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# LINEAR FUZZY CONTROLLER

## Abstract

We consider a process controlled by a controller described by an n-th order linear ordinary differential equation toward its target output. As a special case, the controller is a proportional-integral-derivative (PID) controller. We show how to construct a linear fuzzy controller that gives precisely the same control as the PID controller. It is speculated that nonfuzzy controllers and fuzzy controllers may coincide on an unsuspectingly large class of control problems.

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# Fuzzy Controller Theory

1. Linear FC.
2. Linear Control Rules.

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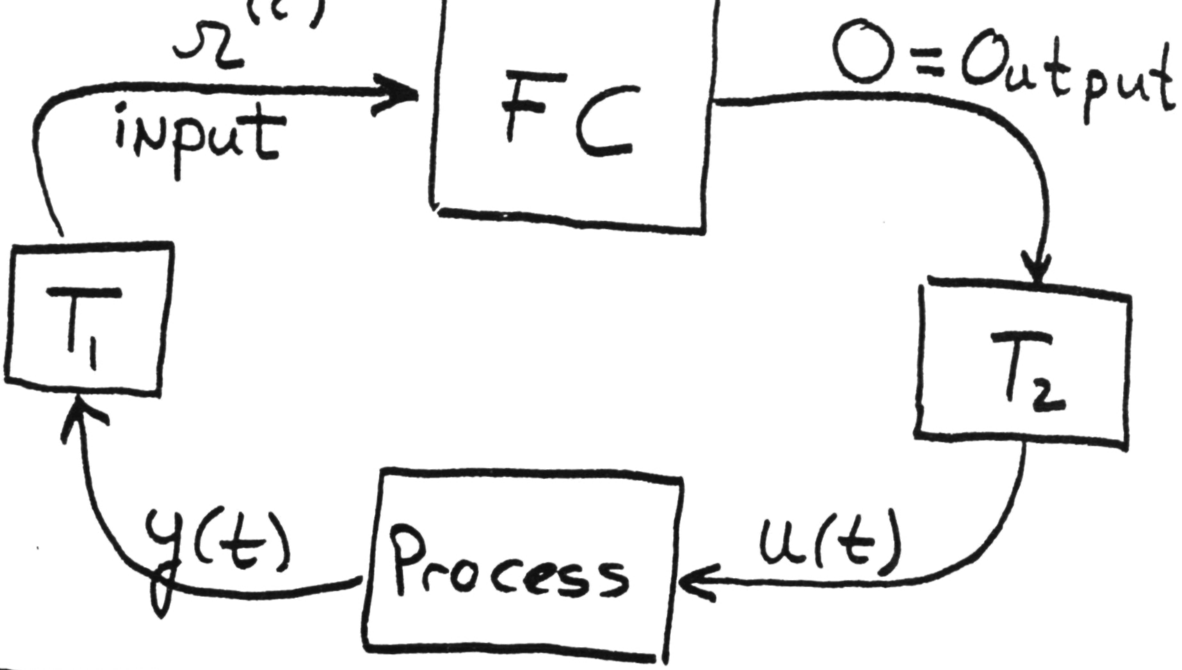


$\boxed{T_1}$ $s$ = set point.

$y^{(i)}(t) = i^{th}$ derivative Error

$= y(t) - s, \quad 0 \le i \le n.$

$y^{(0)}(t) = $ Error.

<u>Scale</u>

$$r^{(i)}(t) = c_i\, y^{(i)}(t) \in [-1, 1],$$
$$0 \le i \le n, \; t \ge 0.$$

input to FC.

$\boxed{T_2}$

$$\bigcirc \xrightarrow{\text{defuzzify}} \delta(t) \in [-1, 1]$$

Output

$$u(t) = u(t-\Delta) + \delta(t)\Delta, \quad t = \Delta, 2\Delta, \dots$$

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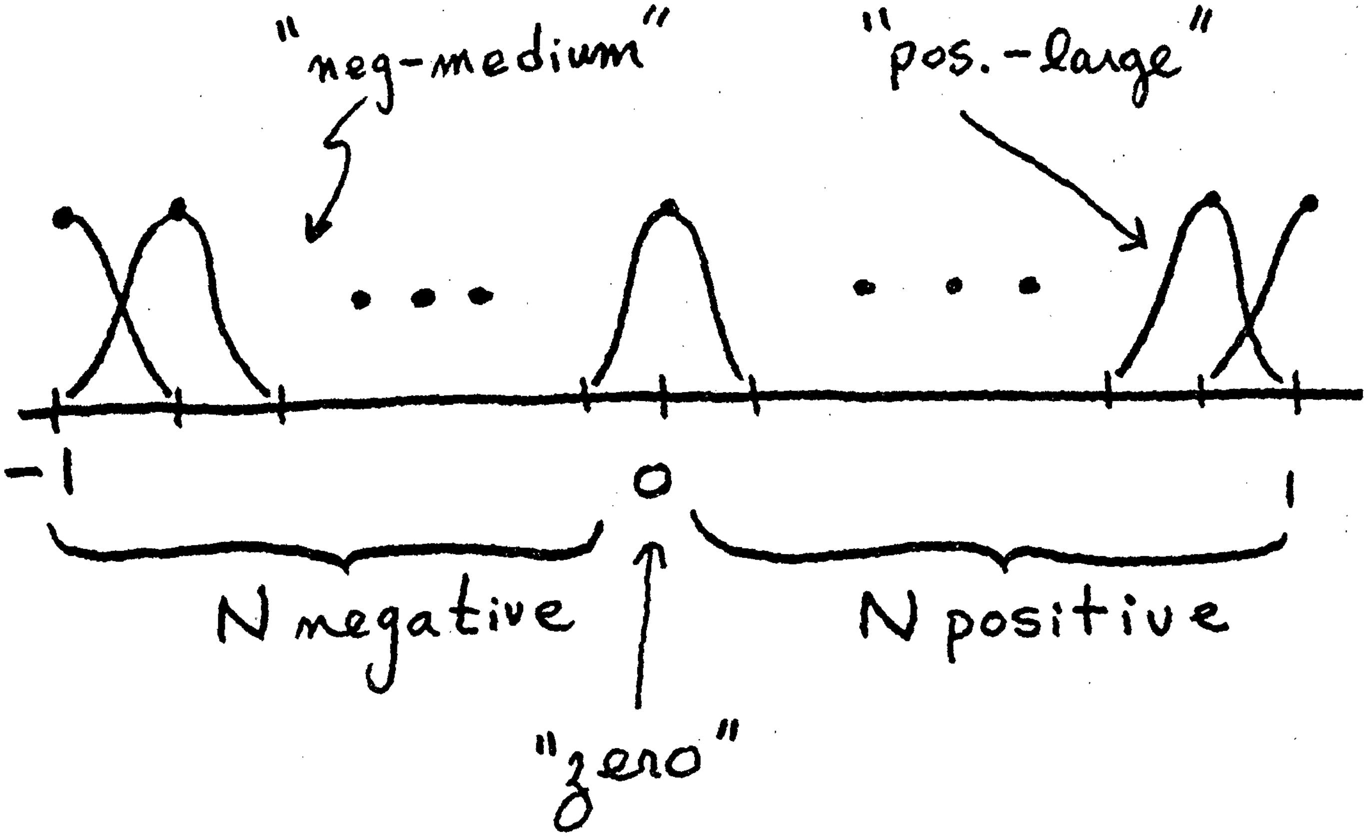
$\boxed{\text{FC}}$

1. Fuzzify — fuzzy numbers for Linguistic variables.
2. Rules, and their evaluation.
3. Defuzzify.

# 1. Fuzzy Numbers:

For each input $r^{(i)}$



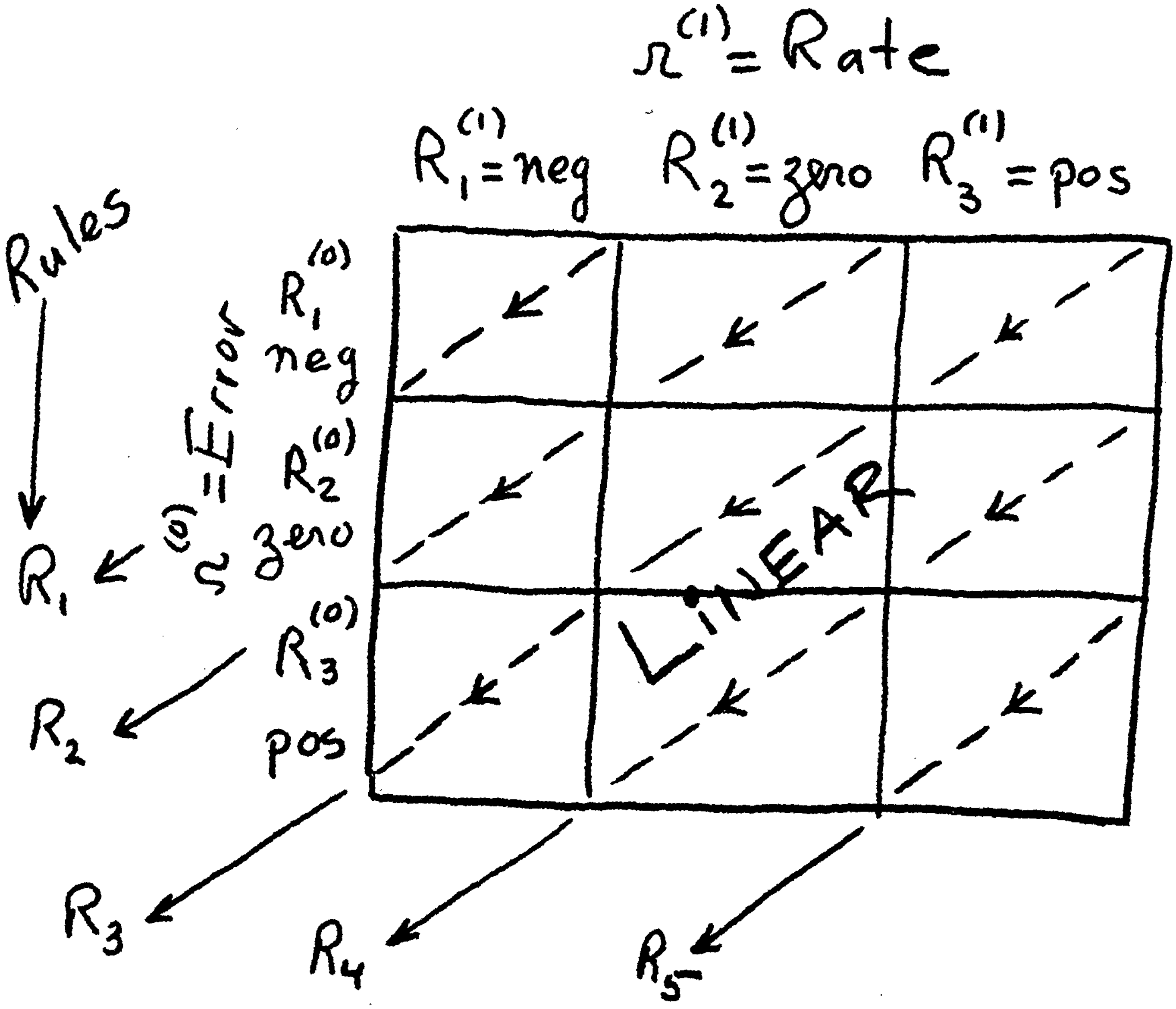

$2N+1$, $N \geq 1$, fuzzy numbers.

Equally spaced. Can be triangles, trapezoids, "normal", ....

Named $R_j^{(i)}$ ← input, $1 \leq j \leq 2N+1$.

## 2. Rules:

$n = N = 1$. Then generalize!

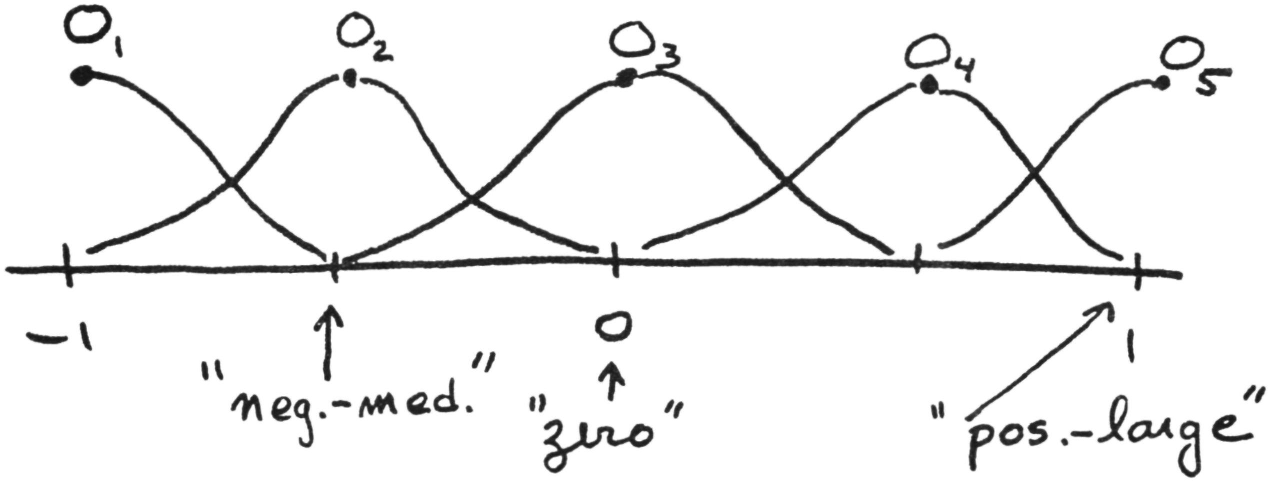



Rule $R_i \to [O = O_{6-i}]$.
$1 \le i \le 5$

Output

fuzzy #

$$O = \text{Output} = \left\{ \frac{\Delta_1}{O_1}, \ldots, \frac{\Delta_5}{O_5} \right\}$$




$R_2$: If[ (Error = Neg) AND (Rate = zero) ]
OR [ (Error = zero) AND (Rate = neg) ],
then $O = O_4$.

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Evaluate all rules:

value LHS $R_i = \Delta_i =$
membership value of $O_{6-i}$.

$$\Delta_1 = T\left(\mu(r^{(0)} \mid Neg),\ \mu(r^{(1)} \mid Neg)\right)$$

$$\Delta_2 = C\Big(T\left(\mu(r^{(0)} \mid neg),\ \mu(r^{(1)} \mid zero)\right),\ T\left(\mu(r^{(0)} \mid zero),\ \mu(r^{(1)} \mid Neg)\right)\Big)$$

$\vdots$

$T$ = any t-norm.
$C$ = any co-t-norm.

Need not be the same from rule to rule.

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for $\Delta_1$, $T$ can be MIN,
for $\Delta_2$, $T$ can be product,

$\vdots$

## 3. Defuzzify:

Defuzzified $O = \delta \in [-1, 1]$

ⓐ $cv_i$ = central value of $O_i$

$$\delta_1 = \frac{\sum_{i=1}^{K} \Delta_i \, cv_{K-i+1}}{\sum_{i=1}^{K} \Delta_i}$$

$K$ = # of rules.

In general, $K = (n+1)(2N) + 1$.

ⓑ All "reasonable" defuzzifiers.

Contains: (i) $\delta_1$,

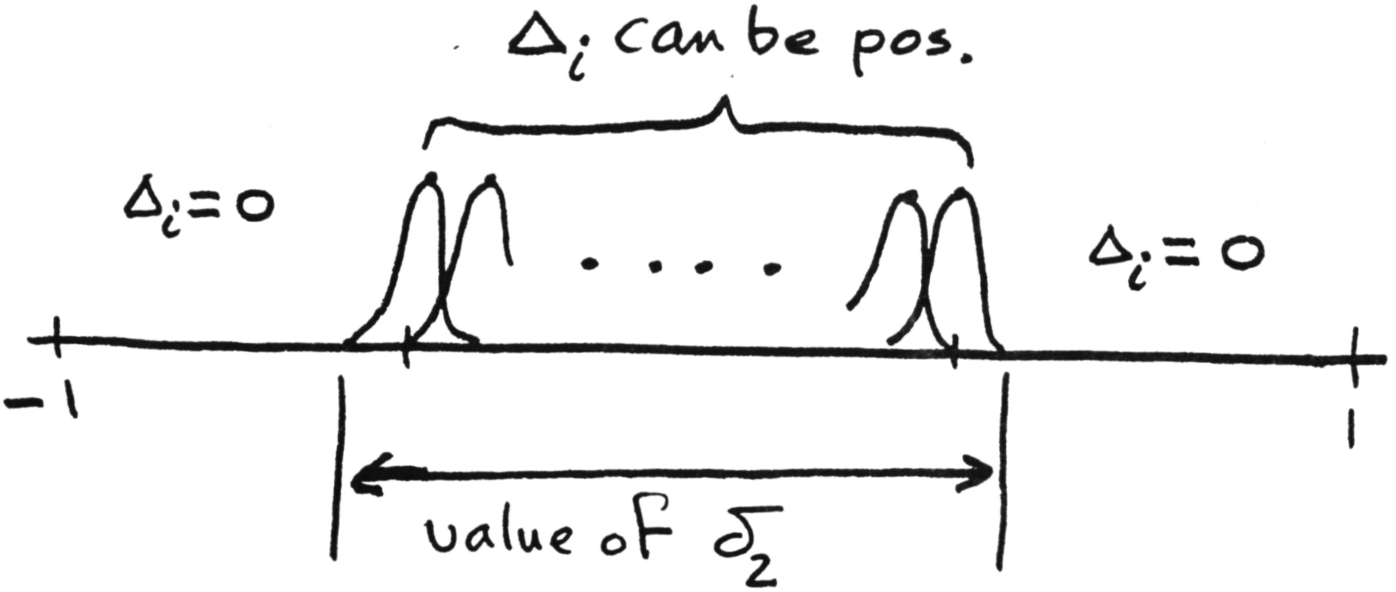
(ii) center of gravity,

(iii) max membership,

⋮

## Output = 0






No specific value of $\delta_2$ need be given for main results.

$\delta_2 \in \bigcup \{$ supports of $O_i$ whose $\Delta_i$ can be pos. $\}$.

# Results:

let $\mathcal{L} = -\sum_{i=0}^{n} \Omega^{(i)} / (n+1)$.

## 1. Linear FC

$\Delta$ fuzzy numbers, $T$ = prob. AND, $C$ = Lukasiewicz OR

$\Rightarrow \delta_1 = \mathcal{L}$ all $n$, $N$.

Always linear!

PI, PID for $n = 1, 2$.

## Refs:


1. Siler and Ying "Fuzzy Control Theory: Linear Case" FSS. Submitted.

Ideas for Linear FC, using different fuzzy logic to evaluate rules, etc. Above result ($\delta_1 = \mathcal{L}$) generalizes one of their results.

2. J.J. Buckley and H. Ying "Linear FC, it is a Linear Non-fuzzy Controller," <u>IJMMS</u>. Submitted. (Proof $\delta_1 = \mathcal{L}$)

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Note

$$\delta_1 \overset{\text{linear}}{\equiv} \sum \Delta_i \, c v_{K-i+1}$$

bec. here

$$\sum \Delta_i = 1 .$$

## 2. Linear Control Rules.

$\delta_2 \longrightarrow \mathcal{L}$ as $N \longrightarrow +\infty$.

Any fuzzy numbers, any T and C, any "reasonable" defuzzifier, all $n$.

Ref:

1. Buckley and Ying "

Automatica. Submitted.

ⓐ So:

$$\delta_2 \approx PI, PID, \cdots (N \text{ large}).$$

ⓑ <u>Rate of convergence</u>:

$$|\delta_2 - \mathcal{L}| \leq \frac{c}{N}, \ c = ?$$

Some results.

ⓒ "linear" rules sufficient but not necessary. N and S condition on rules so that $\delta_2 \to \mathcal{L}$ as $N \to +\infty$ is unknown!

ⓓ <u>Note</u> #Rules $\to +\infty$ as $N \to +\infty$.

(e) $\delta = F(\Omega^{(0)}, \dots, \Omega^{(n)})$

find F for small N.

Some results! How nonlinear is it?

<u>See also</u>:

Buckley "Fuzzy vs Non-Fuzzy Controller", <u>FSS</u>. Submitted.

(f) At other extreme from $N \to +\infty$ is

2 fuzzy numbers, 3 rules

<u>See</u>: Ying, Siler and Buckley, "Fuzzy Control Theory: a Nonlinear Case", <u>NASA Conference</u>.

Also "Expert FC"

I : Theory → FSS. Submitted.
II : Output Strategies → FSS. Sub.
III : Combined Input and Output Strategies → in preparation.
IV : Overall Strategies. Next!

By J.J. Buckley and H. Ying

Fuzzy goals for rise-time, overshoot, ....

$y_i$ = value of fuzzy goal =

$H_i$(scaling constants, $\delta$, rules, fuzzy numbers, ...)

$i = 1, 2, \ldots$

## Objective

$$\max(y_1, y_2, \ldots)$$

Subject to: ________

However $H_i$ unknown!

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Globally optimal FC.

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Decision Theory approach.

Based on our Fuzzy Expert System FLOPS.