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TECHNIQUE FOR CALIBRATING ANGULAR MEASUREMENT DEVICES WHEN CALIBRATION STANDARDS ARE UNAVAILABLE

TOM D. FINLEY

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NASA
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

ABSTRACT

A calibration technique is proposed that will allow the calibration of certain angular measurement devices without requiring the use of an absolute standard. The technique assumes that the device to be calibrated has deterministic bias errors. A comparison device must be available that meets the same requirements. The two devices are compared; one device is then rotated with respect to the other and a second comparison is performed. If the data are reduced using the technique described below, the individual errors of the two devices can be determined.

INTRODUCTION

All normal calibration techniques, whether for length, voltage, pressure, resistance, etc., involve the comparison of the test device against a measurement system or standard of known accuracy. Standard practice requires that a measurement device must be compared to a calibrated standard having an accuracy at least four times better than the test device (ref. 1). This requirement means that ultimately the standard must be calibrated by another laboratory with higher accuracy devices until a national or international standards laboratory is reached.

This is not always possible and it is often desirable to be able to determine the errors in a device when an adequate standard is not available. A technique has been developed that allows the calibration of angular measurement devices that have certain types of errors. If a second similar device is available for comparison, the errors in the two devices can be determined without previous knowledge of the errors in either device. If certain conditions are met, the technique described below shows how to extract individual errors in the two devices.

A technique has been developed in which two angular measurement devices such as protractors, inclinometers, encoders, resolvers, and Hirth couplings (ref. 2, 3) can be compared and the individual errors extracted. The requirements are that both units be stable, the errors in each must be periodic, and a means must be available to compare the two devices. The last requirement may be simple such as the case when two encoders are coupled together, and the comparison involves simply setting the shafts at different angles and reading the outputs of the devices. In other cases, such as the comparison of two Hirth couplings, an external device such as an autocollimator or an accelerometer may be required to monitor the difference between the devices.

ANALYSIS

Assume two angular measurement devices are to be calibrated and that the errors in both are repeatable and periodic. This means the error in the two devices can be represented by:

$$\begin{aligned} e_1 = {} & A_1 \sin(\theta) + A_2 \sin(2\theta) + A_3 \sin(3\theta) + \ldots \\ & + B_1 \cos(\theta) + B_2 \cos(2\theta) + B_3 \cos(3\theta) + \ldots \end{aligned} \tag{1}$$

and,

$$e_2 = A_1'\sin(\theta) + A_2'\sin(2\theta) + A_3'\sin(3\theta) + \ldots + B_1'\cos(\theta) + B_2'\cos(2\theta) + B_3'\cos(3\theta) + \ldots \tag{2}$$

Assume a method is available to measure the deviation between the two devices. The devices are oriented so that each starts at a known angle and a calibration is performed showing the differences between these devices. The set of deviation data obtained at various angles can be converted to the following approximation:

$$\delta_1 = (A_1-A_1')\sin(\theta) + (A_2-A_2')\sin(2\theta) + (A_3-A_3')\sin(3\theta) + \ldots + (B_1-B_1')\cos(\theta) + (B_2-B_2')\cos(2\theta) + (B_3-B_3')\cos(3\theta) + \ldots \tag{3}$$

If one of the devices is rotated with respect to the other by a known amount Φ, and a second comparison calibration is performed, then the new deviation can be characterized by:

$$\begin{aligned}\delta_2 = {} & A_1\sin(\theta) - A_1'\sin(\theta+\Phi) + A_2\sin(2\theta) - A_2'\sin(2(\theta+\Phi)) \\ & + A_3\sin(3\theta) - A_3'\sin(3(\theta+\Phi)) + \ldots \\ & + B_1\cos(\theta) - B_1'\cos(\theta+\Phi) + B_2\cos(2\theta) - B_2'\cos(2(\theta+\Phi)) \\ & + B_3\cos(3\theta) - B_3'\cos(3(\theta+\Phi)) + \ldots\end{aligned} \tag{4}$$

It can be shown that equation (4) can be reduced to:

$$\begin{aligned}\delta_2 = {} & \sin(\theta)\,[A_1 - A_1'\cos(\Phi) + B_1'\sin(\Phi)] \\ & + \sin(2\theta)\,[A_2 - A_2'\cos(2\Phi) + B_2'\sin(2\Phi)] \\ & + \sin(3\theta)\,[A_3 - A_3'\cos(3\Phi) + B_3'\sin(3\Phi)] + \ldots \\ & + \cos(\theta)\,[B_1 - B_1'\cos(\Phi) - A_1'\sin(\Phi)] \\ & + \cos(2\theta)\,[B_2 - B_2'\cos(2\Phi) - A_2'\sin(2\Phi)] \\ & + \cos(3\theta)\,[B_3 - B_3'\cos(3\Phi) - A_3'\sin(3\Phi)] + \ldots\end{aligned} \tag{5}$$

Depending on the amount of offset, Φ, certain values in this series do not produce independent data. For example, if $\Phi = 90$ degrees, then the 4th, 8th, 12th, etc., terms of the series produce the identical information in both sets of data so that the coefficients of these terms cannot be determined. The simplest method to estimate the number of harmonics, N, required to characterize the errors is to examine the initial set of deviations using a Fourier transform, determine the number of harmonics needed to closely fit the data, and then chose the value of Φ by:

$$\Phi = 360/(N+1) \text{ degrees} \tag{6}$$

The technique works best when there are large changes in the patterns from the two sets of measured data. The maximum change occurs when the value of Φ is half of the spacing between the most pronounced cycle of error. Even though selecting a high value for N allows the periodic data to be more precisely fit, care must be taken not to select N too large. If N is high then Φ is small and the shifts in the comparison patterns are slight and the error caused by small random variations becomes large. In the calibration of angular devices N = 3 has been shown to work well. The approximation to the error sets is good, and setting Φ = 90 degrees caused considerable differences to appear between the two sets of data.

If we choose Φ = 90 and limit our approximation to the first three terms, the equation (5) can be simplified to:

$$\delta_2 = (A_1 + B_1')\sin(\theta) + (A_2 + A_2')\sin(2\theta) + (A_3 - B_3')\sin(3\theta) + (B_1 - A_1')\cos(\theta) + (B_2 + B_2')\cos(2\theta) + (B_3 + A_3')\cos(3\theta) \qquad (7)$$

If the coefficients of equations (3) and (7) can be determined then the individual values of the original error terms can be calculated. The coefficients of the sets of measured deviation data can be determined by Fourier analysis, least squares regression, or numerical approximation to continuous Fourier coefficients. Once these coefficients are known, there are 12 equations and 12 unknowns which will permit solving for the coefficients that characterize the errors of the two individual measurement devices.

SAMPLE CALIBRATION

A comparison was performed using two Hirth couplings, each having a manufacturer's accuracy specification of less than 1 arc second, as the test devices and a precision servo accelerometer as an indicator of deviation. The data are shown in table 1. The second set of data, taken with device 2 shifted 90 degrees, appears in column 3 of table 1.

The initial set of data was approximated by a discrete Fourier transform algorithm. The first three terms of the series fit the data with a standard deviation of .06 seconds (see figure 1). The second series was also fit and the standard deviation was also .06 seconds (see figure 2).

The following equations represent the fitted curves. Note that the zero frequency component is included here although it has no meaning since the zero on the dividers is arbitrary.

$$\delta_1 = .2889 + (-.1556)\sin(\theta) + (1.3389)\sin(2\theta) + (-.0111)\sin(3\theta) + (-.0889)\cos(\theta) + (-.2833)\cos(2\theta) + (.1500)\cos(3\theta) \qquad (8)$$

$$\delta_2 = -.1769 + (-.2895)\sin(\theta) + (-.2929)\sin(2\theta) + (-.1476)\sin(3\theta) + (.1799)\cos(\theta) + (.0019)\cos(2\theta) + (-.0091)\cos(3\theta) \qquad (9)$$

We now have 12 equations and 12 unknowns that can be solved to find the equations for the error in the two devices under test:

$A_1 - A_1' = -.1556$ $\quad$ $A_2 - A_2' = 1.3389$ $\quad$ $A_3 - A_3' = -.0111$

$B_1 - B_1' = -.0889$ $\quad$ $B_2 - B_2' = -.2883$ $\quad$ $B_3 - B_3' = .1500$

$A_1 + B_1' = -.2895$ $\quad$ $A_2 + A_2' = -.2929$ $\quad$ $A_3 - B_3 = -.1476$

$B_1 - A_1' = .1799$ $\quad$ $B_2 + B_2' = .0019$ $\quad$ $B_3 + A_3' = -.0091$

This can be solved by matrix manipulation or by simple algebra to determine the following:

$A_1 = -.357$ $\quad$ $A_1' = -.201$

$A_2 = .523$ $\quad$ $A_2' = -.816$

$A_3 = -.159$ $\quad$ $A_3' = -.148$

$B_1 = -.021$ $\quad$ $B_1' = .067$

$B_2 = -.141$ $\quad$ $B_2' = .143$

$B_3 = .139$ $\quad$ $B_3' = -.011$

Thus the error in device #1 is approximately:

$$e_1 = -.357\sin(\theta) + .523\sin(2\theta) - .159\sin(3\theta)$$
$$-.021\cos(\theta) - .141\cos(2\theta) + .139\cos(3\theta)$$

The approximate error in device 2 is:

$$e_2 = -.201\sin(\theta) - .816\sin(2\theta) - .148\sin(3\theta)$$
$$+ .067\cos(\theta) + .143\cos(2\theta) - .011\cos(3\theta)$$

Figure 3 shows the calculated errors in the two test devices.

PROOF OF CONCEPT

A method was devised to demonstrate the ability of this technique to extract individual errors. The method involves the error extraction of two devices with relatively large errors and the subsequent calibration of these devices against calibrated standards. This demonstrates the ability of the technique to separate errors and provides calibration curves that can be compared directly with the calibration curves determined by normal means. The actual values of the errors are large but the technique is based on the shape of the curves so that magnitude is not critical. The ability to extract degree errors to a tenth of a degree would imply the ability to extract arc second errors to a tenth of an arc second.

Two generalized "protractors" (radial spoke patterns with lines about every 10 degrees) were drawn on paper (figures 4 and 5) with intentional errors built in. The errors in each were generated as 3-harmonic sinusoidal series with about 3 degrees peak amplitude but were distinctly different in the two. The protractors were overlaid and aligned at zero degrees. The differences were estimated by eye using a linear scale near the circumference, and are shown in column 2 of table 2. One protractor was then rotated 90 degrees (to the ninth line) with respect to the other and a second set of deviations was recorded (table 2, column 3). The estimated error in making these comparisons was about .25 degrees. The deviation plots are shown in figures 6 and 7.

The data were reduced by the technique described in this paper and the individual errors extracted. The two protractors were each calibrated against a normal protractor to determine their true errors. The accuracy of this comparison was about .25 degrees. The derived curves are compared with the true values in figures 8 and 9.

As can be clearly seen the correction curves determined in the two methods correlate very well. The standard deviation of the differences between the errors using the direct techniques and the new technique was .31 degrees. This proves the concept described in this paper.

CONCLUSIONS

A technique has been developed that will allow certain devices to be calibrated against each other without the need for a standard. If the errors in each are repeatable, periodic, and able to be characterized by a reasonable number of sinusoidal harmonics, the individual errors in the two devices can be determined.

The exact effects of scatter and drift in the test devices and the effect of errors in the comparison measurement have not been thoroughly investigated. This remains as possible future work in this area. Other possible future work involves using this method to intercompare three high-accuracy devices. This would allow the extraction of the data on a common device by comparison to two other devices. This would further verify the method and show how much error should be expected when using this technique when comparing very high accuracy devices.

ACKNOWLEDGEMENT

I would like to express my appreciation for the dedicated efforts of Mr. Alan Smart and Mr. Richard Faison of Wyle Laboratories for their efforts in performing these calibrations.

REFERENCES

(1) Safety, Reliability, Maintainability and Quality Provision for the Space Shuttle, NHB 5300.4 (ID-2), 1979.

(2) Handbook of Dimensional Measurement, Metrology and Inspection Journal, (May 1978).

(3) Francis T. Farago, Handbook of Dimensional Measurements, Second Edition. Industrial Press, Inc., New York, 1982.

Reference angle	deviation (sec) (Phi = 0)	deviation (sec) (Phi = 90)
0	0.00	0.00
10	0.44	-0.26
20	0.95	-0.47
30	1.29	-0.62
40	1.28	-0.55
50	1.27	-0.55
60	1.24	-0.60
70	1.08	-0.51
80	0.82	-0.47
90	0.47	-0.32
100	0.05	-0.20
110	-0.40	-0.25
120	-0.72	-0.34
130	-0.91	-0.36
140	-0.96	-0.32
150	-0.96	-0.32
160	-0.84	-0.37
170	-0.50	-0.32
180	-0.09	-0.35
190	0.39	-0.34
200	0.85	-0.35
210	1.45	-0.32
220	1.89	-0.34
230	1.96	-0.30
240	2.00	-0.19
250	1.66	-0.15
260	1.29	-0.14
270	0.73	-0.07
280	0.10	0.07
290	-0.44	0.18
300	-0.72	0.34
310	-0.92	0.63
320	-1.10	0.64
330	-1.02	0.59
340	-0.82	0.42
350	-0.43	0.14

Table 1. Deviation between Hirth couplers.

Reference angle (deg)	deviation, Phi=0 (deg)	deviation, Phi=90 (deg)
0	0.00	0.00
10	0.61	0.49
20	1.19	0.83
30	1.45	1.20
40	1.33	1.17
50	1.02	1.00
60	0.52	0.43
70	0.00	-0.29
80	-0.26	-0.91
90	-0.03	-1.46
100	0.64	-1.83
110	1.54	-1.83
120	2.67	-1.37
130	3.51	-0.71
140	4.06	0.03
150	3.77	0.46
160	3.05	0.63
170	1.74	0.29
180	0.06	-0.54
190	-1.45	-1.51
200	-2.78	-2.71
210	-3.34	-3.69
220	-3.28	-4.46
230	-2.55	-4.83
240	-1.36	-4.57
250	0.12	-3.86
260	1.36	-3.09
270	2.20	-2.17
280	2.58	-1.31
290	2.29	-0.71
300	1.60	-0.40
310	0.73	-0.23
320	-0.23	-0.34
330	-0.70	-0.49
340	-0.81	-0.54
350	-0.61	-0.46

Table 2. Deviation between protractor readings.

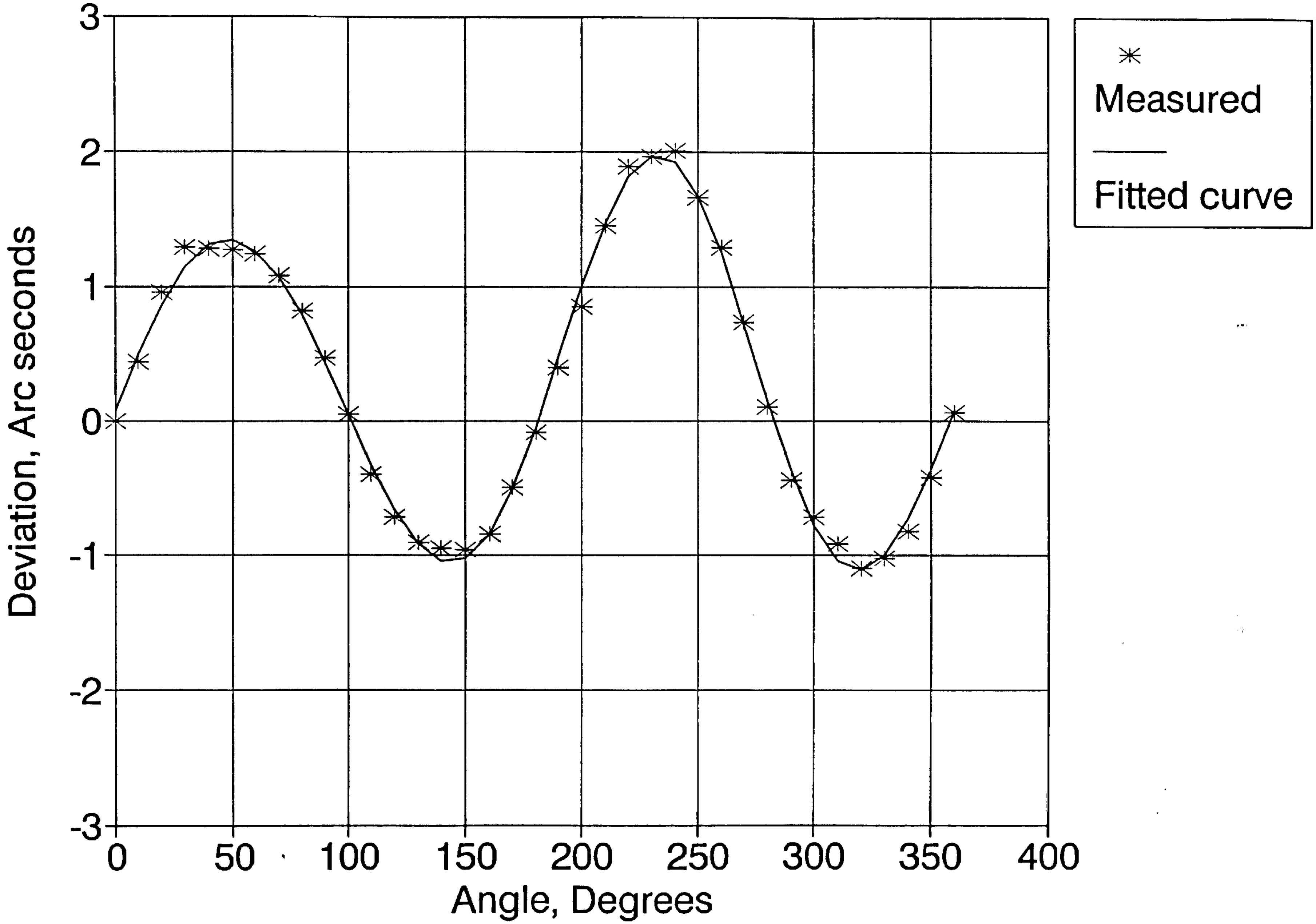

Figure 1. Comparison of unshifted indexers.

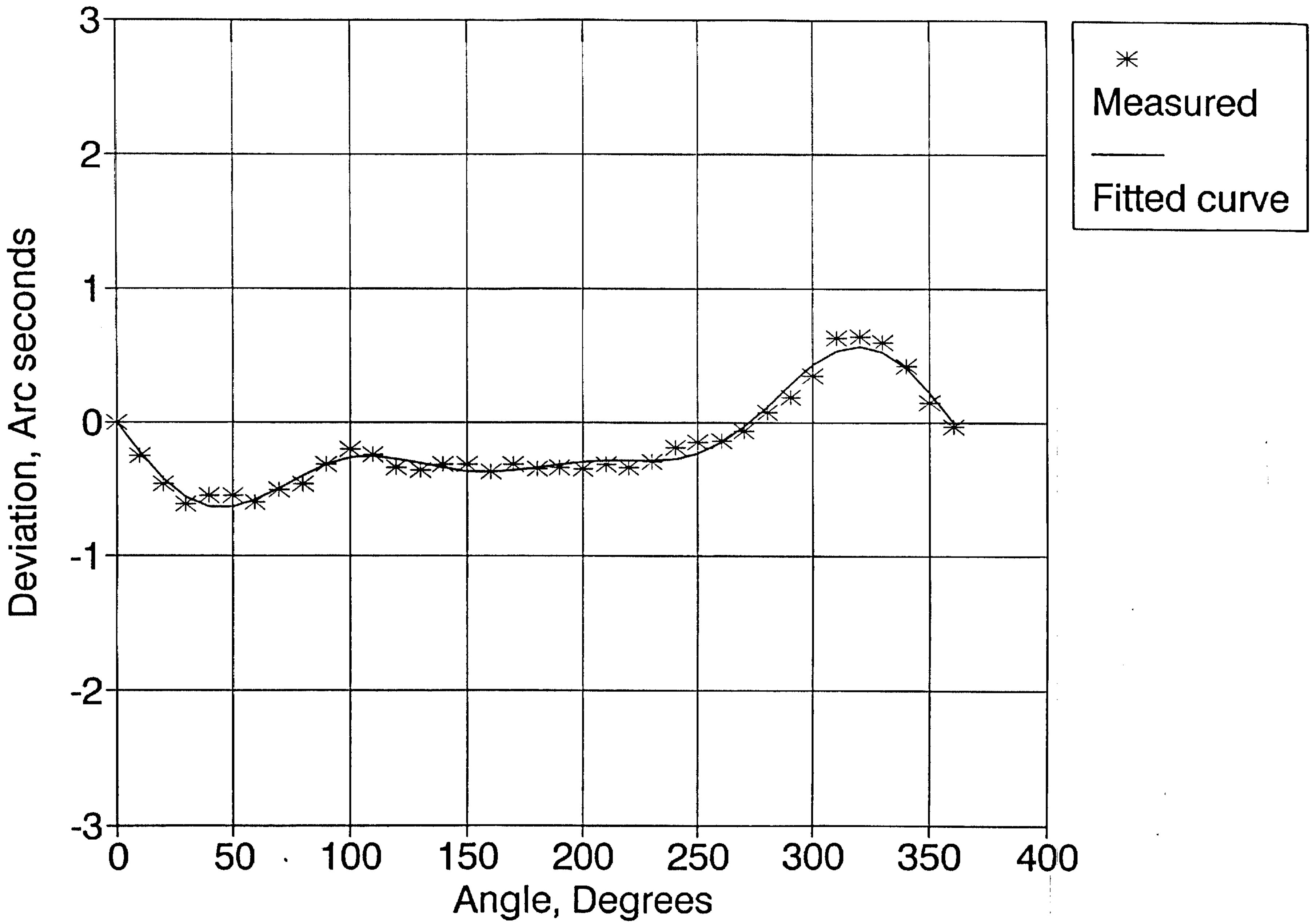

Figure 2. Comparison of shifted indexers.

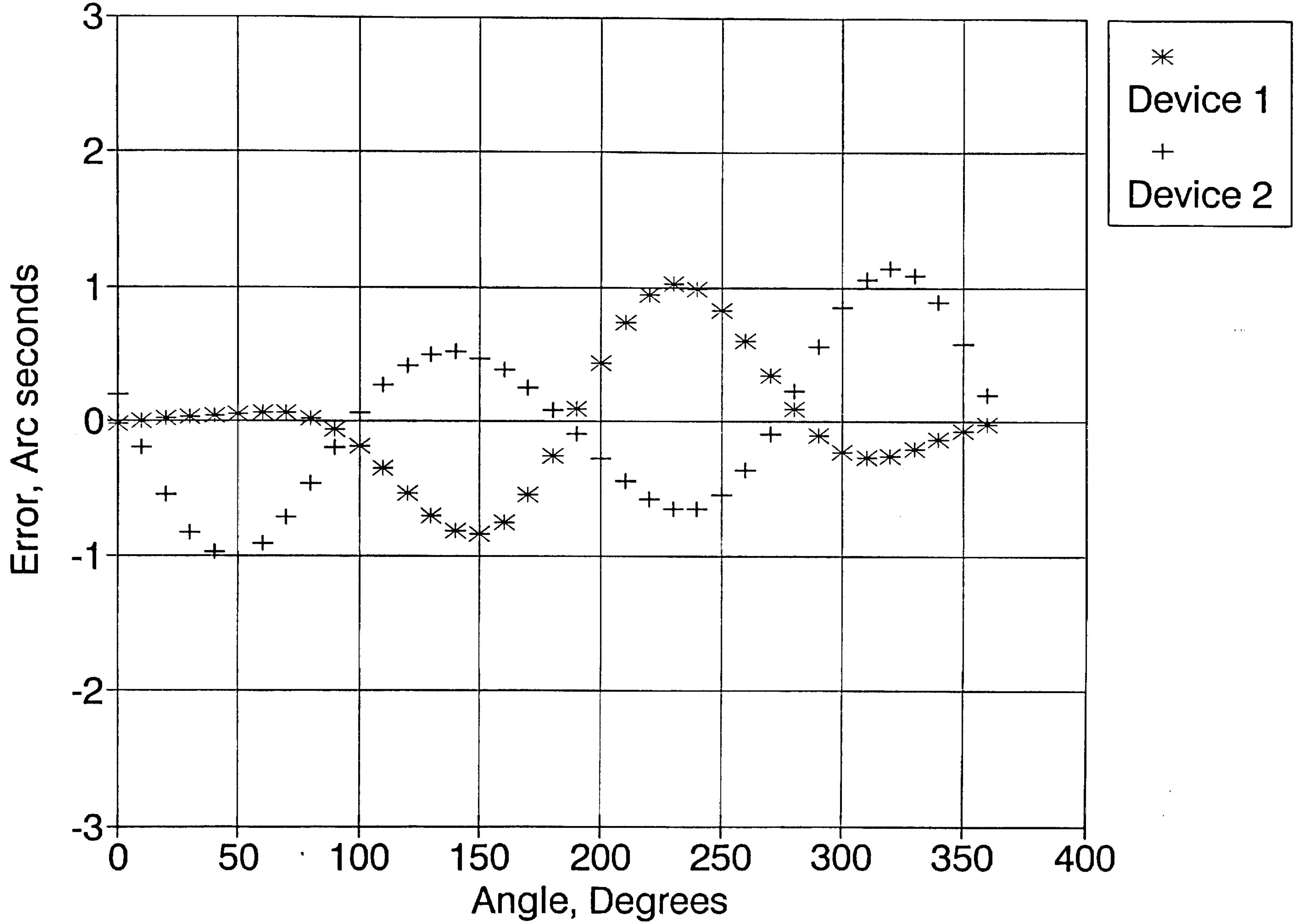

Figure 3. Extracted errors in individual indexers.

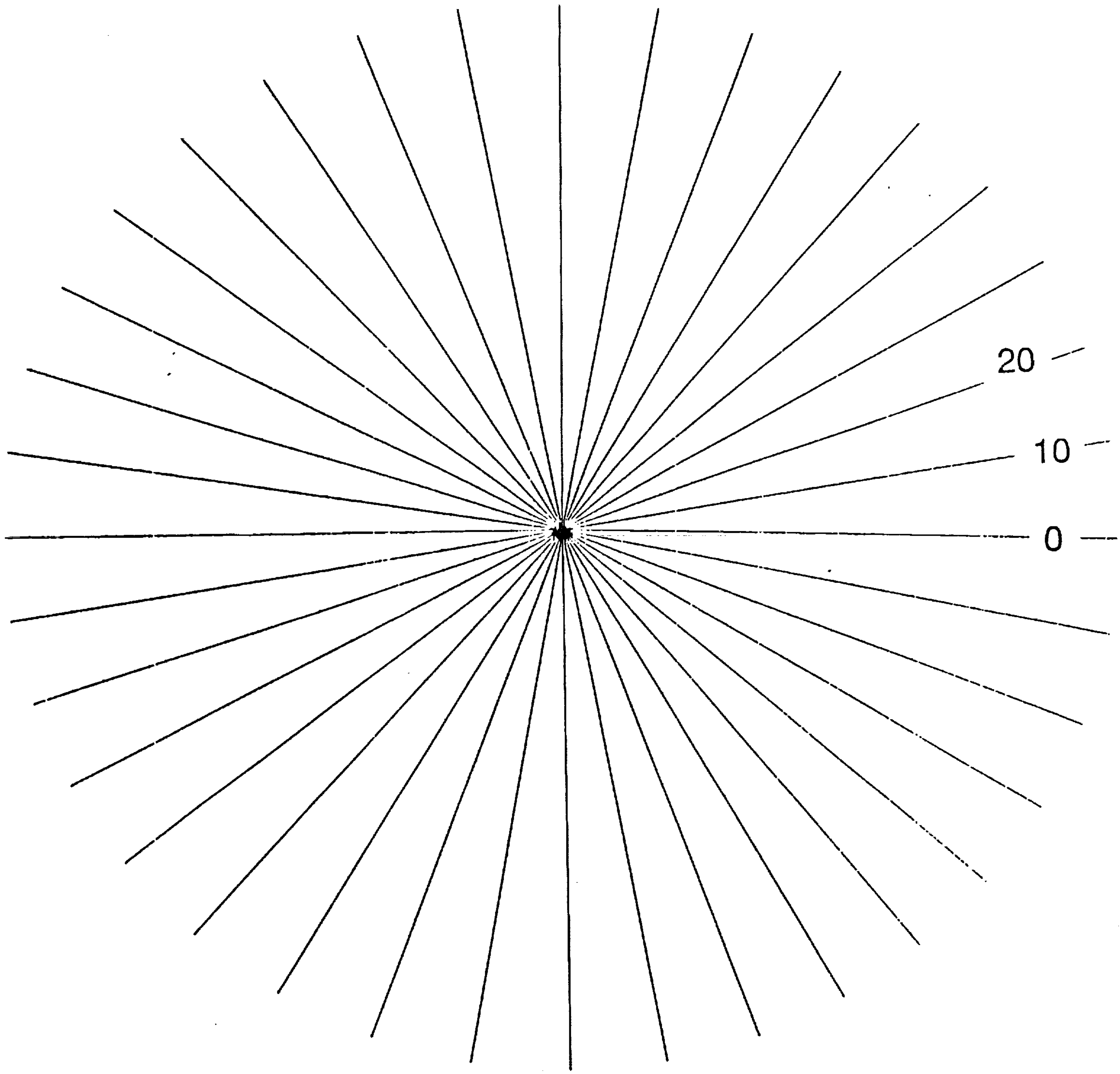

Figure 4. Protractor #1

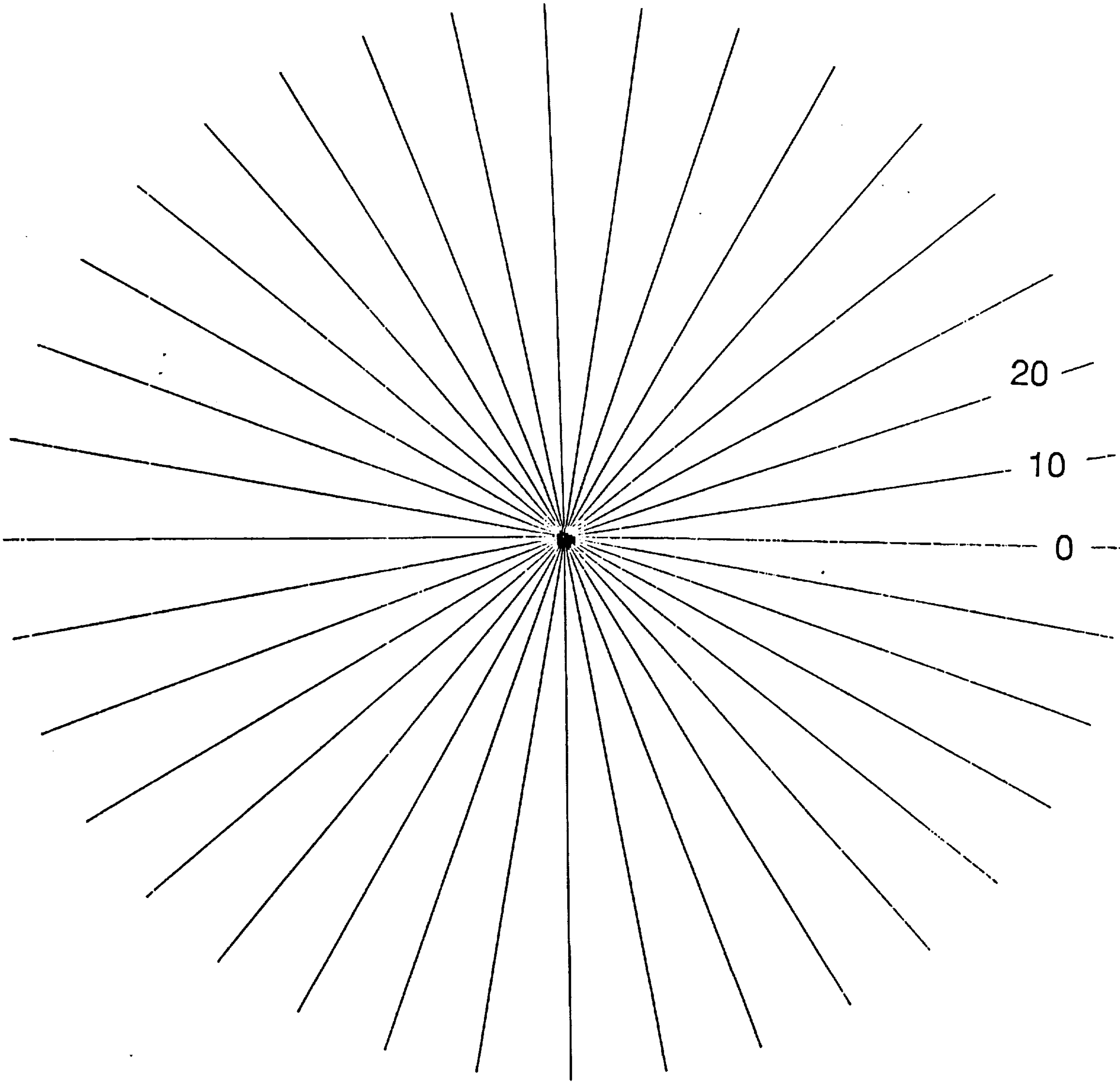

Figure 5. Protractor #2

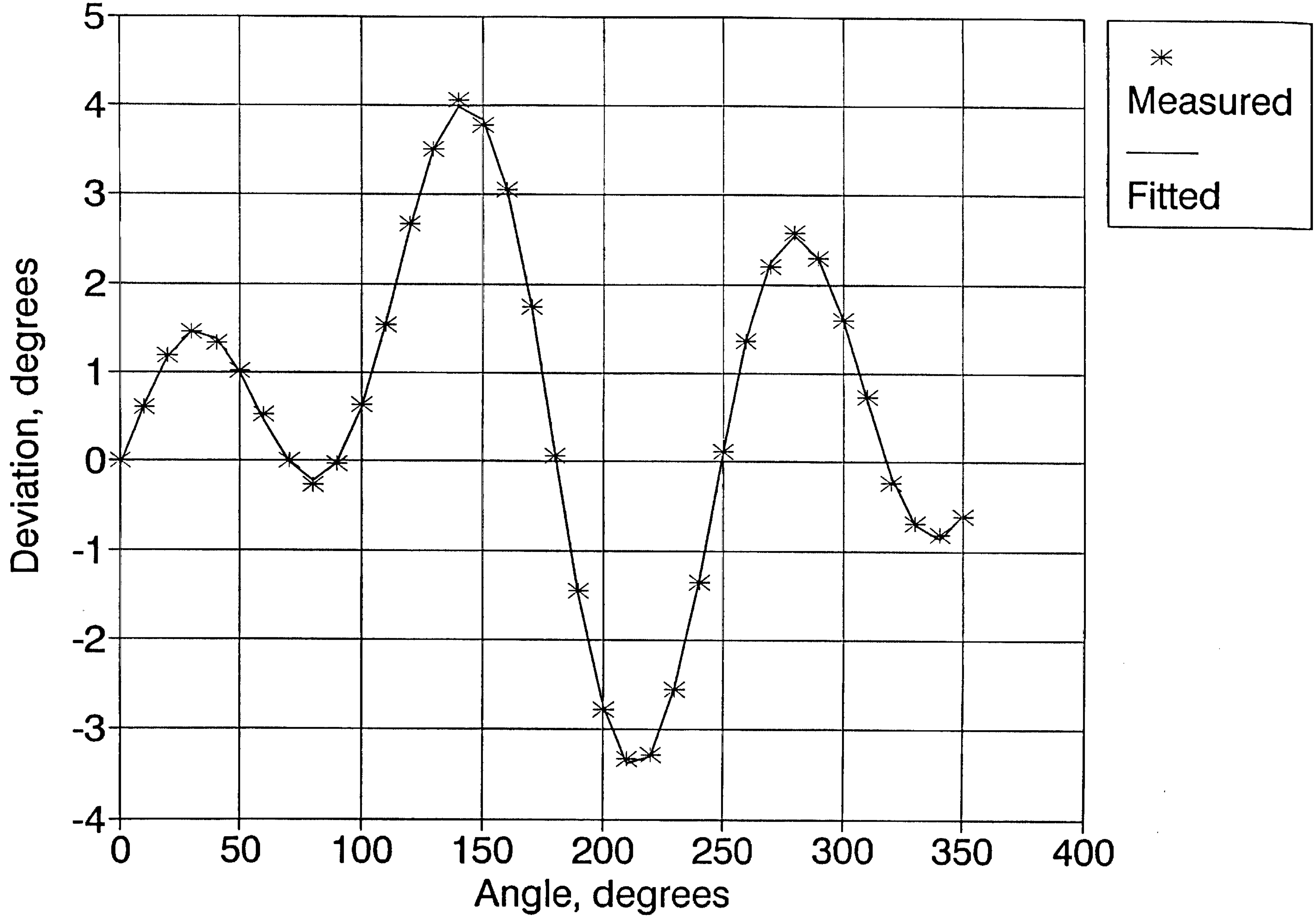

Figure 6. Deviation between protractors (unshifted).

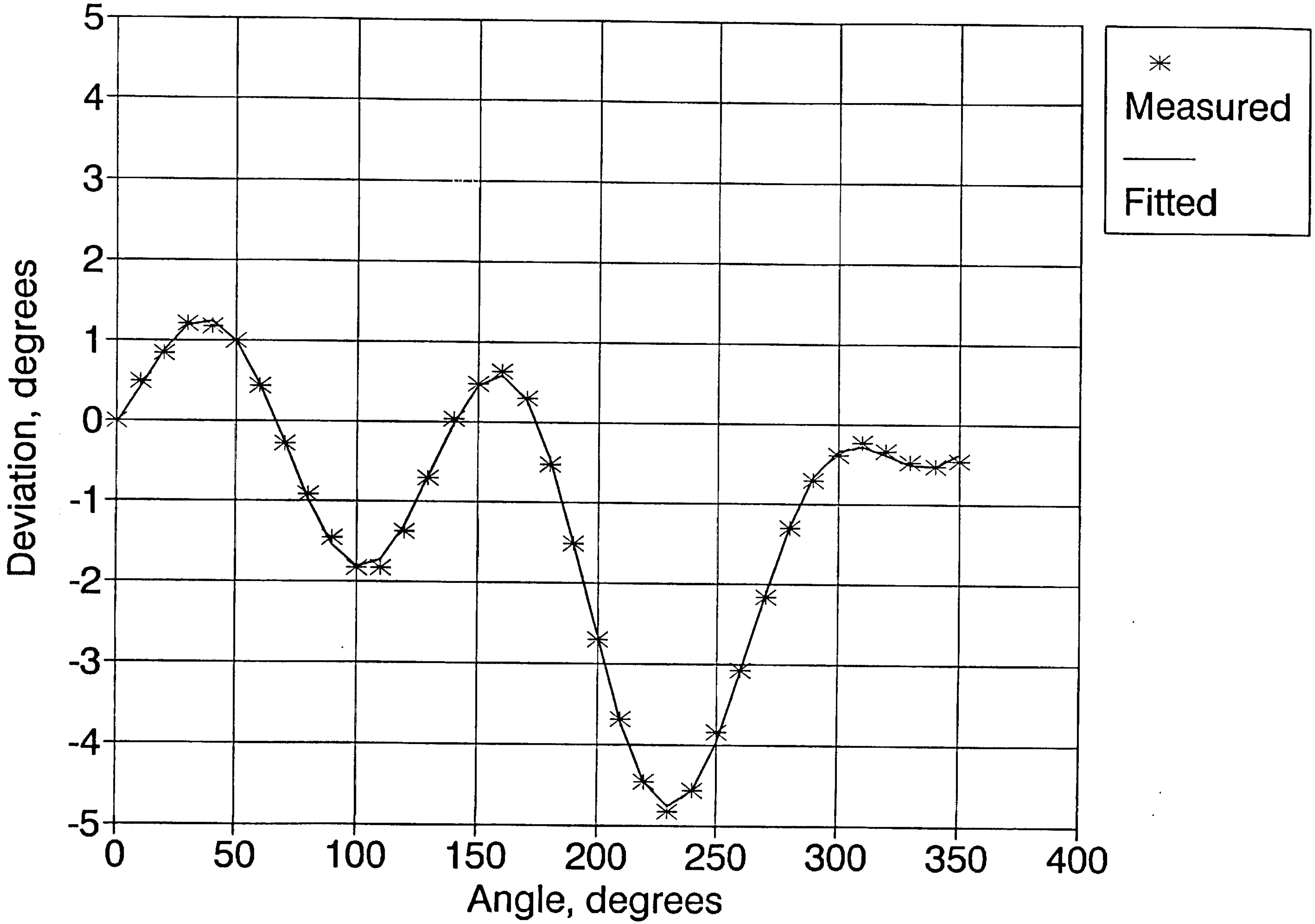

Figure 7. Deviation between protractors (#1 shifted 90 degrees).

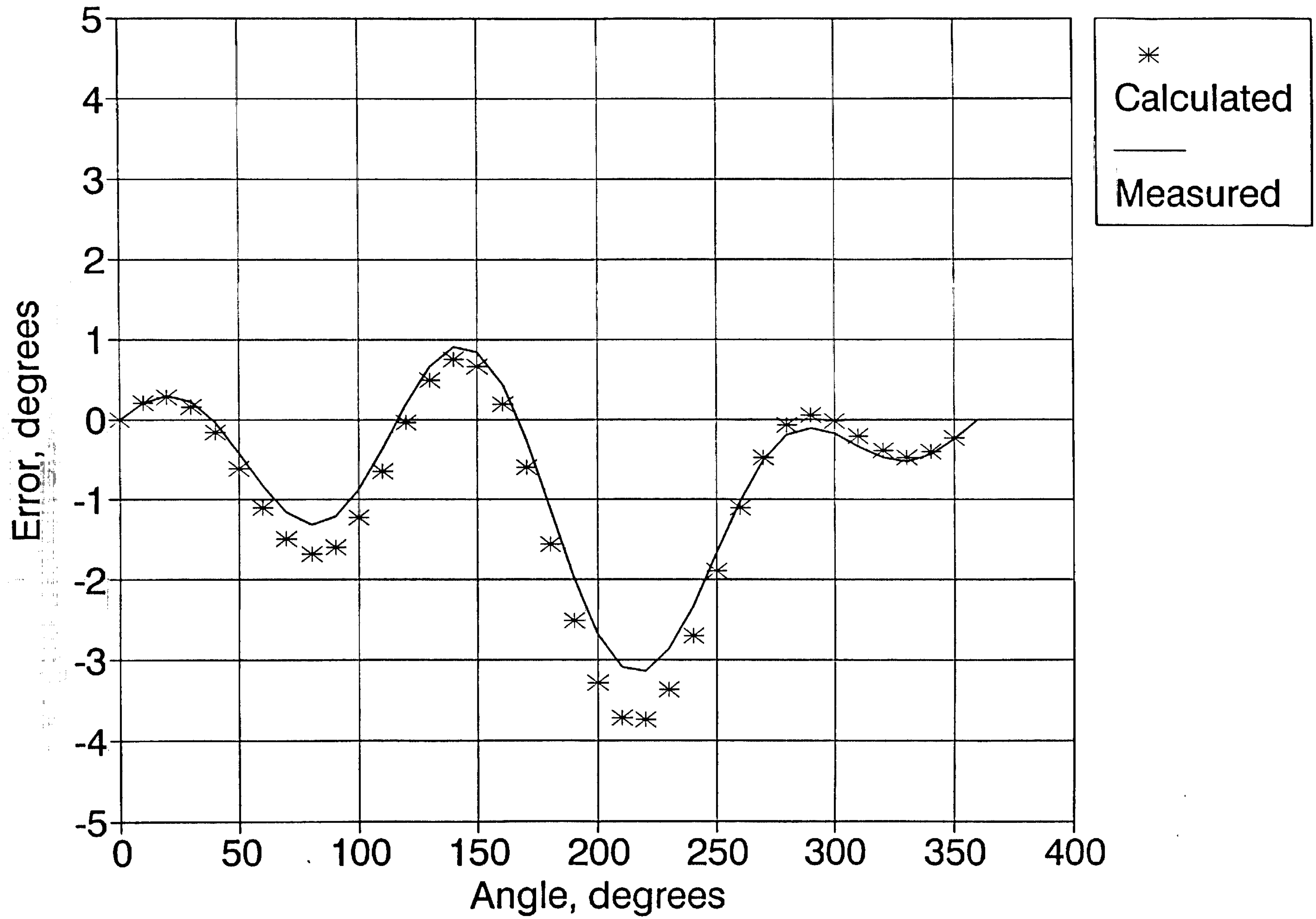

Figure 8. Results of calibrating protractor #1.

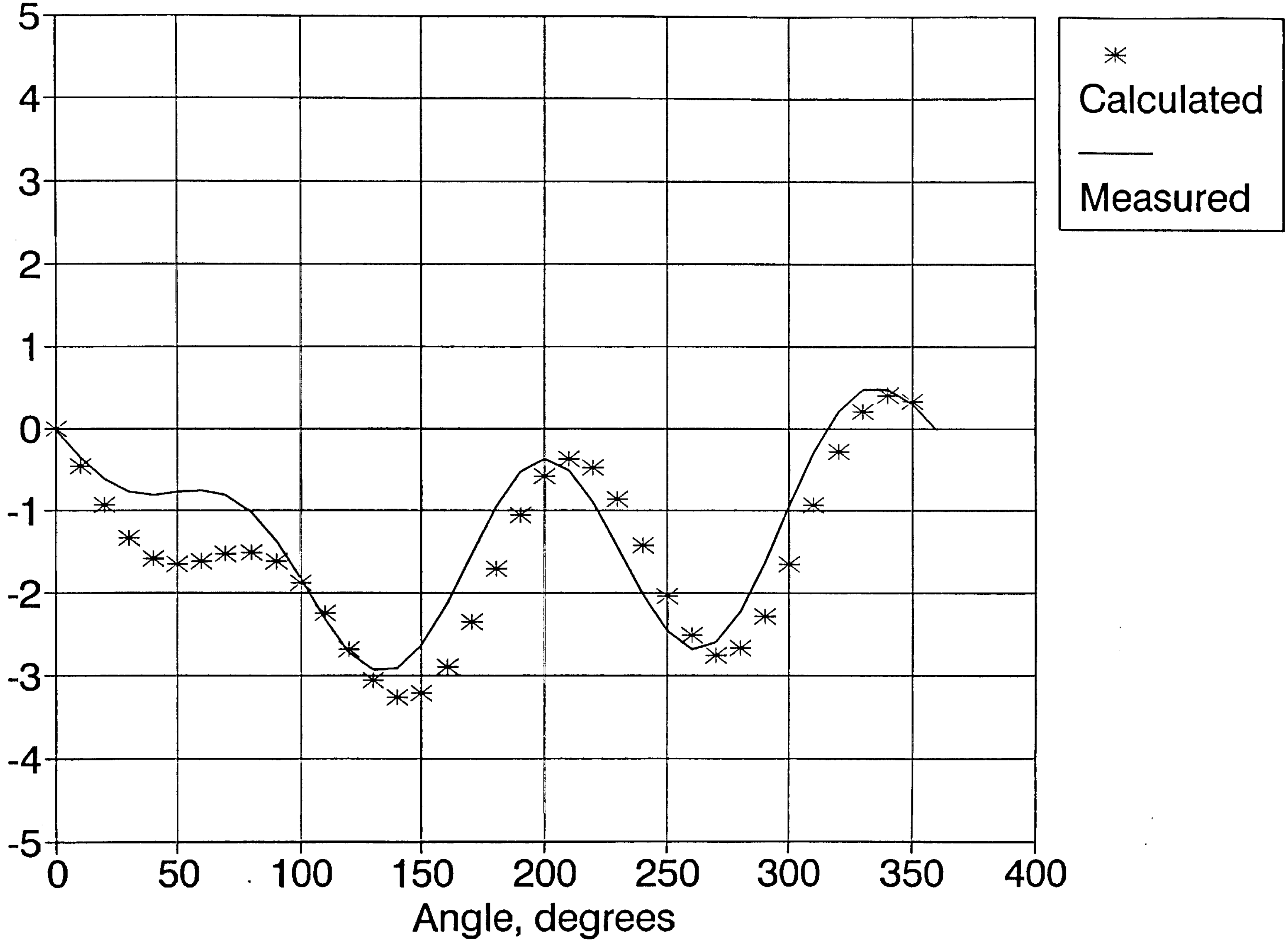

Figure 9. Results of calibrating protractor #2.

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