IN-39
46455
p. 27

NASA CONTRACTOR REPORT 187599

SIMPLIFIED MICROMECHANICAL EQUATIONS FOR THERMAL RESIDUAL STRESS ANALYSIS OF COATED FIBER COMPOSITES

(NASA-CR-187599) SIMPLIFIED MICROMECHANICAL EQUATIONS FOR THERMAL RESIDUAL STRESS ANALYSIS OF COATED FIBER COMPOSITES (Analytical Services and Materials) 27 p CSCL 20K G3/39 N91-32527 Unclas 0046455

R. A. Naik

Analytical Services and Materials, Inc.

Hampton, VA

Contract NASI-18599

SEPTEMBER 1991

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

SIMPLIFIED MICROMECHANICAL EQUATIONS FOR THERMAL RESIDUAL STRESS ANALYSIS OF COATED FIBER COMPOSITES

R. A. Naik

Analytical Services & Materials, Inc.

Hampton, Virginia.

ABSTRACT

The fabrication of metal matrix composites poses unique problems to the materials engineer. The large thermal expansion coefficient (CTE) mismatch between the fiber and matrix leads to high tensile residual stresses at the fiber/matrix (F/M) interface which could lead to premature matrix cracking during cooldown. Fiber coatings could be used to reduce thermal residual stresses. A simple closed form analysis, based on a three phase composite cylinder model, was developed to calculate thermal residual stresses in a fiber/interphase/matrix system. Parametric studies showed that the tensile thermal residual stresses at the F/M interface were very sensitive to the CTE and thickness of the interphase layer. The modulus of the layer had only a moderate effect on tensile residual stresses. For a silicon carbide/titanium aluminide composite the tangential stresses were 20-30% larger than the axial stresses, over a wide range of interphase layer properties, indicating a tendency to form radial matrix cracks during cooldown. Guidelines, in the form of simple equations, for the selection of appropriate material properties of the fiber coating were also derived in order to minimize thermal residual stresses in the matrix during fabrication.

Key Words: composite cylinder, residual stresses, micromechanics, interphase, fiber coatings.

INTRODUCTION

Continuously reinforced metal matrix composites are prime candidates for high temperature aerospace applications. However, the fabrication of these composites poses some unique challenges. The large coefficient of thermal expansion (CTE) mismatch between the fiber and matrix leads to high tensile residual stresses at the fiber/matrix (F/M) interface [1]. Also, reactivity between the fiber and matrix could lead to the formation of brittle reaction products [2] at the F/M interface. For example, in silicon carbide/titanium composites, the fibers are usually coated with a few layers of carbon which act as a barrier between the fiber and the titanium matrix. Besides preventing interdiffusion at the fiber/matrix interface, the carbon layers also lead to a weak interface. A weaker interface, in brittle matrix composites, improves the overall composite strength by deflecting matrix cracks along the fiber/matrix interface, thus, reducing the severity of the cracks and leading to a more stable failure mode [2]. The carbon coating may also affect the residual stresses at the interface. Tensile hoop stresses at the interface could promote matrix cracking, however, compressive radial stresses provide mechanical bonding between the fiber and the matrix [2]. During high temperature operation, the carbon coating on the fibers could be eroded leading to the formation of brittle reaction products (eg. titanium carbides and silicides) at the fiber/matrix interface [2]. These lead to a stronger, more brittle interface with higher thermal residual stresses and do not allow matrix crack deflection along the interface causing lower composite strength.

In titanium aluminide composites, the high residual stresses coupled with the low ductility of the matrix can lead to premature cracking at the F/M interface during processing. Such cracking could, upon exposure to thermal cycling, be exacerbated by oxidation along crack faces and limit the useful life of the material. To minimize matrix cracking during processing, additional metallic fiber coatings are introduced prior to consolidation of the composite [3]. These coatings serve to limit interdiffusion of the reactive elements and act as a compliant interphase layer between the matrix and the fiber [3].

The choice of a coating material for a given F/M combination will first have to be based on its ability to act as a chemical barrier between the fiber and matrix. However, an ideal coating should, besides inhibiting F/M reactions, also lead to the formation of an interphase layer between the fiber and the matrix that causes a reduction in thermal residual stresses at the F/M interface. Thermal residual stress analysis of a fiber/interphase layer/matrix composite would be very useful to the materials engineer in the selection of such a fiber coating material.

An elastic analysis of a three phase composite (fiber/interphase layer/matrix) was first developed by Walpole [4] who demonstrated the pronounced effect of a thin fiber coating on the matrix stresses. Similar three-phase composite cylinder models were also developed by Mikata and Taya [5], Nairn [6], Pagano and Tandon [7], Sideridis [8], Benveniste, et al [9], and Arnold, et al [10] to study the effective properties and stress fields in coated continuous fiber composites under thermo-mechanical loadings. Broutman and Agarwal [11] and Caruso, et al [12] used a finite element model to study the effects of an interfacial layer on composite properties and stresses. The effects of specific interphase layers and matrix particles at the F/M interface in a titanium aluminide composite made from a powder blend were investigated in Ref. 13 using a finite element analysis. These analyses are rigorous but are also very complex and do not lend themselves to simple calculations of thermal residual stresses in a coated fiber composite.

The objectives of the present study were, first, to develop simple micromechanics equations for calculating thermal residual stresses in a fiber/interphase layer/matrix combination; and second, to develop simple guidelines that can be used to select candidate fiber coating materials to reduce thermal residual stresses at the F/M interface.

The derivation of the simplified equations for thermal residual stresses in a coated fiber composite is presented first. Next, a parametric study is presented to compare the influence of different interphase layer properties on residual stresses. Finally, simple equations are presented as guidelines for selecting fiber coating materials that could reduce thermal residual stresses.

NOMENCLATURE

A	material constant given by $[(1+\nu)(1-2\nu)/E]$, GPa^{-1}
B	material constant given by $[(1+\nu)/E]$, GPa^{-1}
B_3, C_3	constants defined in equation (18)
C_1^i, C_2^i	constants used to describe the inplane stresses in cylinder i
E	Young's modulus, GPa
F	material constant given by $(A_f + B_L)$, GPa^{-1}
G	material constant given by $(A_f - A_L)$, GPa^{-1}
K	geometric constant given by (R_L^2/R_f^2)
r	coordinate along the radial direction, m
R	outer radius of a layer, m
t_L	thickness of the interphase layer, m
T_{ij}	thermal parameter given by $\Delta T\ [(1+\nu_i)\ \alpha_i - (1+\nu_j)\ \alpha_j]$
u	radial displacement, m

V_f	fiber volume fraction, (R_f^2/R_m^2)
α	coefficient of thermal expansion, m/m/°C
ΔT	change in temperature, °C
$\Delta\sigma$	percentage reduction in $\sigma_{\theta\theta}^{mL}$ compared to the no layer case
ϵ	strain, m/m
ϵ_a	strain in the fiber axial direction, m/m
ν	Poisson's ratio
σ	stress, MPa

Subscripts/Superscripts

f	fiber
L	interphase layer
m	matrix
mL	matrix and interphase or matrix/interphase interface
rr	radial
zz	axial
$\theta\theta$	tangential

COMPOSITE CYLINDER MODEL

A three phase composite cylinder model is used in the present study to discretely model the fiber, interphase layer and matrix (Fig. 1). Each of the three layers was treated as a homogeneous isotropic medium. The material constants and outer radii for the fiber, layer, and matrix are distinguished by subscripts f, L, and, m, respectively. The outer radius of the matrix (R_m) was chosen to achieve the appropriate fiber volume fraction, $V_f = R_f^2/R_m^2$. A dimensionless parameter K was defined as the ratio (R_L^2/R_f^2) and was used in the analysis. A cylindrical coordinate system centered on the fiber with the z-axis along the longitudinal fiber direction was used in the analysis. Axisymmetry was imposed for the stresses and the displacements in all the cylinders. In the fiber axial direction, a state of generalized plane strain was imposed. Thus, the axial strain ϵ_{zz} was assumed to be constant for all the three regions. Loading was limited to uniform thermal loads. The formulation of the boundary value problem is first presented, followed by (i) an exact implicit solution and (ii) an approximate closed form solution to the equations.

Formulation

The general solution to the plane strain axisymmetric elasticity problem of a thick cylinder is presented in Ref. 14 and the appropriate stresses in cylindrical coordinates have the following form:

$$\sigma_{rr} = C_1 - \frac{C_2}{r^2} , \qquad \text{and} \qquad \sigma_{\theta\theta} = C_1 + \frac{C_2}{r^2} \tag{1}$$

where C_1 and C_2 are constants. The inplane thermal stresses in each of the three regions are assumed to have these functional forms for the generalized plane strain, axisymmetric problem under consideration here. Within the fiber, the functional forms required to ensure finite stresses are

$$\sigma_{rr} = C_1^f \qquad \text{and} \qquad \sigma_{\theta\theta} = C_1^f \tag{2}$$

In the interphase layer, the stresses have the forms

$$\sigma_{rr} = C_1^L - \frac{C_2^L}{r^2} , \qquad \text{and} \qquad \sigma_{\theta\theta} = C_1^L + \frac{C_2^L}{r^2} \tag{3}$$

and in the matrix region, the stresses are

$$\sigma_{rr} = C_1^m - \frac{C_2^m}{r^2} , \qquad \text{and} \qquad \sigma_{\theta\theta} = C_1^m + \frac{C_2^m}{r^2} \tag{4}$$

Since there are no external forces on the model, the outer surface of the matrix is assumed to be a free boundary and, thus, at $r = R_m$ we have $\sigma_{rr} = 0$. Imposing this condition in Eq. (4) gives

$$C_2^m = R_m^2 \, C_1^m \tag{5}$$

Using the constitutive relations for an isotropic, linear thermoelastic material in cylindrical coordinates, the relations between stresses, strains, and, temperature change (ΔT) are

$$\begin{aligned} \epsilon_{rr} &= (1/E)[\sigma_{rr} - \nu\,\sigma_{\theta\theta} - \nu\,\sigma_{zz}] + \alpha\,\Delta T \\ \epsilon_{\theta\theta} &= (1/E)[-\nu\,\sigma_{rr} + \sigma_{\theta\theta} - \nu\,\sigma_{zz}] + \alpha\,\Delta T \\ \epsilon_{zz} &= (1/E)[-\nu\,\sigma_{rr} - \nu\,\sigma_{\theta\theta} + \sigma_{zz}] + \alpha\,\Delta T = \epsilon_a = \text{constant} \end{aligned} \tag{6}$$

The last equation in Eq. (6) imposes $\epsilon_{zz} = \epsilon_a$ = constant to satisfy the generalized plane strain assumption. In Eq. (6), E is Young's modulus, ν is Poisson's ratio, and, α is the coefficient of thermal expansion.

Using the strain-displacement relation, $\epsilon_{\theta\theta} = \frac{u}{r}$ where u is the radial displacement, together with Eqs. (2)-(6), the appropriate functional forms of the in-plane radial displacements can be obtained for each of the three layers. For the fiber we have

$$u_f = r\,[A_f\,C_1^f - \nu_f\,\epsilon_a + (1 + \nu_f)\,\alpha_f\,\Delta T] \tag{7}$$

where, $$A_f = [(1 + \nu_f)(1 - 2\,\nu_f)] \,/\, E_f \tag{8}$$

The radial displacements in the interphase region are

$$u_L = r\,[A_L\, C_1^L + B_L\,(C_2^L/r^2) - \nu_L\,\epsilon_a + (1 + \nu_L)\,\alpha_L\,\Delta T] \qquad (9)$$

where, A_L is obtained from Eq. (8) by replacing the subscript f by L and

$$B_L = (1 + \nu_L) / E_L \qquad (10)$$

The radial displacements in the matrix region are

$$u_m = r\,[C_1^m(A_m + B_m\,(R_m/r)^2) - \nu_m\,\epsilon_a + (1 + \nu_m)\,\alpha_m\,\Delta T] \qquad (11)$$

where, A_m and B_m are obtained from Eqns. (8) and (10), respectively, by replacing all subscripts by m.

Equilibrium at the interface of the fiber and the interphase region $(\sigma_{rr})_f = (\sigma_{rr})_L$ yields

$$C_1^f = C_1^L - (C_2^L/R_f^2) \qquad (12)$$

Similarly, at the interface of the interphase region and the matrix we have $(\sigma_{rr})_L = (\sigma_{rr})_m$ and using Eqn. (5),

$$C_1^m = [R_L^2\, C_1^L - C_2^L]/[R_L^2 - R_m^2] \qquad (13)$$

Since there are no forces in the fiber axial direction, we have

$$\int_0^{R_f}(\sigma_{zz})_f r\,dr + \int_{R_f}^{R_L}(\sigma_{zz})_L r\,dr + \int_{R_L}^{R_m}(\sigma_{zz})_m r\,dr = 0 \qquad (14)$$

Substituting σ_{zz} for each of the three layers from Eqn. (6) and simplifying using Eqns. (5), (12) and (13) we have

$$A_1 \epsilon_a + B_1 C_1^L + C_1 C_2^L = D_1 \qquad (15)$$

where,

$$A_1 = E_f V_f + E_L V_f (K - 1) + E_m (1 - V_f K)$$

$$B_1 = 2 V_f [\nu_f + \nu_L (K - 1) - \nu_m K]$$

$$C_1 = 2 V_f (\nu_m - \nu_f)/R_f^2$$

$$D_1 = \Delta T \, [E_f \alpha_f V_f + E_L \alpha_L V_f (K - 1) + E_m \alpha_m (1 - V_f K)]$$

Displacement compatibility at the fiber/interphase and the interphase/matrix interfaces is imposed by

$$u_f = u_L \quad \text{at} \quad r = R_f \quad \text{and} \quad u_L = u_m \quad \text{at} \quad r = R_L \qquad (16)$$

Substituting Eqns. (7), (9) and (11) into Eqns. (16) and simplifying using Eqns. (12) and (13) yields

$$(\nu_L - \nu_f) \epsilon_a + G C_1^L + F (-1/R_f^2) C_2^L = T_{Lf} \qquad (17)$$

where,

$$G = A_f - A_L$$

$$F = A_f + B_L$$

$$T_{Lf} = \Delta T \, [(1 + \nu_L) \alpha_L - (1 + \nu_f) \alpha_f]$$

and

$$(\nu_m - \nu_L) \epsilon_a + B_3 C_1^L + C_3 C_2^L = T_{mL} \qquad (18)$$

where

$$B_3 = [K V_f(A_L - A_m) - (A_L + B_m)]/(K V_f - 1)$$

$$C_3 = [K V_f(A_m + B_L) + (B_m - B_L)]/[K R_f^2(K V_f - 1)]$$

$$T_{mL} = \Delta T \, [(1 + \nu_m) \alpha_m - (1 + \nu_L) \alpha_L]$$

The only unknowns in Eqns. (15), (17), and, (18) are ϵ_a, C_1^L, and C_2^L. These three unknowns can be obtained by (i) a direct solution of Eqns. (15), (17) and (18) after substituting appropriate values for the coefficients or (ii) an approximate solution of the three equations leading to a closed form equation for the stresses.

Direct Solution

The Eqns. (15), (17), and (18) form a system of linear equations and can be solved simultaneously. Then the stresses in any of the three layers in the model can be obtained by using Eqns. (2)-(6), (12) and (13). Table I shows a comparison of the cooldown stresses in the matrix at the interface of the matrix and the interphase region for a 2 μm interphase layer (K = 1.057) between the fiber and the matrix. Material properties for the constituents were taken from Table II and are typical of silicon carbide/titanium aluminide metal matrix composites [13]. Compared to the average stresses computed using a finite element analysis, there is less than 1.0% error in the stresses given by the composite cylinder model. The finite element analysis used a repeating square array micromechanics model [1], which accounted for fiber interaction, to compute thermal residual stresses. Due to the interaction of the fibers, the stresses around the matrix/interphase layer interface varied somewhat. The peak $\sigma_{\theta\theta}$ and σ_{zz} stresses, from the finite element analysis, differed by less than 5% (Table I) from the respective average stresses, however, the σ_{rr} stress was more sensitive to fiber interaction and its peak was 32% higher than the average stress. Thus, the composite cylinder analysis, which inherently does not account for fiber interaction, is adequate for calculating $\sigma_{\theta\theta}$ and σ_{zz} but does not provide a good estimate for σ_{rr}.

The direct solution described in this section was used to perform a parametric study to investigate the effects of various interphase properties on thermal residual stresses. The results of the parametric study are presented in a later section.

Approximate Closed-Form Solution

An explicit closed-form solution for the stresses in the model can be obtained by using an approximate indirect method to solve Eqns. (15), (17), and (18). This indirect method involves the solution of Eqns. (17) and (18) by assuming plane strain ($\epsilon_a = 0$). This approximation is based on the assumption that the inplane stresses (σ_{rr} and $\sigma_{\theta\theta}$) are insensitive to the imposition of either plane strain or generalized plane strain conditions along the fiber axial direction. In contrast, the σ_{zz} stress is sensitive to the assumption of the strain state in the axial direction and should be calculated using $\epsilon_a \neq 0$.

For $\epsilon_a = 0$ the two unknowns C_1^L and C_2^L, in Eqns. (17) and (18), can be obtained explicitly as

$$C_1^L = [T_{Lf} C_3 R_f^2 + T_{mL} F]/[G\ C_3 R_f^2 + B_3 F]$$

and (19)

$$C_2^L = R_f^2 [T_{mL} G - T_{Lf} B_3]/[G\ C_3 R_f^2 + B_3 F]$$

The inplane stresses $\sigma_{\theta\theta}^{mL}$ and σ_{rr}^{mL} in the matrix at the matrix/interphase region interface can be obtained in closed form by using Eqns. (4)-(5), (13) and (19) as

$$\sigma_{\theta\theta}^{mL} = \left(\frac{\Delta T\ (1 + K\ V_f)}{V_f K\ (A_m - D) + B_m + D}\right)[C\ (1 + \nu_f)\ \alpha_f + (1 - C)(1 + \nu_L)\ \alpha_L - (1 + \nu_m)\ \alpha_m]$$

and (20)

$$\sigma_{rr}^{mL} = \sigma_{\theta\theta}^{mL}[K\ V_f - 1]/[K\ V_f + 1]$$

where

$$C = [A_L + B_L]/[K\ F - G]$$

$$D = [K\ A_L F + B_L G]/[K\ F - G]$$

An approximate value for ϵ_a can now be obtained by substituting C_1^L and C_2^L from Eqn. (19) into Eqn. (15). The axial stress σ_{zz}^m in the matrix region can then be obtained from Eqns. (4), (5), (6), (13), (15) and (19) as

$$\sigma_{zz}^{m} = E_m(\epsilon_a - \alpha_m \Delta T) + 2\ \nu_m\ \sigma_{rr}^{mL}\ K\ V_f/(K\ V_f - 1) \tag{21}$$

Table I shows stresses, computed using closed form Eqns. (20) and (21), in the matrix at the interface of the matrix and the interphase region for a 2 μm interphase layer (K = 1.057) between the fiber and the matrix. Material properties were taken from Table II. Compared to the average stresses computed using the finite element analysis the closed-form results differ by 5.3%, 4% and 2.5% for the σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} stresses, respectively.

For the case in which there is only fiber and matrix with no interphase layer, Eqn. (20) can be used by substituting K = 1, $A_L = A_m$, $B_L = B_m$ and $\alpha_L = \alpha_m$ to obtain

$$\sigma_{\theta\theta} = \left(\frac{\Delta T\ (1 + V_f)}{V_f(A_m - A_f) + B_m + A_f}\right)[(1 + \nu_f)\ \alpha_f - (1 + \nu_m)\ \alpha_m] \tag{22}$$

The expressions for σ_{rr} and σ_{zz} can be obtained from Eqns. (20) and (21) by substituting K = 1.

The closed-form solutions presented in this section were used to develop simplified guidelines to enable the (i) selection of fiber coating materials that will minimize thermal residual stresses and (ii) calculation of the percentage change in residual stresses due to the presence of an interphase layer at the fiber/matrix interface. These guidelines are presented in a later section.

PARAMETRIC STUDY

The composite cylinder model developed in the previous section provides a simple means to study the sensitivity of thermal residual stresses to the thermo-mechanical properties of the interphase region. The direct solution of Eqns. (15), (17) and (18) was used to the determine the three unknowns ϵ_a, C_1^L, and, C_2^L. The stresses in any of the three regions can then be calculated using Eqns. (2)-(6), (12) and (13). Since the stresses in the matrix would be the largest at the matrix/interphase interface, only interfacial matrix stresses were calculated for the present study. The fiber and the matrix properties given in Table II were used in the parametric study and were kept constant while the interphase region properties were varied. All stresses were normalized by the temperature change, ΔT.

Fig. 2 shows the variation of $\sigma_{\theta\theta}$ and σ_{zz} in the matrix at the matrix/interphase interface for a range of α_L/α_m ratios. Since the σ_{rr} stresses in the matrix are compressive (see Table I), they would not lead to premature matrix cracking during cooldown and were therefore not considered to be important to this study. The $\sigma_{\theta\theta}$ and σ_{zz} stresses, however, are tensile (Table I) and could lead to premature matrix cracking. For an interphase (t_L/R_f) ratio of 0.03 (i.e. 2μm coating on a 140μm diameter fiber), $E_L = E_m$ and $\nu_L = \nu_m$, Fig. 2 shows that both $\sigma_{\theta\theta}$ and σ_{zz} decreased rapidly with increasing α_L.

For the same (t_L/R_f) ratio, Fig. 3 shows that $\sigma_{\theta\theta}$ remained unchanged for (E_L/E_m) ratios greater than one and decreased for (E_L/E_m) ratios less than 0.5. The axial stress σ_{zz} decreased with increasing E_L. The (ν_L/ν_m) ratio had virtually no influence on $\sigma_{\theta\theta}$ and σ_{zz} in the matrix at the matrix/interphase interface as shown in Fig. 4. Note that, in Figs. 2-4, the $\sigma_{\theta\theta}$ stress is larger than the σ_{zz} stress by 20-30% over a wide range of interphase layer properties. Thus, for the present composite, radial matrix cracking would be expected to occur in preference to cracking perpendicular to the fiber during cooldown. Micrographs for the silicon carbide/titanium aluminide composite show evidence of radial matrix cracks after consolidation [15] and confirm this prediction.

Fig. 5 shows that the thickness of the interphase layer (t_L) influenced the interfacial matrix stresses significantly. The $\sigma_{\theta\theta}$ and σ_{zz} decreased rapidly for $\alpha_L/\alpha_m = 2.0$ and $E_L/E_m = 0.5$ with increasing (t_L/R_f). Thus, for a "compliant" interphase layer with a high CTE, there is an advantage in having a thick interphase layer. Fig. 6 shows that the stresses increased moderately for $\alpha_L/\alpha_m = 0.5$ and $E_L/E_m = 2.0$ with increasing layer thickness. Thus, for a "brittle" interphase layer with a high modulus and a low CTE, the thickness of the layer should be as small as possible. In Ref. 2, higher thermal residual stresses were measured after the formation of brittle reaction zones at the F/M interface, as would be expected based on the results in Fig. 6.

In summary, the stresses in the matrix at the matrix/interphase interface were strongly influenced by α_L and t_L and moderately influenced by E_L. A thick interphase layer with a CTE greater than the matrix and a Young's modulus less than that of the matrix will, in general, lead to lower tensile thermal residual stresses. Note that the results in this study are based on an elastic analysis. Plastic yielding of the interphase layer or the matrix could alter some of the trends predicted by this analysis [10].

GUIDELINES FOR INTERPHASE PROPERTIES

As shown in the previous section, the properties of the interphase layer influence the interfacial matrix stresses in a rather complicated way. Simple quantitative guidelines for selecting interphase properties that will result in a reduction of tensile interfacial residual stresses will be very useful to the materials engineer. The closed-form equations (20) and (22) derived in this study lend themselves to the derivation of such simple guidelines.

The change in the interfacial matrix stress $\sigma_{\theta\theta}$ due to the addition of an interphase region can be obtained by subtracting Eqn. (20) from Eqn. (22). The first term in braces {} for both these equations are very similar. It can be shown that for $(t_L/R_f) < 0.08$ and $0.1 < (E_L/E_f) < 0.8$ the first terms in Eqns. (20) and (22) differ by only 3.3%. Thus, for most practical purposes it can be assumed that the first terms in the two equations are equal for t_L and E_L within the constraints specified above. The percentage change ($\Delta\sigma$) in the interfacial matrix stress, $\sigma_{\theta\theta}$, due to the addition of an interphase layer to a fiber/matrix interface is given as:

$$\Delta\sigma = \frac{(1 - C)[(1 + \nu_L)\,\alpha_L - (1 + \nu_f)\,\alpha_f]}{[(1 + \nu_m)\,\alpha_m - (1 + \nu_f)\,\alpha_f]} \times 100 \tag{23}$$

where $$1 - C = \frac{H}{H + (1 - \nu_L)(R_f/t_L)} \tag{24}$$

and $$H = 1 + \frac{E_L\,(1 + \nu_f)(1 - 2\nu_f)}{E_f\,(1 + \nu_L)} \tag{25}$$

Eqns. (23-25) can be used to determine, approximately, the percentage reduction in $\sigma_{\theta\theta}$ for a given set of interphase layer properties. Eqn. (23) could also be rearranged to calculate interphase properties for a desired percentage reduction ($\Delta\sigma$) in $\sigma_{\theta\theta}$.

SUMMARY

The fabrication of metal matrix composites poses unique problems to the materials engineer. The high thermal expansion coefficient (CTE) mismatch between the fiber and matrix leads to high tensile residual stresses at the F/M interface which could lead to premature matrix cracking during cooldown. A simple closed form analysis, based on a three phase composite cylinder model, was developed to calculate thermal residual stresses in a fiber/interphase/matrix system. Parametric studies showed that the tensile thermal residual stresses at the F/M interface were very sensitive to the CTE and thickness of the interphase layer. The modulus of the layer had only a moderate effect on the tensile residual stresses. For a silicon carbide/titanium aluminide composite the tangential stresses were 20-30% larger than the axial stresses, over a wide range of interphase layer properties, indicating a tendency to form radial matrix cracks during cooldown. Guidelines, in the form of simple equations, for the selection of appropriate material properties of the fiber coating were also derived in order to minimize thermal residual stresses in the matrix during fabrication.

REFERENCES

1. A. L. Highsmith, D. Shin and R. A. Naik, "Local Stresses in Metal Matrix Composites Subjected to Thermal and Mechanical Loading," Thermal and Mechanical Behavior of Ceramic and Metal Matrix Composites, ASTM STP 1080, J. M. Kennedy, H. H. Moeller, and W. S. Johnson, Eds., American Society for Testing and Materials, Philadelphia, 1990, pp. 3-19.
2. R. A. Naik, W. D. Pollock, and W. S. Johnson, "Effect of a High Temperature cycle on the Mechanical Properties of Silicon Carbide/Titanium Metal Matrix Composites," J. Materials Science, vol. 26, 1991, pp. 2913-2920.
3. C. G. Rhodes and R. A. Spurling, "Fiber-matrix Reaction Zone Growth Kinetics in SiC-Reinforced Ti-6Al-4V as Studied by Transmission Electron Microscopy," ASTM STP 864, J. R. Vinson and M. Taya, Eds., American Society for Testing and Materials, Philadelphia, PA, 1985, pp. 585-599.
4. L. J. Walpole, "A Coated Inclusion in an Elastic Medium," Math. Proc. Camb. Phil. Soc., vol. 83, 1978, pp. 495-506.
5. Y. Mikata and M. Taya, "Stress Field in a Coated Continuous Fiber Composite Subjected to Thermo-mechanical Loadings," J. Comp. Mat., Vol. 19, Nov. 1985, pp. 554-578.
6. J. A. Nairn, "Thermoelastic Analysis of Residual Stresses in Unidirectional, High-Performance Composites," Polymer Composites, vol. 6, No. 2, 1985, pp. 123-130.
7. N. J. Pagano and G. P. Tandon, "Elastic Response of Multi-directional Coated-fiber Composites," Composites Science and Technology, vol. 31, 1988, pp. 273-293.
8. E. Sideridis, "The In-Plane Shear Modulus of Fibre Reinforced Composites as Defined by the Concept of Interphase," Composites Science and Technology, vol. 31, 1988, pp. 35-53.

9. Y. Benveniste, G. J. Dvorak and T. Chen, "Stress Fields in Composites with Coated Inclusions," Mechanics of Materials, vol. 7, 1989, pp. 305-317.

10. S. M. Arnold, V. K. Arya and M. E. Melis, "Elastic/Plastic Analyses of Advanced Composites Investigating the Use of the Compliant Layer Concept in Reducing Residual Stresses Resulting for Processing," NASA TM 103204, Sept. 1990, National Aeronautics and Space Administration, Cleveland, Ohio.

11. L. J. Broutman and B. D. Agarwal, "A Theoretical Study of the Effect of an Interfacial Layer on the Properties of Composites," Polymer Engineering and Science, Aug. 1974, vol. 14, No. 8, pp. 581-588.

12. J. J. Caruso, C. C. Chamis, and H. C. Brown, "Parametric Studies to Determine the Effect of Compliant Layers on Metal Matrix Composite Systems," NASA TM-102465, Jan. 1990, National Aeronautics and Space Administration, Cleveland, Ohio.

13. R. A. Naik, W. S. Johnson and D. L. Dicus, "Micromechanical Thermal Analysis of Interphase Region in a Titanium Aluminide MMC," Titanium Aluminide Composites, P. R. Smith, S. J. Balsone and T. Nicholas, Eds., WL-TR-91-4020, Wright Laboratory, WPAFB, Ohio, Feb. 1991, pp. 563-575.

14. S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, 3rd ed., McGraw-Hill, New York, 1976, pg. 69.

15. J. A. Graves and C. C. Bampton, "Direct Consolidation of Titanium Aluminide Matrix Composites Using Blended Powder," NASP Contractor Report 1051, NASA Langley Research Center, National Aeronautics and Space Administration, Jun. 1989.

Table I. - Evaluation of results from the present analysis ($\Delta T = -555^{o}C$, $R_f = 71.12\ \mu m$, $2\ \mu m$ interphase layer).

Matrix Stress	Finite elements [13]		Direct Solution	Closed Form
	Peak	Average		
σ_{rr}(MPa)	-293.0	-221.6	-224.0	-233.3
$\sigma_{\theta\theta}$(MPa)	496.8	487.7	487.1	507.2
σ_{zz}(MPa)	381.4	363.1	361.8	372.4

Table II. - Material Properties of the constituents [13].

Property	Fiber	Matrix	Interphase
E, GPa	400	132	190
ν	0.25	0.3	0.35
$\alpha \times 10^6$ ($m/m/^{o}C$)	4.86	11.1	6.61
Volume fraction	0.35	0.63	0.02

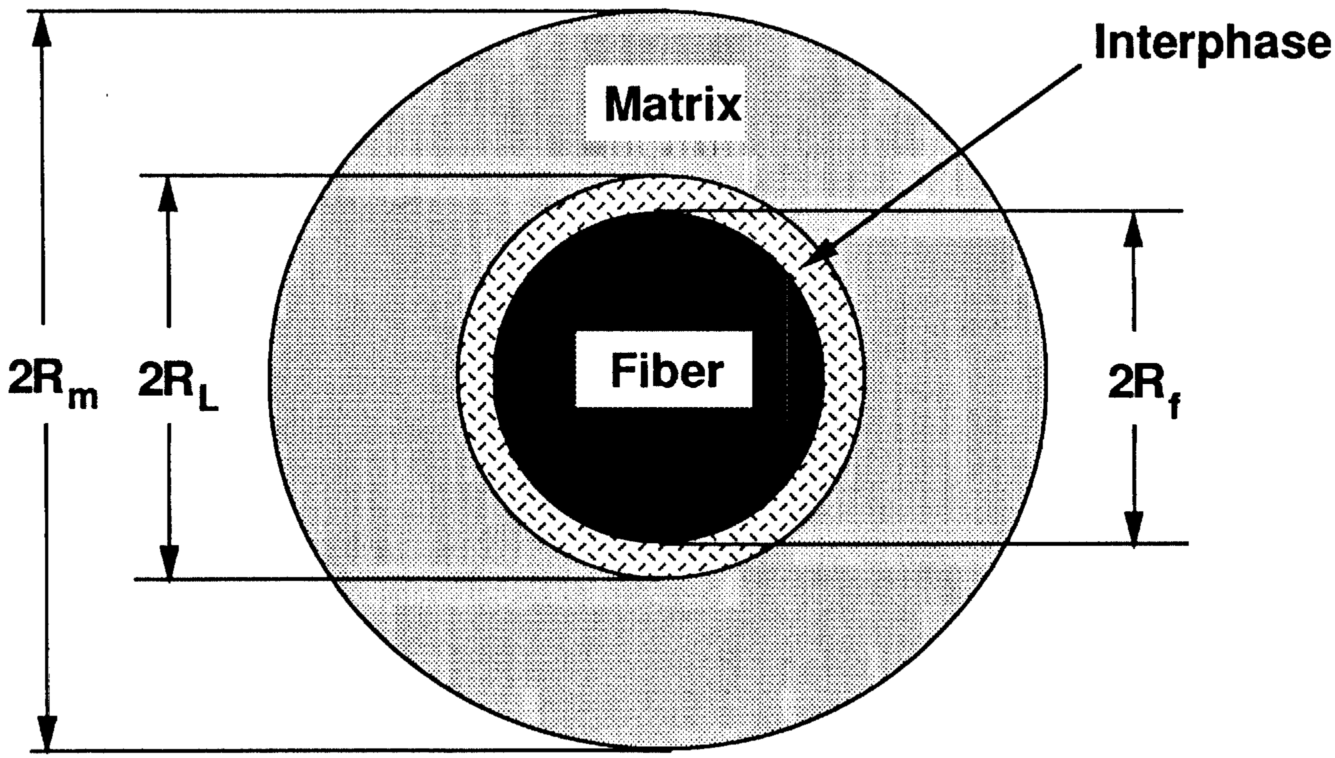

Fig. 1.- Three phase composite cylinder model.

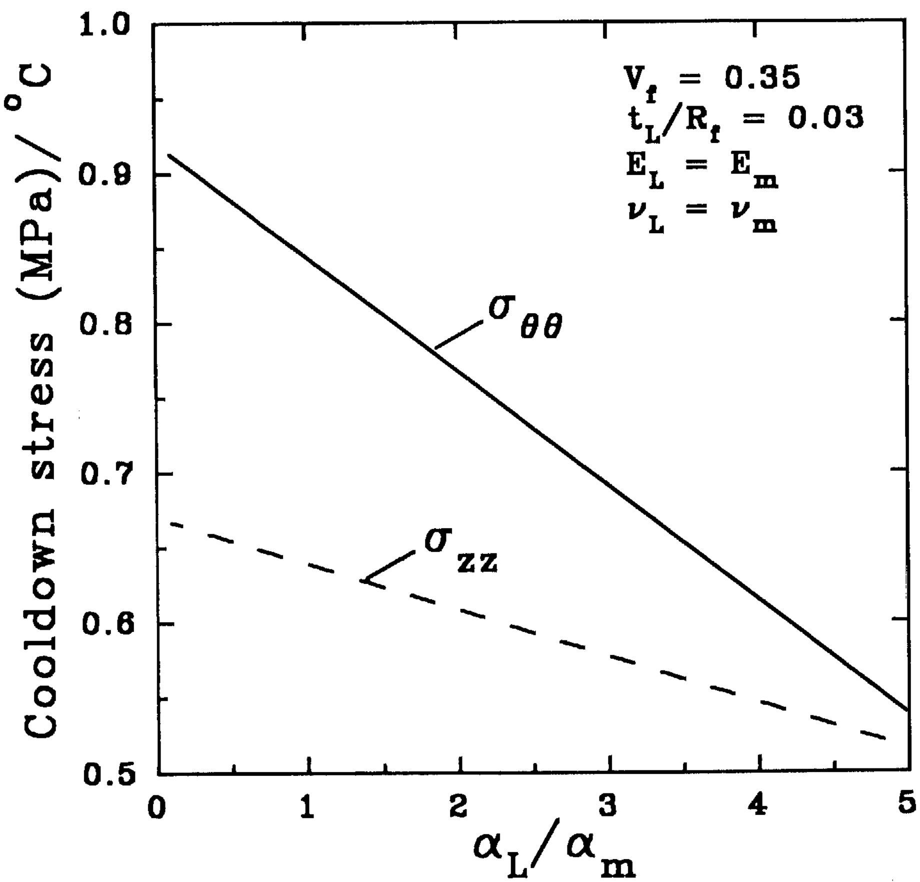

Fig. 2.- Effect of α_L on interfacial matrix stresses.

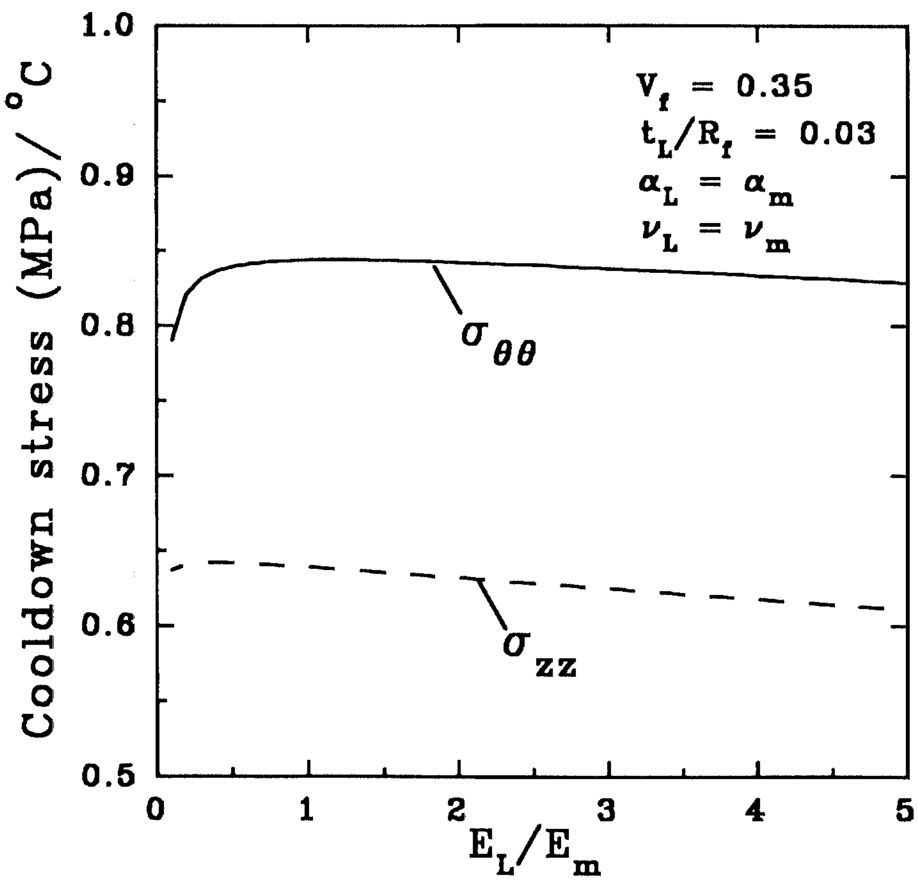

Fig. 3.- Effect of E_L on interfacial matrix stresses.

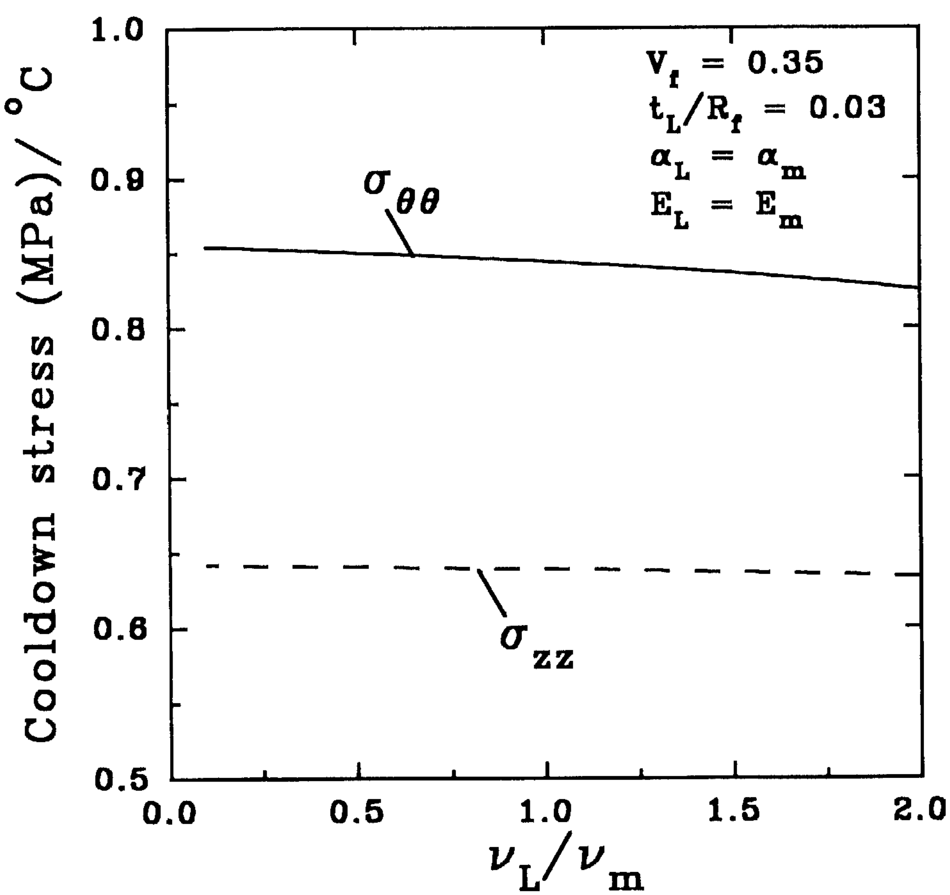

Fig. 4.- Effect of ν_L on interfacial matrix stresses.

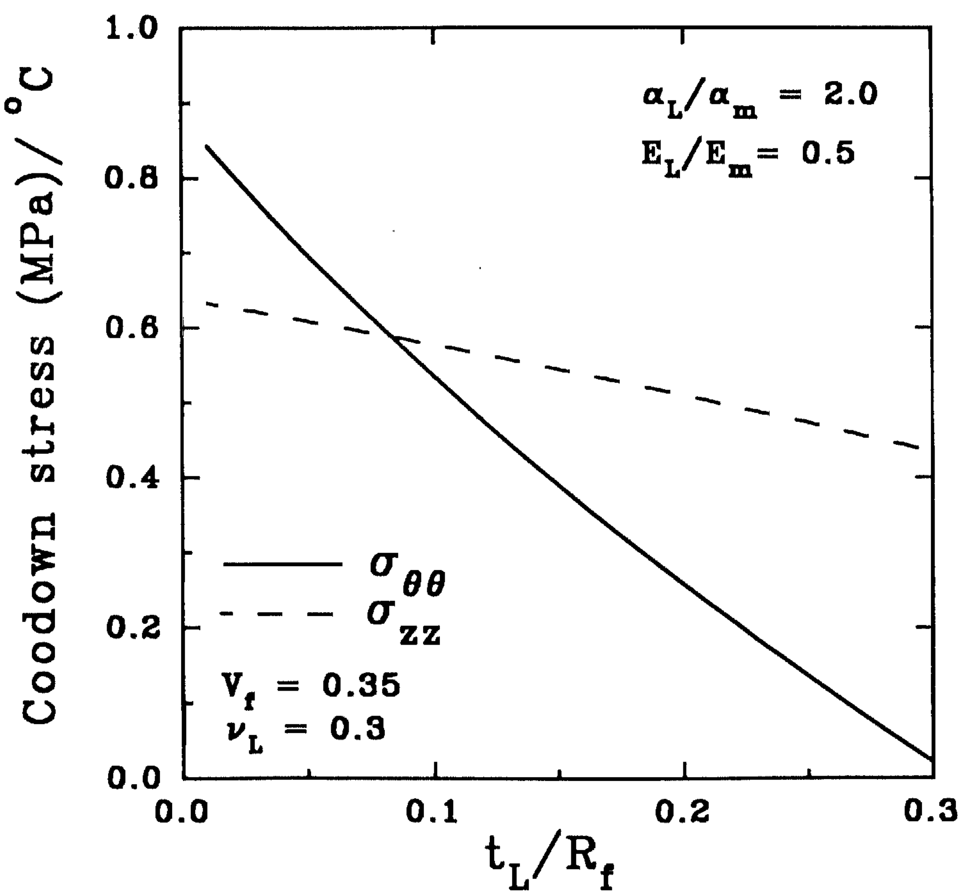

Fig. 5.- Effect of t_L on interfacial matrix stresses for "compliant" layer.

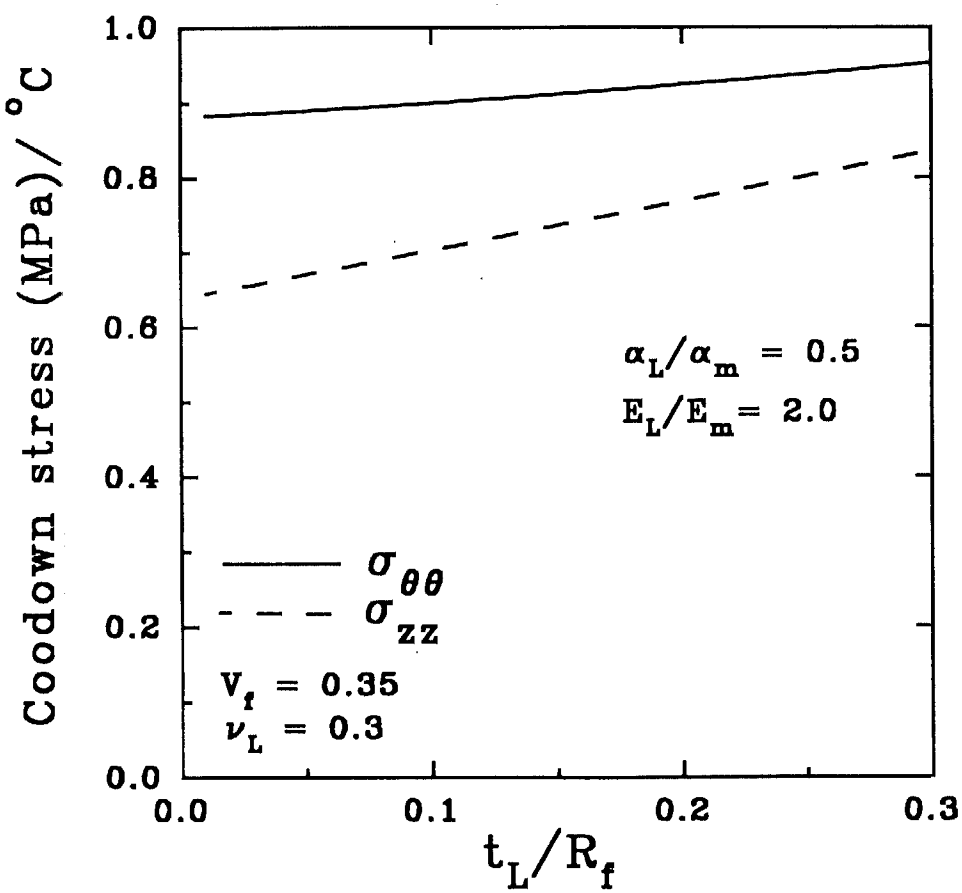

Fig. 6.- Effect of t_L on interfacial matrix stresses for high E, low CTE layer.

NASA National Aeronautics and Space Administration

Report Documentation Page

1. Report No. NASA CR-187599	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Simplified Micromechanical Equations for Thermal Residual Stress Analysis of Coated Fiber Composites		5. Report Date September 1991
		6. Performing Organization Code
7. Author(s) R. A. Naik		8. Performing Organization Report No.
		10. Work Unit No. 505-63-50-04
9. Performing Organization Name and Address Analytical Services and Materials, Inc. Hampton, VA 23665-5225		11. Contract or Grant No. NAS1-18599
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225		14. Sponsoring Agency Code

15. Supplementary Notes

Langley Technical Monitor: C. E. Harris

16. Abstract

The fabrication of metal matrix composites poses unique problems to the materials engineer. The large thermal expansion coefficient (CTE) mismatch between the fiber and matrix leads to high tensile residual stresses at the fiber/matrix (F/M) interface which could lead to premature matrix cracking during cooldown. Fiber coatings could be used to reduce thermal residual stresses. A simple closed form analysis, based on a three phase composite cylinder model, was developed to calculate thermal residual stresses in a fiber/interphase/matrix system. Parametric studies showed that the tensile thermal residual stresses at the F/M interface were very sensitive to the CTE and thickness of the interphase layer. The modulus of the layer had only a moderate effect on tensile residual stresses. For a silicon carbide/titanium aluminide composite the tangential stresses were 20-30% larger than the axial stresses, over a wide range of interphase layer properties, indicating a tendency to form radial matrix cracks during cooldown. Guidelines, in the form of simple equations, for the selection of appropriate material properties of the fiber coating were also derived in order to minimize thermal residual stresses in the matrix during fabrication.

17. Key Words (Suggested by Author(s)) Composite cylinder, Residual stresses, Micromechanics, Interphase, Fiber Coatings	18. Distribution Statement Unclassified - Unlimited Subject Category - 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	26	A03

NASA FORM 1626 OCT 86