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THE EFFECT OF THE EARTH'S OBLATE SPHEROID SHAPE ON THE ACCURACY OF A TIME-OF-ARRIVAL LIGHTNING GROUND STRIKE LOCATING SYSTEM

Paul W. Casper, PE and Rodney B. Bent, Ph.D.

Atmospheric Research Systems, Inc.
2350 Commerce Park Drive, NE, Suite 3
Palm Bay, Florida 32905
U.S.A.

ABSTRACT

The algorithm used in previous technology time-of-arrival lightning mapping systems was based on the assumption that the earth is a perfect spheroid. These systems yield highly-accurate lightning locations, which is their major strength. However, extensive analysis of tower strike data has revealed occasionally significant (one to two kilometers) systematic offset errors which are not explained by the usual error sources. It has been determined that these systematic errors reduce dramatically (in some cases) when the oblate shape of the earth is accounted for. The oblate spheroid correction algorithm and a case example is presented in this paper.

INTRODUCTION

Lightning ground strike tracking systems based on the time-of-arrival (TOA) technique, in combination with wideband waveform detection have been in operation for almost a decade. The accuracy of these systems has steadily improved as more has been learned about the fine characteristics of timing synchronization signals, propagation effects on lightning waveforms and software processing methods for accuracy enhancement. One of the more recent improvements has been accomplished by refinement of the mathematics to more accurately accommodate the oblate shape of the earth spheroid.

Earlier versions of TOA lightning tracking systems used mathematics based on a spherical approximation of the earth. This is usually not seriously in error, especially if the accurate earth's radius for the region of interest is used in the approximation. However, when accuracies otherwise are approaching a few hundred meters, it becomes essential to base all mathematics on an extremely accurate earth model.

Tracking systems using terrestrial timing synchronization signals (e.g., LORAN) are doubly affected by the earth's oblate shape. The following section identifies the affected parameters and discusses the methods for improvement. Subsequent sections contain illustrations of the magnitude of each effect.

MATHEMATICAL PROCEDURE

The oblate shape of the earth spheroid directly affects TOA system accuracy in two ways:

a. Calculation of time clock offsets due to timing reference signal differential propagation delays

b. Calculation of lightning stroke coordinates given a set of accurate receiver time differences

Different mathematical processes are involved in each of these steps. Derivation of the equations is beyond the scope of this paper (consult references [1, 2, 3]), but the methodology for each is described in the following two subsections.

CALCULATION OF TIME CLOCK OFFSETS

Receiver sites and a terrestrial timing reference signal transmitter are at fixed locations. Therefore, the clock offsets due to timing signal differential propagation delays need only be calculated one time and stored in the system software. In order to determine these differential offsets, very accurate geodesic distances between the timing transmitter and each LPATS receiver must be first computed. A geodesic is defined as the curve of minimum length between two points on the surface of a spheroid [1].

Geodesic distances can be estimated fairly accurately by using a perfect sphere model of the earth with a radius equivalent to the earth radius at the mean latitude of the network. However, data to be presented in later sections shows that to achieve systematic location errors of less than several hundred meters, the actual shape of the earth must be properly accounted for.

The non-iterative solution by Sodano [1] provides a highly-accurate means of calculating geodesic distances. Sodano developed a system of equations that are easily programmed and solved in double precision using a personal computer equipped with a mathematic co-processor. The input to this set of equations is only the latitude and longitude (accurate to the fourth decimal place) of the two points of interest. Navigation receivers using the satellite-based Global Positioning System are most convenient for determining the coordinates of receivers to the required accuracy.

Once accurate geodesic distances are available, they must be converted to an equivalent propagation time. For this, we need an accurate ground wave propagation velocity figure, applicable for the high-energy frequencies radiated during the first ten microseconds of a lightning ground stroke. A figure for frequencies in the 50 to 300 KHz range is appropriate, such as the 100 KHz figure provided by the U.S. Coast Guard for the LORAN navigational system. This 100 KHz velocity figure is determined as follows:

$$V' = V/n \qquad (1)$$

where: V' = 100 KHz ground wave velocity

n = index of refraction at the earth's surface for 100 KHz (1.000338)

V = free space velocity (299,792,458 meters/second)

Accurate time clock offsets are then easily calculated:

$$T12 = V' (D1 - D2) \qquad (2)$$

where: $D1$ = geodesic distance from receiver 1 to the timing transmitter

$D2$ = geodesic distance from receiver 2 to the timing transmitter

Equation (2) produces the time offset between clocks in receivers 1 and 2. This figure for each clock pair in the system is stored by the central software and used to correct the time differences actually reported by the receivers.

CALCULATION OF STROKE COORDINATES FROM TIME DIFFERENCES

The second part of the problem is computation of the stroke coordinates given accurate input time differences. Razin [2] describes the mathematics for accomplishing this computation for both the spherical earth approximation and for the more exact oblate spheroid case. However, Razin does not present the equations for the oblate case. Fell [3] fills in this void. The full set of equations is quite complex and extensive, and the reader is referred to both Razin and Fell for the details. The computational procedure will be described here to provide general understanding of the process.

The *first step in the process is to* map the receiver coordinates from the spheroid (earth) onto an osculating sphere, internal to the spheroid, which is tangent to the spheroid at a point P_O. This tangent point is selected near the center of the receiver network. Figure 1 illustrates this arrangement in two dimensions for clarity. Once all the receiver coordinates (P_R) are mapped onto the osculating sphere (P'_R), the solution process proceeds as if the earth is a perfect sphere, using the P'_R set of receiver coordinates. When the stroke coordinates on the osculating sphere are computed (P'_S), they are mapped back to the spheroid (P_S) using a very simple equation pair.

Note that the osculating sphere receiver coordinates need only be computed one time. They can then be stored as fixed constants in the LPATS central computer software and used with the reported time differences to calculate each stroke's coordinates. The burden on the real time central computer software is substantially unchanged from the all-spherical case.

ERROR DUE TO TIME CLOCK OFFSETS

Offsets in the receiver time clocks, because of spherical approximation error in the offset correction constants, can be significant. Here we will examine an actual case to illustrate the point.

The five Florida LPATS network receiver locations are shown in Figure 2, plus the Jupiter, Florida, LORAN transmitter that is used for synchronization. This network has been in operation for several years, and a substantial archive of data has accumulated. In an effort to objectively assess the accuracy of this network, the data base was searched over a complete lightning season in the vicinity of three very tall, attractive objects ("Bithlo towers") to determine if an expected pattern of strikes existed. Each of these towers are over 1,000 feet in height. The location of the three Bithlo towers relative to the network is shown on Figure 2. The pattern of strikes in the vicinity of these towers is shown in Figure 3. Note that there is an unmistakable stroke cluster near each tower, which can only be actual tower strikes. Although the cluster pattern is identical to the tower pattern, there is an apparent systematic offset to the southwest. After investigation of all the usual sources of systematic error (receiver coordinates, computational error, etc.), it was determined that the error was substantially due to time clock propagation offsets and the spherical approximation stroke coordinate solution mathematics. Here we will examine the error due to time clock offsets and address mathematical error in the next section.

Figure 4 is a blow-up of the area around towers #1 and #2. A 200m x 200m grid has been superimposed to aid in scaling distance. The centroid of the cluster near tower #1 is seen to be approximately 590 meters southwest of the tower. The same is true of the #2 tower and stroke cluster. The circles around the tower indicate the approximate attractive radius of each tower due to its height. *It was determined from* the data base that receiver triad 1-4-5 was used to locate virtually all the strokes in the Bithlo tower clusters, which is in fact the triad the system should have used since it is the optimum combination (least affected by timing errors for the Bithlo area). We will concentrate on this receiver triad to examine the spherical approximation time clock offset error.

The centroid of the actual tower #1 cluster is shown in Figure 5 (point A). Point B is a theoretical point, computed as follows:

a. A stroke location at the exact coordinates of the tower was assumed.

b. The exact propagation times to receivers 1, 4 and 5 were computed using the Sodano method.

c. The *relative stroke time* differences seen by each receiver pair were then computed using the propagation times. These time differences should be exactly what the system actually saw, to the extent that it is possible to predict them.

d. The 100 KHz timing signal propagation times from Jupiter to receivers 1, 4 and 5 were computed in two ways: 1) with spherical approximation mathematics, and 2) with Sodano's oblate spheroid mathematics. Secondary correction factors (due to earth conductivity effects) were added to all times.

e. Time clock offset errors were computed by taking the differences between the spherical and oblate times by receiver.

Point B in Figure 5 is what results when actual time differences from step "c" are corrupted with the time clock offset errors predicted in step "e". This very closely approximates the operating condition of the system at the time, which was using time clock offset correction constants computed with spherical approximation mathematics. Point B is 405 meters southwest of the tower, short of the actual 590 meters, but certainly explaining the majority of it. Point B was computed with spherical approximation coordinate solution mathematics, exactly the same as the system was using.

We had to include the time clock offset errors determined in step "e" to simulate actual system conditions and calculate point B. We can just as easily remove the errors and observe the change in system accuracy that should result. Point C indicates the solution location with zero time clock offset error, using the step "c" time differences and spherical approximation coordinate solution mathematics. This point is 415 meters northeast of the tower, which is actually a slight degradation in accuracy. However, there remains a second source of error due to inaccurate solution mathematics, which was based on a spherical earth approximation. This error source is examined in the next section.

ERROR CAUSED BY SOLUTION MATHEMATICS

As noted at the beginning of Section 2, approximating the earth as a perfect sphere affects not only the accuracy of time clock offset calculations, but also the accuracy of stroke coordinate computation given receiver time differences. The magnitude of this error contribution is illustrated here.

Refer again to Figure 5. With the time clock offset errors removed, the solution moves from B to C. Point C was calculated using the same solution mathematics as used by the system at the time, which was based on a spherical earth approximation. If we instead use the oblate spheroid mathematics as described previously, point C moves to D, which is only 205 meters from the tower. The net result of removing the time clock offset and improving the mathematical accuracy is a 50 percent reduction of error from 405 meters (point B) to 205 meters (point D).

The residual error of 205 meters is still under investigation but is believed to be due to approximations in the derivation of the oblate mathematics. Further findings will be presented at the conference.

DISCUSSION

We have seen that moderate systematic error in the TOA LPATS can arise in two ways from using a spherical earth approximation: 1) determination of time clock offsets, and 2) calculation of stroke coordinates from detected time differences. In the example presented in Section 3, removing time clock offset error resulted in an 820-meter shift in the computed coordinates. In this particular example, the radial position error happened to remain about the same, but this was only a fortuitous result. With a different triad of receivers or a stroke in a different location, it is possible that resulting error could be even larger than the original error as long as spherical solution mathematics is still used. Multiple errors can sometimes combine such that remaining error is larger when one error source is removed. It is important to reiterate that time clock offset error is quite easy to avoid in a system with a terrestrial timing reference signal by using accurate propagation time prediction software that accommodates the oblate earth shape (i.e., the method of Sodano). In systems that are synchronized by satellite signals, the earth's oblate shape is handled differently, by including in the line-of-site calculations the elevation of the receiver site above sea level combined with the accurate earth radius for the applicable latitude.

The example illustrated that oblate solution mathematics can also provide a substantial systematic error reduction, in this case 50 percent. Even though the oblate mathematics is more complex than spherical mathematics, almost all of the additional complexity is involved in the initial one-time system set-up constant calculations, and there is practically no additional load on the real time LPATS Central Analyzer software.

The accuracy of all lightning ground strike tracking technologies in current use is affected by the earth's oblate shape. It has been shown in this paper that errors in a TOA system, due to approximating the earth as a perfect sphere, can be significant enough to warrant attention, especially when inherent system accuracy is good enough to be limited in large part by this error source.

REFERENCES

[1] O'Conner, J. J., "Transformations Applicable to Missile and Satellite Technology", Technical Memorandum ETV-73-27, Air Force Eastern Test Range, Patrick Air Force Base, Florida, 26 April 1973 (specifically, Appendix F Part A entitled "Sodano's Noniterative Solution of the Inverse Geodetic Problem").

[2] Razin, S., "Explicit (Noniterative) LORAN Solution", J. of Inst. of Navigation, Vol. 14, No. 3, Fall 1967, pp. 265-269.

[3] Fell, H., "Comments on LORAN Conversion Algorithm", J. of Inst. of Navigation, Vol. 22, No. 2, Summer 1975, pp. 184-185.

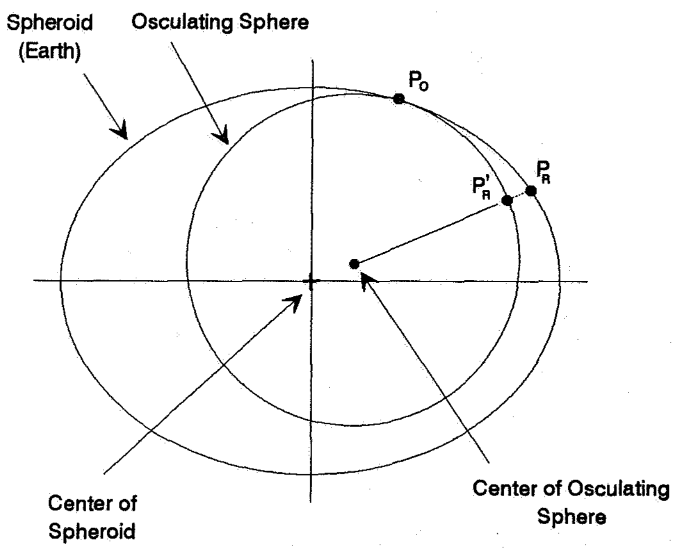


Figure 1: Mapping a Point P_R on the Spheroid to a Point P_R' on an Osculating Sphere.

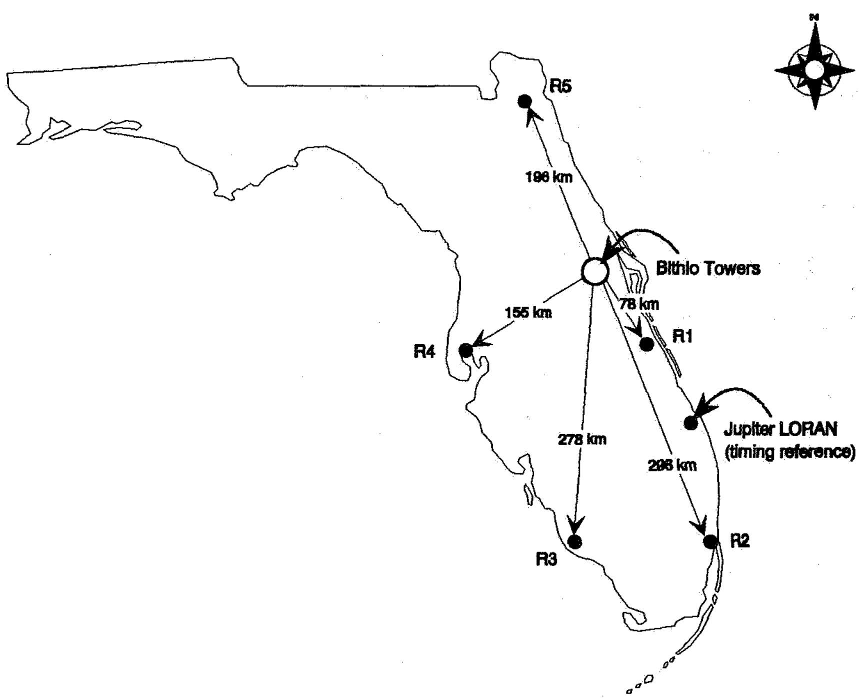


Figure 2: Florida LPATS Network

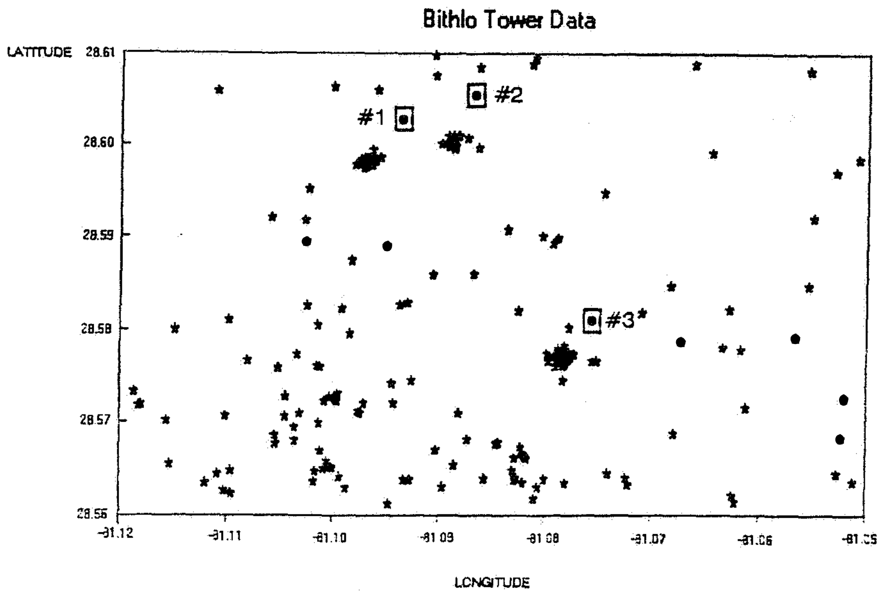
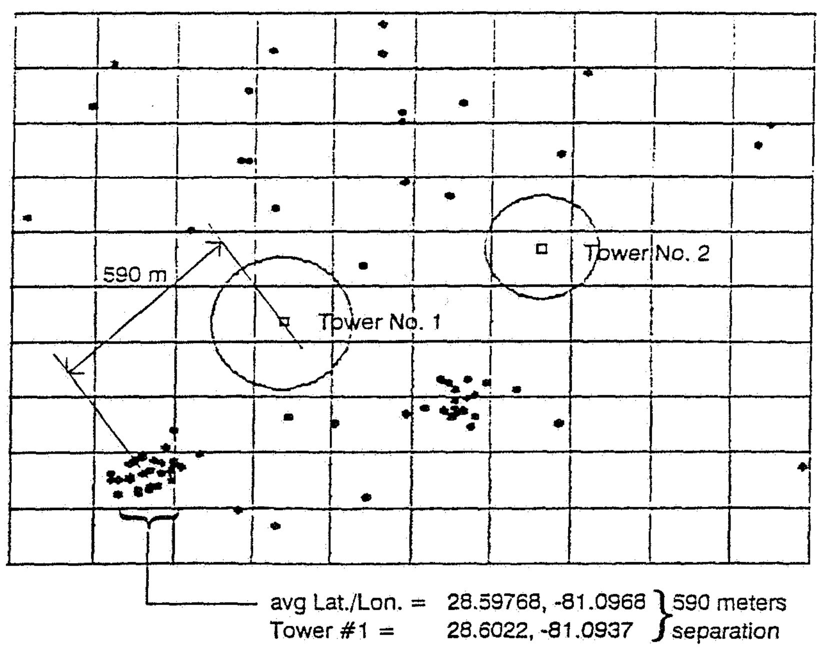


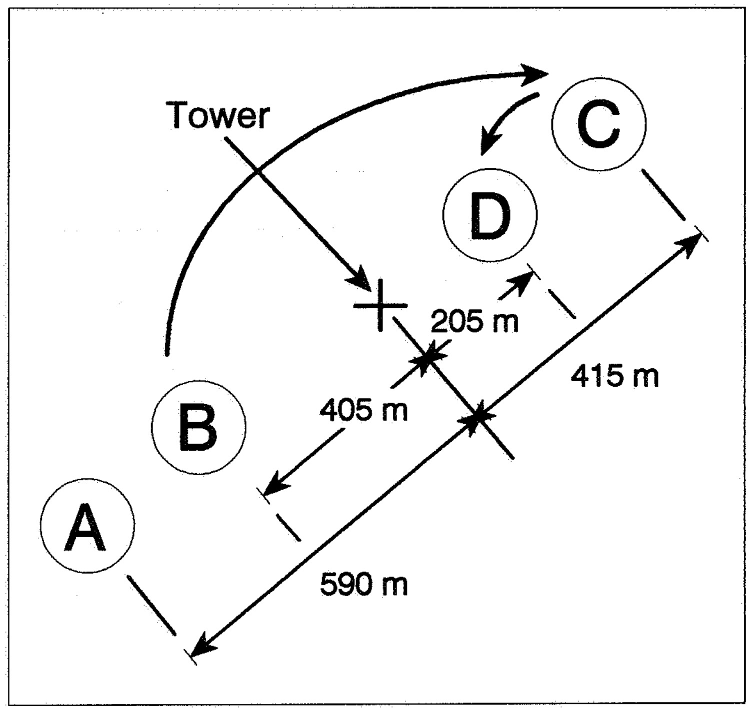
Figure 3: Bithlo Tower Lightning Strikes



TV Towers No. 1 and No. 2

Center of graph:	Latitude: 28.6034 N Longitude: -81.0903 W
Grid size:	2 km x 2 km
Cell size:	200m x 200m

Figure 4: Strikes to Bithlo Towers #1 & #2



A Centroid of the stroke cluster for tower #1.

B Theoretical solution, with predicted timeclock offsets.

C Theoretical solution, with timeclock offsets removed.

D Theoretical solution, timeclock offsets removed and math error removed.

Figure 5. Triad 145 Solutions, as Affected by Timeclock Offset