NASA Technical Memorandum 104036

NASA-TM-104036 19910021824

TWO ALTERNATIVE WAYS FOR SOLVING THE COORDINATION PROBLEM IN MULTILEVEL OPTIMIZATION

Jaroslaw Sobieszczanski-Sobieski

August 1991

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

TWO ALTERNATIVE WAYS FOR SOLVING THE COORDINATION PROBLEM IN MULTILEVEL OPTIMIZATION

Jaroslaw Sobieszczanski Sobieski

August 1991

Summary

The paper describes two new techniques for formulating the coupling between levels in multilevel optimization by linear decomposition, proposed as improvements over the original formulation, now several years old, that relied on explicit equality constraints which were shown by application experience as occasionally causing numerical difficulties. The two new techniques represent the coupling without using explicit equality constraints, thus avoiding the above difficulties and also reducing computational cost of the procedure. The old and the new formulations are presented in detail, illustrated by an example of a structural optimization. A generic version of the improved algorithm is also developed for applications to multidisciplinary system not limited to structures.

Notation

A	vector of cross-sectional areas*, A_i
C_i	cumulative constraint of ith beam
DIS_i	vector defined by eq. (22)*
g_k	vector of constraints* for a beam, e.g., stress limits, and local buckling, $k = 1 \ldots NGB$, partitioned in subsets of lengths NGB_i, each subset corresponding to ith beam
G_k	vector of constraints* for the assembled structure, e.g., displacement limits, $k = 1 \ldots NGA$
I	vector of cross-sectional moments of inertia*, I_i
Li	length of ith beam
NE	number of beams in a framework
N_i	vector of the end forces* for ith beam
NP_i	length of vector P_i
NSS	number of subsystems
NX	length of vector X
P_i	vector of parameters* in optimization of ith beam, comprising elements P_q, $q = 1 \ldots NP_i$
P_{q_i}	qth element of vector P_i
SA	system analysis
SI	input vector of length NSI into SA*
SO	output vector of length NSO from SA*
SSA_i	ith subsystem black box analysis
$\{SSC\}$	vector of geometrical* variables determining the structure overall shape
SSI_i	input vector of length $NSSI_i$ into SSA_i*
SSO_i	output vector of length $NSSO_i$ from SSA_i*
TOL	tolerance parameter set by user

W weight, equivalent to volume in a homogeneous structure

X vector of design variables* at the system level (assembled structure) in optimization by decomposition

Y_i vector of design variables* in ith beam optimization problem

Z vector of design variables* in optimization without decomposition

*{ } brackets identifying vectors are omitted where possible without causing ambiguity.

U and L with X, Y, Z, e.g., XU, XL, denote upper and lower bounds on these variables; other symbols are defined where first used.

Introduction

Large scale optimization problems benefit from decomposition into a set of smaller, more manageable, concurrently-solvable subproblems. In a hierarchic decomposition method, the subproblems form a pyramid with the system problem on top and subsystem problems in the horizontal layers below. These subproblems are coupled through the solution of a coordination problem. A particular procedure for optimization by decomposition introduced in reference 1 solved the coordination problem by enforcing a set of equality constraints between the optimization levels and by using an optimum sensitivity analysis formulated in references 2 and 3. That procedure, referred to as Optimization by Linear Decomposition (OLD), was formulated for two-levels in reference 1 and was demonstrated using a framework as a test case in reference 4 representing a class of skeletal, redundant structures. It was subsequently generalized to an arbitrary number of levels in reference 5. The OLD is a generic method applicable to any system that is amenable to a hierarchic decomposition, e.g., multidisciplinary applications reported in references 6 and 7.

Practical experience with the procedure, and its examination in reference 8 point to the enforcement of equality constraints as the source of numerical difficulties that occasionally make this procedure slow to converge. This observation is consistent with the opinion generally held among the developers and users of optimization methods that addition of equality constraints to an optimization problem tends to make the solution numerically more difficult.

Motivated by the above, and by two new techniques for satisfying equality constraints that were recently introduced in references 9 and 10, this paper defines two alternative modifications to the OLD procedure. Either alternative removes explicit equality constraints from the procedure while, still, achieves their enforcement implicitly.

The two alternative techniques will be introduced by generalizing from an example of the framework structure that was a test case in references 1 and 4. To this end, the framework analysis will be discussed first, followed by the framework optimization problem formulated in a standard manner without decomposition. Next, an abridged description of the OLD will be given, limited to a two-level, structural optimization of the framework test case. With this as a reference, two new alternative modifications to the OLD algorithm will be introduced.

Original Reference Procedure

Analysis

The framework is shown in figure 1. We limit the framework analysis to a two-dimensional case by allowing only in-plane displacements under the action of static loads. The static analysis of this structure may be formulated in two levels by using either a substructuring approach where each of the three beams in the framework is regarded as a separate substructure, or a finite element method in which each beam is a single finite element. Choosing the finite element formulation, the analysis of the assembled framework requires an input and generates an output as defined in table 1.

The derivatives such as the displacement U with respect to the cross-sectional area A_i, $\frac{\partial U}{\partial A_i}$, exist because $U = f(A, I, SSC)$, $N_i = f(A, I, SSC)$, and $L_i = f(SSC)$. These derivatives are obtained by finite differencing or by a quasi-analytical sensitivity analysis embedded in the framework analysis.

Selected data from the above output are entered as input into a local strength analysis of the ith beam. The level of refinement of that analysis is immaterial for the purposes of this discussion, e.g., it might be a finite element analysis whereby the beam is divided into number of finite elements, or an elementary strength of materials analysis. In either case, the input and output are as defined in table 2.

The above approach suggests decomposition of the framework analysis into the assembled framework analysis (system level analysis) and the beam analyses (subsystem level analyses). This decomposition forms a hierarchy in which the former is a parent and the latter are the daughters. The parent-daughter relationship is hierarchic because the information flows from the parent to each daughter and no information is directly transmitted from one daughter to another. Since the daughter analyses are mutually independent, they may be executed concurrently.

To make the system and subsystem level analyses consistent, one has to acknowledge that A_i and I_i are functions of the beam cross-sectional dimensions $\{Y\}_i$. Therefore, when prescribing the values for A_i, I_i, and $\{Y\}_i$, we must satisfy the consistency equations

$$A_i = f_A(\{Y\}_i); \; I_i = f_I(\{Y\}_i), \; i = 1 \ldots NE \tag{1}$$

Keeping these equations satisfied in a multi-level optimization is a part of the so-called coordination problem, and the analysis by decomposition will be exploited to establish a corresponding optimization-by-decomposition scheme that will be described later.

Optimization Without Decomposition

The optimization problem to be solved is

$$\text{"find } Z \text{ such that } F(Z) \text{ is at minimum subject to constraints"} \tag{2}$$

$$G_j(Z) \leq 0, \; j = 1 \ldots NGA; \tag{2a}$$

$$g_k(Z) \leq 0, \; k = 1 \ldots NGB \tag{2b}$$

$$ZL \leq Z \leq ZU \tag{2c}$$

In the static problem at hand, the design variables may include the elements of the $\{SSC\}$ and some, or all, of the cross-sectional dimensions of the beams, $\{Y\}_i$. The constraints G_k comprise the assembled structure displacements and elastic stability, and at the beam cross-section level the g_k constraints entail the allowable stress, beam column buckling, and local buckling.

Optimization by Linear Decomposition (OLD)

The above problem is decomposed into a single problem solved for the assembled structure (system level problem) and NE separate problems solved for each beam (subsystem level problems). It is convenient to describe the subsystem level problem first. The equation underscoring indicates which parts of the formulation will be changed by the two new techniques formulated in this paper.

Subsystem level.

At the subsystem level, an optimization problem for the ith beam is independent of the other problems at that level, hence the beam optimizations may be executed concurrently. The ith beam problem is, taking into account eq. (1):

$$\text{"find } \{Y\}_i \text{ such that } C_i(\{Y\}_i, \{P\}_i) \text{ is at minimum subject to constraints"} \tag{3}$$

$$A_i - f_A(\{Y\}_i) = 0, \tag{3a}$$

$$I_i - f_I\{Y\}_i = 0, \tag{3b}$$

$$\{YL\}_i \leq \{Y\}_i \leq \{YU\}_i; \tag{3c}$$

where $\{YL\}_i$ and $\{YU\}_i$ are the lower and upper side-constraints on $\{Y\}_i$. The $\{P\}_i$ is a vector of parameters comprising A_i, I_i, L_i, and $\{N\}_i$ that are output from the analysis at the system level as defined in table 1 and are passed as input into the beam analysis as shown in table 2. These parameters stay constant in the process of the beam optimization. Hence, for ith beam

$$\{Pq\}_i = \{A_i, I_i, L_i, \{N\}_i\}, \; q = 1 \ldots NP_i \tag{4}$$

The scalar C_i is a cumulative constraint representing the degree of satisfaction, or violation, of the subset $g_k(\{Y\}_i)$, $k = 1 \ldots NGB_i$, pertaining to the ith beam, of the entire set of g_k that appears in eq. (2b). The cumulative constraint may be evaluated as in references 4 and 5 by means of the Kreisselmeier-Steinhauser (KS) function (ref. 11):

$$C_i(g_k) = KS(g_k) = (1/\rho) \ln \left(\sum_k \exp(\rho g_k) \right), \; k = 1 \ldots NGB_i \tag{5}$$

where ρ is a user-controlled coefficient that governs the distance between the KS and $\max(g_k)$. An alternative KS-formulation that avoids generation of large values of the exponential function is

$$C_i(g_k) = \max(g_k) + 1/\rho) \ln \left(\sum_k \exp(\rho(g_k - \max(g_k))) \right), \; k = 1 \ldots NGB_i \tag{5a}$$

The KS function is differentiable and has the property that

$$\max(g_k) < KS(g_k) < \max(g_k) + \ln \; NGB_i/\rho, \; k = 1 \ldots NGB_i \tag{6}$$

approximating the nondifferentiable $\max(g_k)$ with an error dependent on ρ (the larger ρ is, the smaller the error is. However, optimization may be more difficult numerically).

The optimization in eq. (3) alters $\{Y\}_i$ to enforce the consistency eq. (1) by means of the equality constraints in eqs. (3a) and (3b), and to achieve a minimum of C_i equivalent to a minimum of $\max(g_k)$. This optimization produces an optimal solution comprising of $C_{i\min}$ and $\{Y\}_{i\text{opt}}$. It is followed by an optimum sensitivity analysis (ref. 2) to obtain the derivatives of $C_{i\min}$ and $\{Y\}_{i\text{opt}}$ with respect to $\{Pq\}_i$, denoted $\frac{\partial C_{i\min}}{\partial P_{qi}}$ and $\frac{\partial Y_{i\text{opt}}}{\partial P_{qi}}$.

System level.

In the system level problem, the design variable vector X contains A_i, I_i, $i = 1 \ldots NE$, and the elements of $\{SSC\}$

$$\{X\} = \{A_i, I_i, \{SSC\}\}, \; i = 1 \ldots NE; \tag{7}$$

At this level, the assembled structural analysis whose input is defined in table 1 is carried out. The objective function in this case is the structural weight which for a homogeneous material may be replaced with the material volume

$$F(X) = W(X) = \sum_i^{NE} A_i L_i \tag{8}$$

The problem formulation is

$$\text{“find } X \text{ such that } F(X) \text{ is at minimum subject to constraints”} \tag{9}$$

$$G_k(X) \leq 0, \; k = 1 \ldots NGA; \tag{9a}$$

$$C_i(X) \leq \text{TOL}, \; i = 1 \ldots NE; \tag{9b}$$

$$\{YL\}_i \leq \{Y\}_i \leq \{YU\}_i, \; i = 1 \ldots NE; \tag{9c}$$

$$XL \leq X \leq XU; \tag{9d}$$

where TOL is a suitable tolerance parameter.

The constraints $G_k(X)$ pertain to the assembled structure behavior, e.g., displacement limits and overall elastic stability. The $C_i(X)$ is approximated by extrapolation

$$C_i(X) = C_{i\min} + \sum_j \sum_q \frac{\partial C_{i\min}}{\partial P_{qi}} \frac{\partial P_{qi}}{\partial X_j}(X_j - X_{jo}),$$
$$i = 1 \ldots NE, \; j = 1 \ldots NX, \; q = 1 \ldots NP_i \tag{10}$$

where P_{qi} is defined by eq. (4). In the above, $C_{i\min}$ and $\frac{\partial C_{i\min}}{\partial P_{qi}}$ are transmitted from the subsystem level optimization and optimum sensitivity analysis, and the derivative product terms constitute a chain rule differentiation necessary since some of the parameters P_q depend on X_j as noted in discussion of table 1. Since X_j is an element of $\{P\}_i$, we will have $\frac{\partial P_{qi}}{\partial X_j} = 1$ for the coincidences $(P_q)_i = X_j$ that occur in the summation. The values of $(X_j)_o$ are those for which the assembled framework analysis was carried out prior to the current optimization.

When C_i is expressed by a KS function as in eq. (5) or (5a), the extrapolation error in eq. (10) may be significantly reduced, or shown in references 4 and 5, by extrapolating each constraint function g_k that enter C_i

$$g_k(X) = g_{ko} + \sum_j \sum_q \frac{\partial g_k}{\partial P_{qi}} \frac{\partial P_{qi}}{\partial X_j} (X_j - X_{jo}),$$
$$j = 1 \ldots NX, \; q = 1 \ldots NP_i \tag{10a}$$

and, then, computing in extrapolated C_i by means of either eqs. (5) or (5a). This technique removes that part of the extrapolation error that in eq. (10) would be caused by the curvature of the logarithm and exponential functions embedded in the KS function.

An extrapolation similar to eq. (10) is used to approximate $\{Y\}_i$ in eq. (9c):

$$Y_i(X) = Y_{i\text{opt}} + \sum_j \sum_q \frac{\partial Y_{i\text{opt}}}{\partial P_{qi}} \frac{\partial P_{qi}}{\partial X_j}(X_j - X_{jo}),$$
$$i = 1 \ldots NE, \; j = 1 \ldots NX, \; q = 1 \ldots NP_i \tag{11}$$

$$XL \leq X \leq XU$$

Finally, XL and XU, are, respectively, the lower and upper limits on X. These side constraints include the move limits guarding against excessive extrapolation errors in eqs. (10) and (11).

Overall procedure.

The overall OLD procedure is

1. Initialize the overall shape data in $\{SSC\}$, and the beam cross-sectional dimensions $\{Y\}_i$;
2. Enter $\{Y\}_i$ into eqs. (3a) and (3b) to initialize A and I;
3. Execute the assembled structure analysis (table 1);
4. For each beam execute a subsystem level optimization per eq. (3), that refers to the beam analysis per table 2, and carry out the optimum sensitivity analysis with respect to parameters defined in eq. (4);
5. Execute optimization at the system level per eq. (9);
6. If termination criteria set by user are not satisfied, reset X and Y to the new values and repeat from step 3, otherwise exit.

To conclude the description of the OLD procedure at two levels, one should point out that the individual optimizations at both levels are coupled by means of eqs. (3a), (3b), (9b), (9c), (10), and (11). These equations represent the coordination problem which is solved by virtue of converging the overall procedure.

Shortcomings of the OLD Procedure in Need for Improvement.

In the subsystem level formulation in eq. (3), there is a possibility of a conflict between the equality constraints in eqs. (3a), (3b), and the side constraints in eq. (3c). Specifically, it may not be possible to find a feasible solution to the subsystem level problem while satisfying both sets of constraints. To alleviate that conflict, the system level formulation above includes approximate representation of the side constraints on $\{Y\}_i$, eqs. (9c) and (11), to keep the system level optimization from imposing on the ith beam such combinations of the A_i and I_i values that cannot be attained with the physically realizable $\{Y\}_i$. The above potential conflict is one disadvantage of using the equality constraints in eqs. (3a) and (3b).

As mentioned in the Introduction, the other disadvantage is the increased difficulty of solving the optimization problem of eq. (3) brought about by the presence of the equality constraints, as pointed out in reference 8. Hence, the two alternative modifications are introduced next, primarily, to remove these equality constraints.

Proposed Modifications

Two alternative modifications whose introduction is this paper purpose are defined herein. The common feature of both modifications is removal of the equality constraints in eqs. (3a), (3b), (9b), and (9c), and an indirect fulfillment of these constraints by reformulating the subsystem optimization problem. The two modifications differ in the details of that reformulation.

Modification 1

The first modification is based on a technique for locating simultaneous roots of a set of functions using the KS function as described in reference 9. Specifically, if a set of NF functions $F_i(Y) = 0$ for $Y = Y_o$, then a KS function comprising the positive and negative $F_i(Y)$ has a minimum at Y_o. Formally,

$$\text{If } F_i(Y) = 0, \text{ for } Y = Y_o, \text{ then } KS(F_i(Y) - F_i(Y))$$

$$\text{is at minimum for } Y = Y_o; \; i = 1 \ldots NF \tag{12}$$

The above property of the KS function may be used to satisfy the equality constraints in eqs. (3a) and (3b). For brevity, we define nondimensional functions

$$F_{Ai} = (A_i - f_A(\{Y\}_i))/A_i, \; i = 1 \ldots NE$$
$$F_{Ii} = (I_i - f_I(\{Y\}_i))/I_i, \; i = 1 \ldots NE \tag{13}$$

and construct a composite function

$$C_i(\{Y\}_i, \{P\}_i) = KS(F_{Ai} - F_{Ai}, F_{Ii} - F_{Ii})$$
$$= \left(\frac{1}{\rho}\right) \ln\left(\exp(\rho F_{Ai}) + \exp(-\rho F_{Ai}) + \exp(\rho F_{Ii}) + \exp(-\rho F_{Ii})\right) \quad (14)$$

By virtue of eq. (12), this function is a minimum at $\{Y_o\}_i$ where F_{Ai} and F_{Ii} also vanish. We will seek $\{Y_o\}_i$ as $\{Y\}_{i\text{opt}}$ in an optimization that entails $C_i(\{Y\}_i, \{P\}_i)$ defined by eq. (14) as the objective function. This approach was shown to be effective in reference 12, although the details of the function formulation and of the overall procedure defined in that reference were different.

The subsystem level optimization of eq. (3) for the ith beam will now change to the following one in which C_i is defined by eq. (14):

"find $\{Y\}_i$ such that $C_i(\{Y\}_i, \{P\}_i)$ is at minimum subject to constraints" (15)

$$g_k(\{Y\}_i) \leq 0, \; k = 1 \ldots NGB_i \quad (15a)$$

$$\{YL\}_i \leq \{Y\}_i \leq \{YU\}_i \quad (15b)$$

The subsystem level optimization satisfies the local constraints, eqs. (15a) and (15b), and comes as close to $F_{Ai} = 0$ and $F_{Ii} = 0$, that is $f_A = A_i, f_I = I_i$, as possible. However, in contrast to eqs. (3a) and (3b), it is not required to nullify these quantities completely, if that is not yet possible in the process of iterating between the levels in the overall procedure. Hence, the potential conflict among the constraints eqs. (3a), (3b), and (3c) described in the preceding section has been removed. Consequently, *the constraints in eq. (9c) that were needed to alleviate that conflict are now deleted from the system level optimization.* As a result of the deletion of eq. (9c) from the system level optimization, the derivatives $\frac{\partial\{Y\}_{i\text{opt}}}{\partial P_{qi}}$ become unnecessary. Consequently, the optimum sensitivity algorithm from reference 2 may be replaced with the computationally less expensive algorithm from reference 3 for a significant reduction of the computational cost of the entire procedure.

The system level optimization change is limited to deletion of eq. (9c), and to redefinition of C_i from the one given in eq. (5) to that set by eq. (14). The new definition of C_i must also be used in eq. (10).

Even though the equality constraint such as those in eq. (3) do not appear directly in the above subsystem optimization, they are eventually brought to satisfaction within tolerance TOL, indirectly, by virtue of eqs. (14), (15), (15a), (15b), and (9b), when the overall procedure converges.

Modification 2

An algorithm introduced in reference 10 for the purposes of fitting an empirical function to a set of experimental data points may be adapted as a formulation of the subsystem level optimization problem. The algorithm requires augmentation of $\{Y\}_i$ by an additional independent variable, LIM_i, that also doubles for the objective function in a following formulation of the subsystem problem for the ith beam

"find $\{Y\}_i$ and LIM_i such that LIM_i is at minimum subject to constraints" (16)

$$g_k\{Y\}_i \leq 0, \; k = 1 \ldots NGB_i \quad (16a)$$

$$-LIM_i \leq F_{Ai} \leq +LIM_i \quad (16b1)$$

$$-LIM_i \leq F_{Ii} \leq +LIM_i \quad (16b2)$$

$$\{YL\}_i \leq \{Y\}_i \leq \{YU\}_i \quad (16c)$$

$$LIM_i > 0 \quad (16d)$$

The above problem solution produces the values of LIM_{imin} and $\{Y\}_{iopt}$. The LIM_{imin} is eventually reduced to TOL owing to the following changes in the system level optimization of eq. (9): deleting eq. (9c), as in Modification 1 above, and replacing C_i with LIM_i which changes eqs. (9b) and (10) to

$$LIM_i \leq \text{TOL}, \; i = 1 \ldots NE \tag{17}$$

$$LIM_i(X) = (LIM_i)_{\min} + \sum_j \sum_q \frac{\partial LIM_{i\min}}{\partial P_{qi}} \frac{\partial P_{qi}}{\partial X_j}(X_j - X_{jo}),$$
$$i = 1 \ldots NE, \; j = 1 \ldots NX, \; q = 1 \ldots NP_i \tag{18}$$

As in Modification 1, the optimum sensitivity analysis that follows the solution of eqs. (15), (15a), and (15b) may be carried out using the algorithm from reference 3 instead of reference 2. Neither Modification 1 nor Modification 2 change anything in the step-by-step prescription for the overall procedure OLD described previously.

Extension to a Generic System

The above algorithm lends itself to a complete generalization by replacing the variables and terms specific to the framework example with their generic counterparts, while leaving the organization of the decomposed optimization and the formulations of its elements unchanged. This section explains the substitution of the variables and terms in each of the system and subsystem level analyses, subsystem and system level optimizations, and the overall procedure.

System Level Analysis

System Analysis, designated SA, is regarded as a black box that converts an input vector $\{SI\}$ of length NSI into an output vector $\{SO\}$ of length NSO. Vector $\{SI\}$ contains as a subset the vector $\{X\}$ of length NX that comprises the system level design variables. Vector $\{SO\}$ also contains $\{X\}$ as a subset (passed through SA). By definition of SA

$$SO = f(SI) \tag{19}$$

assumed differentiable up to the first derivatives. Consequently, a Jacobian matrix of the first derivatives exists

$$\frac{\partial SO}{\partial SI} = \left[\frac{\partial SO_i}{\partial SI_j}\right], \; i = 1 \ldots NSO, \; j = 1 \ldots NSI \tag{20}$$

Equations (19) and (20) form a functional statement that is a very general one and includes two important special cases: the first derivatives in the above Jacobian, eq. (20), degenerate to zero wherever a particular SO_i is not influenced by a particular SI_j; and the derivatives default to unity wherever, for a particular pair ij, there is $SO_i = SI_j = X_k$.

The Jacobian matrix, eq. (20), is obtainable by finite differencing on SA or by quasi-analytical sensitivity analysis embedded in SA. In either case the Jacobian matrix is regarded as an additional output from SA, separate from SO.

The input defined in table 1 is an example of SI, while the subset of A's and I's in that input is an example of X. The output defined in that table is an example of SO, and the A's and I's present in that output are examples of X as a subset of SO that is being passed through SA unchanged. The derivatives $\partial N/\partial A$ defined in table 1 constitute an example of the elements of $\partial SO/\partial SI$.

Subsystem Level Analysis

It is assumed that there are NSS black boxes representing the next lower level of subsystems. As daughters of the SA parent black box, they receive their input in part from the system level and, in part, from the outside world, but not from each other. Therefore, the subsystem black boxes are mutually independent and may be executed concurrently. The ith black box is designated SSA_i. It converts the input vector $\{SSI\}_i$ of length $NSSI_i$ into an output vector $\{SSO\}_i$ of length $NSSO_i$.

Vector $\{SSI\}_i$ contains selected elements of $\{SO\}$, some of which may be the elements of X. It includes also the vector of the subsystem level design variables $\{Y\}_i$.

At this point, it is convenient to define a vector $\{P\}_i$, $i = 1 \ldots NP_i$, as a subset of those elements of $\{SSI\}_i$ which are functions of X. Naturally, this includes the elements of X, if any are present in $\{SSI\}_i$. An example of such a subset is eq. (4).

We now assume that for the elements of $\{P\}_i$, the $\{SSA\}_i$ contains a functional relation,

$$\{P(X)\}_i = f(\{Y\}_i) \tag{21}$$

An example of the above relation is illustrated by eq. (1). However, eq. (20) is more general than eq. (1) since it recognizes that not only the elements of X but also other data in SSI_i, selected from SO, may be computable as functions of $\{Y\}_i$.

For the ith subsystem optimization, it will be useful to define a "discrepancy" vector $\{DIS\}_i$ defined as

$$\{DIS_j\}_i = (\{P_j\}_i - f(\{Y\}_i))/\{Pj\}_i, \; j = 1 \ldots NP_i \tag{22}$$

Examples DIS_j are FA_i and FI_i in eq. (13). The SSA_i output $\{SSO\}_i$ contains $\{DIS\}_i$ and other behavior variables of interest.

Subsystem Level Optimization

$SSOPT_i$ is an optimization of subsystem i. It can be defined by either one of the two alternative ways, consistent with Modification 1 (eqs. (15), (15a), and (15b)) or Modification 2 (eqs. (16), (16a–d)).

In the first alternative, corresponding to eqs. (15), (15a), and (15b), the design variables are the elements of $\{Y\}_i$, and the objective function C_i is

$$\begin{aligned} C_i(\{Y\}_i, \{P\}_i) &= KS(\{DIS_j\}_i, -\{DIS_j\}_i) \\ &= (1/\rho) \ln \left(\sum_j (\exp(\rho\{DIS_j\}_i) + \exp(-\rho\{DIS_j\}_i)) \right), \; j = 1 \ldots NP_i \end{aligned} \tag{23}$$

In this alternative the optimization problem comprises the above C_i as the objective function and its formulation is as follows:

$$\text{"find } \{Y\}_i \text{ such that } C_i(\{Y\}_i, \{P\}_i) \text{ is at minimum subject to constraints"} \tag{24}$$

$$g_k(\{Y\}_i) \leq 0, \; k = 1 \ldots NGSS_i \tag{24a}$$

$$\{YL\}_i \leq \{Y\}_i \leq \{YU\}_i \tag{24b}$$

In the second alternative, consistent with eq. (16), the design variables are the elements of $\{Y\}_i$ and an additional variable LIM_i. The latter doubles for an objective function so that the optimization problem is

$$\text{"find } \{Y\}_i \text{ and } LIM_i \text{ such that } LIM_i \text{ is at minimum subject to constraints"} \tag{25}$$

$$g_k(\{Y\}_i) \leq 0, \; k = 1 \ldots NGSS_i \tag{25a}$$

$$-LIM_i \le \{DIS_j\}_i \le +LIM_i,\ j = 1 \ldots NP_i \tag{25b}$$

$$\{YL\}_i \le \{Y\}_i \le \{YU\}_i \tag{25c}$$

$$LIM_i > 0 \tag{25d}$$

In both eqs. (24) and (25), the constraint functions g_k are evaluated using the data in SSO_i. The $SSOPT_i$ output is $\{Y\}_{i\text{opt}}$ and, dependently on the choice of eqs. (24) or (25), $F_{i\min} = C_{i\min}$ or $F_{i\min} = LIM_{i\min}$, respectively.

Optimum Sensitivity Analysis and Extrapolation of the Minimum of the Objective Function

The Optimum Sensitivity Analysis, OSA_i, uses the algorithm of reference 3, applied as a post-processor to $SSOPT_i$, to yield for each subsystem i a vector of the derivatives of the minimum objective function $F_{i\min}$ with respect to the parameters P. The vector of these derivatives, $\left\{\frac{\partial F_{i\min}}{\partial \{P\}_i}\right\}$, has the length NP_i. The definition of P (see discussion preceding eq. (21)) and the functional relationship defined by eqs. (19) and (20) imply that

$$\{P_j\}_i = f(X_k),\ j = 1 \ldots \text{NP}_i,\ k = 1 \ldots NX \tag{26}$$

and that the derivatives $\frac{\partial \{P_j\}_i}{\partial X}$ exist. These derivatives default to unity for those pairs jk for which $P_j = X_k$.

Using the above, one may extrapolate F_i by means of the chain rule as an approximate function of X

$$F_i = F_{i\min} + \sum_j \sum_q \frac{\partial F_{i\min}}{\partial P_{qi}} \frac{\partial P_{qi}}{\partial X_j} \left(X_j - X_{jo}\right),\ i = 1 \ldots NSS,\ j = 1 \ldots NX,\ q = 1 \ldots NP_i \tag{27}$$

where $(X_j)_o$ is the X for which the system and subsystem analyses were carried out prior to the subsystem optimization. For an example of eq. (27), see eqs. (10) or (18).

System Level Optimization

Equating F_i either to C_i computed from eq. (23), or to LIM_i defined for eq. (25), dependently on the choice of Modification 1 (eq. (15)) or Modification 2 (eq. (16)) for the $SSOPT$ formulation, the system level optimization, designated $SOPT$, may be formulated so as to accommodate both alternatives. Defining an objective function $FS(X)$ and the vector of constraints $G(X)$, both computed from the elements of SO, the $SOPT$ formulation is as follows:

"find X such that $FS(X)$ is at minimum subject to constraints" (28)

$$G_k(X) \le 0,\ k = 1 \ldots NGS \tag{28a}$$

$$F_i(X) \le \text{TOL},\ i = 1 \ldots NSS \tag{28b}$$

$$XL \le X \le XU \tag{28c}$$

consistent with eq. (9) modified as described in the discussions of the Modifications 1 and 2.

In the above optimization problem, F_i in eq. (28b) is extrapolated by eq. (27), analogous to the extrapolation of C_i in eq. (10) or LIM_i in eq. (18).

Initialization

It is a recommended practice to initialize the entire optimization procedure by first setting the values of $\{Y\}_i$, $i = 1 \ldots NSS$, and then computing X from eq. (21), recalling that, by definition X is a subset of P. This guarantees starting in SA at the system level with the X values that are physically realizable in SSOPT_i by $\{Y\}_i$ within the YL and YU limits. This operation is abbreviated $INIT$.

Generic Two-Level Optimization Procedure

The procedure is the same as the one described previously for the framework example, restated in the generic terms defined in this section.

1. Execute $INIT$;
2. Execute SA;
3. Execute $SSOPT_i$, followed by OSA_i, $i = 1 \ldots NSS$, concurrently, if desired and if the computing equipment permits;
4. Execute $SOPT$;
5. If termination criteria set by user are not satisfied, reset X and Y to X_{opt} and Y_{opt} and repeat from step 2, otherwise EXIT.

The procedure output is the optimal data for X, Y, $G(X)$, $g(X)$, and $FS(X)$. Owing to the formulation of SSOPT_i in eq. (23) or (24), the optimal values of X and Y will satisfy eq. (20) within TOL. The procedure flowchart is depicted in figure 2.

Concluding Remarks

Two new techniques are presented for coupling the levels in optimization by decomposition. The techniques constitute improvements of a previously published algorithm for two-level Optimization by Linear Decomposition (OLD). The OLD algorithm has been summarized and illustrated by an application example to show how the new techniques are implemented by local modifications in that algorithm. The resulting two alternative formulations improve OLD by removing the potential for numerical difficulties that occasionally were caused in the original algorithm by an explicit handling of the equality constraints which constituted the key coupling between optimizations at two levels. Both alternative formulations eliminate the explicit presence of these constraints while satisfying them indirectly. The alternative formulations allow the use of a variant of the optimum sensitivity analysis that does not require second derivatives of behavior and, therefore, is computationally less expensive than the variant used in the original algorithm. Thus, an additional benefit expected from the modified algorithm is a reduction of its overall computational cost. It is shown that the improved algorithm may be generalized to multidisciplinary system applications.

References

1. Sobieszczanski-Sobieski, J.: A Linear Decomposition Method for Large Optimization Problems—Blueprint for Development. NASA TM 83248, February 1982.
2. Sobieszczanski-Sobieski, J.; Barthelemy, J. F.; and Riley, K. M.: Sensitivity of Optimum Solutions to Problem Parameters. AIAA J., Vol. 21, Sept. 1982, pp. 1291–1299.
3. Barthelemy, J. F.; and Sobieszczanski-Sobieski, J.: Optimum Sensitivity Derivatives of Objective Functions in Nonlinear Programing. AIAA J., Vol. 22, No. 6, June 1983, pp. 913–915.
4. Sobieszczanski-Sobieski, J.; James, B. B.; and Dovi, A. R.: Structural Optimization by Multilevel Decomposition. AIAA J., Vol. 23, No. 11, November 1985, pp. 1775–1782.
5. Sobieszczanski-Sobieski, J.; James, B. B.; and Riley, M. F.: Structural Sizing by Generalized, Multilevel Optimization. AIAA J., Vol. 25, No. 1, January 1987, p. 139.
6. Wrenn, G. A.; and Dovi, A. R.: Multilevel Decomposition Approach to the Preliminary Sizing of a Transport Aircraft Wing. AIAA Journal of Aircraft, Vol. 25, No. 7, July 1988, pp. 632–638.
7. Zeiler, T. A.; and Gilbert, M. G.: Integrated Control/Structure Optimization by Multilevel Decomposition. NASA TM 102619, March 1990.

8. Tharcja, R.; and Haftka, R. T.: Numerical Difficulties Associated with Using Equality Constraints to Achieve Multi-level Decomposition in Structural Optimization. AIAA Paper No. 86-0854, AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics, and Materials Conference, San Antonio, Texas, May 1986.

9. Sobieszczanski-Sobieski, J.: A Technique for Locating Function Roots and Satisfying Equality Constraints in Optimization. NASA TM-104037, January 1991.

10. Vanderplaats, G.: Newsletter of VMA-Engineering, May 1990.

11. Kreisselmeier, G.; and Steinhauser, R.: Application of Vector Performance Optimization to a Robust Control Loop Design for a Fighter Aircraft. International Journal of Control, Vol. 37, No. 2, 1983, pp. 251–284.

12. Schmit, L. A.; and Ramanathan, R. K.: Multilevel Approach to Minimum Weight Design Including Buckling Constraints. AIAA J., Vol. 16, Feb. 1973, pp. 97–104.

Table 1. Input and Output of Assembled Framework (System Level) Analysis

	Input
A_i, I_i	cross-sectional area and moment of inertia for ith beam ($i = 1 \ldots NE$, the number of elements in the example is $NE = 3$), both assumed constant along the beam length
$\{SSC\}$	vector of the structure shape coordinates in a reference coordinate system that defines locations of the frame support and corner points
$\{Q\}$	vector of loads Q_j, $j = 1 \ldots NDOF$, applied coincident with the structure unsupported degrees of freedom whose number is $NDOF$
E	the material Young's modulus
	Output
$\{U\}$	vector of displacements U_k, $k = 1 \ldots NDOF$
$\{N\}_i$, $i = 1 \ldots NE$	vector of the end-forces on the beam, at each end there are three such forces: axial force, transverse force, and bending moment
L_i	length of the ith beam

A_i, I_i, and E as an input passed through the analysis to output;

Derivatives: $D(U, A)$, $D(U, I)$, $D(U, SSC)$, $D(N, A)$, $D(N, I)$, $D(N, SSC)$, $D(L, SSC)$

Table 2. Input and Output of ith Beam (Subsystem Level) Analysis

	Input
NAL and SAL	normal and shear allowable stresses, respectively
Input selected from the Output in table 1:	
Ni, $i = 1 \ldots$NE	vector of the end-forces on the beam
L_i	length of the ith beam
A_i, I_i, and E;	
$\{Y\}_i$	vector of the beam cross-sectional dimensions Y_k, $k = 1 \ldots \mathrm{NY}_i$, shown in the inset, Fig. 1.
	Output
$\{SN\}_i$ and $\{SS\}_i$	vectors of the normal and shear stresses, respectively, at judiciously chosen points on the end cross sections
$\{UB\}_i$	beam displacements in the beam local coordinate system
$\{NCR\}_i$ and $\{SCR\}_i$	vectors of the normal and shear critical stresses, respectively, for evaluation of the beam column and local buckling constraints

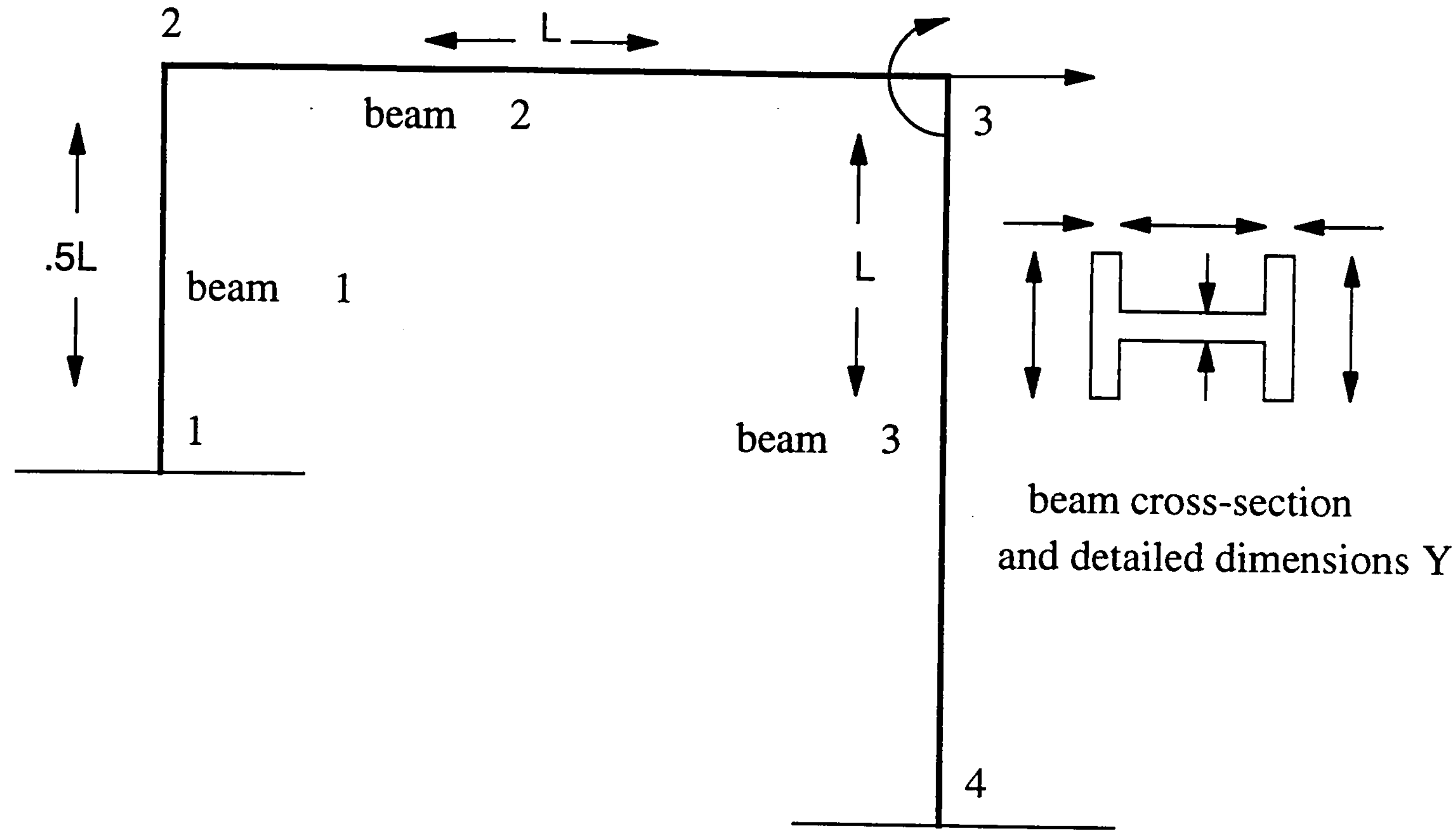

Fig.1 A portal framework.

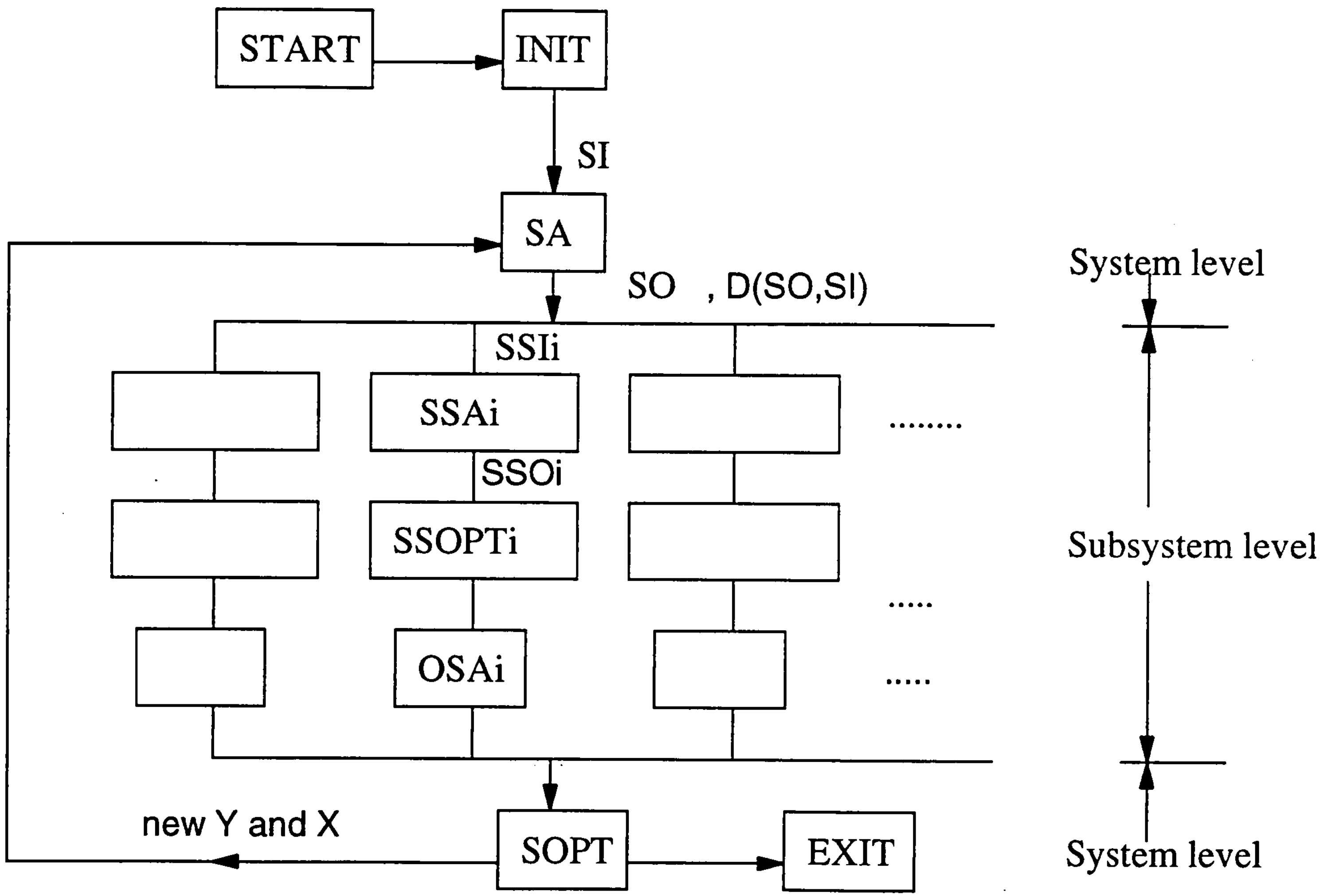

Fig.2 Flowchart of the generic procedure.

NASA

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-104036		

4. Title and Subtitle	5. Report Date
Two Alternative Ways for Solving the Coordination Problem in Multilevel Optimization	August 1991
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Jaroslaw Sobieszczanski-Sobieski	
	10. Work Unit No.
	505-63-36
9. Performing Organization Name and Address	11. Contract or Grant No.
NASA Langley Research Center Hampton, VA 23665	
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
National Aeronautics and Space Administration Washington, DC 20546-0001	Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The paper describes two new techniques for formulating the coupling between levels in multilevel optimization by linear decomposition, proposed as improvements over the original formulation, now several years old, that relied on explicit equality constraints which were shown by application experience as occasionally causing numerical difficulties. The two new techniques represent the coupling without using explicit equality constraints, thus avoiding the above difficulties and also reducing computational cost of the procedure. The old and the new formulations are presented in detail, illustrated by an example of a structural optimization. A generic version of the improved algorithm is also developed for applications to multidisciplinary system not limited to structures.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
cumulative constraint optimization geometrical	Unclassified - Unlimited Subject Category - 05

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	16	A03

NASA FORM 1626 OCT 86