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BENDING WAVES AND ORBITAL INCLINATIONS.

Wm. R. Ward, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, and San Juan Capistrano Research Institute, San Juan Capistrano, CA 92675.

Disc tides may play an important role in the formation of a planetary system. Modification of protoplanet semi-major axes and eccentricities through density waves was first suggested by Goldreich and Tremaine (1979, 1980) and calculations of linear and non-linear protoplanet-nebula interactions have been carried out by a number of researchers (e.g., Goldreich and Tremaine, 1979; Papaloizou and Lin, 1984; Lin, Papaloizou, and Savonije, 1990; and Ward, 1986, 1990). The possible significance of radial drift to the accretion process has been discussed by Hourigan and Ward (1984) and Ward (1986, 1989a), while the evolution of orbital eccentricities has been studied by Goldreich and Tremaine (1980, 1981) and Ward (1989b) for a perturber orbiting external to a ring and by Ward (1988) for a perturber embedded in the ring. Here we stress the possible importance of bending waves to the early evolution of protoplanet inclinations.

Resonant interactions between a disc and a perturber in an inclined orbit occur at locations of vertical resonance, which are analogues of Lindblad resonances and for which the disc disturbance takes the form of a spiral bending wave (e.g., Shu et al., 1983; Shu, 1984). The perturbing vertical acceleration can be Fourier decomposed into terms of the form $g_p = \mathrm{Re}\{f_m \exp i(m\theta-\omega t)\}$. The strongest terms have frequencies $\omega_\epsilon = (m-\epsilon)\Omega_p$ and amplitudes

$$(1) \qquad f_m = \epsilon \frac{GM_s}{2r_s^2} b^m_{3/2}(\gamma) \sin I \, e^{i\pi/2}$$

where r_s, Ω_s, and M_s are the semi-major axis, mean motion, and mass of the perturber, I is its inclination, and $b^m_{3/2}(\gamma)$ is a Laplace coefficient with argument $\gamma = r/r_s$. For each m there are two forcing terms given by $\epsilon = \pm 1$. Resonance occurs when the vertical oscillation frequency, Ω_z, equals the Doppler shifted forcing frequency, i.e., when

$$(2) \qquad D = \Omega_z^2 - (m\Omega - \omega)^2 = 0$$

where Ω is the local disc orbital frequency. In a Keplerian disc about a primary, M_p; $\Omega = \Omega_z = (GM_p/r^3)^{1/2}$, and resonances fall at $\gamma \approx 1+4\epsilon/3m$.

Borderies et al. (1984) determined the effects of the reaction torque from an m^{th} order resonance on the inclination of the perturber and found that it increased at a rate

$$(3) \qquad \frac{dI}{dt} = \frac{\pi^2}{4}\left(\frac{M_s}{M_p}\right)\Omega\left(\frac{\Omega^2}{|\mathcal{D}|}\right)\left(\frac{\sigma r^2}{M_p}\right)(b^m_{3/2})^2 \sin I$$

where $\mathcal{D} = r\,dD/dr$, and σ is the surface density of the disc. Both inner and outer resonances contribute to excitation. For a Keplerian disc, $|\mathcal{D}| = 3|m+\epsilon|\Omega^2$. If $|1-\gamma| \ll 1$, Laplace coefficients can be approximated by modified Bessel functions, $b^{m}_{3/2}(\gamma) \approx (3m^2/2\pi)K_1(m|1-\gamma|)$. These are to be evaluated at resonance, i.e., $m|1-\gamma_V| = 4/3$. For $m >> 1$, eqn (3) becomes, $(\csc I)dI/dt = 0.0235(M_s/M_p)\Omega(\sigma r^2/M_p)m^3$. To find the *total* change in I, eqn (3) must be summed over all resonances.

As an application, consider a Jovian size protoplanet forming from a circumstellar disc. The protoplanet is assumed to have terminated its gas accretion phase by tidally truncating the disc as proposed by Lin and Papaloizou (1979). Truncation will abort radial drift and damp the eccentricity, provided corotation torques from the edges of the gap dominate Lindblad torques (e.g., Goldreich and Tremaine, 1980, 1981; Ward, 1989b). However, Borderies et al. (1984) point out that there are no analogous corotation torques in the vertical resonance problem and thus no analogous mechanism to halt inclination growth. Resonances are summed to $m_{max} = (4/3)(r/w)$, where w is the gap half-width. The resulting rate implies a characteristic inclination growth time, $\tau \sim \sin I/(dI/dt)$, of

$$(4) \qquad \tau \sim \frac{27}{\Omega}\left(\frac{M_p}{M_s}\right)\left(\frac{M_p}{\sigma r^2}\right)\left(\frac{w}{r}\right)^4$$

The minimum size gap will be on the order of the scale height of the disc, $h \sim c/\Omega \sim .025r(aT_2)^{1/2}$, where a is in AU and T_2 is the disc temperature in 10^2 °K. Note that perturbation wavelengths, $\lambda \sim 2\pi r/m \gtrsim h$, so that the thin disc model used to derive eqn (3) should be roughly applicable. Adopting a one solar mass primary, $M_p \sim 2 \times 10^{33}$ gm., a Jovian size protoplanet, $M_s/M_p \sim 10^{-3}$, at 5.2 A.U., and a nebula surface density and temperature of, 400 gm/cm^2, and 160°K, respectively; eqn (4) reads, $\tau \sim 10^3\ (w/h)^4$ years. The expected lifetime of the nebula is of order 10^7 years (Adams and Shu, 1986; Walter, 1986). A characteristic growth time this long would require a gap width of order 10 scale heights.

The critical protoplanetary mass necessary to open a gap against viscous diffusion depends on the turbulent viscosity, ν, of the disc, i.e., $M_c/M_p \sim (\nu/r^2\Omega)^{1/2}(c/r\Omega)^{3/2}$ (e.g., Hourigan and Ward, 1984; Papaloizou and Lin, 1984). The mass required to sustain the edge at a distance $w > h$, exceeds M_c by a factor $(w/h)^{3/2}$. Thus, a gap width $w \sim 10h$ could be maintained by a protoplanet $\gtrsim 30$ times that needed to truncate the disc. However, since disc truncation will inhibit further gas accretion, it is not obvious how to reconcile these requirements. Borderies et al. (1984) speculated that the sign of dI/dt may change when $I \gtrsim w/a$, citing a similar behavior in the eccentricity case found by Goldreich and Tremaine (1981). Nevertheless, a minimum width gap would still permit inclination growth to $I \sim h/r \approx 4°$ for both Jupiter and Saturn, which seems inconsistent with their small mutual inclination. Alternatively, eqn (4) could imply that much of the gas accretion phase of the giant planets was correlated with disc dispersal so that their co-existence was for an interval significantly shorter than the disc's lifetime.

We should point out that the excitation rate of eqn (3) was calculated for a thin, self-gravitating particle disc. By contrast, a minimum mass solar nebula is a pressure dominated disc with a finite thickness in hydrostatic equilibrium with the vertical component of the solar gravity. However, in the case of density waves, it is well known that the same angular momentum flux is carried by both gravity and pressure waves and that the torque at a Lindblad resonance is independent of the restoring force responsible for the wave action. We suspect this is true for bending waves as well and that the torque strength will prove rather insensitive to the details of the wave process. The application discussed here underscores the importance of developing a more rigorous treatment of this problem.

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