two experiments, the Homestake ^{37}Cl experiment and the Kamiokande neutrino-electron scattering experiment. In the ^{37}Cl experiment, the observed neutrino flux=2.2±0.3 SNU (Solar Neutrino Unit, 1 SNU=10^{-36}capture events/target nucleus/sec)[1], compared with the predicted 7.9 SNU by the Standard Solar Model (SSM) of Bahcall *et al*[1] or 5.8 SNU of the French solar model.[2] Although many attempts have been made to assign an estimate of the uncertainty[3,4] in the theoretical numbers, the spread between the French and US models is probably as good as any estimate (it should be noted that the two models do approximately agree when the same choice of input parameters are made but differ primarily because of different choice as to the preferred input parameters). In the Kamiokande II experiment, the measured fluxes=$0.45 \pm 0.06_{stat} \pm 0.06_{syst}$ relative to the predicted value of Bahcall *et al.*[5] The Kamiokande and Homestake experiments are mainly sensitive to the higher energy ^{8}B neutrinos with the latter also have some admixture of ^{7}Be neutrinos.[1] Furthermore, two new experiments using ^{71}Ga (GALLEX and SAGE) which can see the main-line *pp* neutrinos have recently begun operating, while preliminary results from SAGE also appear reduced relative to theory.[6] These reports remain to be established following subsequent calibration studies using ^{51}Cr sources to verify that ^{71}Ge produced by neutrino capture on ^{71}Ga can indeed be quantitively detected.

Vacuum neutrino mixing was proposed to solve the problem, but it required large vacuum mixing angles relative to the Cabbibo angle ($\theta_c = 13°$) to provide a maximal flux reduction of $1/N_\nu$, where N_ν is number of neutrino flavors involved in mixing.[1] The Mikheyev-Smirnov-Wolfenstein (MSW) matter mixing effect[7,8] enables neutrinos to flip flavor while propogating in matter even with a small vacuum mixing angle. Thus, it provides a more natural solution to the solar neutrino problem.[9] In the two-flavor mixing case, it has been found that in order to get the observed reduction with the MSW effect, only a triangular area in parameter space $\Delta = (m_2^2 - m_1^2)$ $vs. sin^2 2\theta_{12}/cos 2\theta_{12}$ is allowed.[1] Full three-flavor MSW effects have also been investigated previously by Kuo and Pantaleone[10] and Barger *et al*[11]. In this paper we reexamine the full three-flavor mixing in the light of the present experimental situation. Since we now know from LEP[12] and from cosmology[13] that there are indeed 3 neutrino families, it seems only reasonable that the formalism used should take into account this realty. However, we will see that the basic two-neutrino picture still gives a reasonable understanding of most of the action. We will also assume $m_3 \gg m_2 > m_1$, which, if neutrinos have mass, is a reasonable theoretical expectation. In section II, we derive the probability of solar neutrinos flipping their flavors on the way to the earth. In section III we calculate numerically the allowed parameter space from the data of the ^{37}Cl and Kamiokande II experiments assuming three-flavor matter mixing.

Mixing between three othogonal neutrino states can be generally described by a unitary operator U

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle, \quad \alpha = e, \mu, \tau, \quad i = 1, 2, 3 \tag{1}$$

where ν_α denotes flavor eigenstates and ν_i mass eigenstates.

Before looking at the complication of three-flavor mixing, let us review the standard two-flavor formalism.[14,15,16] In the special case of two neutrino mixing between ν_e and ν_x (ν_x might be a linear combination of ν_μ and ν_τ),

$$U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}. \tag{2}$$

The Schrödinger Eq. of a neutrino travelling through matter in flavor basis is

$$i\frac{d}{dt}\begin{pmatrix} C_e(t) \\ C_x(t) \end{pmatrix} = M \begin{pmatrix} C_e(t) \\ C_x(t) \end{pmatrix}. \tag{3}$$

Let $\Delta = m_2^2 - m_1^2$,

$$M = \frac{1}{4E}\begin{pmatrix} -\Delta cos2\theta + 2A & \Delta sin2\theta \\ \Delta sin2\theta & \Delta cos2\theta \end{pmatrix} = \frac{\Delta_M}{4E}\begin{pmatrix} -cos2\theta_M & sin2\theta_M \\ sin2\theta_M & cos2\theta_M \end{pmatrix} \tag{4}$$

where $A = 2\sqrt{2}G_F n_e E$ is the induced mass of the electron neutrino due to the MSW effect. G_F is the Fermi constant, n_e is electron density of the matter and E the neutrino energy. A term proportional to the identity matrix has been subtracted. In the second expression, Δ_M and θ_M are counterparts in matter of Δ and θ, where

$$\Delta_M = [(\Delta cos2\theta - A)^2 + (\Delta sin2\theta)^2]^{1/2} \tag{5}$$

$$tan2\theta_M = tan2\theta/[1 - (A/\Delta)sec2\theta]. \tag{6}$$

When $A = \Delta cos2\theta$, $\theta_M = \pi/4$, Δ_M reaches its minimum, the two mass eigenstates are almost degenerated and the neutrino encounters a resonance.

An interesting case occurs if neutrinos propagate through matter which has a slow and monotonically varying electron density. In that case the neutrinos can pass through the resonance adiabatically and flip their flavor (Fig.1). But generally, neutrinos evolve adiabatically in regions away from resonance. At resonance, they have some probability to jump between two mass eigenstates and thus don't change their flavor completely. Such jump probability can be obtained by solving the Schrödinger eq.(3) assuming linear variation of electron density at the resonance region. The result is[17]

$$P_{jump} = \exp[\frac{-\pi\Delta sin^2 2\theta}{4Ecos2\theta}(\frac{n_e}{|dn_e/dr|})_{res}]. \tag{7}$$

The probability for a ν_e produced at a high density region, passing through resonance, reaching the vacuum region, to maintain its flavor is[14]

$$P = \frac{1}{2} + (\frac{1}{2} - P_{jump}) cos2\theta_M cos2\theta \tag{8}$$

where θ_M is the mixing angle at which the ν_e is produced. If the density at which ν_e is produced is sufficiently large, $\theta_M \to \pi/2$ (see eq.(6)) and eq.(8) becomes

$$P \approx sin^2\theta + P_{jump} cos2\theta. \tag{8'}$$

To obtain the observed neutrino flux reduction, a large fraction of the ν_e's produced at the center of the Sun have to change their flavor. This requires the following: (a) part of the solar neutrinos have to pass through a resonance; (b) from eq.(8) and (8′), P_{jump} must be sufficiently small; (c) also from eq.(8) and (8′) $\sin^2\theta$ has to be sufficiently small. These conditions yield constraints on neutrino masses and mixing angles and limit them to a triangle in parameter space Δ $vs. sin^2 2\theta/cos2\theta$.[1] Each region of the triangle produces a different spectrum for the resultant neutrino flux to be observed. On the horizontal region, only high energy neutrinos go through resonace and change flavor; thus, the high energy part of neutrino spectrum gets "cut off". This region is effectively ruled out because the Kamiokande II experiment which is only sensitive to high energy neutrinos ($E_{threshold}$ of recoil electron=7.5MeV[5]) detected a larger fraction of its theoretical flux than the ^{37}Cl experiment ($E_{threshold}$=0.814MeV[1]) did. On the diagonal region, the lower energy neutrinos go through resonance with a smaller P_{jump}, they are thereby supressed more than the higher energy ones. This is consistant with experiments. On the vertical part of the triangle the entire spectrum is uniformly suppressed. This is also not contradictary to the observation at the 3σ level.

II. Matter mixing for three-neutrino mixing:

The three-neutrino matter mixing is more complicated. It involves three mixing angles. The unitary operator

$$U = e^{i\psi\lambda_7} e^{-i\phi\lambda_5} e^{i\omega\lambda_2} \tag{9}$$

where $\lambda_2, \lambda_5, \lambda_7$ are the generators of SU(3),

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \tag{10}$$

In our problem, we are only interested in how much the ν_e flux is reduced without caring about which flavor it is converted to. We can rotate the flavor basis by $\exp(-i\psi\lambda_7)$ to get rid of ψ. The new basis is ν_e, $\nu_{\mu'}$, $\nu_{\tau'}$ and

$$U = e^{-i\phi\lambda_5} e^{i\omega\lambda_2} = \begin{pmatrix} cos\phi cos\omega & cos\phi \sin\omega & -sin\phi \\ -sin\omega & cos\omega & 0 \\ sin\phi cos\omega & sin\phi sin\omega & cos\phi \end{pmatrix}. \tag{11}$$

In general there are two resonances. These resonances can be coupled so it is hard to see when they happen and how neutrinos behave while crossing these resonances. However, if $m_3 \gg m_2 > m_1$, the resonance between two lower mass eigenstates can be well decoupled from the higher one. The assumption that $m_3 \gg m_2 > m_1$ is reasonable from see-saw model consideration.[18] This also would be anticipated if ν_τ, the heaviest neutrino, is associated with Hot Dark Matter where $m_{\nu_\tau} \sim 25\mathrm{eV}$.[19] Kuo and Pantaleone[10] found that under such assumption the lower resonance happens when

$$A cos^2\phi = \Delta cos2\omega \tag{12}$$

and the higher resonance occurs when $A = m_3^2 cos2\phi$. We will follow the above assumption to solve the Schrödinger equation for three-neutrino mixing. We also assume that the higher resonance requires a much higher density than that of the Sun and becomes irrelavant. To find P_{jump} for the lower resonance, we further rotate the basis by $\exp(i\phi\lambda_5)$. The new basis becomes $\nu_{e'}$, $\nu_{\tau''}$ (both linear combinations of ν_e and $\nu_{\tau'}$) and $\nu_{\mu'}$. The associated Schrödinger equation is

$$i\frac{d}{dt}\begin{pmatrix} C_{e'}(t) \\ C_{\mu'}(t) \\ C_{\tau''}(t) \end{pmatrix} = M \begin{pmatrix} C_{e'}(t) \\ C_{\mu'}(t) \\ C_{\tau''}(t) \end{pmatrix} \tag{13}$$

with[10]

$$M = \frac{1}{4E}\begin{pmatrix} -\Delta cos2\omega + 2A cos^2\phi & \Delta sin2\omega & -A sin2\phi \\ \Delta sin2\omega & \Delta cos2\omega & 0 \\ -A sin2\phi & 0 & 2m_3^2 - m_2^2 - m_1^2 + 2A sin^2\phi \end{pmatrix}. \tag{14}$$

Under the assumption $m_3^2 \gg m_2^2$ and m_1^2, we can see that $C_{\tau''}$ oscillates so fast that it averages out (if we take the see-saw model, $m_3/m_2 \sim m_\tau^2/m_\mu^2 \sim 10^2$ using leptons (or $\sim m_t^2/m_c^2 \sim 10^4$ using quarks), $C_{\tau''}$ would oscillate $\sim 10^4(10^8)$ times faster than the other two components). Then the transition probability of the lower resonance P_{jump} is given by eq.(7).

If we let

$$\Phi_{1,2\ or\ 3}(t_1,t_2) = \exp\Big[-i\int_{t_1}^{t_2} E_{1,2\ or\ 3}(t)dt\Big], \tag{15}$$

E_1, E_2, E_3 are energy eigenvalues of three mass eigenstates, the time evolution of a ν_e produced at time t_i, passing through lower resonance, detected at time t_f is

$$U_{e1}(t_i)\Phi_1(t_i,t_r)|\nu_1(t_i)\rangle + U_{e2}(t_i)\Phi_2(t_i,t_r)|\nu_2(t_i)\rangle + U_{e3}(t_i)\Phi_3(t_i,t_r)|\nu_3(t_i)\rangle \quad \text{(before resonance)} \tag{16}$$

and

$$\Phi_1(t_r,t_f)[a_1U_{e1}(t_i)\Phi_1(t_i,t_r) - a_2^*U_{e2}(t_i)\Phi_2(t_i,t_r)]|\nu_1(t_f)\rangle$$

$$+\Phi_2(t_r,t_f)[a_1^*U_{e2}(t_i)\Phi_2(t_i,t_r)+a_2U_{e1}(t_i)\Phi_1(t_i,t_r)]|\nu_2(t_f)\rangle$$

$$+\Phi_3(t_r,t_f)a_3U_{e3}(t_i)\Phi_3(t_i,t_r)|\nu_3(t_f)\rangle \qquad \text{(after resonance)} \quad (16')$$

where $|a_2|^2 = P_{jump}cos^2\phi$ (remember the P_{jump} we calculated earlier was in the basis $\nu_{e'}, \nu_{\mu'}$ and $\nu_{\tau''}$, a rotation of basis $\nu_e, \nu_{\mu'}$, $\nu_{\tau'}$ by angle ϕ) , $|a_1|^2 = 1 - P_{jump}cos^2\phi$, and $|a_3|^2$, from the above decoupling approximation, is 1. The amplitude of detecting a ν_e at t_f is

$$A_{e,e} = [a_1U_{e1}(t_i)\Phi_1(t_i,t_f) - a_2^*U_{e2}(t_i)\Phi_1(t_r,t_f)\Phi_2(t_i,t_r)]U_{e1}(t_f)$$

$$+[a_1^*U_{e2}(t_i)\Phi_2(t_i,t_f) + a_2U_{e1}(t_i)\Phi_1(t_i,t_r)\Phi_2(t_r,t_f)]U_{e2}(t_f)$$

$$+a_3U_{e3}(t_i)U_{e3}(t_f)\Phi_3(t_i,t_f). \qquad (17)$$

After averaging over the position of production, i.e. t_i, and the position of detection t_f, the probability P of survival of ν_e is

$$P = |a_1U_{e1}(t_i)U_{e1}(t_f)|^2 + |a_2^*U_{e2}(t_i)U_{e1}(t_f)|^2 + |a_1^*U_{e2}(t_i)U_{e2}(t_f)|^2$$

$$+|a_2U_{e1}(t_i)U_{e2}(t_f)|^2 + |a_3U_{e3}(t_i)U_{e3}(t_f)|^2. \qquad (18)$$

Taking expression of U_{ei} from eq.(11)

$$U_{e1} = cos\phi cos\omega, \quad U_{e2} = cos\phi sin\omega, \quad U_{e3} = -sin\phi \qquad (19)$$

we get

$$P = [\frac{1}{2} + (\frac{1}{2} - P_{jump}cos^2\phi)cos2\omega_M cos2\omega]cos^2\phi_M cos^2\phi + sin^2\phi_M sin^2\phi$$

$$\approx [\frac{1}{2} + (\frac{1}{2} - P_{jump}cos^2\phi)cos2\omega_M cos2\omega]cos^4\phi + sin^4\phi \qquad (20)$$

where ϕ_M and ω_M are $\phi(t_i)$ and $\omega(t_i)$, the mixing angles at the position of production; $\phi(t_f)$ and $\omega(t_f)$ are ϕ and ω if we detect in vacuum. We can see that the ν_e flux will suffer an additional overall supression when $\phi \neq 0$.

From eq.(20) we can calculate the experimental constraints on parameter space Δ *vs.* $sin^22\omega/cos2\omega$, which reduces to Δ vs. $sin^22\theta/cos2\theta$ when $\phi = 0$.

III. Numerical Result:

Figures 2–4 show the allowed parameter space for different ϕ with 1σ and 3σ errors. from the results of both ^{37}Cl and the Kamiokande II experiments (by convention, we replace ω by θ). Fig.2(b) is basically consistent with Chen and Cherry's result.[20] We can see that the horizontal region is still ruled out, in agreement with our previous discussion of the two-flavor mixing case. It is interesting to note that at the 1σ level (Fig.2(a) to

4(a)) there is no overlap for the vertical region either. This is understandable: for the 1σ error, the lower bound of the Kamiokande II is $0.45 - \sqrt{0.06^2 + 0.06^2} = 0.365$, which is higher than the upper bound from ^{37}Cl experiment $2.5/7.9 \approx 0.32$; besides, on the vertical region, the neutrino spectrum is uniformly suppressed. Thus, there can't be any overlap. So at the 1σ level the only possible parameter space is the diagonal region.

From Fig.2(b) to Fig.4(b) we find at the 3σ level that the vertical region is then allowed. However, part of this region will show day-night effect because of the regeneration of electron-neutrino flux in the earth.[21] So, after considering the Kamiokande II's limit on the absence of a day-night effect, region $\sin^2 2\theta/\cos 2\theta > 0.02$ and $\Delta = 2\times10^{-6}$–$10^{-5}\mathrm{eV}^2$ is also excluded at 90% C. L. when $\phi = 0$.[22] At large ϕ, the overall suppression factors in eq.(20) which are not affected by neutrinos travelling through the earth (because of its low density) will smooth out the regeneration. Therefore we expect the excluded region to shrink.

When ϕ=0.1rad (Fig.3), we find no significant change in parameter space. This case would be interesting if the reported 1% mixing of 17keV neutrino in β decay turn out to be true (then the 17keV neutrino would serve as the heaviest neutrino species).[23] At large ϕ , Fig.4 shows that the allowed parameter space is shifted, but we still find the same situation with respect to the vertical and diagonal regions as in ϕ=0 case.

The ongoing ^{71}Ga experiments have a very low threshold 0.233MeV. Thus, they will bring more severe constraints on the parameter space. As mentioned earlier, at present SAGE gives a preliminary low result.[6] This would suggest that the central diagonal region would be the final answer if the MSW effect is the solution to the solar neutrino problem.

In standard Solar Models, the ^{8}B neutrino flux is very sensitive to the temperature at the center of the Sun ($\propto T_c^{18}$) whereas the other neutrino sources are less sensitive.[1,24] Therefore the theoretical expected fluxes calculated for current ^{8}B dominated solar neutrino experiments depend drastically on what T_c is calculated in the Solar Model. This gives rise to greater astrophysical uncertainty for the determination of neutrino parameter space. To try to estimate the sensitivity to the uncertainty we have calculated the allowed parameter spaces for different central temperatures. If we assume the central temperature is only slightly varied, the predicted solar ^{8}B neutrino flux would be changed by some factor while other neutrino fluxes are almost unaffected. Fig.5(a) and (b) show the overlap of two experimental results under different solar ^{8}B fluxes at 1σ error. We see that a change of the solar ^{8}B flux by a full factor of 2 will only shift the overlaped region back and forth without inducing any new allowed parameter space (after the completion of this work, we notice that Smirnov also calculated the allowed parameter space for different T_c, but at 2σ level[25]). We also find that at 1σ error, in order for the neutrino to have any allowed MSW parameter space, the predicted ^{8}B neutrino flux for the ^{37}Cl experiment must be higher than $\sim$2.5 SNU when $\phi = 0$ and $\sim$5 SNU when $\phi = 0.5$rad. For the ^{71}Ga experiments, such changes in the the solar ^{8}B neutrino flux produces neglegible effect because Gallium experiments are dominated by the pp neutrino flux.

Based on Figures 2–5, we can conclude that if ^{71}Ga does give a very low flux, $\Delta = m_2^2 - m_1^2$ will lie in 10^{-7} to 2×10^{-6}eV^2, and $sin^2 2\theta$ will be 0.01–0.3, which is naturally in agreement with Bahcall and Bethe's prediction[26] (using only two neutrino families and a similar analysis of the ^{37}Cl and Kamiokande implications) and consistent with the Cabibbo angle θ_c ($sin^2 2\theta_c = 0.2$). This corresponds to a ν_μ mass of $\sim 10^{-3}$eV, which is more than 10^3 times larger than that of some SUSY see-saw models, and ~ 5 times smaller than the lowest m_{ν_μ} value in the SO(10) see-saw model, both of which are discussed by Bludman *et al.*[27] Thus our m_{ν_μ} is within the admittedly large theoretically predicted range. Generally, if we take the see-saw inspired relationship $m_{\nu_\tau}/m_{\nu_\mu} = m_t^2/m_c^2 \times C(C \sim \mathcal{O}(1)$ but larger than 1)[27], using $m_c = 1.55$GeV, $m_t = 124 \pm 34$GeV,[28] we obtain $m_{\nu_\tau} \sim 10$eV. This would make ν_τ a natural candidate for the Hot Dark Matter (see also discussion in ref. 29). Similarly, we obtain from the see-saw model $m_{\nu_e} \sim 10^{-8} eV$. These numbers are consistent with the current experimental bounds on the neutrino masses and the neutrino oscillation parameters (Fig.6).[30,31]

IV. Conclusion:

After taking into account the three-neutrino matter mixing under the assumption that $m_3 \gg m_2 > m_1$, we find that for various values of the second mixing angle ϕ, the neutrino mixing parameters most probably lie in the diagonal region of parameter space $\Delta vs. sin^2 2\theta/cos 2\theta$. The vertical region is less probable but hasn't been ruled out at 3σ level (except those parts ruled out by the lack of a day-night effect). The horizontal region of the parameter space is excluded by current experiments. The Gallium experiment may give us a final resolution between the diagonal area and vertical area. Changing the high energy ^{8}B neutrino flux will shift the allowed parameter space, but will not change the shape of allowed space significantly. The basic robustness of two-flavor MSW mixing results is not weakened by taking into account the full three neutrino flavors.

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Figure Captions:

Fig.2–4

The doted region is allowed by the ^{37}Cl experiment; the crossed region is allowed by the Kamiokande II. Percentages at the lower left show expected flux for the ^{71}Ga experiment relative to SSM.

2(a) ϕ=0, 1σ error. 2(b) ϕ=0, 3σ error.

3(a) ϕ=0.1, 1σ error. 3(b) ϕ=0.1, 3σ error.

4(a) ϕ=0.5, 1σ error. 4(b) ϕ=0.5, 3σ error.

Fig.5

This figure shows the parameter space allowed by both the ^{37}Cl experiment and Kamiokande II with different predicted solar ^{8}B neutrino fluxes for the ^{37}Cl experiment assumed.

5(a) $\phi = 0$, 1σ error. 5(b) ϕ =0.5, 1σ error.

Fig.6

The current experimental constraints on neutrino oscillation parameters. [1] $\nu_\mu \rightarrow \nu_e$; [2] $\nu_\mu \leftrightarrow \nu_\tau$; [3] $\nu_e \leftrightarrow \nu_\tau$. [4] $\bar{\nu}_e \rightarrow \nu_x$; [1], [2] and [3] are from accelerator experiments, [4] is from reactor experiments. The left side of the lines is excluded at 90% C. L..

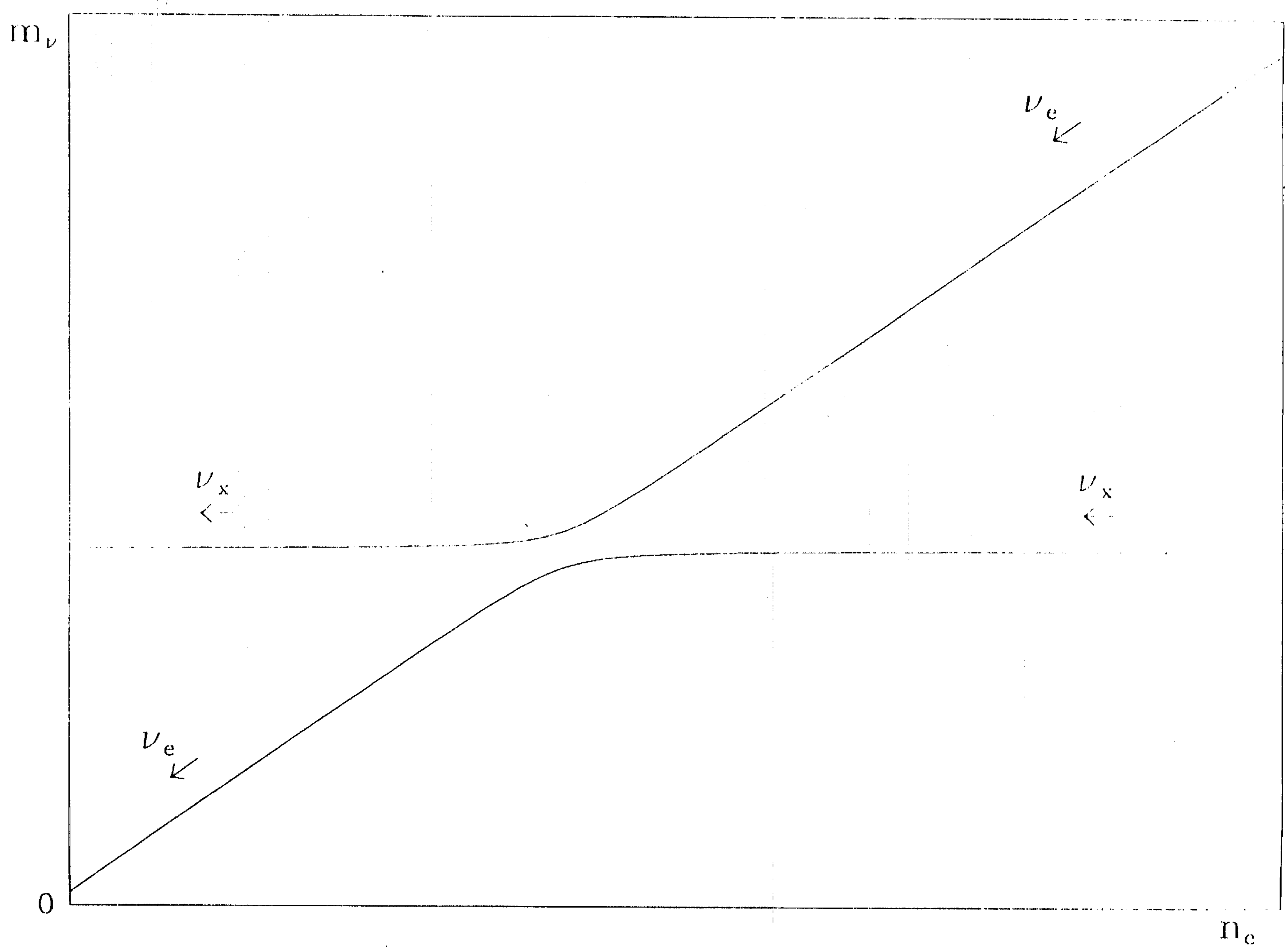
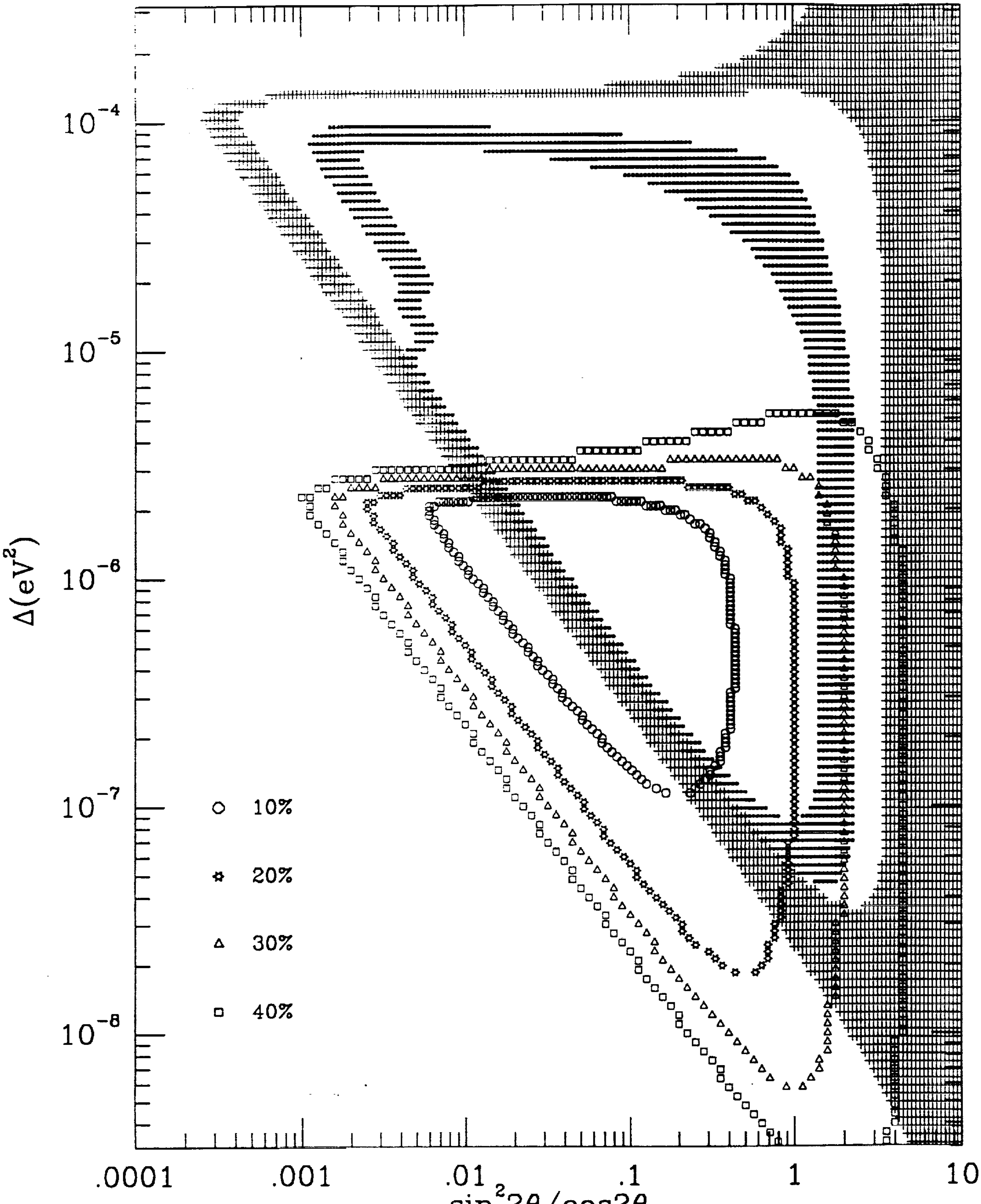
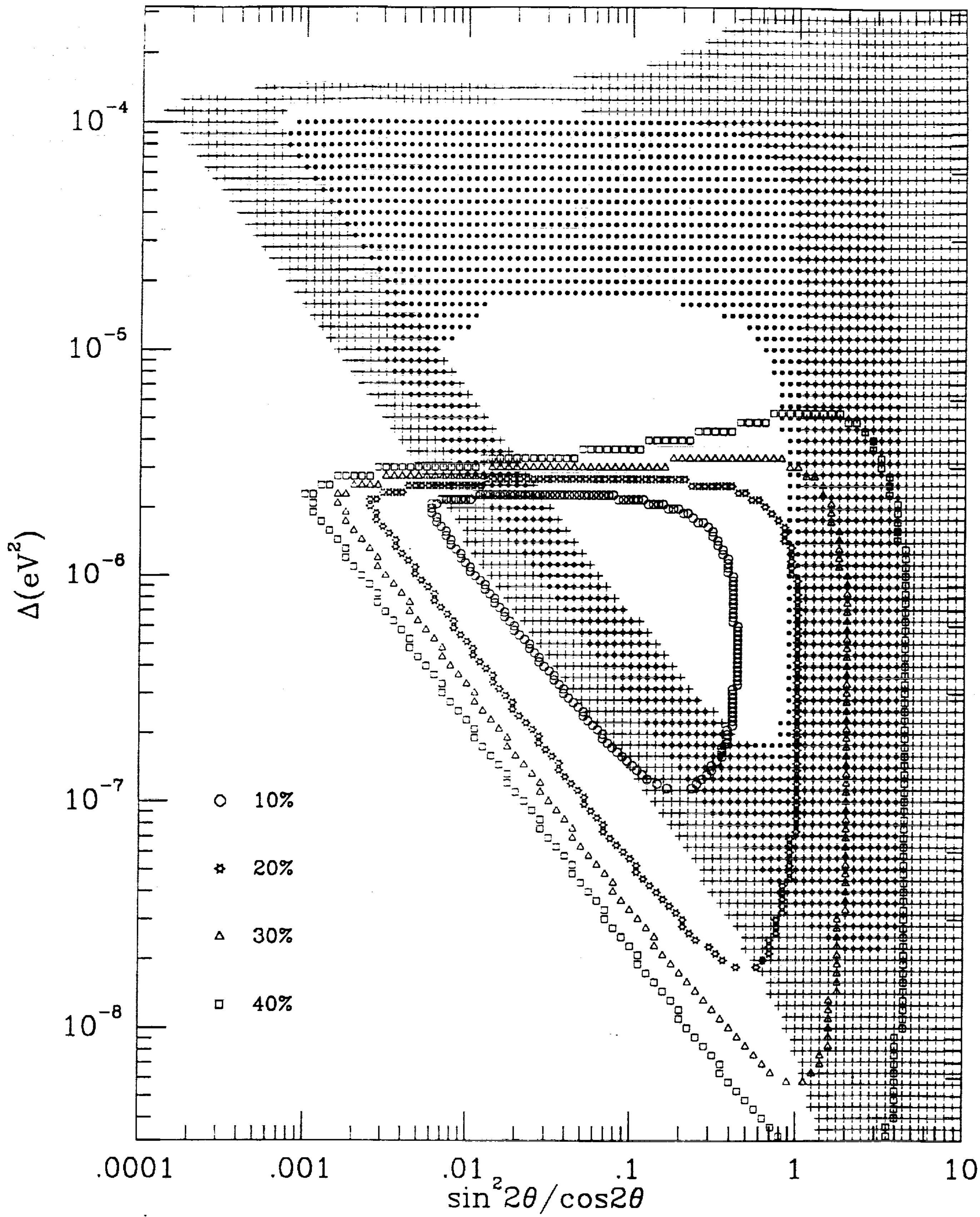


Fig.1



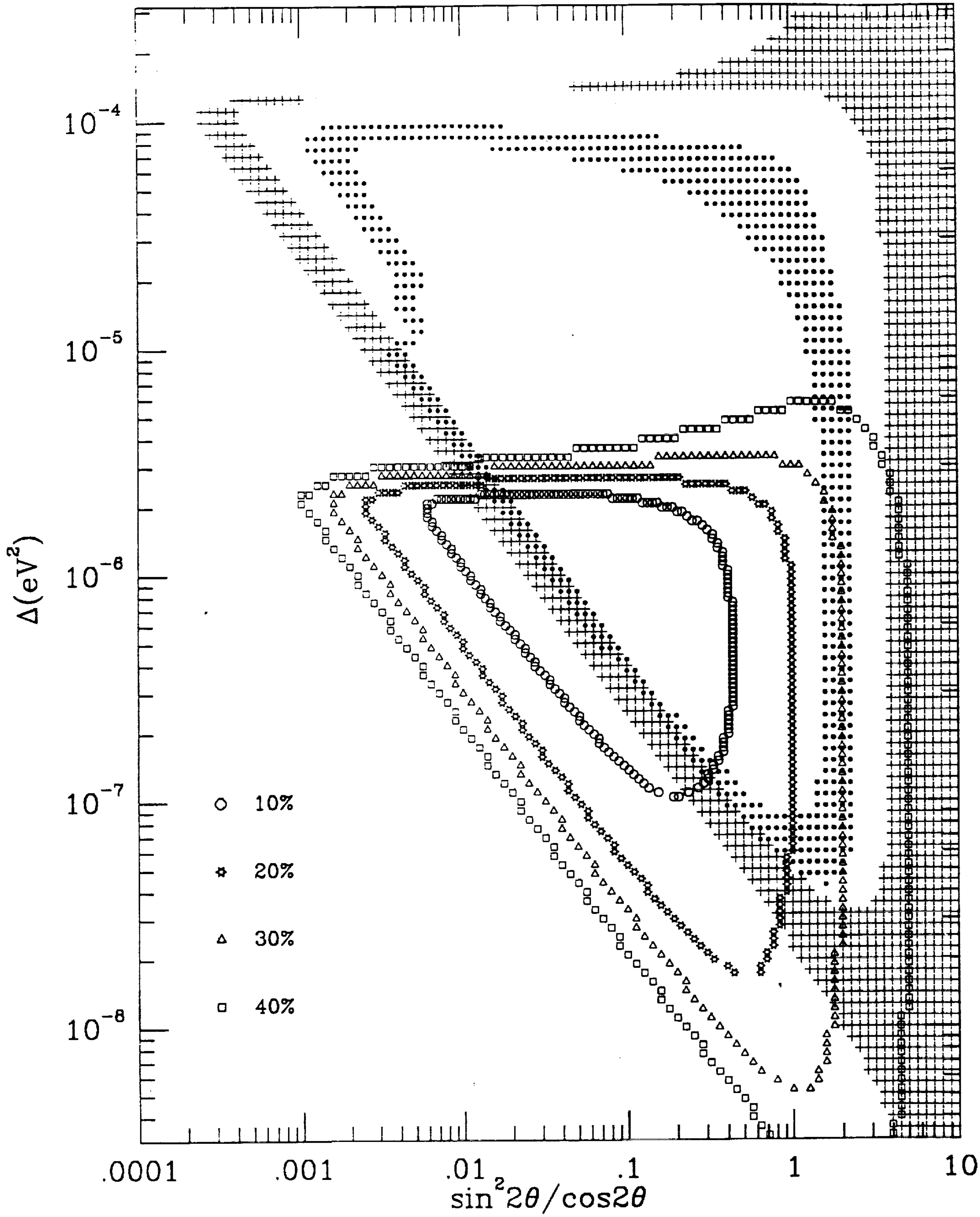
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta(eV^2)$
10%
20%
30%
40%
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$

Fig.2(a)



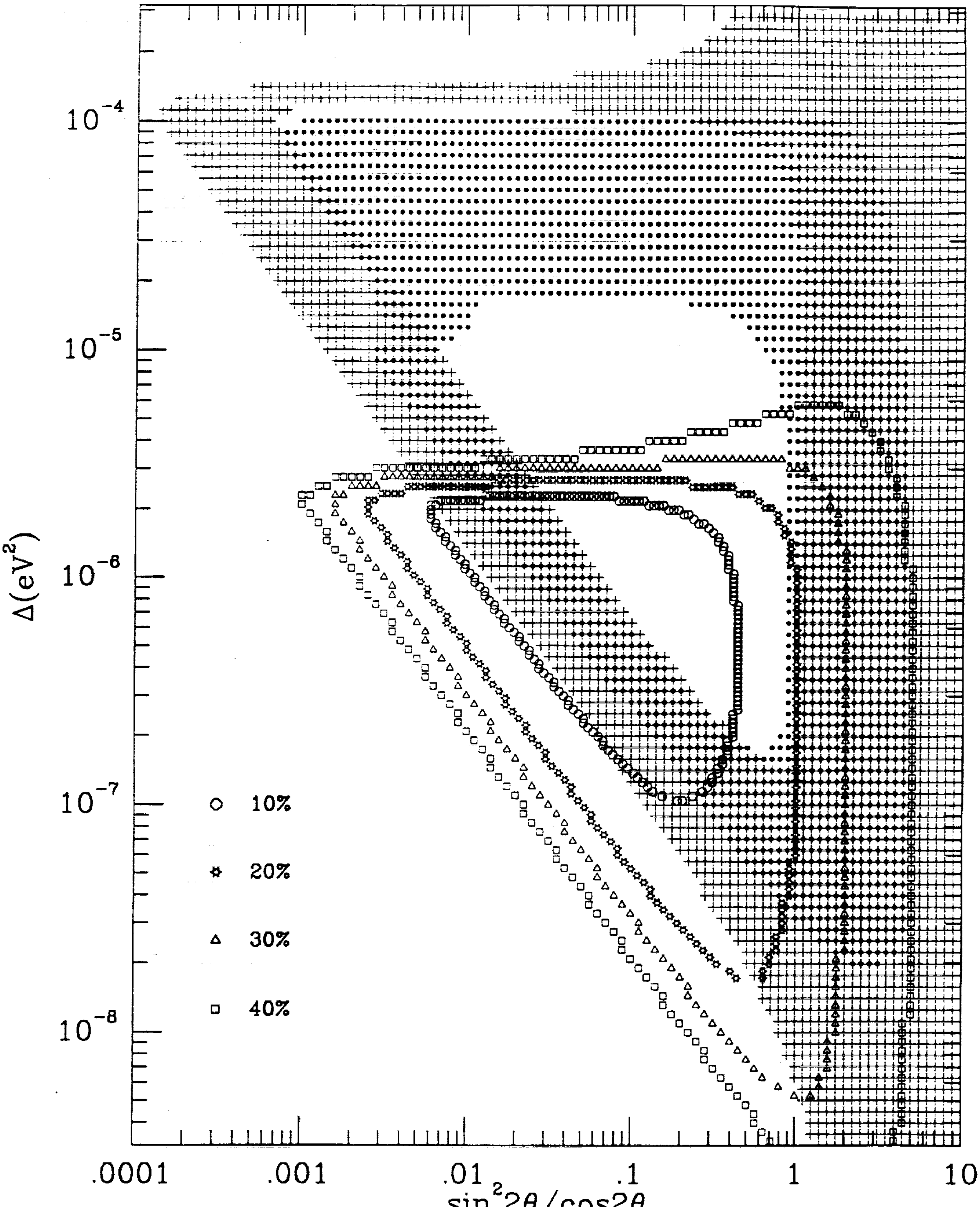
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta(eV^2)$
○ 10%
✡ 20%
△ 30%
□ 40%
.0001
.001
.01
.1
1
10
$\sin^2 2\theta / \cos 2\theta$

Fig.2(b)



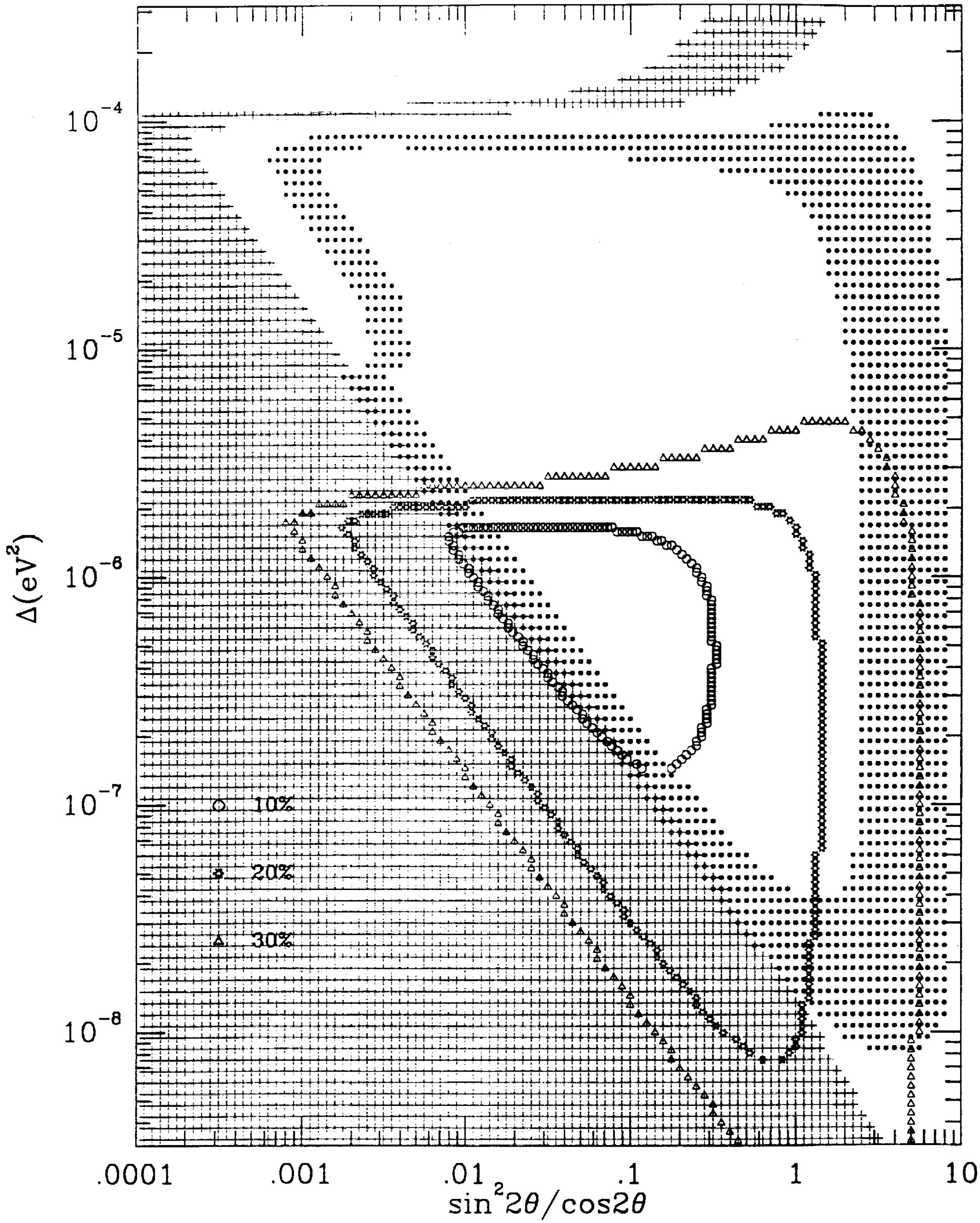
$\Delta(eV^2)$
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$
10%
20%
30%
40%

Fig.3(a)



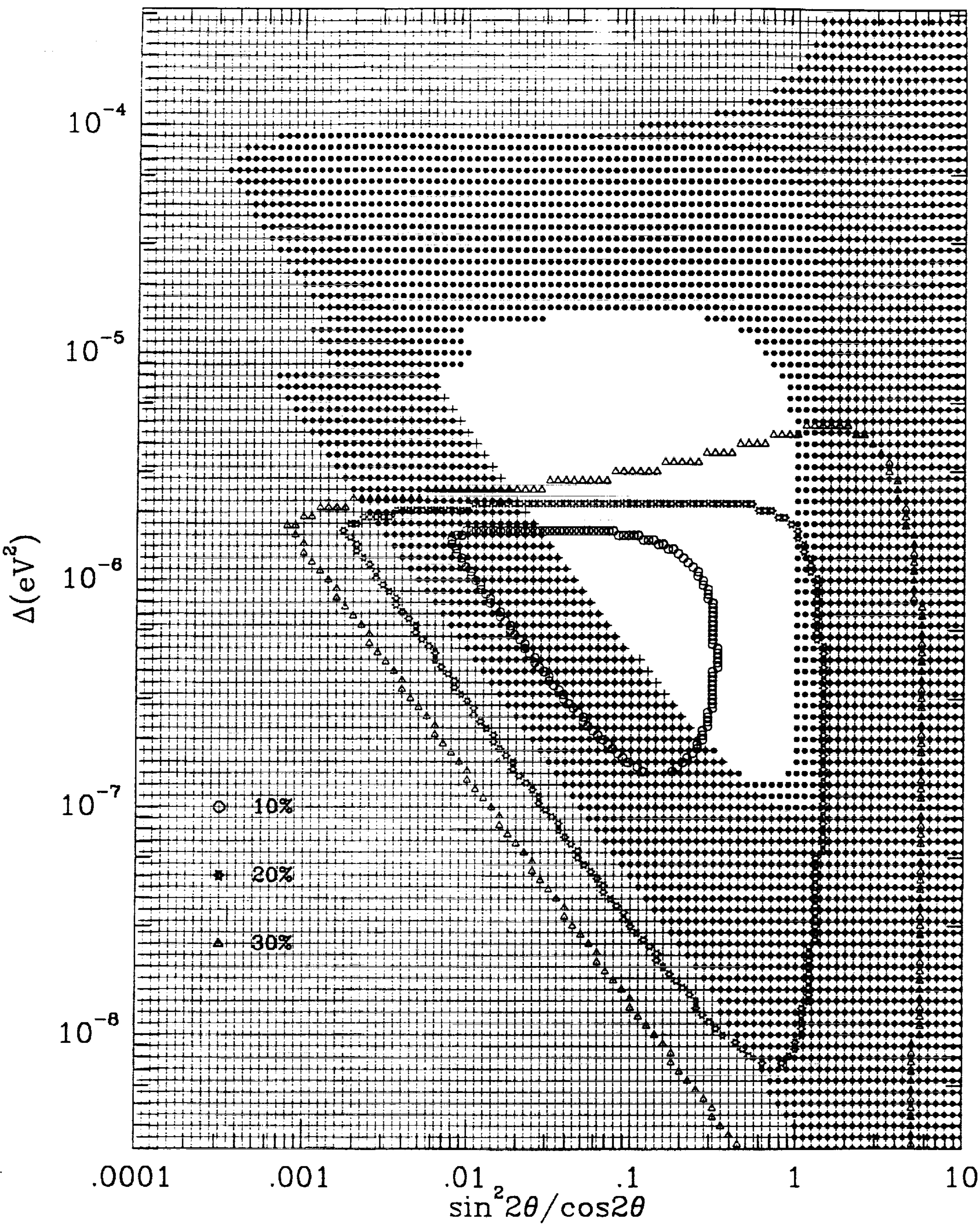
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta(eV^2)$
10%
20%
30%
40%
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$

Fig.3(b)



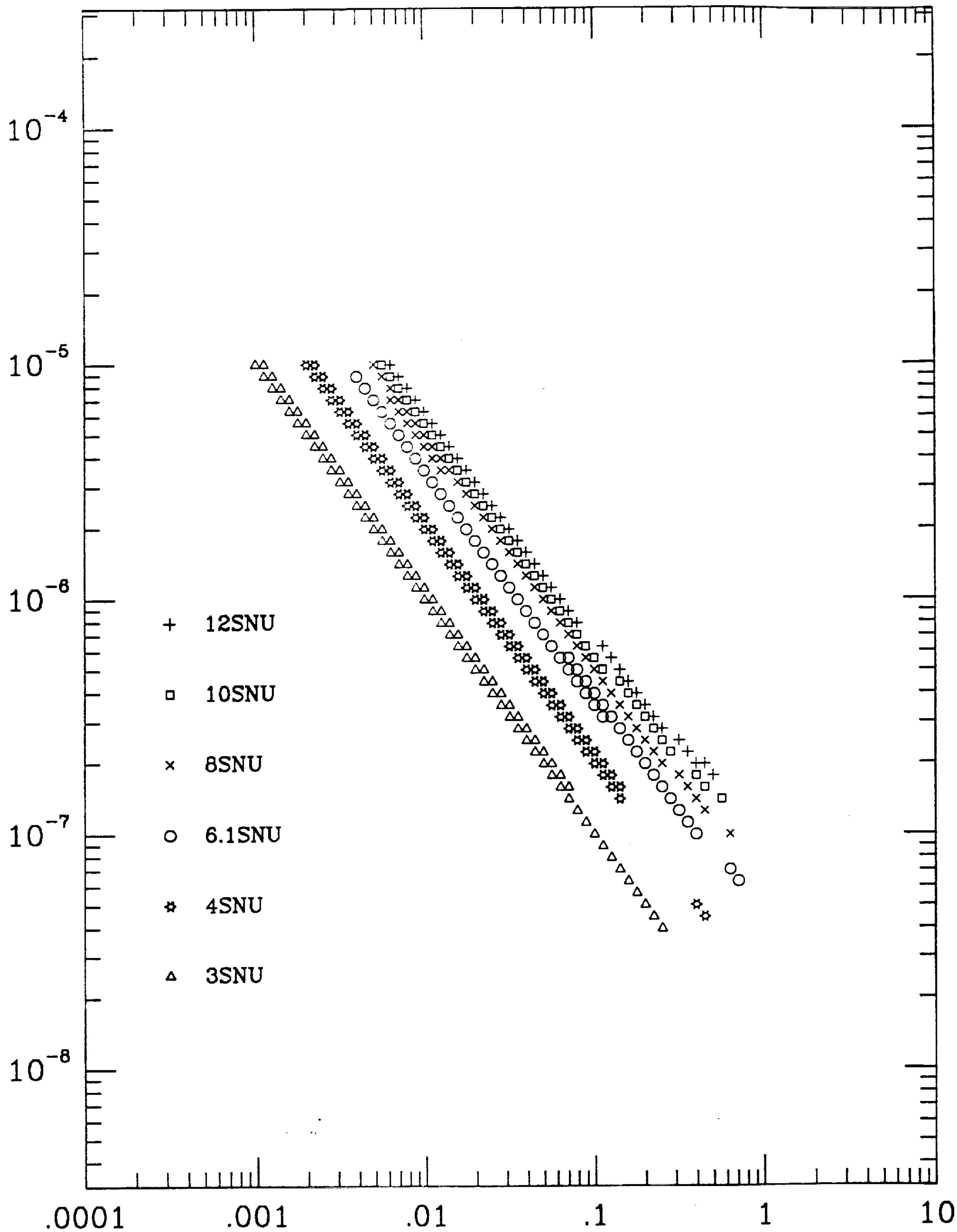
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta(eV^2)$
10%
20%
30%
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$

Fig.4(a)



10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Delta(eV^2)$
10%
20%
30%
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$

Fig.4(b)



10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
12SNU
10SNU
8SNU
6.1SNU
4SNU
3SNU
.0001
.001
.01
.1
1
10

Fig.5(a)

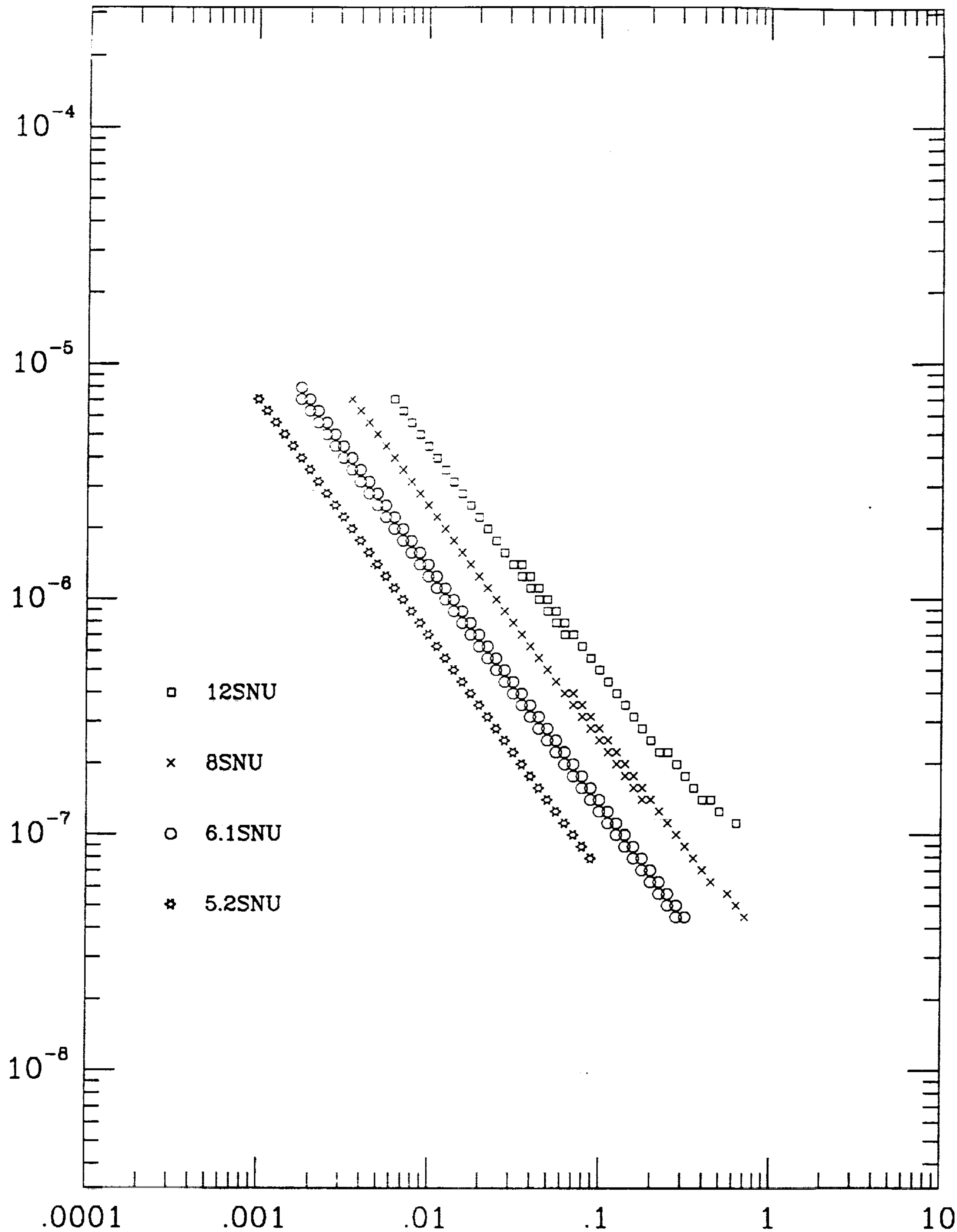


Fig.5(b)

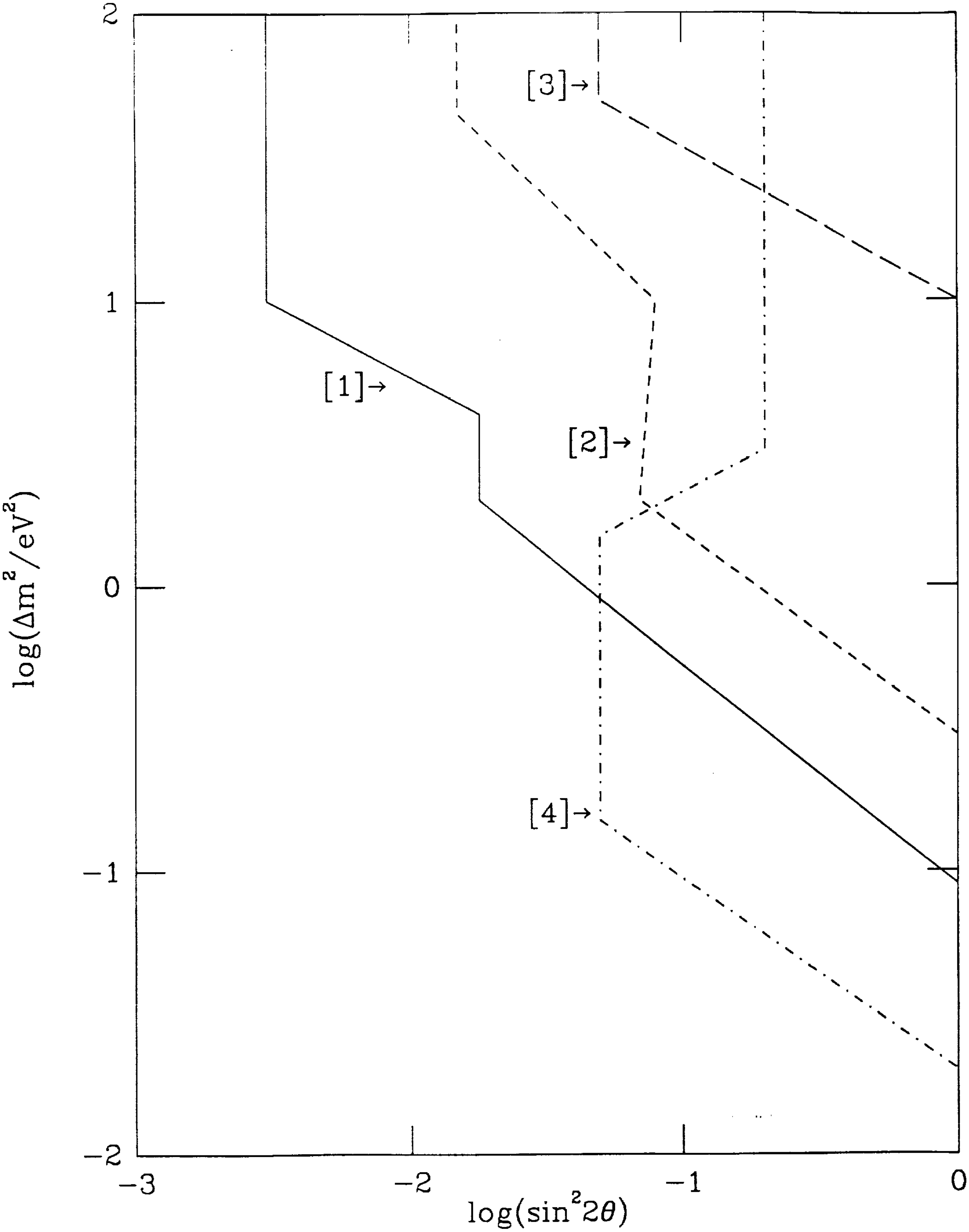


Fig.6