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ENHANCEMENT OF SURFACE DEFINITION AND GRIDDING IN THE EAGLE CODE

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NSF Engineering Research Center
for Computational Field Simulation
of
Mississippi State University
P. O. Drawer 6176
Mississippi State, MS 39762

ENGINEERING RESEARCH CENTER FOR
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COMPLEX GEOMETRY / COMPLEX PHYSICS

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Joe F. Thompson, Director
NSF Engineering Research Center
for Computational Field Simulation
Mississippi State University
P. O. Box 6176
Mississippi State, MS 39762
(601) 325–8278
FAX: (601) 325–7692
email: joe@erc.msstate.edu

Abstract

A method for locally fairing uniform cubic B–Spline curves and surfaces will be presented. The algorithm uses an automated technique which detects undesirable geometric characteristics by using a local fairness criterion. The geometric entity is then smoothed using a method based on the repeated removal and insertion of the spline knots in the vicinity of the geometric irregularity.

1. Fairness

In order to determine what constitutes an error in the representation of a curve or surface, a means of judging its integrity must be defined. An accurate representation of a geometry should exhibit discontinuities only at locations intended by the designer and nowhere else. This is to say that a curve or surface should be fair in regions absent of intended geometric discontinuities. Defining exactly what constitutes a fair curve is quite subjective; however, Dill [5] defines a fair curve as having a curvature plot consisting of smoothly varying monotone segments. A similar definition is also offered by Farin [3], and by Su and Liu [6].

Using the aforementioned definition of fairness, a method must now be formulated by which the geometry can be interrogated to determined regions needing to be smoothed. Several means of interrogation have been suggested. For detection of irregularities in surfaces, Kaufmann and Klass [7] use reflection lines. Beck [8] uses a variety of methods including contour plots, shaded images, and maps of principal curvature. Hoschek [9] uses k–orthotomic curves for interrogation of planar curves, and Renz [10] uses second divided differences. Farin suggests the use of curvature plots (Refs. [1], [2], and [3]) since they are highly sensitive to changes in curve shape and they allow easy detection and location of inflection points. However, signed curvature applies only to planar curves and is, by definition, nonnegative for space curves. As a more general approach, third derivatives may be used Ref [4].

In the present work, third derivatives will be used as the basis for judging the degree of fairness of a curve. This quantity will be suitable for both planar and space curves and is quite sensitive to small changes in curve shape. A separate criterion for surfaces will not be defined since smoothing of a surface will be accomplished by smoothing the curve net defining the surface.

Since a fair curve should be composed of segments with smoothly varying third derivatives, location of third–derivative discontinuities will provide a means of determining where smoothing is needed. One method of locating discontinuities is to calculate the left and right hand limits of third derivatives at each spline knot and then compare them. This will be the basis for a quantity known as *local fairness* and leads to the following definition:

Definition: Let x(t) be a C^2 parametric cubic piecewise curve with t as the global parameter. The the local fairness ϵ is defined as

$$\epsilon = \| \mathbf{x}'''(t_+) - \mathbf{x}'''(t_-) \| .$$

Note that ϵ is a local quantity since it may vary with the parameter value of the parameter t. It is reasonable to say that the point most in need of smoothing is the point with the largest value of ϵ. A question now arises concerning how the local fairness is to be calculated. Since the present work limits the smoothing algorithm to the treatment of B–Spline curves, use will be made of the B–Spline basis functions. However, it should be noted that the calculation of local fairness is only necessary at the spline knots. This is due to the fact that each B–Spline segment is a polynomial and therefore is differentiable an infinite number of times. Hence, derivative discontinuities can only exist at points where spline segments meet (the spline knots).

2. Curve Fairing

In order to fair or smooth at a particular spline knot, a method must be developed which is local in nature. That is, when the method is applied, it only affects the curve in a small region surrounding the point which was smoothed. One such method is proposed by Farin [3]. First, the local fairness is calculated at each knot in the spline; then the knot with the largest value of ϵ is chosen as the knot at which smoothing will take place. Let this knot be the knot associated with the B–Spline control point $\mathbf{d}_i$. The knot is then removed from the knot sequence and a new location $\overline{\mathbf{d}}_i$ for the control point $\mathbf{d}_i$ is calculated. The knot is then reinserted into the knot sequence so that the number of spline segments remains the same as in the original curve.

The criteria for the selection of a new location for the control point $\mathbf{d}_i$ will be third derivative continuity at the knot. Mathematically, this amounts to equating the left and right hand limits of the third derivative at the spline knot (e.g. driving ϵ to zero). The left and right hand limits of the third derivative of a B–Spline curve at the ith knot are given by

$$\mathbf{P}_i^{\prime\prime\prime}(0) = -\mathbf{d}_{i-1} + 3\mathbf{d}_i - 3\mathbf{d}_{i+1} + \mathbf{d}_{i+2}, \tag{1}$$

$$\mathbf{P}_{i-1}^{\prime\prime\prime}(1) = -\mathbf{d}_{i-2} + 3\mathbf{d}_{i-1} - 3\mathbf{d}_i + \mathbf{d}_{i+1}. \tag{2}$$

Setting Equation (1) equal to E0zquation (2) and solving for $\mathbf{d}_i$ the new location for $\mathbf{d}_i$, $\overline{\mathbf{d}}_i$ is given by

$$\overline{\mathbf{d}}_i = \frac{1}{2} l_i + \frac{1}{2} r_i, \tag{3}$$

with the points $\mathbf{l}_i$ and $\mathbf{r}_i$ given by

$$\mathbf{l}_i = \frac{4}{3}\mathbf{d}_{i-1} - \frac{1}{3}\mathbf{d}_{i-2}, \tag{4}$$

$$\mathbf{r}_i = \frac{4}{3}\mathbf{d}_{i+1} - \frac{1}{3}\mathbf{d}_{i+2}. \tag{5}$$

Figure 1 illustrates the process given by Equations (3) through (5).

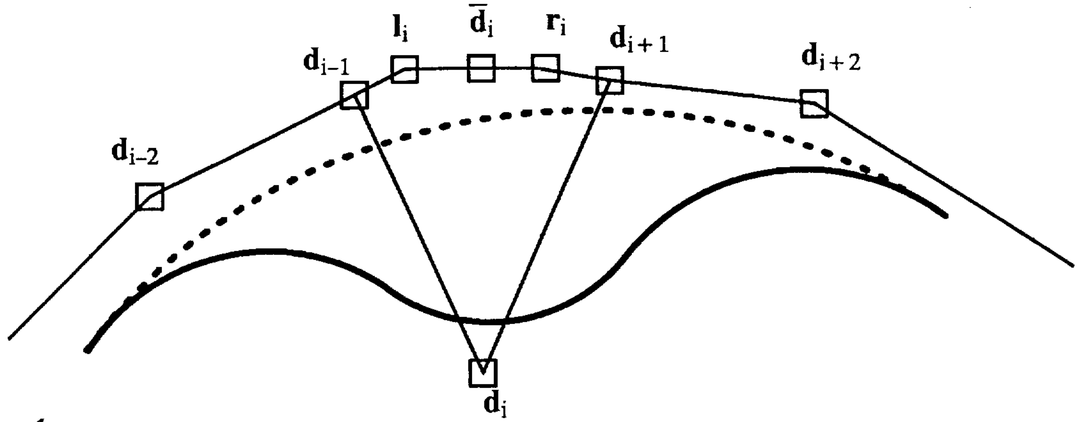

Figure 1.

Because movement of the B–Spline control vertices results in changing the shape of the original curve, measures must be taken to assure that the original geometry is not disturbed beyond some working tolerance. This is accomplished by storing the original curve and measuring the shape perturbation due to the smoothing process. If at a given knot, the new location of the control vertex is farther away than problem tolerances allow, the vertex is moved in the direction of the new location, but the distance is constrained by the prescribed tolerance.

4. Surface Fairing

Fairing of surfaces is accomplished by using a tensor product method. This amounts to smoothing the curve net that defines the surface. Curves are functions of only one parameter, u. However, surfaces are functions of two parameters, u and v. In the tensor product approach, each curve of constant v is smoothed and the results stored. Then, using the result of the first smoothing pass, each curve of constant u is smoothed. This procedure constitutes one smoothing pass for a surface. The resulting surface will be smoother than the original. The tensor product method is discussed in detail in Ref. [3], and its application to surface smoothing is discussed in Refs. [2] and [3].

5. Results

The smoothing algorithms discussed here have been applied to both curves and surfaces. The following figures illustrate effects of smoothing on shape and geometrical qualities.

Figures 2 through 4 are of a NACA 0012 symmetrical airfoil. As illustrated by Figure 2, the results of smoothing are, at best, very difficult to detect from a distance. However, the zoomed view provided in Figure 3 provides a better picture of the shape change due to smoothing. Figure 4 is a comparison of signed curvature calculations on the original and smoothed curves. This is a good illustration of how virtually imperceptible changes in the curve geometry can result in dramatic changes in the character of its derivatives.

Figures 5 through 8 are of a waisted body configuration. Figure 5 is, again, a plot of the B–Spline control polygon for the geometry before and after smoothing. Figure 6 is a comparison of curvature plots for the smoothed and unsmoothed data. Figure 7 shows the effect of smoothing on the metric coefficient η_y. Metric coefficients are important to flow calculations provided by CFD codes. Perturbation of the geometry due to smoothing is also an important consideration. Figure 8 provides some indication of the change in shape due to smoothing. The point displacement values are determined by measuring the magnitude of the distance each point on the smoothed geometry is from its corresponding point on the original. The values for point displacement are given in percent of total arclength of the curve defining the body.

The next set of figures provides some idea of the problems associated with digitized data. Figure 9 is a curve produced from digitized F–15 fighter aircraft surface data. Rough spots on the curve are easily visible; in addition, the curvature plot in Figure 11 reveals large discontinuities in the third derivative. Figure 10 is the same curve after the smoothing algorithm has been applied. Not all points on the curve were smoothed, only those chosen by the automatic interrogation algorithm. The dark curve in Figure 11 is a plot of curvature after smoothing. As can be seen by comparison of both the curves and the curvature plots, the smoothed curve is a much more aesthetic entity.

Surface smoothing capability is illustrated by Figures 12 and 13. Figure 12 is a plot of are F–15 fighter aircraft fuselage. The data for this surface was digitized. Figure 13 is the same surface after the smoothing algorithm has been applied. In this case, no tolerances on point movement were set. This was to allow the results the smoothing process to be readily visible. In practice, however, this degree of geometry perturbation would almost never be allowed.

Conclusion

Experience has shown that this algorithm works well and is quite robust. In addition, it has shown promise for smoothing sets of data points in addition to the B–spline control nets for which it was formulated. However, this has yet to be proved formally and is based purely on experience and conjecture on the part of the author. This algorithm will now be incorporated into the EAGLE grid generation system.

References:

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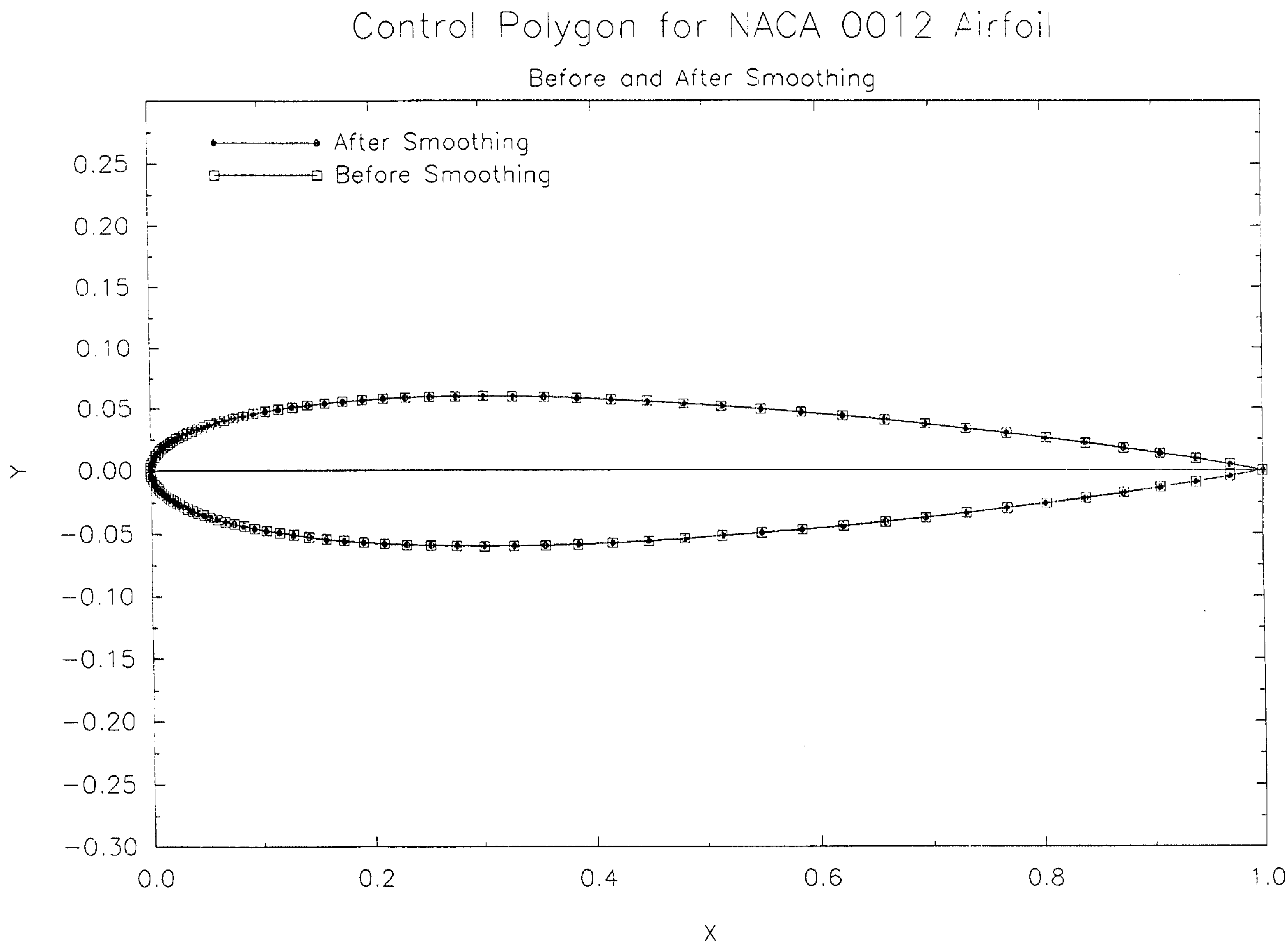
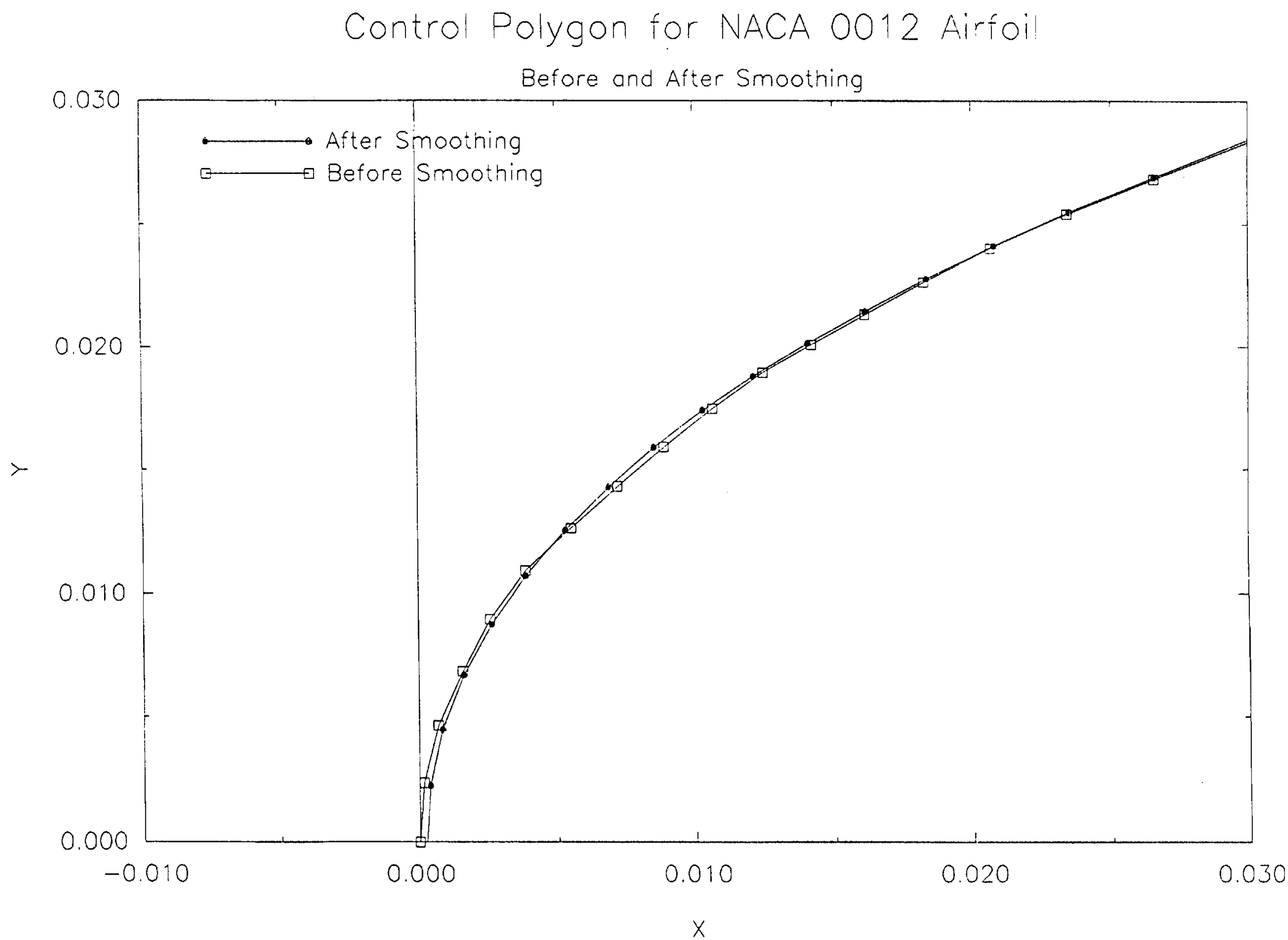

Control Polygon for NACA 0012 Airfoil
Before and After Smoothing
After Smoothing
Before Smoothing
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
Y
0.0
0.2
0.4
0.6
0.8
1.0
X

Figure 2



Control Polygon for NACA 0012 Airfoil
Before and After Smoothing
After Smoothing
Before Smoothing
0.030
0.020
0.010
0.000
Y
−0.010
0.000
0.010
0.020
0.030
X

Figure 3

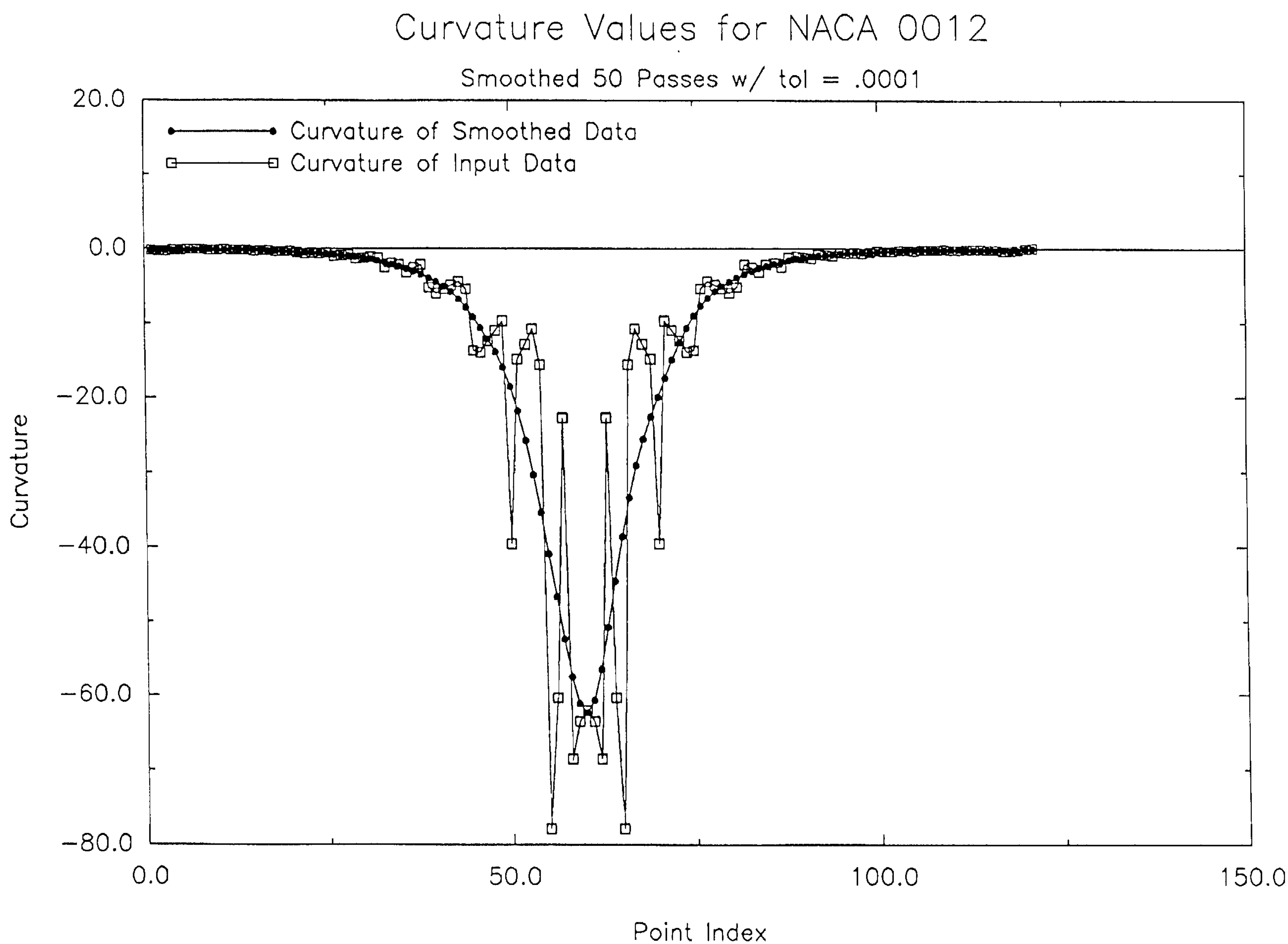

Curvature Values for NACA 0012
Smoothed 50 Passes w/ tol = .0001
Curvature of Smoothed Data
Curvature of Input Data
20.0
0.0
-20.0
-40.0
-60.0
-80.0
Curvature
0.0
50.0
100.0
150.0
Point Index

Figure 4

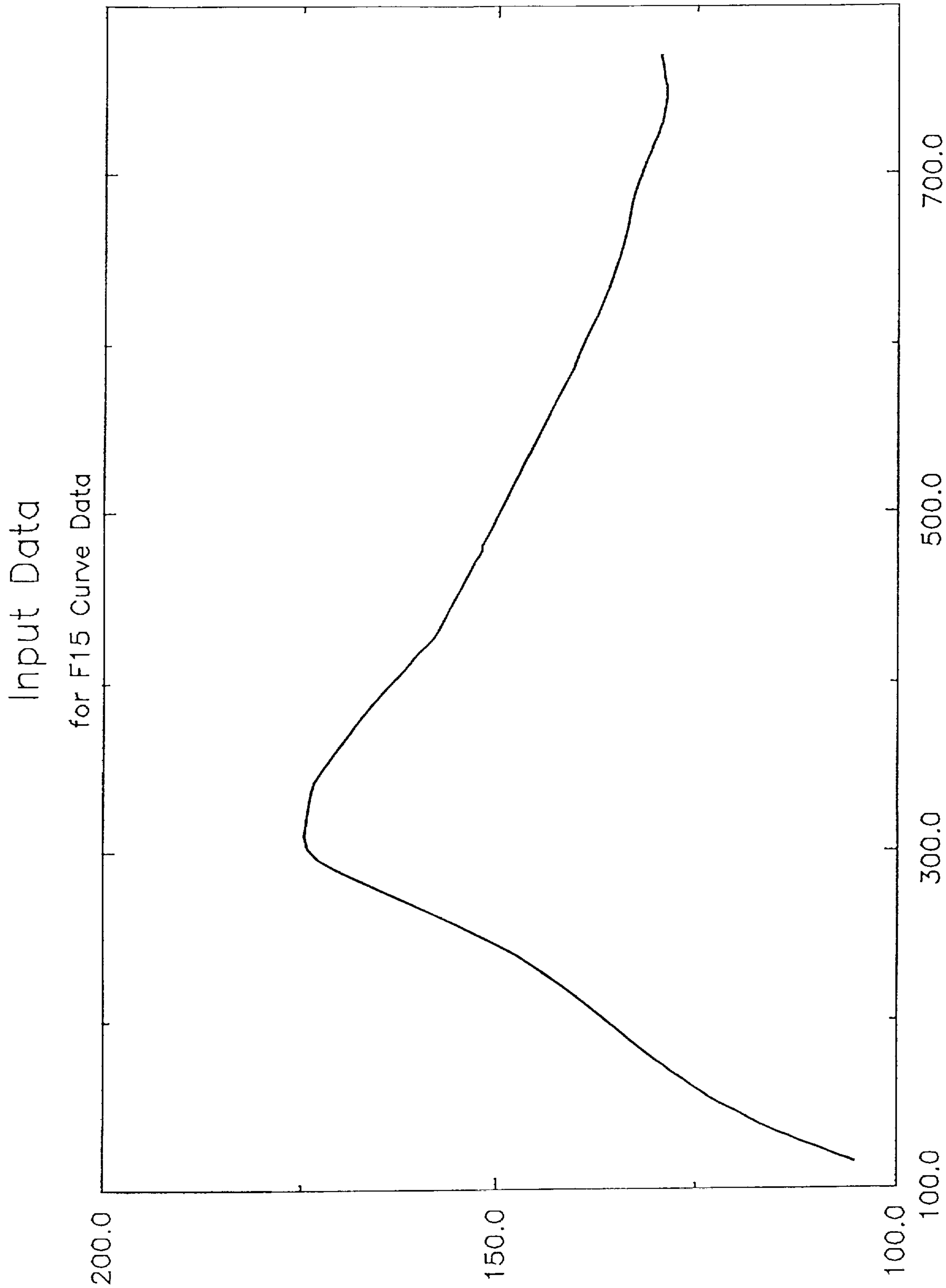


Figure 9

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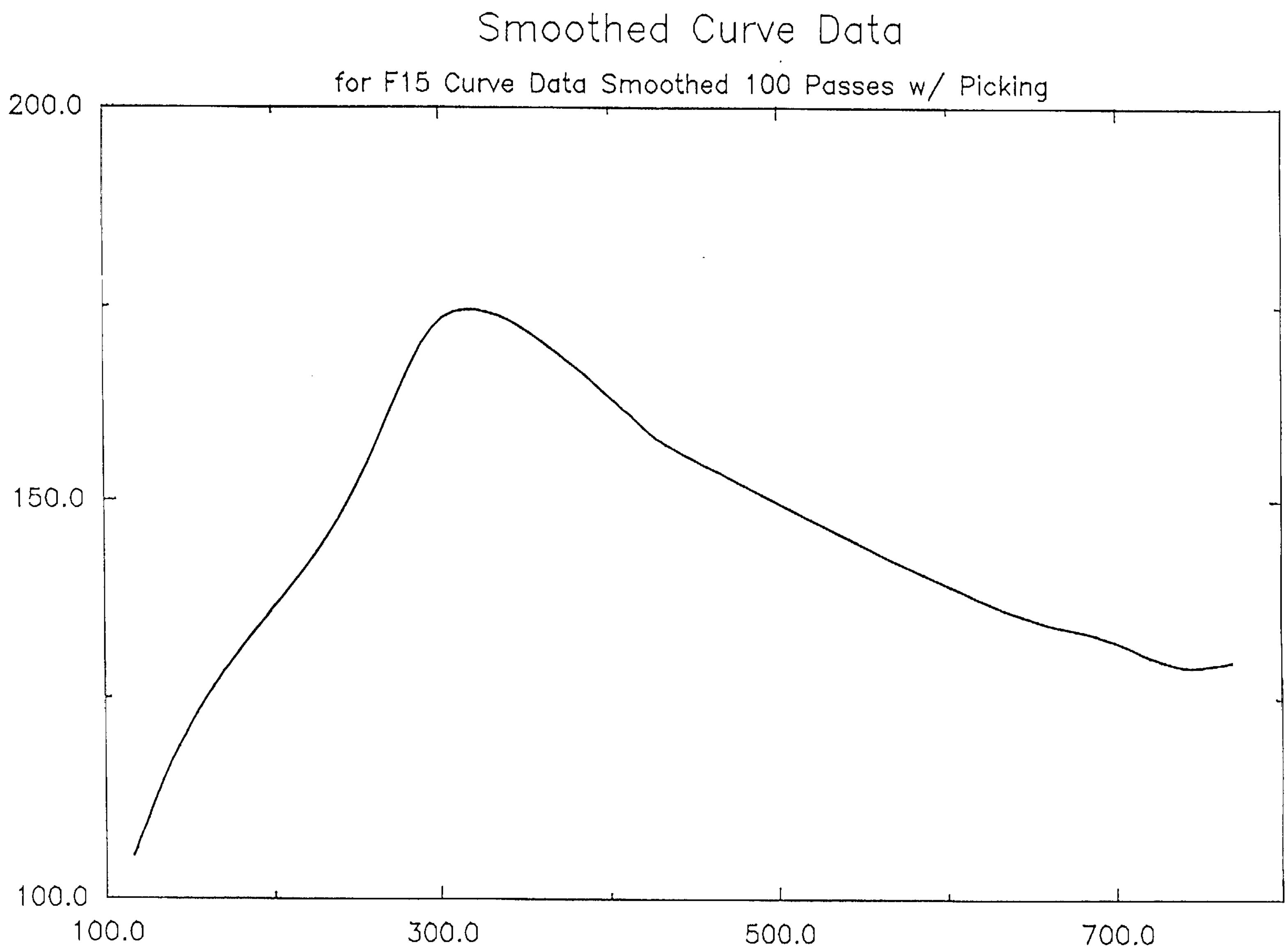
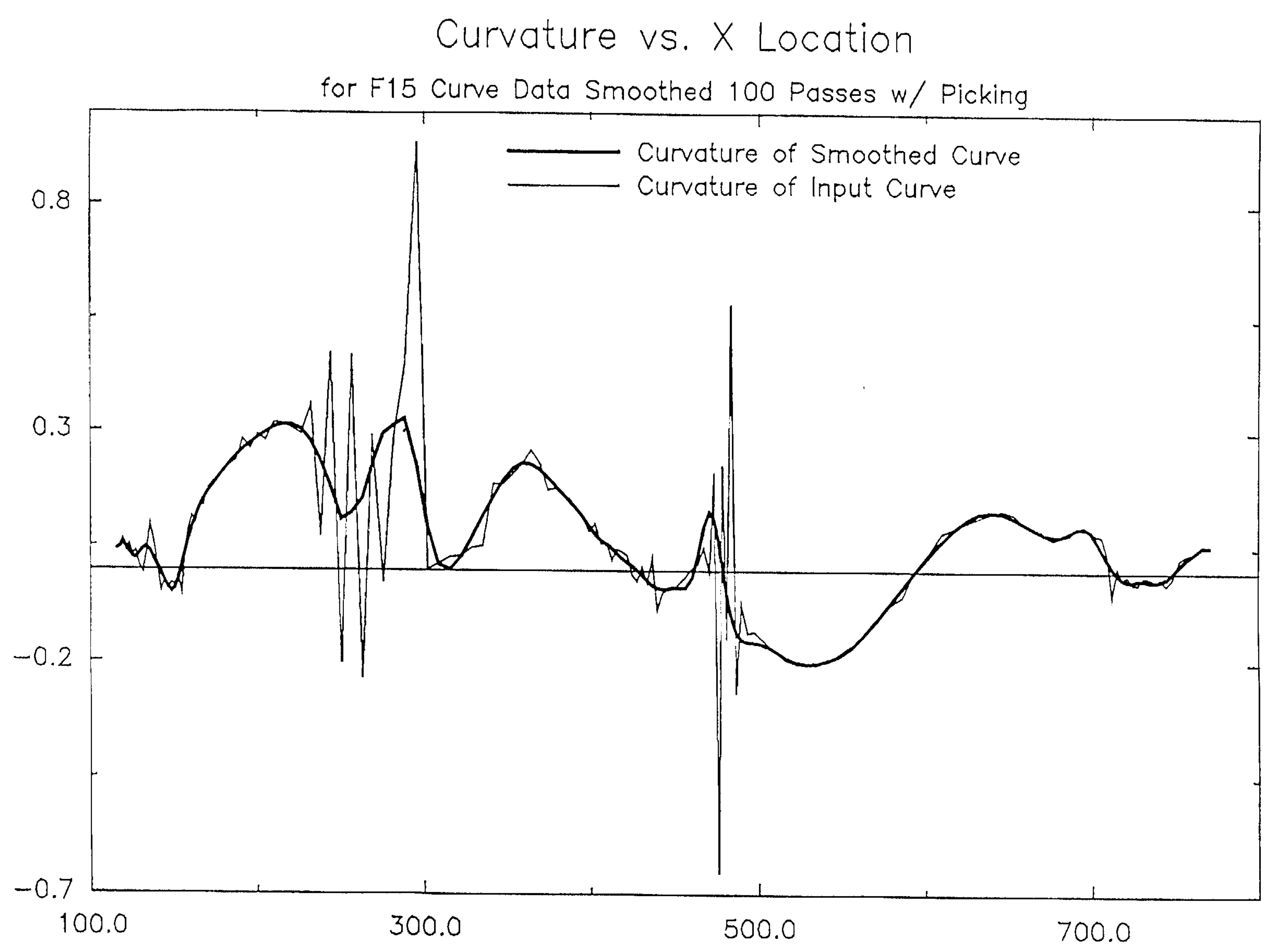

Smoothed Curve Data
for F15 Curve Data Smoothed 100 Passes w/ Picking
200.0
150.0
100.0
100.0
300.0
500.0
700.0

Figure 10



Curvature vs. X Location
for F15 Curve Data Smoothed 100 Passes w/ Picking
Curvature of Smoothed Curve
Curvature of Input Curve
0.8
0.3
-0.2
-0.7
100.0
300.0
500.0
700.0

Figure 11

Original Surface Geometry

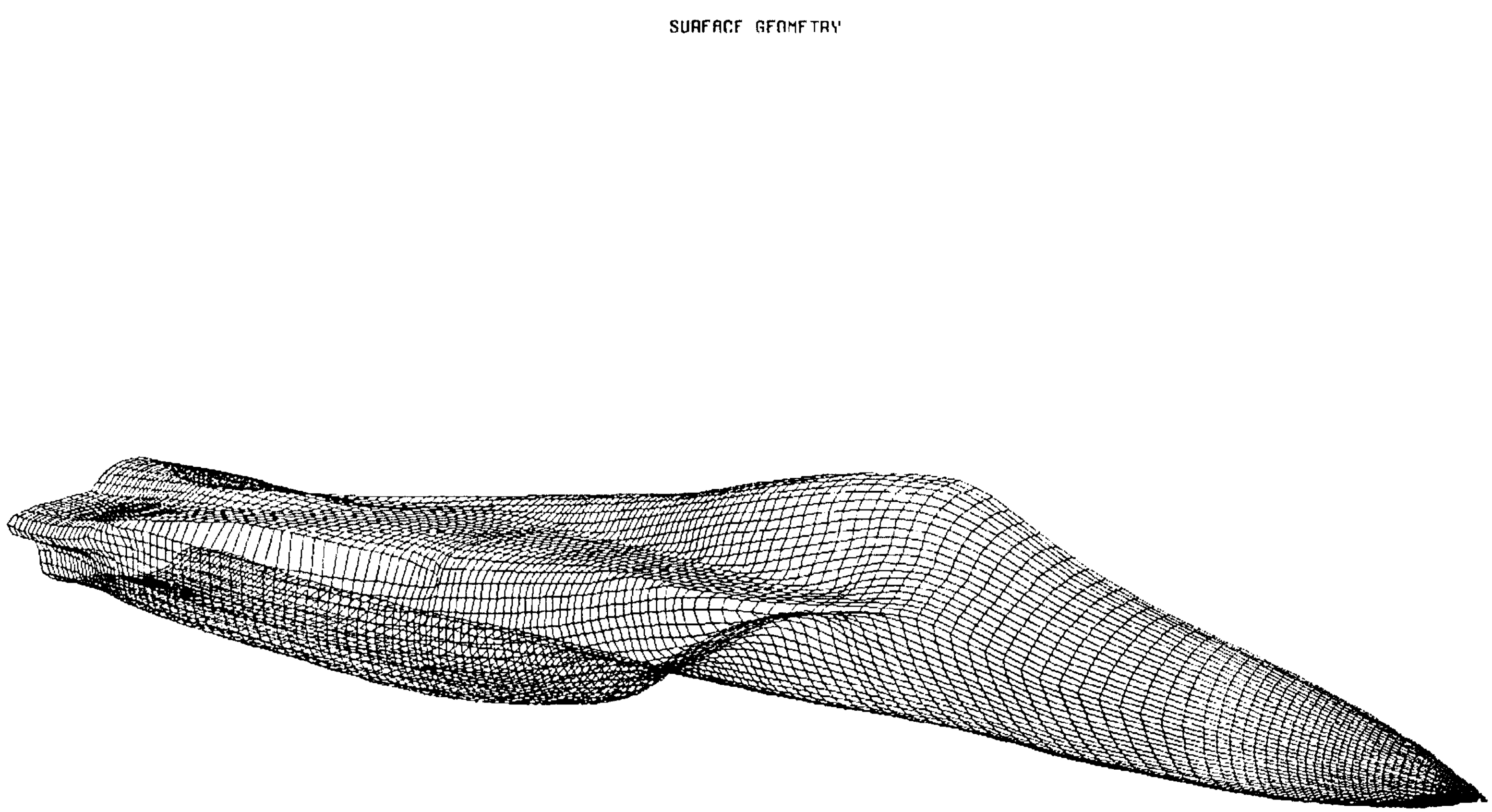


Figure 12

Surface Geometry After 10 Smoothing Passes

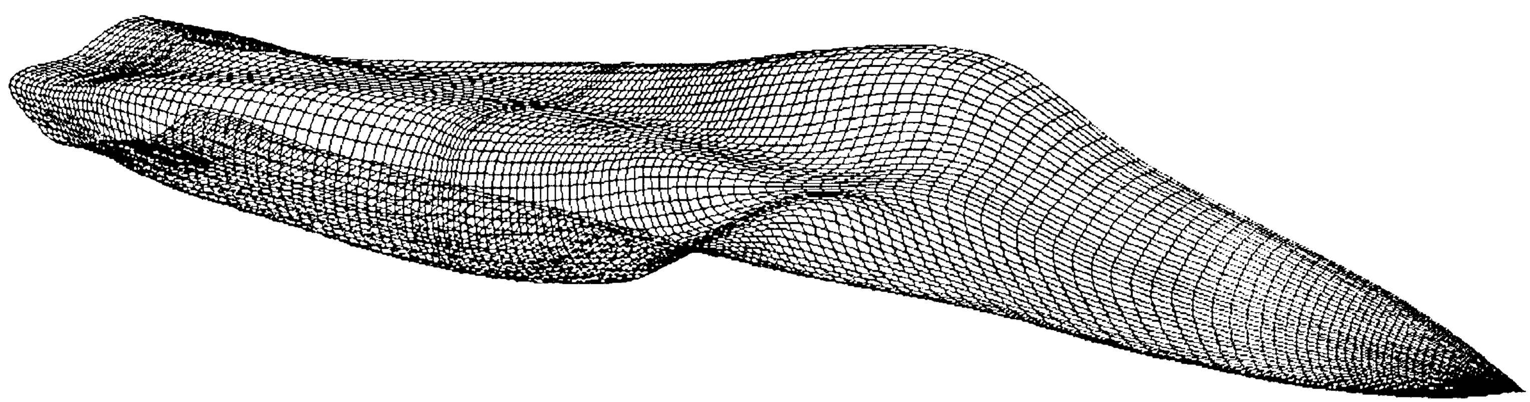


Figure 13