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A WAY TO COMPARE COSTS OF BUILDING ROTATING STRUCTURES IN SPACE

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In order to compare the costs of constructing in space large rotating shells of different shapes containing atmosphere, surrounded by a layer of non-rotating radiation shield, and suitable for habitation, quantitative parameters are defined for two of the most useful shapes and a simple cost model is set up. Formulas are presented for structural mass, shielding mass, atmospheric mass, projected area, and cost of construction of a sphere and a torus (ring shape) in such a way as to permit comparison of the merits of the two geometries under various conditions. For simplicity only stressed skin structures and tensile stresses are considered.

Size of a Rotating Structure

Rotation with angular velocity ω of a shell of radius R about an axis of symmetry produces a pseudogravitational acceleration $g = R\omega^2$. This parameter g and the rotation rate ω are basic design choices. Their selection determines R and thus the overall size of the structure.

Projected Area

Structures are built to provide area on which people may live. In an environment where there are forces such as gravity, houses and buildings must align with them. This fact means that the useful area is a surface perpendicular to g in the structure. Building on curved surfaces by terracing will not make available any more surface than is contained in the surface perpendicular to g. The area of this surface available in a given structure is called the projected area A. Its location and size depend upon the magnitude and location of the maximum value of g in the structure and in what range of variation of g habitation is acceptable.

For simplicity, however, let us take for A the largest surface perpendicular to g in the structure. Then for a torus with major radius R and minor radius ηR (where η is an aspect parameter), $A_t = 4\pi\eta R^2$ the area of a belt of width $2\eta R$ passing through the middle of the tube of the torus around the axis of rotation at radius R. For a sphere $A_s = 2\pi R^2$ which is the area of a belt of width $\sqrt{2}\,R$ passing around the axis of rotation at a radius $R/\sqrt{2}$.

If R is fixed, specification of A as a design parameter determines how many structures of a given shape will be required to provide the desired amount of projected area. Obvious difficulties arise if a fraction of a structure is called for. In practice a complete structure must be built producing excess capacity. The economics of the problem of excess capacity must be considered as a problem separate from the construction cost of a unit of projected area.

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Structural Mass

A torus of major radius R and minor radius ηR, built with a stressed skin of a material of density ρ, and containing a gas at a pressure P_A will need a thickness of wall $t = \dfrac{P_A R \eta}{\sigma - \rho g R}$ where σ is the working stress of the structural material. Assuming the wall thickness is small compared to ηR, the structural mass will be the product of the surface area of the torus, the density of the structural material and the wall thickness $m_t = \dfrac{4\pi^2 \rho P_A \eta^2 R^3}{\sigma - \rho g R}$

The wall thickness and related structural mass of a sphere can be found to be

$$t_{sp} = \frac{P_A R}{2\sigma - \rho g R} \quad \text{and} \quad m_{sp} = \frac{4\pi P_A R^3 \rho}{2\sigma - \rho g R}$$

It is useful to note that if $R \ll \sigma/\rho g$

$$m_t \approx 4\pi^2 \rho P_A \eta^2 R^3 / \sigma \quad \text{and} \quad m_{sp} \approx 2\pi \rho P_A R^3 / \sigma$$

This approximation neglects the effects of rotation. Its use in what follows introduces errors of < 11% as long as $R \rho g/\sigma < .1$. The approximation also makes evident the functional dependence of cost on various quantities as will be seen below.

Shielding Mass

The mass of a radiation shield of thickness t_s and density ρ_s surrounding a torus of major radius R and minor radius ηR is for $t_s \ll \eta R$

$\bar{m}_t = \rho_s t_s 4\pi^2 \eta R^2$. For a sphere it is $\bar{m}_{sp} = \rho_s t_s 4\pi R^2$.

Atmospheric Mass

An atmosphere of density ρ_A will correspond in a torus to a total mass $\rho_A 2\pi^2 \eta^2 R^3$; in a sphere the mass will be $\rho_A 4\pi R^3/3$.

Cost

Projected area is the end product sought. Structural mass, shielding mass and mass of atmosphere are the main resources paid to obtain a given projected area. The cost of these expenditures can be measured in man-hours or dollars or whatever units seem suitable. Let P_1 be the cost of a unit of structural mass; let P_2 be the cost of a unit of shielding mass; let P_3 be the cost of a unit of mass of atmosphere. The cost of a unit of projected area for a torus is then $C_t = \dfrac{P_1 \rho \pi P_A \eta R}{2\sigma} + P_2 \rho_s t_s + \dfrac{P_3 \eta \pi R \rho_A}{2}$ and for a sphere

$$C_s = \frac{P_1 \rho P_A R}{\sigma} + 2P_2 \rho_s t_s + \frac{P_3 2R \rho_A}{3}$$

Comparative Analysis

Construction costs are the same for the two shapes when $C_t = C_s$. Defining $y = \frac{P_2 \rho_s t_s \sigma}{P_1 \rho P_A R}$ and $x = \frac{P_3 \sigma \rho_A}{P_1 P_A \rho}$ we can rewrite $C_t = C_s$ as

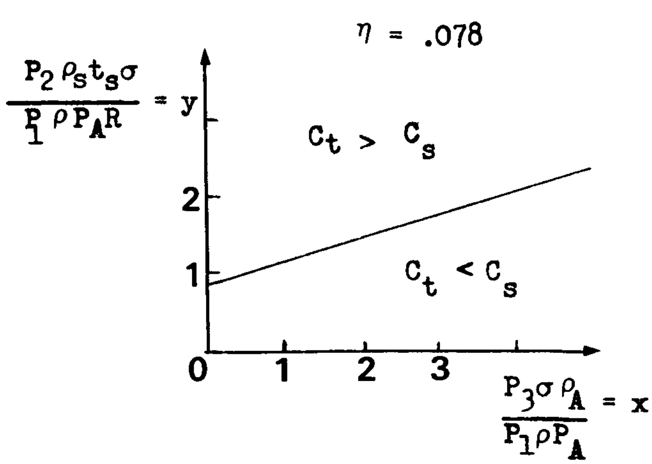
$y = \frac{2 - \pi\eta}{\pi - 2} + \frac{4 - 3\pi\eta}{6(\pi-2)} x$. For a given choice of η, the equation divides the x-y plane into two regions: $C_t > C_s$ and $C_t < C_s$. A plot of the x-y equation for a given value of η gives a convenient way to determine the relative costs of the two kinds of construction. After calculating the values of x and y for a given configuration, we can read off the graph which mode is the more expensive way to provide a unit of projected area.

Within the limits of the approximation under which the equation was derived the graph also makes evident the dependence of cost on the various parameters of the structures. From the definitions of x and y we see that changes in P_2, ρ_s, t_s, or R will displace a point describing the difference in cost parallel to the y axis. Similarly, changes in P_3 will result in displacements along the x axis. Changes of ρ, σ, or P_1 will produce displacements along a line of unit slope through the point. (The ratio ρ_A/P_A appearing in the definition of x can be replaced by $M/R_g T$ where M is the molecular weight of the gas, R_g is the gas constant, and T is the absolute temperature. Consequently, varying P_A or ρ_A does not affect x; only changes in T or M will affect x.)

Example

Consider the Stanford Torus (1) for which $\eta = .078$ and R = 830 m. Then with $P_A = 50$ kPa, $\sigma = 200$ MPa (29 000 psi), $\rho_s = 2.65$ g/cm^3, $\rho = 2.7$ g/cm^3, and $t_s = 4.5$ m the x-y equation can be written $21.3\ P_2/P_1 = .768 + .436\ P_3/P_1$.

If $P_3 = 0$, i.e. if the atmospheric mass costs nothing, then only if P_2/P_1 is less than .036 will toruses be cheaper to build than a sphere. Otherwise, although the structural mass required for the toruses sufficient to produce a desired projected area A will be much less than for a sphere, this benefit will be offset by the extra shielding which a torus requires in the ratio of π:2 for each unit of A.



On the other hand, if the atmosphere contains an appreciable amount of nitrogen which will have to be brought from Earth, P_3 will not be zero. In fact P_3/P_1 will be somewhere between 10 and 100. (1,2) If, for example, $P_3/P_1 = 10$, then $C_t < C_s$ for $P_2/P_1 < .24$.

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Summary

A discussion of the costs of construction in terms of the masses required per unit of projected area leads to a description which makes evident the dependence of the difference in cost of construction of toroidal and spherical shapes upon their respective structural parameters. The formulation permits a simple graphical representation which further facilitates interpretation.

References

(1) Space Colonization: A Design Study, Report of the 1975 NASA/Ames - ASEE - Stanford University Summer Design Study Group. NASA-SP (In preparation, Mountain View, California).

(2) O'Neill, G. K., Science, December 5, 1975, pp. 943-947.

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