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APPENDIX

MEASUREMENTS OF THE TEMPERATURE PROFILE IN THE STRATOSPHERE

from

CHANIN, HAUCHECORNE
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If the backscattered light is only due to Rayleigh and Mie scattering, the lidar equation can be written :

$$N(z_i) = \frac{N_0 A\, K\, R_q\, T^2(z_0, z_i)}{4\pi (z_i - z_0)^2} \cdot [n_r(z_i)\, \beta_r + n_m(z_i)\, \beta_m(z_i)]\, \Delta z \qquad (1)$$

$N(z_i)$	is the number of detected photons for one laser pulse, from a layer of thickness Δz centered at the height z_i
N_0	is the number of photons emitted for each laser pulse
A	the telescope area
K	the optical efficiency of the lidar system (including the optical transmission through the transmitter and receiver)
R_q	the quantum efficiency of the photomultiplier
$T(z_0, z_i)$	the atmospheric transmission between the altitude of the lidar site and the height of the emitting layer z_i
n_r and $n_m(z_i)$	the air molecules and aerosols concentrations
β_r and $\beta_m(z_i)$	the Rayleigh and Mie backscattering cross-sections.

In the height range when the Mie contribution is negligible (i.e. between 35 and 80 km) the atmospheric density is given by the expression :

$$\rho(z) = C\, [S_L(z) - B(z)] / T^2(z, \infty) \qquad (2)$$

$S_L(z)$ is the signal coming from the altitude z, in a constant solid angle (i.e. multiplied by $(z - z_0)^2$) and eventually corrected for the non linearity of the photomultiplier if this one is close to saturation

B (z) is the background signal due to dark current and sky background, extrapolated linearly for the altitude z*

T (z, ∞) is the atmospheric transmission between z and the top of the atmosphere, evaluated at the laser wavelength, taking into account ozone and Rayleigh attenuation.

C a normalisation constant, depends upon N_0, K, R_a and T (z_0, z_i) as defined above and may vary with time (mainly under the influence of fluctuations in the laser energy output and the atmospheric transmission). For each period of measurement C is evaluated by fitting the density measured either with a model (CIRA 1972) between 30 and 35 km, or with radio-sonde data at 30 km obtained from the nearest meteorological site.

The relative uncertainty on the density determination is given by :

$$\frac{\Delta \rho}{\rho} = \frac{\Delta S_L (Z)}{S_L (z) - B (z)} \qquad (3)$$

TEMPERATURE DETERMINATION

The temperature profile is computed from the density profile assuming that the atmosphere obeys the perfect gas law and is in hydrostatic equilibrium. This second assumption implies that atmospheric turbulence does not affect the mean air density, which is the case considering the temporal and spatial resolutions of the lidar data. The constant mixing ratio of the major atmospheric constituents (N_2, O_2 and Ar) and the negligible value of the H_2O mixing ratio justify the choice of a constant value M for the air mean molecular weight. The air pressure P (z), density ρ(z) and temperature T (z) are then related by :

$$P (z) = \frac{R \rho (z) T (z)}{M} \qquad (4)$$

$$dP (z) = - \rho (z) g (z) dz \qquad (5)$$

where R is the universal gas constant and g (z) the acceleration of gravity. The combination of Eq. (4) and Eq. (5) leads to :

$$\frac{dP (z)}{P (z)} = \frac{M g (z)}{R T (z)} dz = d (\mathrm{Log}\, P (z)) \qquad (6)$$

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* *The recovery time of a photomultiplier exposed to a high level of light may be a source of error in estimating the background coming from the dark current. To reduce such a source of error, a shutter should be used during the return of the low altitude echo ; both mechanical and electronic shutter have been used successfully.*

If the acceleration of gravity and the temperature are assumed to be constant in the i^{th} layer, the pressure at the bottom and top of the layer are related by :

$$\frac{P\ (z_i - \Delta z/2)}{P\ (z_i + \Delta z/2)} = \exp \frac{M\ g\ (z_i)}{R\ T\ (z_i)} \Delta z \qquad (7)$$

and the temperature is expressed as :

$$T\ (z_i) = \frac{M\ g\ (z_i)\ \Delta z}{R \operatorname{Log} [P\ (z_i - \Delta z/2)\ /\ P\ (z_i + \Delta z/2)]} \qquad (8)$$

The density profile is measured up to the n^{th} layer (about 80 km). The pressure at the top of this layer is fitted with the pressure of the CIRA 1972 model, $P_m\ (z_n + \Delta z/2)$, for the corresponding month and latitude. The top and bottom pressures of the i^{th} layer are then :

$$P\ (z_i + \Delta z/2) = \sum_{j=i+1}^{n} \rho\ (z_j)\ g\ (z_j)\ \Delta z + P_m\ (z_n + \Delta z/2) \qquad (9)$$

$$P\ (z_i - \Delta z/2) = P\ (z_i + \Delta z/2) + \rho\ (z_i)\ g\ (z_i)\ \Delta z \qquad (10)$$

Let X be :

$$X = \frac{\rho\ (z_i)\ g\ (z_i)\ \Delta z}{P\ (z_i + \Delta z/2)} \qquad (11)$$

The temperature is then :

$$T\ (z_i) = \frac{M\ g(z_i)\ \Delta z}{R \operatorname{Log}\ (1 + X)} \qquad (12)$$

The statistical standard error on the temperature is :

$$\frac{\delta T\ (z_i)}{T\ (z_i)} = \frac{\delta \operatorname{Log}\ (1 + X)}{\operatorname{Log}\ (1 + X)} = \frac{\delta X}{(1 + X) \operatorname{Log}\ (1 + X)} \qquad (13)$$

with

$$\left(\frac{\delta X}{X}\right) = \left|\frac{\delta\rho\ (z_i)}{\rho\ (z_i)}\right|^2 + \left|\frac{\delta P\ (z_i + \Delta z/2)}{P\ (z_i + \Delta z/2)}\right|^2 \tag{14}$$

$$\delta P\ (z_i + \Delta z/2)^2 = \sum_{j=i+1}^{n} (g\ (z_j)\ \delta\rho\ (z_j)\ \Delta z)^2 + (\delta P_m\ (z_n + \Delta z/2))^2 \tag{15}$$

The uncertainty on the extrapolated pressure at the top at the profile is evaluated to be 15 %. Its contribution to the temperature uncertainty decreases rapidly with altitude and is smaller than 2 %, at 15 km from the top, and smaller than 1 %, 5 km lower. It is important to notice that the term X represents a ratio of experimental density values and consequently the constant of normalisation disappears. The temperature determination is then absolute as soon as one can neglect the term due to the pressure at the top, even though the density is only measured on a relative way.