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One-Way Nesting for a Primitive Equation Ocean Model

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Prognostic numerical models for atmospheric and oceanic circulations require initial fields, boundary conditions, and forcing functions in addition to a consistent set of partial differential equations, including a state relation and equations expressing conservation of mass, momentum and energy. Depending on the horizontal domain to be modeled, the horizontal boundary conditions are either physically obvious or extremely difficult to specify consistently. If the entire atmosphere is modeled, periodic horizontal boundary conditions are appropriate. On the other hand, the physical horizontal boundaries on the entire ocean are solid walls. Obviously, the normal velocity at a solid wall is zero while the specification of the tangential velocity depends on the mathematical treatment of the horizontal viscous terms. Limitations imposed by computer capacity and cost, as well as research interests, have led to the use of limited area models to study flows in the atmosphere and ocean. The limited area models do not have physical horizontal boundaries, merely numerical ones. Correctly determining these open boundary conditions for limited-area numerical models has both intrigued and frustrated numerical modelers for decades.

One common approach is to use the closed or solid wall boundary conditions for a limited-area model. The argument given for this approach is that the boundary conditions affect flow near the walls but that none of these effects are propagated into the interior. Therefore, one chooses a big enough domain that the central region of interest is not corrupted by the boundary flow. Research in progress to model the North Atlantic circulation (J. D. Thompson, private communication) vividly illustrates the pitfalls of this approach. The area covered by the Atlantic Ocean model lies between longitudes 0 and 100W and between latitudes 60N and 20S with the continental boundaries in place as appropriate and the open water boundaries artificially closed. Two model runs are compared: (A) The southern boundary at 20S between latitudes 0 and 40W is artificially closed and (B) the same boundary is specified as open with an inward transport of 15 *Sv* (determined from a global model with the same physics) uniformly spread across the boundary. Comparison of runs A and B shows significant differences. For example, the maximum eddy kinetic energy (divided by the mean density) is 700 cm^2/sec^2 in run A while that for run B is 1900 cm^2/sec^2. The Gulf Stream in run B detaches from the eastern boundary of the United States at the correct latitude of approximately 40N while the Gulf Stream in run A never truly flows along the eastern boundary of the United States at all. The circulation in the tropics and along the eastern boundary of South America also differs radically between the two runs. There are regions in the two runs where there is no difference but such regions are small and of little interest, i.e. they have very low eddy kinetic energy. These studies and others indicate that the interior flow of limited-area models can be dramatically affected by the incorrect use of closed boundary conditions.

A second common approach is to "nest" the limited-area model inside of another numerical model which covers a much larger domain. The outer domain model then supplies the boundary conditions at the open boundaries of the inner domain or limited-area model. As an example, the North Atlantic model described above could have boundary information supplied by a global ocean model which has physical, solid walls or closed boundaries. The outer domain model usually has a larger time step and coarser mesh size than the inner domain model. If the inner and outer domain models are described by the same differential equations and assumptions, then the nesting problem is homogeneous. Otherwise, the nesting problem is heterogeneous. The nesting is described as two-way if information passes from the outer domain to the inner domain and vice-versa. If the outer domain model passes information to the inner domain but the inner domain information is not passed into the outer domain, then the nesting is one-way. Only one-way nesting with a homogeneous system of numerical models is presented here although future work with two-way (or coupled) nesting and with heterogeneous model systems is planned.

In general, nesting involves two separate problems. The first is the interpolation of information from a coarse mesh, outer domain, to a finer mesh, inner domain. The second is the modification of the information supplied by the outer domain before it is applied to the boundary of the inner domain. Much of the research done to date has not distinguished between these two separate problems.

Linear interpolation is the easiest interpolation method to use. However, linear interpolation alters the long wavelength information contained in the original fields and adds short wavelengths that are not present at all in the original fields. Thus, linear interpolation alters the energy distribution of the original fields. To avoid these problems, a variation of the resampling method commonly used by engineers in the time-frequency domain (B.E. Eckstein, private communication) has been tested. A fast Fourier transform (D.N. Fox, private communication) has been modified so that the output fields, after the inverse Fast Fourier Transform, have the required fine grid mesh, although the input fields were supplied on the coarse grid mesh. After testing, this technique was modified (A. Wallcraft, private communication) to handle irregular coastal geometry, which also has to be interpolated. This interpolation scheme has been used extensively with the Pacific Basin numerical models to avoid the lengthy and expensive new spin-ups required whenever the mesh size is changed. (Further discussion of the Pacific Basin research can be found in Hurlburt et al. [1]).

The effects of changing the mesh size are similar in many ways to those found by changing the coefficient of horizontal eddy viscosity, A_H. Therefore, in order to avoid interpolation effects, the open boundary conditions are studies using models with different coefficients of horizontal viscosity. There are three model runs to be considered here. The *applied* run is made with the large outer domain and with a large value of A_H. The *nested* run is made with the small inner domain and a small value of A_H. The *true* run is made with the large outer domain and with a small value of A_H. The boundary conditions applied on the open boundaries of the small domain are taken from the matching grid points on the outer domain and "adjusted" as described below.

The numerical ocean model used for both the inner and outer domain is a reduced gravity, one active layer, primitive equation model with the hydrostatic approximation used. The fluid is assumed to be incompressible with uniform density in each layer. The effects of the density difference between the two layers is ignored except when multiplied by the earth's gravitational acceleration. The prognostic equations for the horizontal components of momentum are written in transport form while the continuity equation is the prognostic equation for the layer-depth of the upper, active layer. A spherical coordinate system is used and the effects of the earth's rotation are included. For further details of these equations in analytic form, see Hurlburt and Thompson [2]. This ocean model will be referred to as the NOARL model.

The outer domain used is a rectangle. The wind forcing is analytic and drives a double gyre in the ocean model. This choice permits the placement of the inner domain to isolate various types of flow: normal or tangential to the open boundary, strong or weak, or flow which changes along the open boundary either spatially or temporally (for time-varying forcing). The work presented here has only one open boundary, either on the western or northern boundary of the inner domain, and the other three boundaries are closed, matching the outer domain.

The NOARL ocean model uses a staggered grid to increase the computational accuracy. If solid walls (closed boundaries) are used, then the eastward velocity, u, and the northward velocity, v are set to zero along the solid walls. It follows that the eastward transport, U, and the northward transport, V, must be zero also on the solid walls. For solid walls, no boundary condition for the layer depth, h, is required. needed. If a boundary is open, then initial conditions for all five variables u, v, U, V, and h must be specified to obtain a numerical solution. However, arbitrary specification of these five variables on the open boundary will in general overspecify the solution. In general and in this research, if the inner domain open boundary values are supplied directly from the outer domain with no modification or adjustments, the inner domain model will eventually "blow up", much less give the correct solution.

If the open boundary condition cannot be specified exactly, then the goal is to prevent reflections at the open boundary which quickly destroy the interior solutions. Most nesting work uses some combination of four basic techniques (Koch and McQueen [3]): blending, filtering, damping, and radiation. Damping refers to an increase in the coefficient of eddy viscosity near the open boundary. Filtering, which is used in many numerical models without open boundaries, is the replacement of a calculated value at a given gridpoint with a weighted combination of the calculated value and the surrounding values. Blending is the replacement of the calculated prognostic term near the boundary of the inner grid with a combination of the prognostic term from the inner grid and that from the outer grid. The radiation technique (Sommerfeld [5] and Orlanski [4]) calculates the boundary values, assuming a wave is passing through the boundary. The first three techniques tend to destroy the small scale structure of the inner grid parameters which defeats the main purpose of running the inner grid with increased horizontal resolution. The radiation technique tends to let the waves pass out but is limited by the problem of calculating the phase speed needed. The question arises as to how the phase speed should be calculated if there are several types of waves present.

The goal of this research is to produce a nesting technique which does not destroy the inner grid solution or reduce any improvements made in the solution by using the finer grid. Therefore, no blending nor any additional damping or filtering has been used on the inner domain. The radiation technique has been modified from that used by Sommerfeld [5] and Orlanski [4]. The wave equation is used, not with an inner grid variable, but with a new variable that is the difference between the inner domain and the outer domain variable, i.e., $Q(inner) - Q(outer)$. The actual open boundary condition used on the open boundary is the sum of the outer domain solution and the q found from the wave equation:

$$\partial q/\partial t + c\partial q/\partial n = 0,$$

where c is the phase speed and n is the direction normal to the boundary. The phase speed used is determined from the mean outflow and the inflow phase speed is set to zero. The mass exchange along the boundary is the same for the inner and outer domains.

The quality of the nesting technique is measured by how well the inner domain solution (the *nested* run) compares with the *true* run (with the outer domain) solution. This difference is compared to the difference between the *true* outer domain solution and the *applied* outer domain solution. The first tests were done with steady forcing and nearly normal outflow. For these cases the differences between the *true* and *nested* solution after a year are less than five percent of the differences between the *true* and the *applied* solutions everywhere except for a very small area near part of the open boundary where the values go up to 20%. This small portion of the open boundary is where both the non-normal flow is the largest and the normal flow reverses sign. Note that this region is confined close to the boundary and does not propagate into the interior of the inner domain. Model runs have been extended for five years. Although the differences between *true* and *applied* runs increase with time, the differences between the *true* and *nested* runs increase much more slowly. Therefore, the percentages cited above actually decrease with longer model runs.

Ongoing research includes testing open boundaries with non-normal flow, strong jets, and reversal of flow with time. Also, the nesting technique is being tested with actual ocean models with irregular coastlines included. Specifically, a tropical Pacific Ocean model has been nested into a Northern Pacific Basin Model for testing.

The results to date include:

- Open boundary conditions that can handle both inflow and outflow grid points.
- Phase speed selection is not crucial for regime tested.
- Horizontal interpolation is more critical than temporal interpolation.

- Five-year nested model runs have been completed.
- Strong tangential flows require both modified h and non-normal treatment of phase speed.
- Differences in variable values between *true* and *nested* runs are, in general, less than 5% of those between *true* and *applied* runs.

References

[1] Hurlburt, H., J. Kindle, E.J. Metzger and A. Wallcraft, 1989: Results from a global ocean model in the Western Tropical Pacific. *Proceedings of the Western Pacific International Meeting and Workshop on TOGA COARE*, by J. Picaut, R. Lukas, and T. Delcroix (Editors), 343–354.

[2] Hurlburt, H., and J.D. Thompson, 1982: The dynamics of the Loop Current and shed eddies in a numerical model of the Gulf of Mexico. *Hydrodynamics of semi-enclosed seas*, by J.C.J. Nihoul (Editor), Elsevier, 243–298.

[3] Koch, S.E., and J.T. McQueen, 1987: A survey of nested grid techniques and their potential for use within the MASS weather prediction model. *NASA Technical Memorandum* 87808, 25 pp.

[4] Orlanski, I., 1976: A simple boundary condition for unbounded hyperbolic flows, *J. Comp. Phys.,* **21**, 250–257.

[5] Sommerfeld, A., 1949: *Partial Differential Equations: Lectures on Theoretical Physics*, **6**, Academic Press.