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Global analysis of fermion mixing with exotics

Enrico Nardi[a,1], Esteban Roulet[b] and Daniele Tommasini[a,2]

[a] *International School for Advanced Studies, I-34014 Trieste, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy*
[b] *NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory P.O. Box 500, Batavia, IL 60510-0500*

Abstract

We analyze the limits on deviations of the lepton and quark weak–couplings from their standard model values in a general class of models where the known fermions are allowed to mix with new heavy particles with exotic $SU(2)\times U(1)$ quantum number assignments (left-handed singlets or right-handed doublets). These mixings appear in many extensions of the electroweak theory such as models with mirror fermions, E_6 models, etc. Our results update previous analyses and improve considerably the existing bounds. As experimental constraints we use the new results on M_Z, Γ_Z, on the Z partial decay–widths and on the asymmetries measured at the Z resonance, as well as updated results on the W mass, on deep-inelastic ν–q and ν–e scattering and on atomic parity violation. Present constraints on lepton universality, unitarity of the quark mixing matrix and induced right-handed currents are also included. A global analysis of all these data leads to upper limits on the mixing factors $s^2 \equiv \sin^2\theta_{\rm mix}$. When just one mixing is constrained at a time, we obtain for most of the fermions the tight limits $s^2 \lesssim 0.002 \div 0.01$ at 90 % c.l.. For u_R, c_R and ν_τ the bounds are $s^2 < 0.03$, however if ν_τ mixes with an ordinary heavy neutrino the constraint is $s^2 \lesssim 0.1$ and a signal of non–zero mixing at 90 % c.l. is found. For s_R and b_R we find the much weaker bounds $s^2 \lesssim 0.35$. The constraints are weakened by a factor between 2 and 5 if accidental cancellations among different mixings are allowed to occur.

Addresses after November 1st:
[1] *Randall Lab. of Physics, Univ. of Michigan, Ann Arbor, MI 48109-1120 U.S.A.*
[2] *Dep. de Física Teòrica, Univ. de València, 46100 Burjassot, Valencia, Spain.*

1. Introduction

In the last few years the ever increasing accumulation of precise electroweak experiments have been regularly employed to check the consistency of the standard model (SM), to determine $\sin^2\theta_W$ and to make predictions for the still unknown value of the top mass. Possible indirect signatures of physics beyond the SM, such as the effects of additional gauge bosons or of mixings of the standard fermions with exotic ones, as well as the contributions of non–decoupled physics to radiative corrections, have also been constrained by these measurements.

The first pre–LEP analyses [1,2,3] used the available information on gauge boson masses from colliders, neutral current (NC) data on ν scattering, parity violation, fermion asymmetries in e^+e^- annihilation below the Z resonance, and in some cases charged current (CC) constraints. By now the situation has improved considerably. A remarkable improvement has been achieved in the determination of the W boson mass from UA2 and CDF [4]. In the NC sector, there are new measurements on atomic parity violation in Cs [5] and new calculations of the atomic matrix elements involved [6], there are new results on $\nu_\mu e$ scattering [7,8,9] as well as new and updated analyses of the c and b asymmetries in γ–Z interference processes at PEP and PETRA [10,11,12]. In the CC sector, new constraints are available on the universality of the lepton couplings and on the unitarity of the quark mixing matrix, and the problem of the charm quark threshold [13], that affects the νq CC cross section used to normalize the deep-inelastic NC experiments, has been studied in more detail. The really new input, however, comes from the large set of accurate measurements carried out at the Z–peak at LEP and SLC. Besides M_Z, that is now very precisely known, the determination of the total and of the partial Z–widths and of the on–resonance forward–backward and τ polarization asymmetries has provided very precise informations about the fermion couplings to the Z.

Some of these data have been recently used to update the predictions on m_t [14] and to constrain extensions of the SM with extra U(1) gauge bosons [15], as well as technicolour models, strongly interacting Higgs–bosons and other kinds of heavy physics that could manifest itself through radiative corrections [16].

It is our purpose here to update the bounds on possible mixings between the known fermions and new exotic ones. There have been several earlier analyses of the limits on fermion mixings [17], and the first (pre-LEP) global analysis of this kind of new physics was done by Langacker and London [3]. Subsequently, two of us [18] showed that the very first LEP data already improved some bounds significantly, more recently, Langacker, Luo and Mann [19] have also discussed the sensitivity to some exotic mixings that will be attained with the forseeable precision of the ongoing or planned precision electroweak experiments.

The existence of new fermions with exotic weak couplings is a quite common feature in most of the extensions of the SM, being the 'superstring inspired' E_6 models well known and still popular examples [20] of these. A mixing between ordinary and exotic fermions is allowed whenever their $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{em}$ quantum numbers are the same. If at the same time the new fermions have non canonical $SU(2)_L$ assignments, the couplings of the light states with both the W and the Z vector bosons will be modified, leading to deviations from the SM expectations. This is the kind of effects we aim to constrain by means of a careful analysis of the available experimental results.

The general formalism to describe fermion mixing that was introduced in [3] will be briefly surveyed in section II. We then present in section III the theoretical expressions for the different observables that we have used to work out the constraints. A brief description of each measurement and a discussion of the experimental data are also given in this section. In section IV we comment on the results of the global analysis. The results are presented as 90 % c.l. upper limits on the ordinary–exotic mixing parameters, both in the case in which only one fermion is allowed to mix at a time and in the case were all mixings are simultaneously present so that accidental cancellations may occur. Finally, in section V we draw our conclusions.

2. Formalism

The lack of observation of new particles in the last accelerator runs indicates that if possible new fermions exist, they will generally have large masses ($> 50-100$ GeV). Even if these particles cannot be directly produced with the experimental facilities available at present, it is still possible that their effects are indirectly detected as small deviations of the observed fermion couplings from the standard ones. In particular, this happens if the exotic fermions have non–canonical $SU(2)_L \times U(1)$ assignments and they mix with the ordinary ones. We consider a fermion to have canonical $SU(2)_L$ quantum numbers if it is a left–handed $SU(2)_L$ doublet or a right–handed $SU(2)_L$ singlet. These are called ordinary fermions while fermions with non–canonical quantum numbers are classified as exotics. Exotic fermions can appear in mirror models [21] in which generally whole mirror generations with R-doublets and L-singlets are introduced, in models with vector doublets (singlets) where both left and right fermions have the same transformation properties under weak–isospin, or as singlet Weyl neutrinos. Fermions with exotic charges or colour assignments cannot mix with the known quarks and leptons and thus we will not consider them.

In order to describe the mixing between ordinary and exotic charged fermions, we introduce [3] two vectors for the left and right-handed ordinary and exotic weak eigenstates $\Psi^o_{L(R)} = (\Psi^o_O, \Psi^o_E)^T_{L(R)}$, and two different vectors for the light and heavy mass eigenstates $\Psi_{L(R)} = (\Psi_l, \Psi_h)^T_{L(R)}$. The weak and mass eigenstates are related by unitary transformations

$$\Psi^o_{L(R)} = U_{L(R)} \Psi_{L(R)}. \tag{2.1}$$

It is convenient to decompose the matrix U as

$$U_{L(R)} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_{L(R)}, \tag{2.2}$$

where A and F describe the overlap of the light eigenstates with the ordinary and exotic fermions respectively. Since we have included fermions of sequential families or canonical members of vector multiplets as ordinary states, the labels 'light' and 'heavy' should be taken as suggestive only. From the unitarity of U it follows that

$$A^\dagger A + F^\dagger F = AA^\dagger + EE^\dagger = I. \tag{2.3}$$

Hence, the matrix A describing the mixing with the ordinary fermions is non–unitary by small terms quadratic in the ordinary–exotic fermion mixings present in F.

The fermion current coupling to the Z is

$$\frac{1}{2} J^\mu_Z = \sum_f \bar{\Psi}^o_f \gamma_\mu (t^f_3 I_L P_L + t^f_3 I_R P_R - Q^f s^2_W I) \Psi^o_f, \tag{2.4}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the L and R chiral projectors, $s_W^2 \equiv \sin^2\theta_W$, Q^f and t_3^f denote charge and third isospin component of f and $I_{L,R}$ project onto the subspaces of the ordinary and exotic weak doublets of Ψ^o, *i.e.*

$$I_L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad I_R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}. \tag{2.5}$$

Hence, (omitting the label l) the light fermions–Z vertex is given by the Lagrangian

$$\mathcal{L}_Z = -\left(\sqrt{2}G_F M_Z^2\right)^{1/2} \sum_f \bar{\Psi}_f \gamma_\mu (L^f P_L + R^f P_R)\Psi_f \; Z^\mu, \tag{2.6}$$

with

$$\begin{aligned} L^f &= t_3^f A_L^{f\dagger} A_L^f - Q^f s_W^2 \\ R^f &= t_3^f F_R^{f\dagger} F_R^f - Q^f s_W^2. \end{aligned} \tag{2.7}$$

Although the matrices $A^\dagger A$ and $F^\dagger F$ are in principle quite general, non–vanishing off diagonal terms would induce FCNC that are experimentally known to be very suppressed [3]. Hence, we will assume that different light mass eigenstates do not mix with the same exotic partner, in which case the absence of FCNC is automatically guaranteed. With this assumption one gets

$$(F_a^\dagger F_a)_{ij} = (s_a^i)^2 \; \delta_{ij} \,, \qquad a = L, R, \tag{2.8}$$

where $(s_a^i)^2 \equiv 1 - (c_a^i)^2 \equiv \sin^2\theta_a^i$, and $\theta_{L(R)}^i$ is the mixing angle between L(R) light and heavy partners.

Then the neutral–current couplings for the light fermions can be written as

$$\begin{aligned} \hat{\epsilon}_L(f_i) &\equiv (L^f)_{ii} = t_3^{f_i}(c_L^{f_i})^2 - Q^{f_i} s_W^2 \\ \hat{\epsilon}_R(f_i) &\equiv (R^f)_{ii} = t_3^{f_i}(s_R^{f_i})^2 - Q^{f_i} s_W^2, \end{aligned} \tag{2.9}$$

and we see that while the L–mixings reduce the strength of the isospin current, the presence of R–mixings induces a right–handed current. Clearly the electromagnetic current is left unchanged. The vector and axial–vector couplings in the presence of mixing are (omitting the generation index i)

$$\begin{aligned} v_f &\equiv \hat{\epsilon}_L(f) + \hat{\epsilon}_R(f) = t_3^f \left[(c_L^f)^2 + (s_R^f)^2\right] - 2Q^f s_W^2 \\ a_f &\equiv \hat{\epsilon}_L(f) - \hat{\epsilon}_R(f) = t_3^f \left[(c_L^f)^2 - (s_R^f)^2\right]. \end{aligned} \tag{2.10}$$

Henceforth v_f and a_f will always denote the true couplings of the light fermions including mixing terms.

The charged current between light states is

$$\frac{1}{2}J_W^\mu = \bar{\Psi}_u\gamma^\mu(V_L P_L + V_R P_R)\Psi_d. \tag{2.11}$$

The first 3 components of the vectors Ψ_u, Ψ_d represent the standard quarks, while the remaining $n-3$ 'light' fields correspond to possible extra sequential, or vector doublet, quarks. $V_L = A_L^{u\dagger} A_L^d$ and $V_R = F_R^{u\dagger} F_R^d$ generalize the SM Cabibbo-Kobayashi-Maskawa (CKM) quark mixing. In particular, the matrix V_L is non unitary due to the mixing with the exotic quarks, however it can be decomposed as

$$V_{L\,ij} = c_L^{u_i} c_L^{d_j} K_{L\,ij}, \tag{2.12}$$

where K_L is unitary [3]. For the induced right–handed currents, it is convenient to introduce the parameters

$$\kappa_{ij} \equiv \frac{V_{R\,ij}}{K_{L\,ij}}, \tag{2.13}$$

which are quadratic in the light–heavy mixings.

For the neutral fermions the situation is more complicated because in the presence of Majorana mass terms three kinds of neutral fields with different isospin assignments can mix at the same time, and also because due to the lack of experimental constraints the assumption on the absence of FCNC must be released. Besides the ordinary neutrinos that appear in L–doublets $(n_O^o,\ e_O^{o-})^T$, we can have exotic states that appear in the CP conjugates of $SU(2)$ R-doublets $(E_E^{o+} n_E^o)_L^T$, (these can mix with n_O^o through $\Delta\mathrm{L} = \pm 2$ Majorana mass terms) and also exotic singlets n_{SL}^o can be present.

In analogy with the charged fermion case, we write the weak and mass eigenstates as

$$n_L^o = \begin{pmatrix} n_O^o \\ n_E^o \\ n_S^o \end{pmatrix}_L \quad , \quad n_L = \begin{pmatrix} n_l \\ n_h \end{pmatrix}_L . \tag{2.14}$$

These states are related through $n_L^o = U_L n_L$. The unitary matrix U can be decomposed as

$$U_L = \begin{pmatrix} A & E \\ F & G \\ H & J \end{pmatrix}_L , \tag{2.15}$$

with A, F, H describing the overlap of the light neutrinos with n_O^o, n_E^o and n_S^o respectively.

Note that we do not distinguish between left handed neutrinos and antineutrinos, they are all described by fields n_L and the right handed fields will be denoted as $n_R^c = C\bar{n}_L^T$. Clearly $n_R^{oc} = U_R n_R^c$ with $U_R = U_L^*$.

The LEP measurements of the number of light neutrino species implies that if neutrinos with large exotic n_E^o components exist, their masses must be heavier than $M_Z/2$. Light singlets, however, as in the case of Dirac neutrino masses, could be present and a mixing with exotic doublets would allow them to couple to the Z boson. For simplicity we will not consider this case, but our results are largely independent of this restriction. In conclusion we will assume the light neutrinos to be mainly ordinary states so that we will consider the elements of F and H as small light–heavy mixings.

We will chose the flavour basis such that the charged lepton flavour eigenstates coincide with the charged mass eigenstates up to light–heavy mixing effects. Hence, the charged current between light mass eigenstates reads

$$J_W^\mu = \bar{n}_L\gamma^\mu A_L^{\nu\dagger} c_L^e e_L + \bar{n}_R^c\gamma^\mu F_R^{\nu\dagger} s_R^e e_R. \tag{2.16}$$

The first term in this equation is the usual left–handed current with the overall strength reduced by the effect of light–heavy mixing, while the second term corresponds to an induced right–handed current that can produce neutrinos of the wrong helicity in weak decays. This term is present when both the light neutrino and charged lepton mix with the components of an exotic doublet.

It is convenient to write $A^{\nu\dagger} = K^\nu \mathcal{A}^{\nu\dagger}$, where the matrix K^ν is unitary and, being the leptonic analog of the CKM matrix, it is non–trivial if the light neutrinos have masses and ordinary mixings. The exotic mixings appear only in $\mathcal{A}$, which deviates from the identity only by terms of $O(s^2)$. In the charged current processes that we will consider, a sum has to be taken over the unobserved final neutrino mass eigenstates (the kinematical effects of ν masses are negligible) and thus the information in K^ν is lost. In weak decays, for example, the mixings induce a change in the decay rate with respect to the SM rate Γ_o that, to $O(s^2)$ and restricting ourself to the primary vertex, can be written as

$$\frac{1}{\Gamma_o}\sum_i \Gamma(e_a \to n_i) = (c_L^{e_a})^2 (\mathcal{A}_L^\nu \mathcal{A}_L^{\nu\dagger})_{aa} + O(s^4), \tag{2.17}$$

where $(A^\nu A^{\nu\dagger})_{aa} = (\mathcal{A}^\nu \mathcal{A}^{\nu\dagger})_{aa} \equiv (c_L^{\nu_a})^2$ accounts for the neutrino light-heavy mixing. As we see, the sum over the final undetected states allows us to take just one mixing angle per neutrino flavour to describe the exotic mixings, although in general the matrix $\mathcal{A}^{\nu\dagger}\mathcal{A}^\nu$ is not diagonal.

The weak neutral current for the light neutrino states is

$$J^\mu_{\nu Z} = \frac{1}{2}\bar{n}_L\gamma^\mu(A_L^{\nu\dagger}A_L^\nu - F_L^{\nu\dagger}F_L^\nu)n_L, \tag{2.18}$$

where $A_L^{\nu\dagger}A_L^\nu$ and $F_L^{\nu\dagger}F_L^\nu$ originate respectively from the ordinary n_O^o and exotic n_E^o neutrinos that have opposite isospin assignments.

Up to mixing effects in the target, and summing again over the undetected light n_i neutrinos, the scattering process $n_a \to n_i$ is modified with respect to the normal case as

$$\begin{aligned}\frac{1}{\sigma_o}\sum_i \sigma(n_a \to n_i) &= \frac{1}{(c_L^{\nu_a})^2}(A^\nu(A^{\nu\dagger}A^\nu - F^{\nu\dagger}F^\nu)^2A^{\nu\dagger})_{aa} \\ &= 1 - 2\left(K^{\nu\dagger}(2F^{\nu\dagger}F^\nu + H^{\nu\dagger}H^\nu)K^\nu\right)_{aa} + O(s^4),\end{aligned} \tag{2.19}$$

where the factor $(c_L^{\nu_a})^2$ in the denominator comes from the normalization of the n_a produced in the weak decay of e_a. In (2.19) we have used the unitarity of U_L as well as the decomposition of the matrix A into the unitary K matrix, and we have neglected terms $O(s^4)$. Defining now $(K^\dagger F^\dagger FK)_{aa} \equiv \lambda_F^a(s_L^{\nu_a})^2$ and $(K^\dagger H^\dagger HK)_{aa} \equiv \lambda_H^a(s_L^{\nu_a})^2$ we finally get

$$\frac{1}{\sigma_o}\sum_i \sigma(n_a \to n_i) = 1 - \Lambda_a(s_L^{\nu_a})^2 + O(s^4). \tag{2.20}$$

Since the sum of the λ^a' s is constrained to be ≤ 1 from the unitarity of U_L in (2.15), the value of the effective parameter $\Lambda_a \equiv 4\lambda_F^a + 2\lambda_H^a$ must lie between 0 and 4, depending on the mixing involved. If the light states are mixed with ordinary states (that will be mainly heavy) then the couplings are not affected and $\Lambda_a = 0$. If only singlet states n_S^o mix with the known neutrinos then $\Lambda_a = 2$ while $\Lambda_a = 4$ describes mixings involving only exotic states n_E^o.

The decay rate of the Z boson into undetected neutrinos is proportional to the sum of the square of the neutrino neutral–current couplings. Using the same approximations as in the previous case we find

$$\mathrm{Tr}(A^{\nu\dagger}A^\nu - F^{\nu\dagger}F^\nu)^2 = 3 - \sum_a \Lambda_a(s_L^{\nu_a})^2 + O(s^4), \tag{2.21}$$

and we see that the effective parameter Λ_a could largely influence the reduction in the decay rate.

3. Observables

In this section we discuss the measurements that we have used to constrain the fermion mixing angles. In comparing the experimental results with the corresponding theoretical expressions some care is needed, since indirect effects of the mixings that depend on the particular experimental procedure used to extract the data could be present.

To match the precision reached by the 'last–generation' experiments, 1-loop effects should be taken into account in the evaluation of the theoretical expressions. We have followed the general attitude of including only the set of SM radiative corrections, which are nowadays completely known, neglecting the effect of mixings in them as well as the contribution of the additional states in the loops. We should stress however that the large number of exotic fermions present in the models under investigation could give rise to non negligible higher order effects [16] especially in the case of non-degenerate doublets [22]. QCD corrections have also been included in all the relevant cases when hadronic final states were involved.

Our set of fundamental input parameters consists of the QED coupling constant α measured at $q^2 = 0$, the mass of the Z boson M_Z and the Fermi coupling constant G_F. The numerical values of α and M_Z as extracted from experiments are not affected by the mixings. The position of the resonance–peak does not depend on the exact form of the fermion couplings with the Z (the shape and height of the peak, in contrast, are modified by the mixings) and the standard set of QED corrections needed to reconstruct the exact peak–position can be safely applied, since also the electromagnetic current is not modified.

Throughout this work we will fix the Z–mass at the value $M_Z = 91.175$ GeV [23] since the theoretical uncertainties induced by the present experimental error of ± 21 MeV are negligible.

In contrast with the previous two parameters, the Fermi coupling constant extracted from the measured life–time of the μ-lepton, $G_\mu = 1.16637(2) \times 10^{-5}\,\mathrm{GeV}^{-2}$, is affected by fermion mixings. The relation between G_F and the effective μ-decay coupling constant (neglecting the $O(s^4)$ effect of induced right handed currents (RHC)) is

$$G_\mu = G_F c_L^{\nu_e} c_L^{\nu_\mu} c_L^e c_L^\mu. \tag{3.1}$$

Clearly, this indirect dependence on the light lepton mixing angles is propagated in all the expressions that contain G_F. This is the case for example for the W boson mass, for which no other explicit dependence on mixings appears.

In addition, also the value of the top-quark m_t and Higgs boson M_H masses must be specified, since they enter the expressions via loop corrections. The dependence on M_H is soft, and we keep its value fixed at 100 GeV. In contrast, varying the value of m_t can induce sizeable effects. We have chosen to fix the top mass at the value $m_t = 120$ GeV that corresponds approximately to the minimum of our χ^2 function when all the mixing parameters are set to zero.

Whenever some other experimental parameter enters our theoretical expressions, we have used those experimental determinations for which mixing effects are absent or negligible. This will be the case *e.g.* of the strong coupling constant $\alpha_s(M_Z^2)$, of the semileptonic branching ratio $Br(b \to \ell + X)$ and of the $B^0 - \bar{B}^0$ mixing parameter χ_B on which we will further comment in the following.

Experimental errors have been evaluated by adding statistical and systematic uncertainties in quadrature and correlations have been taken into account in all the relevant cases.

The W mass

The standard way of computing the value of the W mass is to compare the amplitude for W exchange at $q^2 \simeq 0$ in μ decay with the effective strength of the Fermi interaction. Radiative corrections are large and must be included [24]. The theoretical expression for the W mass reads

$$M_W^2 = \frac{\rho M_Z^2}{2}\left[1+\sqrt{1-\frac{G_\mu}{G_F}\frac{4\mathcal{A}}{\rho M_Z^2}\left(\frac{1}{1-\Delta\alpha}+\Delta r^{rem}\right)}\right], \tag{3.2}$$

where $\mathcal{A} = \pi\alpha/\sqrt{2}G_\mu$. The $1/(1-\Delta\alpha)$ term renormalizes the QED low energy coupling to the M_Z scale and resums to all orders the large logs contained in the photon vacuum polarization function. The leading top effects, quadratic in m_t, are included in the parameter $\rho \simeq 1 + 3G_\mu m_t^2/8\sqrt{2}\pi^2$ [22]. We have taken $\rho = 1$ at the tree level, that corresponds to the absence of non–doublet Higgs VEV's and of extra $U(1)$ gauge bosons with non–zero mixing with the standard Z. The non-leading top effects, Higgs and other small corrections are included in the Δr^{rem} term. We refer to [25] for a detailed discussion of all these corrections.

The expression for M_W in (3.2) is affected by the mixings only indirectly via the G_μ/G_F ratio. We note that increasing values of both the mixing angles and of the top mass tend to increase M_W. Since the same interdependence enters also the expression for the effective weak–mixing angle that defines the neutral–current couplings of the fermions, a sizeable anticorrelation between m_t and the light lepton mixings is to be expected, resulting into a stronger constrain for larger values of m_t.

Experimentally the value of the W mass, as measured by CDF, is $M_W = 79.91 \pm 0.39$ GeV [4]. The UA2 collaboration has measured the ratio of the W and Z masses, for which many systematic errors cancel, obtaining $M_W/M_Z = 0.8831 \pm 0.0055$ [4]. Using the LEP value for M_Z and averaging the two results yields

$$M_W = 80.13 \pm 0.31 \mathrm{GeV}. \tag{3.3}$$

Charged currents

i) Lepton universality

The ratios g_μ/g_e and g_τ/g_e of the leptonic couplings to the W boson, which in the SM are predicted to be unity (universality), are modified by fermion mixings according to

$$\left(\frac{g_i}{g_e}\right)^2 = \frac{(c_L^{l_i})^2(c_L^{\nu_i})^2 + (s_R^{l_i})^2(s_R^{\nu_i})^2}{(c_L^{e})^2(c_L^{\nu_e})^2 + (s_R^{e})^2(s_R^{\nu_e})^2} \simeq \frac{(c_L^{l_i})^2(c_L^{\nu_i})^2}{(c_L^{e})^2(c_L^{\nu_e})^2}, \qquad i = \mu, \tau. \tag{3.4}$$

Experimentally these ratios can be determined by comparing two different leptonic decay processes. In table I we give the values of $(g_i/g_e)^2$ extracted from several experiments:

1) from the ratios of the partial cross sections

$$\frac{\sigma(p\bar{p} \to W)B(W \to l_i\nu_i)}{\sigma(p\bar{p} \to W)B(W \to e\nu_e)} \qquad i = \mu, \tau \tag{3.5}$$

as measured by UA1 and UA2 [26];

2) from the ratios of the τ and μ decay rates

$$\frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\tau \to e\nu\bar{\nu})}, \qquad \frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})}, \tag{3.6}$$

that can be evaluated using the experimental values of the τ branching fractions into e and μ and the measured τ–lifetime [27]. Combining the averages of [27] with two very recent measurement of the L3 and OPAL collaborations [28] we obtain $Br(\tau \to \mu\nu_\mu\nu_\tau) = 0.175 \pm 0.003$ and $Br(\tau \to e\nu_e\nu_\tau) = 0.178 \pm 0.003$ from which the values in table I are derived.

3) Finally, the ratios

$$\frac{\Gamma(\pi \to \mu\nu)}{\Gamma(\pi \to e\nu)}, \qquad \frac{\Gamma(K^+ \to \mu^+\nu)}{\Gamma(K^+ \to e^+\nu)}, \tag{3.7}$$

computed from the values reported in [27], have been also used for constraining universality of the μ and e couplings.

TABLE I. Charged Current experimental constraints on lepton universality (g_i/g_e), unitarity of the quark mixing matrix V_{ij}, and induced hadronic RHC (κ_{ij}).

Quantity	Experimental value	Correlation	Processes
$(g_\mu/g_e)^2$	1.00 ± 0.20		$W \to l\nu$
$(g_\tau/g_e)^2$	1.00 ± 0.12		
$(g_\mu/g_e)^2$	1.016 ± 0.026	0.40	$\tau \to l\nu\bar{\nu}$ and
$(g_\tau/g_e)^2$	0.952 ± 0.031		$\mu \to e\nu\bar{\nu}$
$(g_\mu/g_e)^2$	1.014 ± 0.011		$\pi \to l\nu$
"	1.013 ± 0.046		$K \to l\nu$
$\sum_{i=1}^{3} \lvert V_{ui}\rvert^2$	0.9981 ± 0.0021		hadrons decays
$\sum_{i=1}^{3} \lvert V_{ci}\rvert^2$	1.08 ± 0.37		ν-d scatt. and D_{e3}
$\mathrm{Re}(\kappa_{ud})$	0 ± 0.0037		$K \to 3\pi, 2\pi$
$\mathrm{Re}(\kappa_{us})$	0 ± 0.0037		

ii) CKM unitarity

Fermion mixings lead to violations of the 3-generation unitarity of the observable CKM matrix V_{ij}, $(i,j=1,2,3)$, as is apparent from eq. (2.11–2.13). Thus, a measurement of the deviation from unity of the sum of the $|V_{ij}|^2$, for each matrix row, puts constraints on the mixings. V_{ud} and V_{us} are obtained by dividing by G_μ the measured vector coupling in β decay and in K_{e3} and hyperon decays, respectively. Hence [3],

$$V_{ui} = \frac{G_F}{G_\mu}(V_{Lui} + V_{Rui})c_L^e c_L^{\nu_e}, \qquad i = d, s. \tag{3.8}$$

The value of $|V_{ub}|$, obtained from the analysis of semileptonic B decays, is negligibly small for our purposes. Using the unitarity of the matrix K_L introduced in (2.12), and neglecting terms of $O(s^4)$ and $O(s^2 \sum_{i=4}^n |K_{Lui}|^2)$,

$$\sum_{i=1}^{3} |V_{ui}|^2 = \left(\frac{G_F}{G_\mu} c_L^e c_L^{\nu_e}\right)^2 \left\{ (c_L^u)^2 - \sum_{i=4}^{n} |K_{Lui}|^2 + \sum_{i=1}^{2} |V_{ui}|^2 \left[2\mathrm{Re}(\kappa_{ui}) - (s_L^{d_i})^2\right] \right\}, \tag{3.9}$$

where we have approximated $|K_{Lui}|^2$ with the experimental values $|V_{ui}|^2$ in the coefficients of the $O(s^2)$ terms.

V_{cd} is determined from the di-muon production rate of charm off valence d-quarks [29] while V_{cs} is extracted from D_{e3} decays [27]. Hence

$$|V_{cd}|^2 \simeq (c_L^c)^2 |K_{Lcd}|^2, \qquad |V_{cs}|^2 \simeq |K_{Lcs}|^2 [(c_L^c)^2 (c_L^s)^2 + 2\mathrm{Re}(\kappa_{cs})] \tag{3.10}$$

where, due to the comparatively large uncertainty affecting these measurements, those mixings that are more effectively constrained by other experiments have been neglected. Taking the sum, and neglecting also $|V_{cb}|^2$ and $O(s^4)$ terms, we obtain

$$\sum_{i=1}^{3} |V_{ci}|^2 \simeq (c_L^c)^2 - \sum_{i=4}^{n} |K_{Lci}|^2 + [2\mathrm{Re}(\kappa_{cs}) - (s_L^s)^2]|V_{cs}|^2. \tag{3.11}$$

For the $|V_{ij}|$'s we use the values given in ref. [27], and the experimental constraints from unitarity are listed in table I. [1]

iii) Right handed currents

[1] Recent analyses [30] have reduced the error on V_{ud}. However, still unsettled theoretical issues concerning atomic corrections affect these conclusions [31], so we used the more conservative values of [27]. Similar considerations apply also for V_{cs}.

The RHC's induced by mixings with exotic quarks allow to constrain the κ_{ij} parameters of the hadronic sector.

Very stringent limits on the $\bar{u}_R\gamma^\mu d_R$ and $\bar{u}_R\gamma^\mu s_R$ RHC's were set [32] from the observation that the $K \to 3\pi$ amplitude and slope parameters are predicted, within an accuracy of $\sim 10\%$, by PCAC and by the measured $K \to 2\pi$ matrix elements. The resulting limits

$$|\mathrm{Re}\,\kappa_{ud}|, |\mathrm{Re}\,\kappa_{us}| < \frac{8 \times 10^{-4}}{|V_{ud}||V_{us}|} \simeq 0.0037 \tag{3.12}$$

will be treated as 1σ experimental constraints [3] on $\mathrm{Re}(\kappa_{ud})$ and $\mathrm{Re}(\kappa_{us})$.

Limits on the $\bar{c}_R\gamma^\mu d_R$, $\bar{c}_R\gamma^\mu s_R$ RHC's were set by the CDHS collaboration [29] from the analysis of the y distribution in the di-muon charm production ($\nu d, \nu s \to \mu^- c$), from which the following 95% confidence level bound was obtained

$$\frac{|\kappa_{cd}|^2 + |\kappa_{cs}|^2 \left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}}{1 + \left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}} < 0.07. \tag{3.13}$$

Here U, D, S denote the quark content of the isoscalar target [29], and $|V_{cd}|$ and $|V_{cs}|$ have been introduced to approximate $|K_{Lcd}|$ and $|K_{Lcs}|$. Using the experimental estimate [29] for $\left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}$ we can derive from this bound an experimental constraint on the parameters κ_{cd} and κ_{cs}.

The leptonic RHC's are limited by the measurements of the μ and τ Michel parameters, as well as by the electron polarization in β decays. The corresponding bounds on parameters such as $s_R^{e_e} s_R^{\nu_e}$ have not improved with respect to those obtained in [3], so we will not repeat them here.

Neutral currents

Neutral–current experimental results are conveniently given as fits to the couplings appearing in the effective Lagrangians that describe the corresponding four–fermion processes [27]. The form of these effective Lagrangians relies only on the assumption of spin–one gauge boson exchange and of massless left–handed neutrinos, and thus the experimental values of the phenomenological parameters are essentially model independent.

We will treat separately the $\nu - q$, $\nu - e$ and the parity–violating $e - q$ sectors. For clarity we will only display the tree level expressions, but in our numerical computations the SM radiative corrections [33,34] have been included as well.

i) Neutrino - quark sector

The effective Lagrangian for the neutral current interaction of the light neutrinos with quarks is

$$-\mathcal{L}^{\nu q} = \frac{G_\mu}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \; [\epsilon_L(q)\bar{q}\gamma_\mu(1-\gamma_5)q + \epsilon_R(q)\bar{q}\gamma_\mu(1+\gamma_5)q] \, . \tag{3.14}$$

The values of the quark couplings $\epsilon_{L,R}(q)$ are extracted from deep–inelastic scattering experiments off isoscalar and proton targets, normalized to the charged current cross sections, i.e. from the ratios

$$R_\nu = \frac{\sigma(\nu_\mu N \to \nu X)}{\sigma(\nu_\mu N \to \mu^- X)} \quad , \quad R_{\bar{\nu}} = \frac{\sigma(\bar{\nu}_\mu N \to \bar{\nu} X)}{\sigma(\bar{\nu}_\mu N \to \mu^+ X)}. \tag{3.15}$$

In comparing the experimental results with the theoretical expressions, the effect of the mixings in the normalization factors has to be taken into account as well [3], since the fermion charged–current couplings are modified according to (2.11) and, as discussed in the previous subsection, also the value of the CKM element V_{ud} obtained from β-decay experiments is affected.

Using now (2.6), (2.10) and (2.20) and taking the normalization effects properly into account, the values of the quark couplings as extracted from experiments correspond to

$$\epsilon_{L,R}(q) = \frac{1}{2} F_1(s^2, \kappa)(v_q \pm a_q), \tag{3.16}$$

where

$$F_1(s^2,\kappa) = \frac{1 - \Lambda_\mu (s_L^{\nu_\mu})^2/2}{1 - (s_L^\mu)^2 - (s_L^{\nu_\mu})^2 - \mathrm{Re}(\kappa_{ud})}. \tag{3.17}$$

In this factor the numerator comes from the modified NC ν-couplings while the denominator comes from the experimental normalization.

The experimental values [27] are given in table II in terms of

$$g_a^2 \equiv \epsilon_a(u)^2 + \epsilon_a(d)^2 \quad , \quad \theta_a \equiv \tan^{-1}\left[\frac{\epsilon_a(u)}{\epsilon_a(d)}\right] \qquad a = L, R \tag{3.18}$$

that have negligible correlations.

ii) Neutrino - electron sector

The effective Lagrangian for the $\nu - e$ sector is

$$-\mathcal{L}^{\nu e} = \frac{G_\mu}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \; \bar{e}\gamma_\mu(g_V^e - g_A^e\gamma_5)e. \tag{3.19}$$

The electron vector and axial–vector couplings are extracted from $\nu_\mu - e$ scattering experiments that, as in the previous case, are normalized with the ν_μ-hadron charged–current cross sections. For high–energy neutrinos like those of CERN and FERMILAB, the CC deep–inelastic process leads to the same normalization factor as in the $\nu - q$ sector, so that the relation between the couplings extracted from experiments and the theoretical ones is

$$g_V^e = F_1\, v_e \quad , \quad g_A^e = F_1\, a_e. \tag{3.20}$$

For the low-energy neutrinos of BNL, the CC scattering is a quasi–elastic process so that the factor $F_1(s^2, \kappa)$ in eq. (3.20) is replaced by [3]

$$F_2(s^2) = \frac{1 - \Lambda_\mu (s_L^{\nu_\mu})^2/2}{1 - (s_L^\mu)^2 - (s_L^{\nu_\mu})^2}. \tag{3.21}$$

In table II we list the values of $g_{V,A}^e$ that we have used. The recent CHARM II [8] results, as well as the CHARM I [7] and BNL [9] data on both ν_μ and $\bar\nu_\mu$ scattering off electrons have all been included. In particular, CHARM II has measured g_V^e/g_A^e from the ratio between ν and $\bar\nu$ NC cross sections, leading to a clean measurement of v_e/a_e since the F_1 factor cancels.

iii) Electron - quark sector

iii) The measurements of parity violation effects in atoms and the polarized $e - D$ scattering experiments are sensitive to weak–electromagnetic interference effects and allow the determination of the $e - q$ parity violating couplings $C_{1,2}$. These parameters appear in the effective Lagrangian

$$-\mathcal{L}^{eq} = -\frac{G_\mu}{\sqrt{2}} \sum_i (C_{1i} \bar e \gamma_\mu \gamma^5 e\ \bar q^i \gamma^\mu q^i + C_{2i} \bar e \gamma_\mu e\ \bar q^i \gamma^\mu \gamma^5 q^i), \tag{3.22}$$

where $i = u, d$. Their relation with the theoretical couplings is

$$C_{1i} = 2\left(\frac{G_F}{G_\mu}\right) a_e v_i \quad , \quad C_{2i} = 2\left(\frac{G_F}{G_\mu}\right) v_e a_i. \tag{3.23}$$

For the determination of the coefficients C_1, parity violating transitions in Cs are quite effective since for heavy nuclei the vector couplings of the quarks are coherently enhanced, in addition since Cs has only a single electron outside a completely filled shell, rather clean theoretical calculations for the atomic effects are available [6]. The

TABLE II. Neutral Current experimental constraints.

Deep-inelastic ν-q		
g_L^2	0.2977 ± 0.0042	
g_R^2	0.0317 ± 0.0034	
θ_L	2.50 ± 0.03	
θ_R	$4.59 \begin{smallmatrix} + 0.44 \\ - 0.27 \end{smallmatrix}$	
ν-e scattering		**experiment**
g_V^e/g_A^e	0.047 ± 0.046	CHARM II
g_V^e	-0.06 ± 0.07	CHARM I
g_A^e	-0.57 ± 0.07	"
g_V^e	-0.10 ± 0.05	BNL
g_A^e	-0.50 ± 0.04	"
e-q parity violation		**correlation**
C_{1u}	-0.249 ± 0.066	-0.99 -0.95
C_{1d}	0.391 ± 0.059	0.95
$C_{2u} - \frac{1}{2}C_{2d}$	0.21 ± 0.37	

results are expressed in terms of the weak charge $Q_W = -2(C_{1u}(2Z + N) + C_{1d}(Z + 2N))$ whose value is [5] $Q_W({}^{133}_{55}Cs) = -71.04 \pm 1.58 \pm 0.88$ (the second error comes from atomic theory). The particular combination $C_{2u} - \frac{1}{2}C_{2d}$ has been also measured in the SLAC polarized $e - D$ scattering experiment [35]. The values of the parity–violating coefficients listed in table II have been derived from the quoted value of Q_W, and from the results given in table 1 of ref. [35].

Physics at the Z–peak

The recent experiments performed at the LEP and SLC Z–factories have provided us with a set of high precision measurements that are very sensitive to the values of the fermion couplings to the Z–boson.

Besides the accurate determination of the value of the Z–mass, that together with α and G_F completes the set of fundamental input parameters of the SM, also the total Z width and the partial decay widths into hadronic final states and into each of the three lepton flavours have been measured at LEP with very high precision. Less accurate results have been obtained also for the b and c partial widths.

The measurements of the different Γ_f's are sensitive to the particular combination of couplings $v_f^2 + a_f^2$, while the independent combination $v_f a_f/(v_f^2 + a_f^2)$ enters the expressions of the on–resonance forward–backward asymmetries $A_f^{\rm FB}$, that have been measured for $f = e, \mu, \tau, c, b$, and of the τ polarization asymmetry A_τ^{pol}. Already the first experimental results of LEP led to a drastic improvement of the bounds on the mixings of the heavy quarks and the τ–lepton [18], that were otherwise poorly constrained [3]. The present accuracy leads to a general improvement of the limits on all the mixing angles.

i) Z decay widths

All the four LEP collaborations have measured the total Z–width Γ_Z as well as the hadronic and the three flavour–dependent leptonic partial widths Γ_h, Γ_e, Γ_μ and Γ_τ [23].

Due to the very high experimental accuracy, radiative corrections have to be carefully taken into account in all the theoretical expressions. At 1–loop, the partial decay width of the Z–boson to f–flavour fermions reads [25]

$$\Gamma_{Z\to f\bar{f}} = N_c^f \frac{M_Z}{12\pi}\sqrt{2}G_F M_Z^2 \rho_f (a_f^2 + v_f^2)(1+\delta_{QED}^f)(1+\delta_{QCD}^f), \tag{3.24}$$

where $N_c^f = 3(1)$ for quarks (leptons). δ_{QCD}^f is the gluonic correction for hadronic final states ($\delta_{QCD}^f \simeq \alpha_s(M_Z^2)/\pi$ in leading order). For the strong coupling constant we have used the value $\alpha_s(M_Z^2) = 0.118 \pm 0.008$ determined from jet analysis in hadronic Z decays [36], and we have neglected the theoretical uncertainty related to the error on this parameter. δ_{QED}^f is a (small) additional photonic correction. Electroweak corrections appear in the ρ_f term as well as in the effective weak mixing angle that renormalizes the vector-coupling v_f :

$$\rho_f = \rho + \Delta\rho_f^{rem}, \tag{3.25}$$

$$s^2_{eff}(f) = \frac{1}{2}\left[1 - \sqrt{1 - \frac{G_\mu}{G_F}\frac{4\mathcal{A}}{\rho M_Z^2}\left(\frac{1}{1-\Delta\alpha} + \Delta\bar{r}_f^{rem}\right)}\right]. \quad (3.26)$$

In these equations the ρ–term is universal and includes the potentially large heavy–top effects. $\Delta\rho_f^{rem}$ and $\Delta\bar{r}_f^{rem}$ contain, among others, non universal flavour–dependent contributions arising from vertex form factors. These contributions are generally small, except in the case of b–quarks final states for which loops involving the top–quark appear in the correction to the Zbb vertex [37]. Finite mass effects not displayed in eq. (3.24) have been also taken into account. Analytical formulae for these corrections can be found in [25]. We note that, since G_μ/G_F enters the definition of the effective weak–mixing angle, all the LEP measurements contribute indirectly to bound the four mixings involved in the μ–decay.

The experimental values of the five width Γ_Z, Γ_h, Γ_ℓ ($\ell = e,\ \mu,\ \tau$) as measured by the four LEP collaborations are affected by common systematic errors. For the weighted averages listed in table III we have assigned to Γ_Z a common systematic error of 5 MeV from point–to–point error in the LEP energy calibration, and to the partial widths a 0.5% error from luminosity [38]. Experimentally the widths are determined by fitting simultaneously the data for the reactions $e^+e^- \to hadrons$, e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, and thus they are expected to have correlations that cannot be neglected, but that unfortunately are not always given. To overcome this inconvenience, we have adopted the following procedure. A second set of experimental quantities equivalent to $\{\ \Gamma_Z,\ \Gamma_h,\ \Gamma_e,\ \Gamma_\mu,\ \Gamma_\tau\ \}$, but with much cleaner correlations, is provided by $\{\ \Gamma_Z,\ \sigma_h^0,\ R_e,\ R_\mu,\ R_\tau\ \}$ where σ_h^0 is the peak hadronic cross section corrected for the effect of initial state radiation, and $R_l = \sigma_h^0/\sigma_l^0 = \Gamma_h/\Gamma_l$ ($l = e, \mu, \tau$). The correspondence between the two sets is given by

$$\sigma_f^0 = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}. \quad (3.27)$$

A remarkable property of this second set of quantities is that their systematic errors have in general different origins and that the correlation arising from their functional relation to the measured observables is negligible, with the exception of the one between Γ_Z and σ_h^0. In order to make a definite ansatz we have assumed an anticorrelation between these two quantities of –25%. The correlation matrix for the set of widths, shown in table III, has been worked out via an iterative procedure by requiring that it reproduces this anticorrelation (together with vanishing small off–diagonal coefficients for the other entries) when we transform to the second set by means of eq. (3.27). We have explicitly checked that this procedure leads to quite acceptable results when confronted with the available correlations [23]

A direct measurement of the Z invisible width by single photon counting [39] $\Gamma_{\text{inv}} = 500 \pm 76$ MeV has also been included in our analysis.

ii) Leptonic asymmetries.

Forward–backward asymmetries are defined as follows

$$A_f^{\text{FB}} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{3.28}$$

where σ_F (σ_B) is the cross section for events with the f- fermion scattered into the forward (backward) hemisphere with respect to the electron beam direction.

On resonance this gives

$$A_f^{\text{FB}} = 3\,\frac{v_e a_e}{v_e^2 + a_e^2}\,\frac{v_f a_f}{v_f^2 + a_f^2}, \tag{3.29}$$

where again $v_{e,f}$ should be expressed in terms of the effective weak mixing–angle (3.26). For quarks, final state QCD corrections must also be included (see *e.g.*[40]). We have taken into account the bulk of the effects of QED initial–state radiation, that are known to yield large corrections, by convolving the $e^+e^- \to f\bar{f}$ differential cross sections with a suitable "radiator" kernel [41]. The convoluted complete s-dependent formulae [41] (instead of just eq. (3.29)) have then been used to fit the asymmetries.

Even if statistical errors are still large if compared with the uncertainties on the leptonic partial widths, the leptonic FB asymmetries constitute an additional important set of quantities for testing universality of the lepton couplings to the Z. In fact, while the Γ_ℓ's are mainly sensitive to the squared axial couplings, the asymmetries are sensitive to the ratios v_ℓ/a_ℓ. The combined measurement of these two sets of quantities allows for an independent determination of v_ℓ and a_ℓ, and turns out to be quite effective for constraining both the right and left mixing angles even in the "joint fits" where all the mixings are allowed to be present simultaneously. In table III we give the values of the peak asymmetries averaged over the results of the LEP collaborations, but for the leptonic asymmetries we have also included in our analysis the data at ± 1 GeV around resonance in order to increase the statistics. Whenever available we have used the asymmetries determined from a maximum likelihood fit to the angular distribution $d\sigma_f/d\cos\theta \sim 1+\cos^2\theta+\frac{8}{3}A_f^{\rm FB}\cos\theta$ (where θ is the scattering angle), otherwise we have used the direct countings of the events and we have corrected for the relevant angular range of detection in the theoretical expressions.

The τ polarization asymmetry [42] has been also measured at LEP in τ pair production, using the distributions of its decay products [43]. At the Z resonance this quantity reads

$$A_\tau^{pol} = \frac{-2v_\tau a_\tau}{v_\tau^2 + a_\tau^2} \tag{3.30}$$

and it is very sensitive to the τ vector coupling to the Z, having the advantage with respect to the forward–backward asymmetry that it is not suppressed by the small electron vector coupling. Experimentally, the τ polarization is inferred from the slope of the energy distribution of the decay products under the assumption of pure V − A coupling with the $W^\pm$ bosons [42]. The presence of mixing–induced RHC could then in principle affect the quoted experimental results, but this effect is O(s^4) and can thus be neglected. The weighted average of the ALEPH and OPAL results [43] is given in table III.

iii) Heavy flavours

For the measurement of the width of the Z decay into b quarks, different methods have been used by different collaborations. ALEPH, L3 and OPAL at LEP and MARK II at SLC [44] used high momentum and high–$p_{\rm T}$ muons and/or electrons to tag the b quark, thus they measure the quantity $\Gamma_b/\Gamma_h\, Br(b\to \ell\nu X)$ for $\ell = e,\, \mu$. A value for the b–branching ratio into electrons and muons is then needed for the determination of Γ_b. This branching has been measured by the L3 collaboration [45] by analysing the ratio of the events where both b's decay semileptonically to the single lepton events. The L3 measurement can be combined with the PEP and PETRA determination of $Br(b\to \ell\nu X)$ (quoted in [45]) to obtain a value that is largely independent of assumptions on the b neutral–current couplings since the first result is, in first order, independent of Γ_b, and at the c.m. energies of the latter experiments weak effects contribute only a few percent to b–production. We have used the resulting value $Br(b\to \ell\nu X) = 0.119 \pm 0.006$ [45] for deriving Γ_b/Γ_h from the average of the first four measurements. The strategy adopted by DELPHI [44] in order to identify the b quarks is based on the fact that, due to the larger B–hadrons mass, a greater "sphericity" is expected for the corresponding jets. This measurement gives a further

TABLE III. Results on Z–partial widths (in MeV) and on–resonance asymmetries. The values displayed for the leptonic asymmetries correspond to the peak–data and have been corrected only for angular acceptance. Also displayed are the values of the b and c axial–vector couplings extracted from the off–resonance $A_{b,c}^{\gamma Z}$ asymmetries (used only in the single fits) and the charm asymmetry measured through D^*–tagging (used in the joint fits).

Quantity	Experimental value	Correlation			
Γ_Z	2487 ± 10	0.52	0.52	0.29	0.25
Γ_h	1739 ± 13		−0.15	0.55	0.48
Γ_e	83.2 ± 0.6			−0.08	−0.07
Γ_μ	83.4 ± 0.9				0.26
Γ_τ	82.8 ± 1.1				
$A_e^{\mathrm{FB}}(peak)$	-0.019 ± 0.014				
$A_\mu^{\mathrm{FB}}(peak)$	0.0070 ± 0.0079				
$A_\tau^{\mathrm{FB}}(peak)$	0.099 ± 0.096				
A_τ^{pol}	-0.121 ± 0.040				
Γ_b	367 ± 19				
Γ_c	299 ± 45				
A_b^{FB}	0.123 ± 0.024				
A_c^{FB}	0.064 ± 0.049				
$a_b^{\gamma Z}$	-0.405 ± 0.095	(for single fits)			
$a_c^{\gamma Z}$	0.515 ± 0.085				
$A_{c,D^*}^{\gamma Z}$ (29 GeV)	-0.101 ± 0.027	(for joint fits)			
$A_{c,D^*}^{\gamma Z}$ (35 GeV)	-0.161 ± 0.034				

direct determination of the Γ_b/Γ_h ratio.

From the overall average of these quantities and using the experimental value of Γ_h (table III), we obtain $\Gamma_b = 363 \pm 19$ MeV, where the overall uncertainty is dominated by the error on the b–semileptonic branching ratio.

The Z partial decay width into charmed quarks has also been measured, but due to the greater difficulties in the identification of the primary c quarks the accuracy achieved is worse. ALEPH [46] uses high p and p_T electrons while OPAL uses muons [46]. Averaging these two measurements and using the value $Br(c \to \ell\nu X) = 0.096 \pm 0.006$ determined at PEP and PETRA and quoted by L3 in [45] a first value for Γ_c/Γ_h is obtained.

A second determination of the $c\bar{c}$ production rate has been performed by OPAL (last paper in [46]) with a different method. The Γ_c/Γ_h ratio is determined from the analysis of the reconstructed D^* momentum distribution produced in Z decays. The same ratio has been determined by the DELPHI collaboration [46] from the inclusive analysis of charged pions from $D^{*+} \to \pi^+ D^0$ decay. In combining these two measurements, the common systematic error coming from the $c \to D^*$ hadronization probability has been taken into account. The result of the average of the four measurement is given in table III.

Three measurements of the b–quark forward–backward asymmetry have been reported [47]. In each case the b channel is selected using electronic and muonic b decays, with the requirement of high p and p_T for the final leptons, the consequent reduction in the statistics leads to rather large experimental errors. In addition the effect of $B^0 - \bar{B}^0$ mixing has to be taken into account. This effect tends to reduce the asymmetry since the neutral B^0 meson can transform into its charge conjugate before it decays. The relation between $A_b^{\rm FB}$ and the *observed* asymmetry is [48]

$$A_b^{\rm FB} = \frac{A_{obs}^{\rm FB}}{1 - 2\chi_B}, \tag{3.31}$$

where χ_B is a measure of the probability of a B^0 meson to oscillate into a $\bar{B}^0$ meson. Several measurements of the B–mixing parameter have been performed [49]. The method adopted, which is largely independent of the b–quark neutral couplings, is to count the ratio between like–sign to opposite–sign b–originated di–lepton events, since two leptons of the same charge are a signature that one B^0 meson has oscillated into its CP conjugate.

The result quoted in table III for the b forward–backward asymmetry has been obtained by adjusting all the measurements to $\chi_B = 0$, and then correcting the average with $\chi_B = 0.146 \pm 0.016$ that was obtained by averaging the ALEPH, L3 and UA1 measurements [49]. [2]

[2] We have not included the ARGUS and CLEO results [50] since these observations stem

The forward–backward asymmetry for charmed quarks has been measured only by the ALEPH collaboration [47] in a simultaneous fit with $A_b^{\rm FB}$. Their result is displayed in Tab. III.

We have included in our analysis also the FB b and c asymmetries $A_{b,c}^{\gamma Z}$, measured in the γ–Z interference region at PEP and PETRA. These asymmetries are essentially determined by the product of the axial–vector couplings $a_e a_{b(c)}$, that is a different combination from what is measured on top of the resonance. High p and p_T leptons have been used for tagging both the heavy quarks [10,12], leading to non negligible correlations between the two asymmetries. For the c quark also the D^* tagging technique (largely independent of the b couplings) have been used [11,12]

In our individual fits we have used the PEP/PETRA averages for a_b and a_c quoted in ref. [10]. In the joint analysis it is no more consistent to include these results since each axial coupling is determined while keeping the others fixed at their SM value, so in this case we have restricted our set of data by including only the c FB asymmetry measured with the D^*–tagging technique [11,12].

The measurements of the leptonic asymmetries in weak–electromagnetic interference do not improve significantly the constraints, and have not been included in our fit.

4. Results

To obtain the constraints on the mixing parameters s_i^2 we have confronted the theoretical expression X_α^{th} for each observable with the corresponding experimental result $X_\alpha^{exp} \pm \sigma_\alpha$ by constructing a χ^2 function

$$\chi^2 = \sum_{\alpha,\beta} \frac{(X_\alpha^{th} - X_\alpha^{exp})}{\sigma_\alpha} (C^{-1})_{\alpha\beta} \frac{(X_\beta^{th} - X_\beta^{exp})}{\sigma_\beta} \tag{4.1}$$

where the C represents the matrix of correlations.

from the analysis of $\Upsilon(4S)$ decays were only B_d mesons are produced, while at LEP the relative abundance of B_s to B_d mesons is estimated to be of 0.3-0.4.

Some care must be paid in the interpretation of the confidence levels from the χ^2 since the variables s_i^2 are bounded in [0,1]. For each parameter we then assume a probability distribution

$$P(s_i^2) = N_i \; e^{-\chi^2(s_i^2)/2} \tag{4.2}$$

with $N_i^{-1} = \int_0^1 \exp(-\chi^2(s_i^2)/2) \; ds_i^2$. For the joint fits, in which all mixing parameters are allowed to vary simultaneously, the χ^2 function in the expression for $P(s_i^2)$ is minimized with respect to all the remaining parameters for each value of s_i^2.

The 90% c.l. upper bounds $\bar{s}_i^2$ are computed by requiring

$$\int_0^{\bar{s}_i^2} P(s_i^2) \; ds_i^2 = 0.90 \tag{4.3}$$

under the additional condition $\chi^2(\bar{s}_i^2) > \chi^2(0)$ that, if not satisfied, would be a signature for non–zero mixing angles at 90% c.l..

Although there are more than 20 mixing parameters, the large number of observables allows to constrain all of them. The inclusion of the recent results from LEP, together with the updated NC and CC results, have considerably improved almost all the previous limits [3,18]. Our results for the 90 % c.l. bounds obtained in the individual and joint analyses are collected in table IV.

For simplicity we have assumed $\Lambda_e = \Lambda_\mu = \Lambda_\tau$ (corresponding to ordinary-exotic mixings of the same kind for the three neutrinos) but these parameters could in principle differ. In the individual analysis, since only the bounds on the neutrino mixings may depend on the value of Λ, we just show the results for $\Lambda = 2$. Furthermore, since the electron and muon neutrino mixings are mainly constrained by CC measurements, they are largely independent of the value of Λ. In contrast, for the τ neutrino different values of Λ led to different bounds, since in this case the LEP measurement of Γ_Z gives an important constraint. The *upper* bounds for ν_τ are respectively $(s_L^{\nu_\tau})^2 < 0.098, 0.032, 0.015$ for $\Lambda_\tau = 0, 2, 4$ corresponding to neutrino mixings with heavy ordinary doublets in sequential or vector doublets, with heavy singlets and with exotic doublets respectively. For the joint bounds, we present all the results for $\Lambda = 0$, 2 and 4.

One possibility that we have not considered for simplicity is the presence of *light* neutrinos that are mainly singlets. These could appear for instance in models where the light neutrinos are Dirac particles. These light singlets could mix with exotic doublets and hence couple to the Z, giving rise to a new invisible decay channel. The additional parameters s_R^ν describing these mixings would then be constrained by the measurement of Γ_Z. This would affect mainly the bounds on the τ neutrino mixings in the joint analysis (for $\Lambda_\tau \neq 0$), while the other bounds would essentially remain unmodified.

In the last column of table IV we list, for each mixing angle, the observables that are more important for establishing the constraints. Often a tight constraint set by some accurate measurements can be evaded in the joint analysis, since the deviations caused by the mixing under consideration may be canceled in these observables by adjusting other mixings to non–zero values. Other observables, for which the possibilities of cancellations are more restricted, can then become important in the determination of the joint bounds even if they were not decisive for the individual fits. In table IV these observables are labeled with a (*). This happens *e.g.* for Γ_Z and Γ_{had}, that are crucial for the single bounds but that should be supplemented by other constraints in the joint analyses since they depend on several mixing parameters. Hence, the large number of measurements at our disposal plays a crucial rôle for setting the limits.

A look at table IV makes apparent that the measurements of the Z partial and total widths contribute to the limits on all the exotic mixings. The bounds on the leptons and b–quark mixings receive further contributions from the on–resonance asymmetries, while PEP and PETRA off–resonance asymmetries help to strengthen the bounds for the c quark.

For the fermions of the first generation and for the μ_L and ν_L^μ leptons, both the 'low-energy' NC constraints (especially ν–q scattering and to a smaller extent the e–q sector) and the CC constraints on the unitarity of the CKM matrix are also important. These last quantities also bound the mixings $|V_{ui}|$ and $|V_{ci}|$ with sequential or vector doublets, as well as the parameters κ_{ij}, that are further constrained by direct searches of induced hadronic RHC's.

Due to the fact that the presence of the mixings modify the fermion couplings, the various determinations of the effective weak angle cannot be used as direct measurements. However since the theoretical expression for s_{eff} depends on the ratio G_F/G_μ, besides the direct constraints, the combination of all the LEP and NC experiments put also important indirect constraints to the mixings that appear in μ decay. These indirect bounds are quite effective for the electron and muon neutrino mixings, and are of some relevance also for $(s_L^\mu)^2$ in the joint analyses. These two indirect sources of constraints have been denoted respectively with s_{eff}^{LEP} and s_{eff}^{NC} in table IV.

The W boson mass, that also constrains the ratio G_μ/G_F, is not very important in the individual analysis due to its present experimental error. However, it gains relevance in the joint fits since it does not allow for accidental cancellations between different mixings, as usually happens for LEP measurements.

For the left handed charged leptons and neutrinos, the constraints on lepton universality are also crucial. Some peculiarities arise in the $\tau - \nu_\tau$ sector, since to some extent the τ-decay measurements are better accounted for with non-universal CC lepton couplings (see e.g. ref. [51]). In particular, non–vanishing τ_L and/or ν_L^τ mixings weaken the $W\tau\nu_\tau$ coupling, allowing for a longer τ lifetime, as is favoured by experiments. The excellent agreement of the accurate LEP measurements with the SM predictions forces the overall probability distribution to be consistent with vanishing values for the τ and ν_τ mixings. However, if ν_τ mainly mixes with an ordinary sequential or vector doublet neutrino ($\Lambda_\tau \simeq 0$), the NC experiments are ineffective for constraining this mixing. In this case, both in the individual and in the joint analyses, we find that the value $s_L^{\nu_\tau} = 0$ falls out of the 90% confidence regions, that are respectively $0.0075 < s_L^{\nu_\tau} < 0.098$ and $0.0057 < s_L^{\nu_\tau} < 0.097$. However, within two standard deviations the data are consistent with zero mixing.

Another complication in the analysis is due to a peculiarity in the behaviour of the observables involving the d_R-type quark mixings. Indeed for $(s_R^q)^2 \simeq 0.3$ ($q = d,\ s,\ b$), the s^4 terms cancel against the quadratic ones inside both $v_q a_q$ and $v_q^2 + a_q^2$, that are the only combinations of couplings measurable at the Z–peak. Since the constraints on s_R^s and s_R^b are provided essentially by LEP experiments, the corresponding χ^2 distributions are characterized by two equivalent minima, one lying around vanishing value for the mixing and a second one near 0.3 and as a consequence the confidence intervals are split into two disjoint regions. Actually, due to the low central value of the b-quark axial–vector coupling as extracted from the asymmetries in the γ–Z interference region [10], the value $(s_R^b)^2 \simeq 0.3$ is even in slightly better agreement with the data. For the bounds in table IV we have conservatively integrated over both regions, however a restriction to the interval consistent with zero mixing gives $(s_R^s)^2 \lesssim 0.09$ and $(s_R^b)^2 \lesssim 0.10$, *i.e.* about three times tighter limits.

We can add a final comment about the interplay between the mixings and the bounds on the top mass. A fit to m_t leaving all the mixings free to vary, indicates

TABLE IV. 90 % c.l. upper bounds on the ordinary-exotic fermion mixings for the individual fits, where only one parameter is allowed to vary, and for the joint fits where cancellations between different mixings can occur. The observables that mainly contribute to determine the numerical values of the bounds are listed in the last column (those labeled by an asterisc (*) are effective only in the joint analyses). s_{eff}^{LEP} and s_{eff}^{NC} refer to the effective weak mixing angle, from Z-peak and NC experiments, which contribute indirectly to constrain the mixings in G_F/G_μ.

	Individual	Joint			Source
		$\Lambda=2$	$\Lambda=0$	$\Lambda=4$	
$(s_L^e)^2$	0.0047	0.015	0.0090	0.015	$\Gamma_e, M_W^*, A_\mu^{FB*}, eq^*, g_e^*$
$(s_R^e)^2$	0.0062	0.010	0.0082	0.010	$\Gamma_e,\ A_e^{FB},\ A_\mu^{FB*},\ \nu e^*$
$(s_L^\mu)^2$	0.0017	0.0094	0.0090	0.011	$V_{ui}^2, \nu q, g_\mu, \Gamma_\mu, s_{eff}^{LEP*}, M_W^*$
$(s_R^\mu)^2$	0.0086	0.014	0.014	0.013	$\Gamma_\mu,\ A_\mu^{FB}$
$(s_L^\tau)^2$	0.011	0.017	0.015	0.017	$\Gamma_\tau,\ A_\tau^{FB}, g_\tau,\ A_\tau^{pol*}$
$(s_R^\tau)^2$	0.011	0.012	0.014	0.012	$\Gamma_\tau,\ A_\tau^{pol}, A_\tau^{FB},\ g_\tau^*$
$(s_L^u)^2$	0.0045	0.019	0.015	0.019	$V_{ui}^2,\ \Gamma_h,\ \Gamma_Z, eq, \nu q$
$(s_R^u)^2$	0.018	0.024	0.025	0.024	$\nu q,\ \Gamma_h,\ \Gamma_Z,\ eq$
$(s_L^d)^2$	0.0046	0.019	0.016	0.019	$V_{ui}^2,\ \Gamma_h,\ \Gamma_Z,\ \nu q$
$(s_R^d)^2$	0.020	0.030	0.028	0.029	$eq,\ \Gamma_h,\ \Gamma_Z,\ \nu q$
$(s_L^s)^2$	0.011	0.038	0.039	0.041	$\Gamma_h,\ \Gamma_Z,\ V_{ui}^2$
$(s_R^s)^{2\,\dagger}$	0.36	0.67	0.63	0.74	$\Gamma_h,\ \Gamma_Z$
$(s_L^c)^2$	0.013	0.040	0.042	0.042	$\Gamma_h,\ \Gamma_Z,\ \Gamma_c^*,\ A_c^{\gamma Z*}$
$(s_R^c)^2$	0.029	0.097	0.10	0.099	$\Gamma_h, \Gamma_Z, A_c^{\gamma Z*}, \Gamma_c^*, A_c^{FB*}$
$(s_L^b)^2$	0.011	0.070	0.072	0.069	$\Gamma_h,\ \Gamma_Z,\ \Gamma_b,\ A_b^{FB*}$
$(s_R^b)^{2\,\dagger}$	0.33	0.39	0.40	0.39	$\Gamma_b,\ \Gamma_Z, \Gamma_h,\ A_b^{\gamma Z},\ A_b^{FB*}$
$(s_L^{\nu_e})^2$	0.0097	0.015	0.016	0.014	$s_{eff}^{LEP}, g_e, s_{eff}^{NC}, M_W^*$
$(s_L^{\nu_\mu})^2$	0.0019	0.015	0.0087	0.011	$V_{ui}^2, g_\mu, \nu q, s_{eff}^{LEP}, M_W^*$
$(s_L^{\nu_\tau})^{2\,\dagger}$	0.032	0.064	0.097	0.035	$\Gamma_Z,\ g_\tau$
$\sum_{i=4}^n K_{ui}^2$	0.0048	0.014	0.010	0.018	V_{ui}^2
$\lvert\kappa_{ud}\rvert$	0.0011	0.0059	0.0060	0.0058	V_{ui}^2, νq, RHC's, νe^*
$\lvert\kappa_{us}\rvert$	0.0054	0.0061	0.0061	0.0061	V_{ui}^2, RHC's
$\sum_{i=4}^n K_{ci}^2$	0.53	0.76	0.76	0.76	V_{ci}^2
$\lvert\kappa_{cd}\rvert$	0.31	0.31	0.31	0.31	RHC's
$\lvert\kappa_{cs}\rvert$	0.24	0.29	0.29	0.29	V_{ci}^2, RHC's

† For some peculiarities that occur for s_R^s and s_R^b, and for a discussion of the bounds on $s_L^{\nu_\tau}$, see text.

that the preferred value is shifted downwards by about 25 GeV with respect to the case when all the mixings are set to zero. As already noted this is mainly due to the anticorrelation with the mixings that appear inside the ratio G_μ/G_F. As a consequence of the large number of free parameters the error is larger, and the upper bound on m_t is slightly relaxed. However, considering that also the loop effects of the new heavy fermions are expected to lower the upper bound on m_t, we can conclude that in general a 'light' top is preferred in models of this kind.

5. Conclusions

As a summary, we have analysed the limits on the mixings of the known leptons and quarks with possible heavy fermions with exotic $SU(2)_L$ assignments. We have obtained significant constraints on a very large set of mixing parameters by performing a detailed global analysis of the available electroweak data.

In order to guarantee the experimentally observed absence of FCNC, we have assumed that each ordinary charged fermion mixes with a unique exotic state, in which case just one mixing angle per degree of freedom is enough to describe the effects of the mixing. For the neutrinos there is no experimental evidence of FCNC suppression, but since one has to sum over the flavor of the unobserved final ν states, again just one mixing angle per neutrino flavor allows to describe the mixings, with the addition of an effective parameter (Λ) that takes into account the type of exotic neutrinos involved.

In order to constrain the mixing angles we have analyzed the effects that they could induce in the couplings between the light fermions and the weak bosons. Very accurate measurements of the fermion weak–couplings are provided by several experiments, as for example tests on CC universality and on the unitarity of the CKM matrix, limits on induced right-handed currents, collider measurements of M_W, low-energy NC experiments (ν-scattering, atomic parity violation and polarized $e - D$ scattering) and in particular the huge amount of data obtained at LEP and SLC from experiments at the Z-resonance. The results of our analysis are collected in table IV.

If only one mixing angle is considered at a time, for most of the mixing factors $(s^f)^2$ the limits are below the 1 % level, with the exception of the mixings of u_R, d_R, c_R and $\nu_{\tau L}$ that are of the order of a few percent and those of s_R and b_R that are still poorly constrained to values $\lesssim 1/3$. Allowing for accidental cancellations among different mixings the constraints are relaxed by a factor between 2 and 5.

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