Discussion Summary: Fictitious Domain Methods

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Fictitious Domain methods are constructed in the following manner: Suppose a partial differential equation is to be solved on an open bounded set Ω in two or three dimensions. Let R be a rectangular domain containing the closure of Ω. The partial differential equation is first solved on R. Using the solution on R, the solution of the equation on Ω is then recovered by some procedure.

The advantage of the fictitious domain method is that in many cases the solution of a partial differential equation on a rectangular region is easier to compute than on a non-rectangular region. Not only do more accurate algorithms exist for rectangular regions but they are also more computationally efficient. The difficulty in the method, of course, is the procedure that is used to tie the "global" solution on R to the "local" solution on Ω. This is generally where the inefficiencies of the method creep in and where most of the current research on the method is being done. A classic application of a fictitious domain method is the computation of the solution of an elliptic partial differential equation on a general region. Here, the global solution on a rectangular region can be computed by a fast Poisson solver. These solvers are quite effecient for rectangular regions but not for other geometries. The global solution is tied to the local solution on the general region via ideas in capcitance matrix techniques. A discusion of this approach is given in [1]. Other uses of fictitious domain methods are given in the remaining references.

Fictitious domain methods for solving elliptic PDEs on general regions are also very efficient when used on a parallel computer. The reason for this is that one can use the many domain decomposition methods that are available for solving the PDE on the fictitious rectangular region. In domain decomposition methods, the global rectangle is decomposed into sub-rectangles of equal size and elliptic PDEs are solved on each sub-rectangle to provide approximations to the elliptic PDE on the global rectangle This process is iterated upon several times to get successively better approximations, see references in [4]. This is significant in that the approximations on the sub-rectangles can often be computed simultaneously and, thus, can be carried out in parallel on the individual processors of a multiprocessor. Moreover, because the approximations to the global equation are furnished by the solutions of PDEs on sub-rectangles of equal size, they can be calculated in the same amount of time. This is advantageous because the load balancing and synchronization overhead incurred from managing the tasks of computing the approximations on the processors becomes almost non-existent.

The discussion on fictitious domain methods began with a short talk by Roland Glowinski where he gave some examples of a variational approach to fictitious domain methods for solving the Helmholtz and Navier-Stokes equations. After Glowinski's introduction to the subject, the question of the usefulness of the method was tossed out to the audience. A

comment was made that the fictious domain method seems to be the only reasonable way of solving three-dimensional partial differential equations on general domains. This statement was justified by the admission by several people that the implementations of standard finite element or finite difference techniques to 3-D problems is a horrendous task. There was a considerable amount of discussion on this point but, in the end, the audience agreed that the fictitious domain method is a viable approach to three-dimensional problems and more research is needed in this area.

References

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