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PERTURBATIONS FROM STRINGS DON'T LOOK LIKE STRINGS!*

Andreas Albrecht and Albert Stebbins
NASA/Fermilab Astrophysics Center, P.O.B. 500
Batavia, IL 60510, USA

ABSTRACT

A systematic analysis is challenging popular ideas about perturbations from cosmic strings. One way in which the picture has changed is reviewed here.

1. The Conventional View

Cosmic strings have enjoyed a great deal of attention as possible seeds for the formation of galaxies and larger scale structure in the universe. Strings may form as defects in a cosmic phase transition, resulting in a network of long strings and a distribution of loops. As time goes on, the strings slowly "equilibrate" away via the chopping off of loops from the long string, and via the eventual decay of these loops into gravity waves.

Discussions of the evolving string network have focused on the self-similar "scaling solution", according to which the gross features of the strings are equivalent at any two times, up to an overall rescaling. At any given time there is a network of long strings, each of which looks like a random walk with step-size ξ. On scales smaller than ξ, the long strings are fairly straight, although small scale wiggles may effectively renormalize the mass per unit length of the long string. At any given time there is also a distribution of loops which have chopped off the network, but have yet to decay. Most of these loops have sizes much smaller than ξ.

The scaling picture is very convenient: To understand the string network at all times one need only have a "snapshot" of the network at one time, and know the correct scaling law. We are finding, however, that the scaling picture is perhaps *too* convenient. Earlier work on string seeded structure formation has placed a lot of emphasis on the "snapshot" of the network at a given time, and has lead to the incorrect analysis of processes which inherently proceed over long *periods* of time. In earlier work, structure formation was studied by investigating the collapse of matter around the string network "seeds" present at a particular time. Usually it was argued that this time should be taken around the time of matter-radiation equality, and that the resulting perturbations would be the dominant ones.

We have found, at least in the case where the dark matter is cold, that this snapshot approach is completely incorrect. To correctly account for the perturbations

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induced by the strings, one must take into account events occurring over essentially the entire history of the universe. One of the interesting consequences of this is that distinctive "stringy" features, such as the wake of a single long string or the matter collapsed around an individual loop, do not dominate the perturbations.

2. Our Analysis

We have analysed (in linear theory) the effects of a string network on a flat Friedmann-Robertson-Walker universe dominated by cold dark matter. (See our latest paper[1] for a more complete discussion and other references to the literature.) The quantity relevant to the perturbation theory is $\delta(x) \equiv (\rho_d(x) - \bar{\rho}_d)/\bar{\rho}_d$, where $\rho_d(x)$ is the density of the dark matter today, and the overbar represents the spatial average. We use a Green function solution of the form:

$$\delta(x, \eta') = 4\pi(1 + z_{eq}) \int d\eta \int d^3x' T(x - x', \eta, \eta')\Theta_+(x', \eta), \tag{1}$$

where η is conformal time. The string source density is given in terms of the string stress-energy density: $\Theta_+(x) \equiv \Theta_{00}(x) + \Theta_{ii}(x)$. A useful quantity is the power spectrum, $P(k)$, which is given by $(2\pi)^3 P(k)\delta^{(3)}(\vec{k} - \vec{k}') = \langle\delta(\vec{k})\delta(-\vec{k}')\rangle$, where $\delta(\vec{k})$ is the Fourier transform of $\delta(x)$. The power spectrum represents the contribution from the kth Fourier component of $\delta(x)$ to the spatial average of $\delta^2(x)$.

We find that (in linear theory) the power spectrum today is well approximated by the expression:

$$P(k) = 16\pi^2(1 + z_{eq})^2 \int \left|\tilde{T}(k, \eta, \eta')\right|^2 F(k, \eta) d\eta \tag{2}$$

where $\tilde{T}(k)$ (the transfer function) represents the growth of perturbations due to gravitational collapse, and $F(k, \eta)$ is a "structure function" which accounts for the string source at time η. (We use the "LX" string parameters[1] here, with h=100.)

3. Discussion

One thing which is clear from the above equations is that perturbations induced by cosmic strings do not manifestly originate at a particular moment in time. The time integrals represent the cumulative contributions of many different string motions to a given perturbation. The only circumstance under which a given time can be singled out is when the integrand is peaked in time. This is in fact the case, as one can see from Fig 1 where the integrand of Eq 2 is plotted versus η.

Although each curve in Fig 1 is peaked, the peaks occur at different times for different scales. This fact has important implications: For example, one can note that the peak contribution to the power for $\lambda = 100Mpc$ comes at $\eta \approx 100Mpc$. A snapshot of the string network at that time would have long strings forming random walks with step size $O(\eta)$ and mean separation $O(\eta)$, and a distribution of smaller loops. Although the main contribution to $P(k = 2\pi/100Mpc)$ comes form the sheet-like wakes of these long strings, one can *not* expect to see $100Mpc$ sheets standing out among the perturbations. For such a sheet to stand out, a given wake has to dominate on a whole *range* of scales. Instead, the perturbations on 10Mpc, for example, are produced mainly by the string motions at $\eta \approx 10Mpc$, which have little to do with $100Mpc$ sheet-like wakes. Although the $100Mpc$ sheets are technically "there", they are lost in the "noise" of other perturbations produced at earlier times. (See Fig 2)

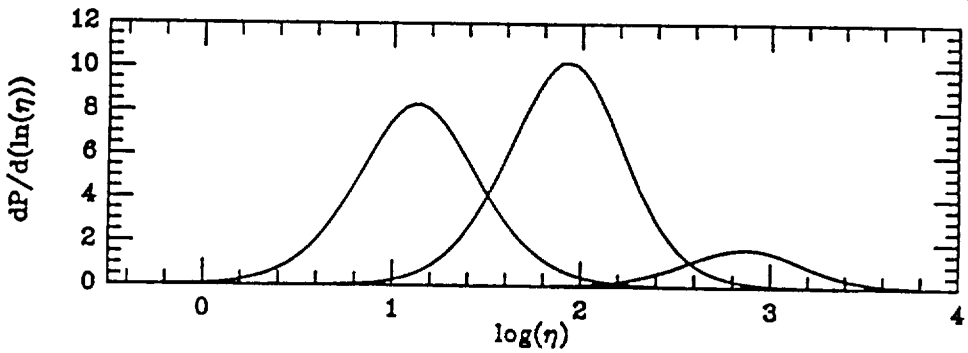

Figure 1: The integrand of Eq 2 ($\times\eta$) vs $\log(\eta)$. The three curves (from left to right) correspond to $k = 2\pi/10Mpc$, $k = 2\pi/100Mpc$, and $k = 2\pi/1000Mpc$.

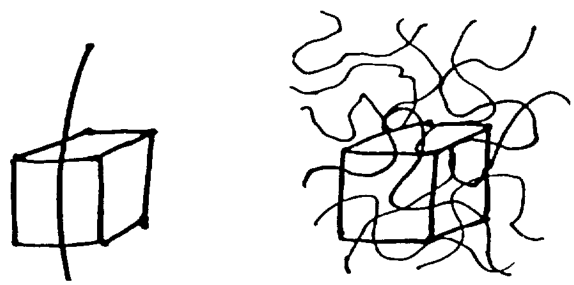

Figure 2: The region around a single string wake (left) was once perturbed by a more dense network (right). The earlier perturbations dominate on the scale pictured.

We conclude that, while the scaling properties of cosmic strings figure significantly in the analysis, care must be taken when thinking in terms of single-time "snapshots". The process of seeding density perturbations is not fundamentally localized in time, and this fact can wash out many of the details which appear in a single snapshot.

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References

1. A. Albrecht and A. Stebbins. Perturbations from cosmic strings in cold dark matter. Fermilab Preprint FERMILAB-Pub-91/17-A, 1991.